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Chance gets measured
Laying the foundations
Measure theoretic probability
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Random graphs and matrices

Randomness and Structure

Takis Konstantopoulos

17 November 2010

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Randomness occupied humans thousands of years ago...

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Where from and how old are these dice?

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Made 7000 years ago

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Achilles and Ajax playing dice

ca. 520–510 BC, Museé du Louvre

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Tyche (*TYXH*): Tutelary deity of fortune

Chance was a mystery...

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People invented a goddess for it:

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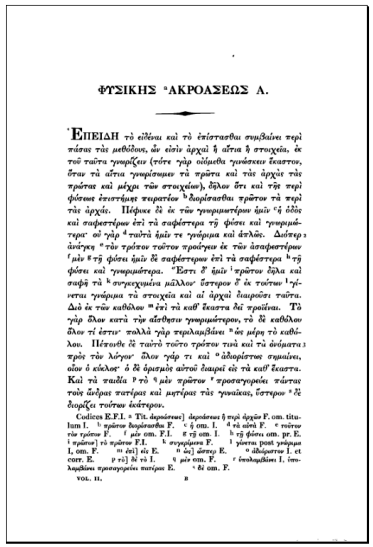

ΦΥΣΙΚΗΣ ΑΚΡΟΑΣΕΩΣ Α.

ΕΠΕΙΔΗ τὸ εἰδέναι καὶ τὸ ἐπίστασθαι συμβαίνει περὶ πάσας τὰς μεθόδους, ἃν εἰσὶ ἀρχαὶ ἢ αἴτια ἢ στοιχεῖα, ἐκ τοῦ ταῦτα γνωρίζειν (τότε γὰρ οἰόμεθα γινώσκειν ἕκαστον, ὅταν τὰ αἴτια γνωρίσωμεν τὰ πρῶτα καὶ τὰς ἀρχὰς τὰς πρώτας καὶ μέχρι τῶν στοιχείων), δῆλον ὅτι καὶ τῆς περὶ φύσεως ἐπιστήμης πειρατέον [b]διορίσασθαι πρῶτον τὰ περὶ τὰς ἀρχάς. Πέφυκε δὲ ἐκ τῶν γνωριμωτέρων ἡμῖν [c]ἡ ὁδὸς καὶ σαφεστέρων ἐπὶ τὰ σαφέστερα τῇ φύσει καὶ γνωριμώτερα· οὐ γὰρ [d]ταὐτὰ ἡμῖν τε γνώριμα καὶ ἁπλῶς. Διόπερ ἀνάγκη [e]τὸν τρόπον τοῦτον προϊέναι ἐκ τῶν ἀσαφεστέρων [f]μὲν [g]τῇ φύσει ἡμῖν δὲ σαφεστέρων ἐπὶ τὰ σαφέστερα [h]τῇ φύσει καὶ γνωριμώτερα. Ἔστι δ' ἡμῖν [i]πρῶτον ὗλα καὶ σωφὴ τὰ [k]συγκεχυμένα μᾶλλον· ὕστερον δ' ἐκ τούτων [l]γίνεται γνώριμα τὰ στοιχεῖα καὶ αἱ ἀρχαὶ διαιροῦσι ταῦτα. Διὸ ἐκ τῶν καθόλου [m]ἐπὶ τὰ καθ' ἕκαστα δεῖ τρωπέον. Τὸ γὰρ ὅλον κατὰ τὴν αἴσθησιν γνωριμώτερον, τὸ δὲ καθόλου ὅλον τί ἐστιν· πολλὰ γὰρ περιλαμβάνει [n]ὡς μέρη τὸ καθόλου. Πέπονθε δὲ ταὐτὸ τοῦτο τρόπον τινὰ καὶ τὰ ὀνόματα· πρὸς τὸν λόγον· ὅλον γάρ τι καὶ [o]ἀδιόριστον σημαίνει, οἷον ὁ κύκλος· ὁ δὲ ὁρισμὸς αὐτοῦ διαιρεῖ εἰς τὰ καθ' ἕκαστα. Καὶ τὰ παιδία [p]τὸ [q]μὲν πρῶτον [r]προσαγορεύει πάντας τοὺς ὁμοίους πατέρας καὶ μητέρας τὰς γυναῖκας, ὕστερον [s]δὲ διαρίζει τούτων ἑκάτερον.

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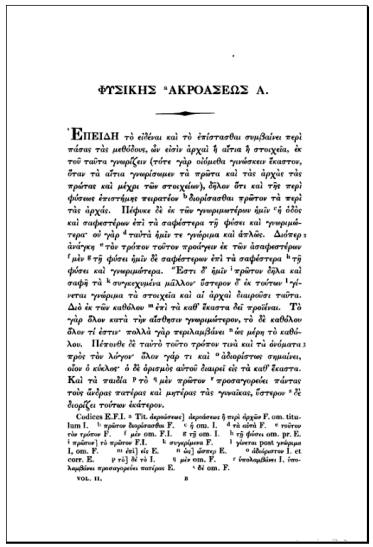

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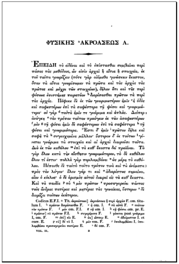
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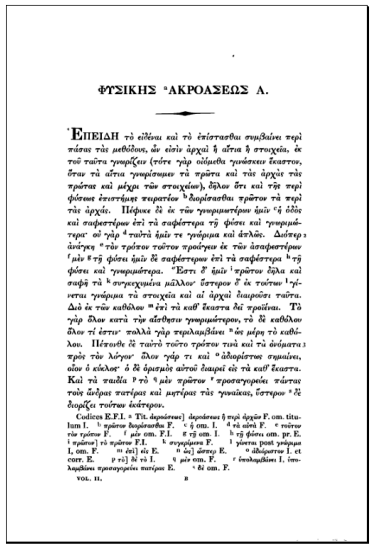

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N.B. Aristotle was the favorite child of the Christian Church for long time: his views, they said, were compatible with the new religion.

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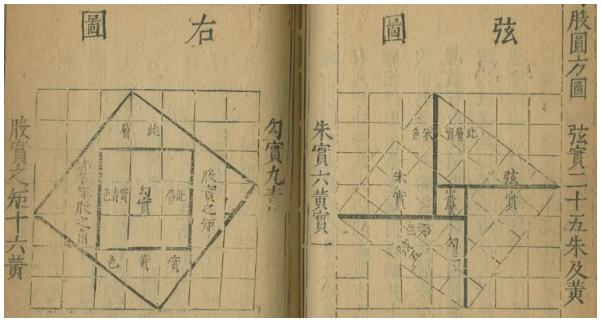
Yet, they knew how to think and compute, through the Science of Mathematics



The *Zhou bi suan jing* book 周髀算经

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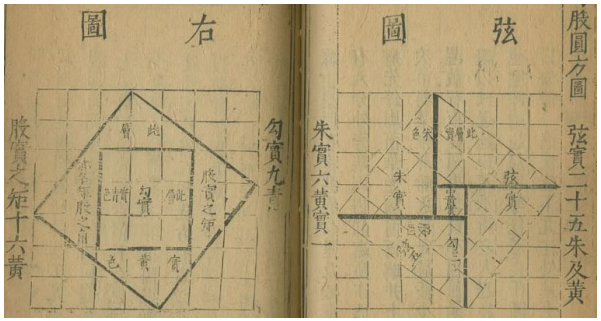

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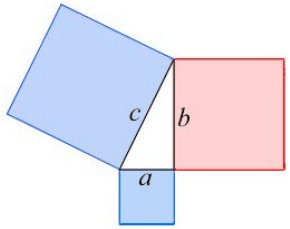

$$a^2 + b^2 = c^2$$

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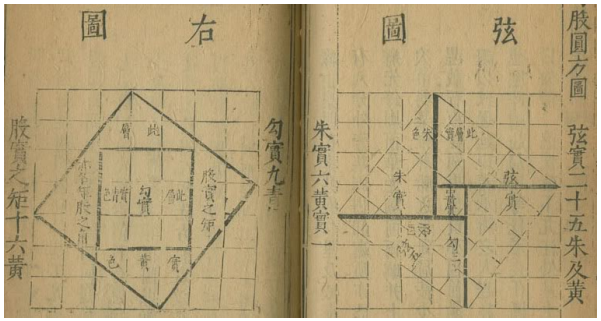

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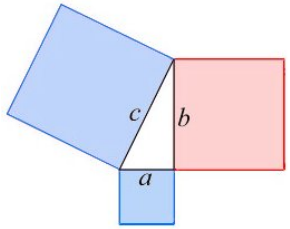

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Otherwise, they wouldn't have built this

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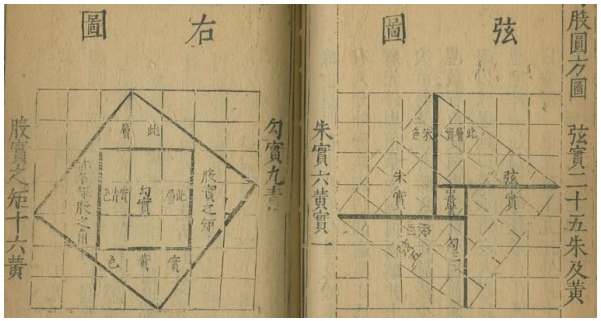

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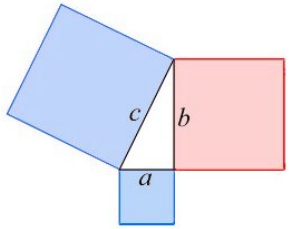

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A: Toss a die 4 times; bet on the appearance of a six
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Pierre de Fermat (1601?-1665) and Blaise Pascal (1623-1662) *computed:*
$P(A) = 0.518, \quad P(B) = 0.491.$

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But they were wrong

Girolamo Cardano (1501-1576), *Liber de Ludo Aleae* (1526, publ. 1663): First systematic treatment of probability. Perhaps, based on work of Luca Pacioli (1446-1517).

Chevalier de Méré (mid 17th c.), nobleman, salon theorist, writer, bon-vivant: *The problem of points.*
A: Toss a die 4 times; bet on the appearance of a six
B: Toss a pair of dice 24 times; bet on the appearance of a pair of sixes

Pierre de Fermat (1601?-1665) and Blaise Pascal (1623-1662) *computed:*
$P(A) = 0.518, \quad P(B) = 0.491.$

Abraham de Moivre (1667-1754)
De Mensura Sortis (1711)
On the Doctrine of Chances (1718)

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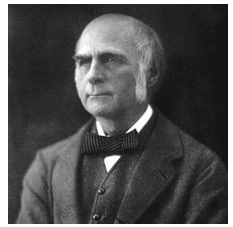
And then things start moving fast

Pierre-Simon de Laplace

Carl Friedrich Gauss



Sir Francis Galton

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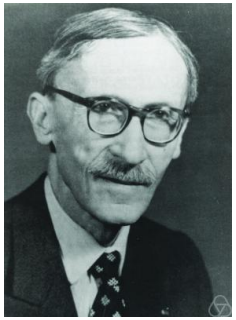
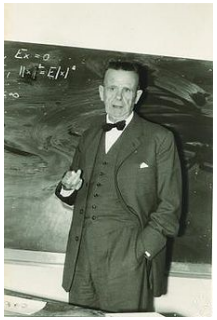
The 20th century

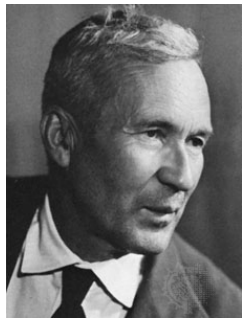
Constantin Carathéodory (1873-1950)



Paul Lévy (1886-1971)



Harald Cramér (1893-1985)



Andrei Nikolaevich Kolmogorov (1903-1987)

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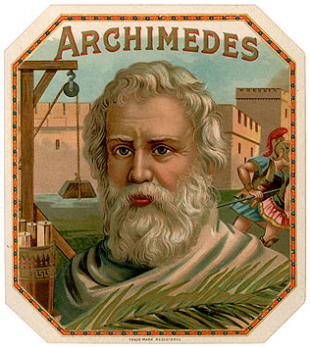
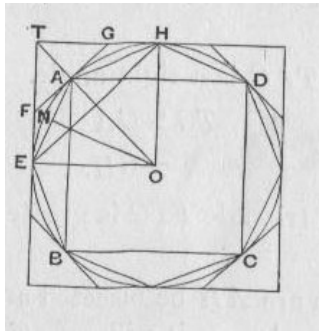
The concept of measure



measures area of



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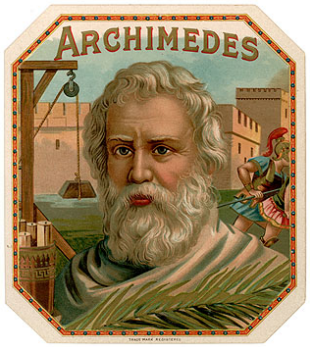

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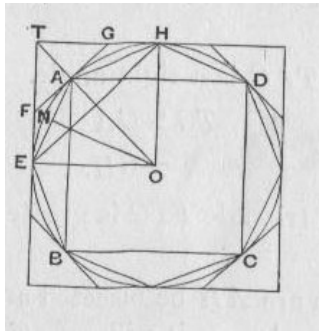

And 1700 years later...

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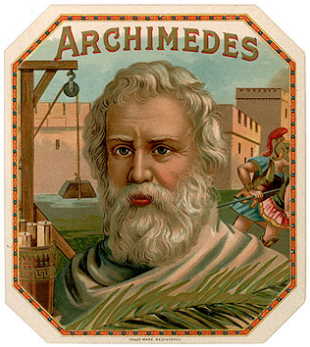

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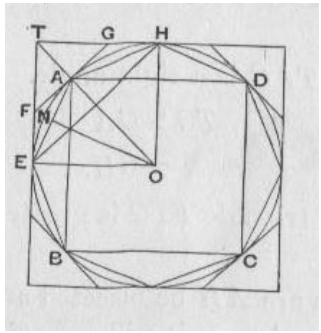

And 1700 years later...

defines AREA


VORLESUNGEN ÜBER
REELLE FUNKTIONEN

Göttingen, Dezember 1917.

C. Carathéodory.

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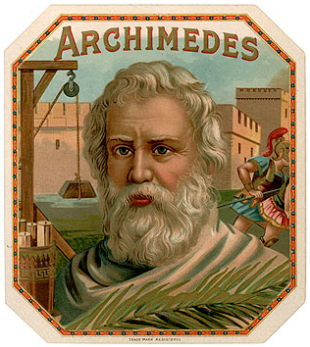

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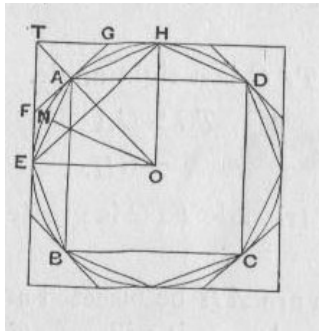

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defines AREA


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and prepares the ground for the foundations of Probability Theory

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A.N. Kolmogorov (1929). General measure theory and the calculus of probabilities. *Trudy Kommunistichekoĭ Akad. Razdel Mat.* **1**, 8-21 (in Russian).

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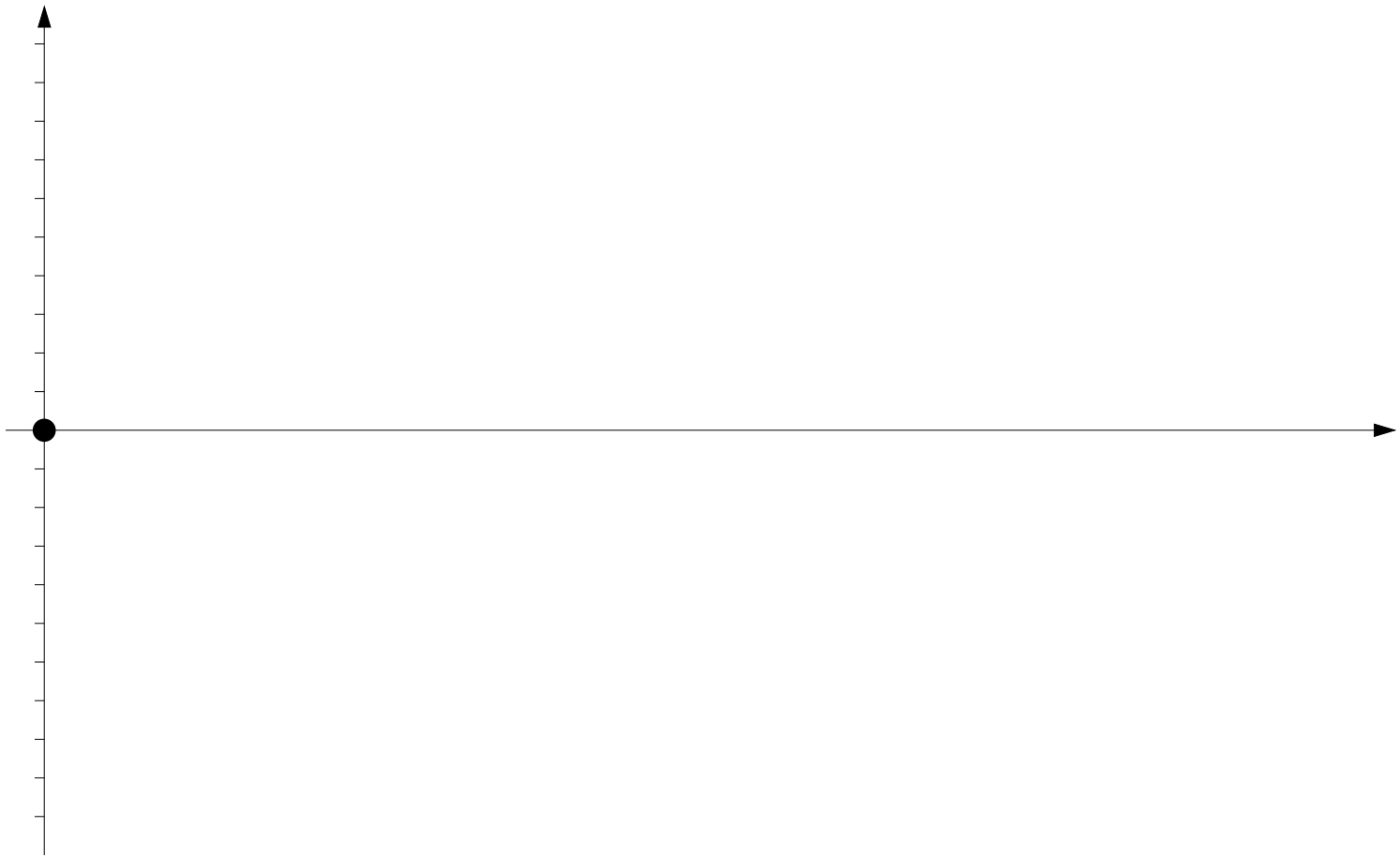

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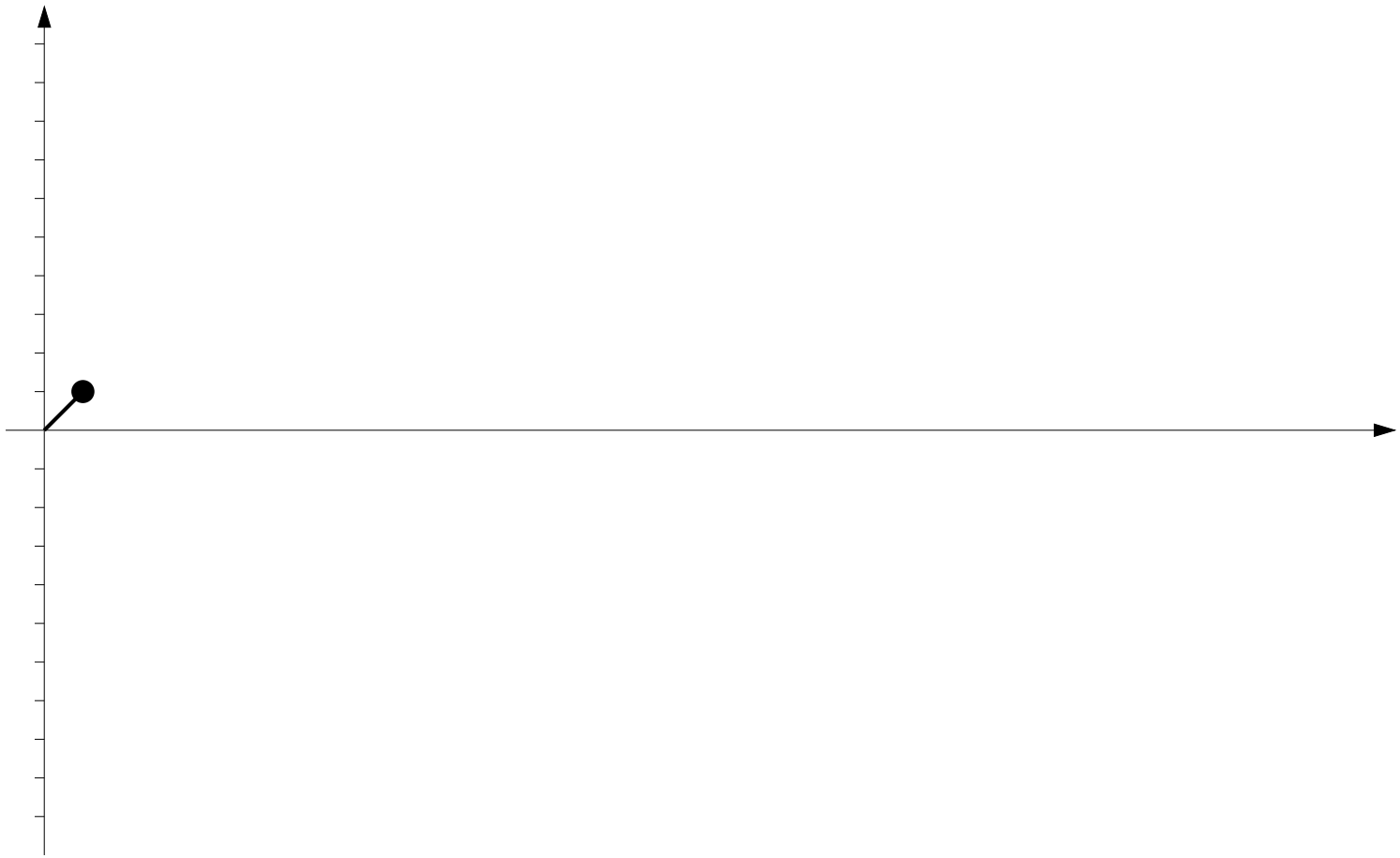

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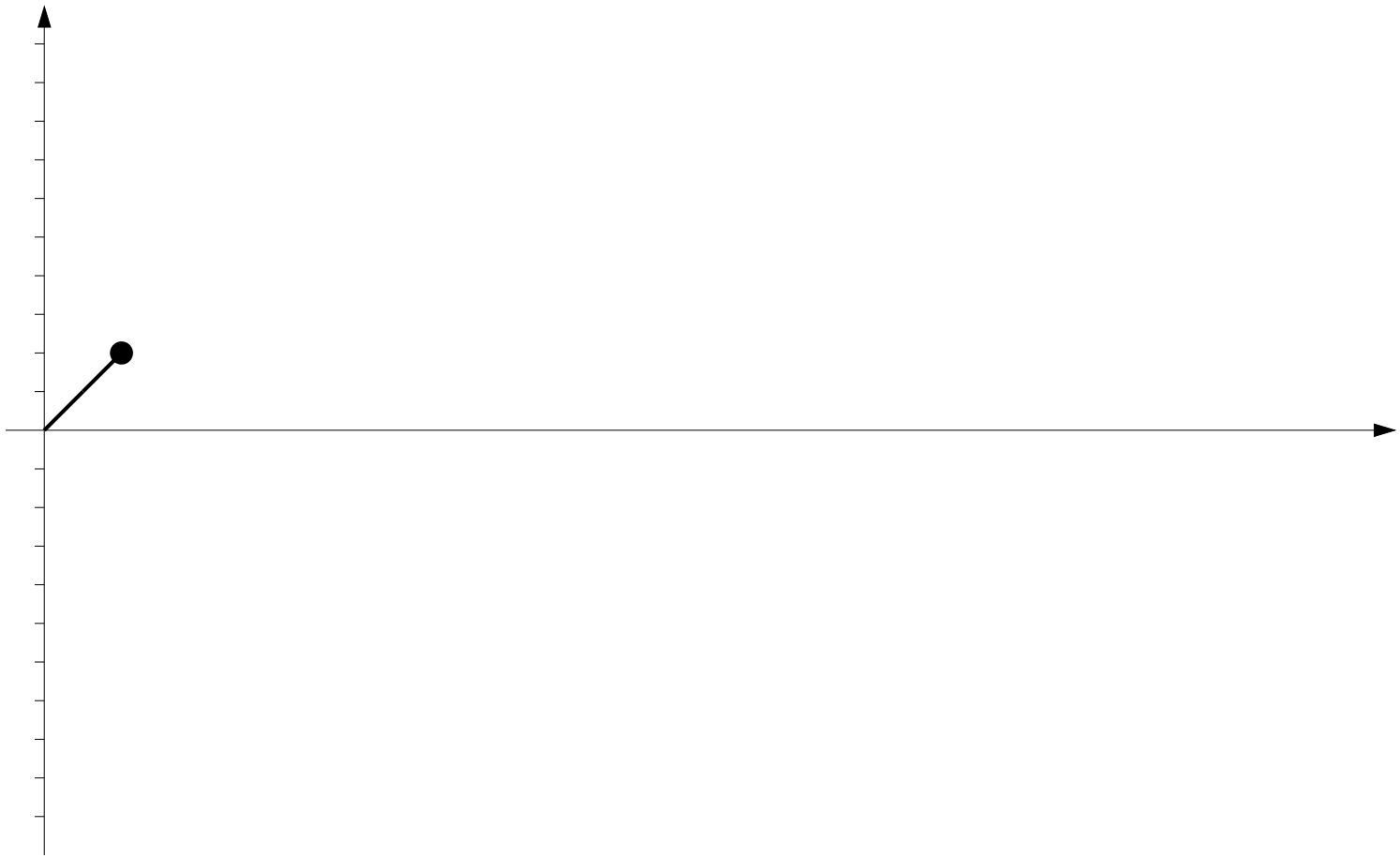

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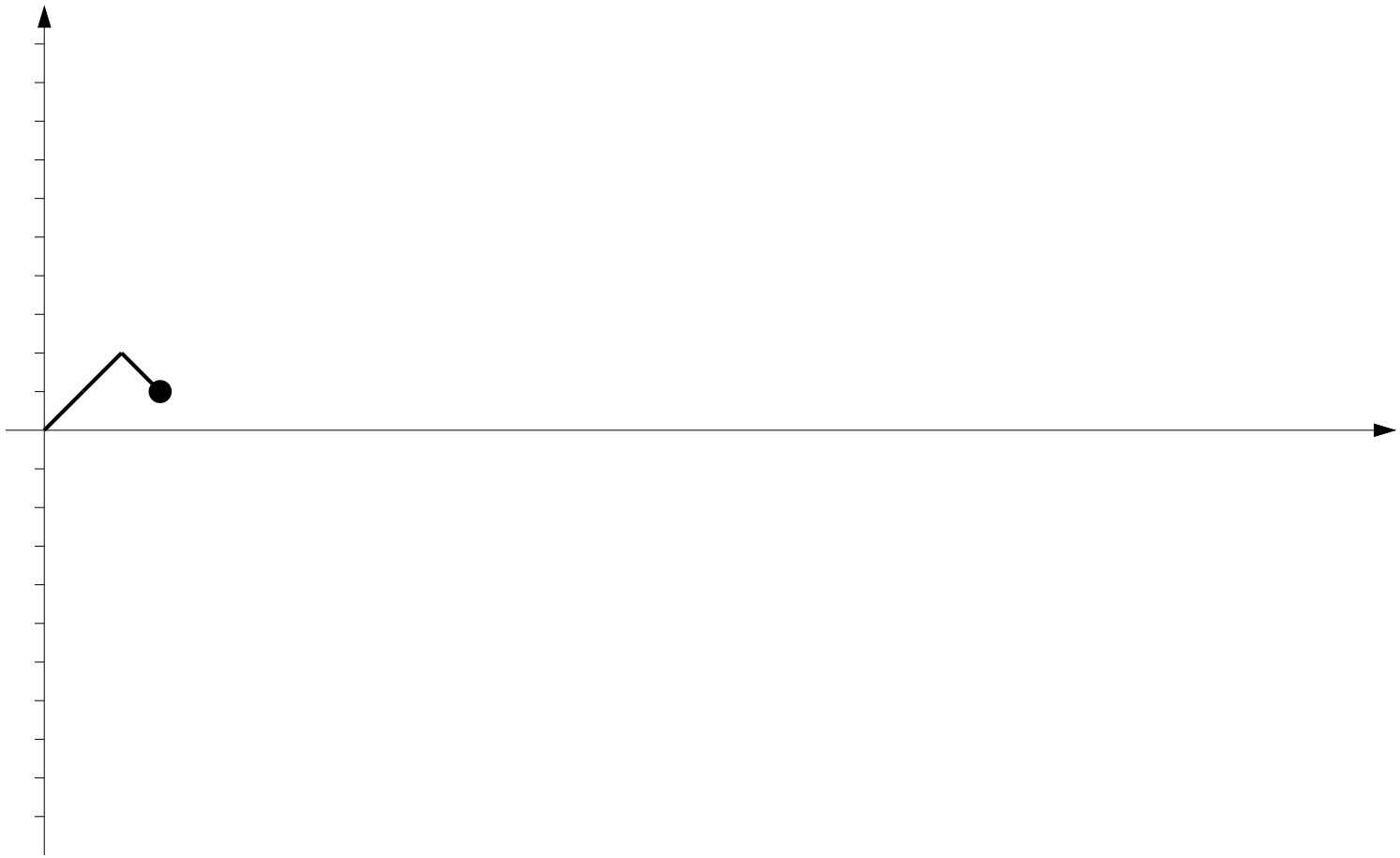

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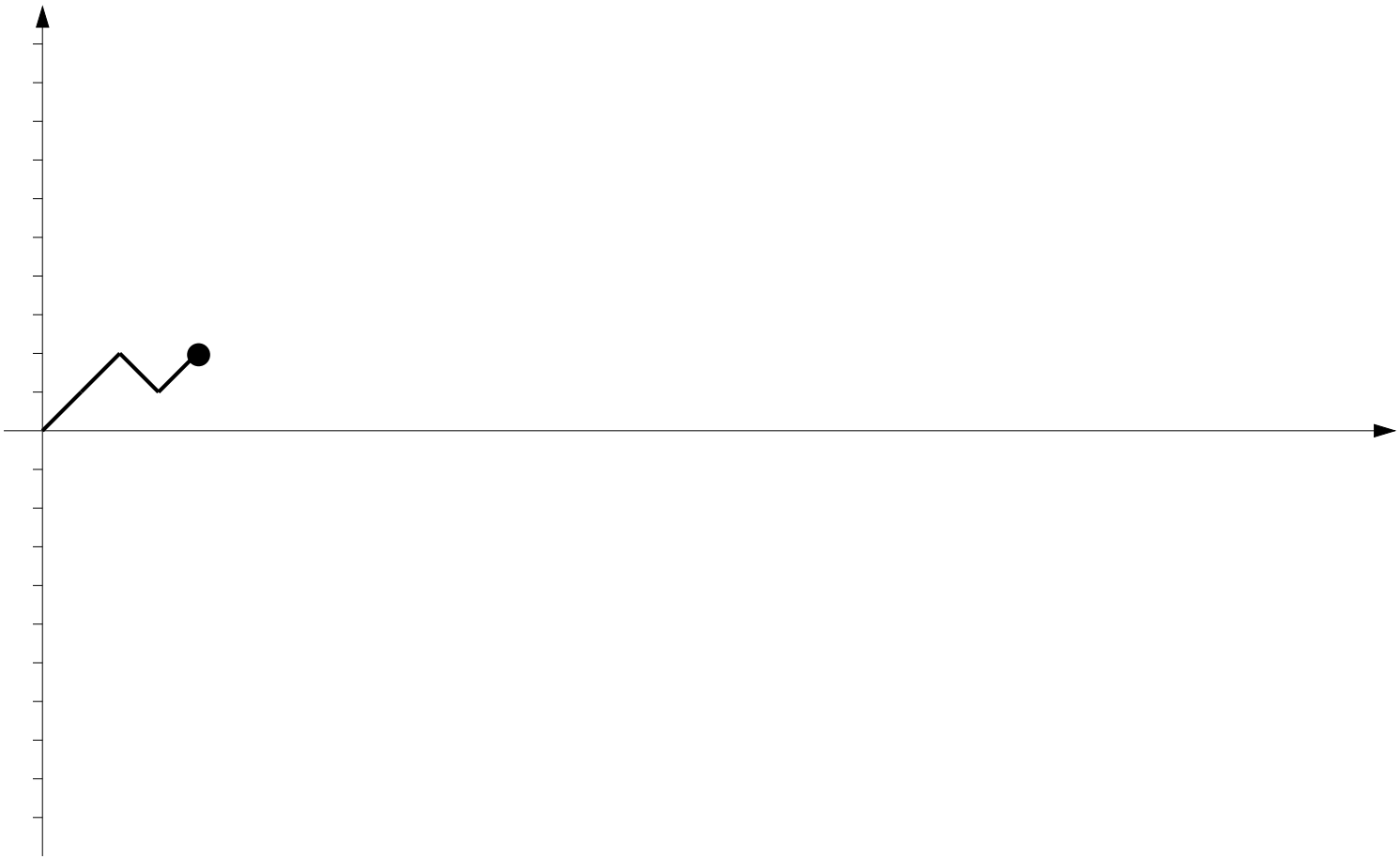

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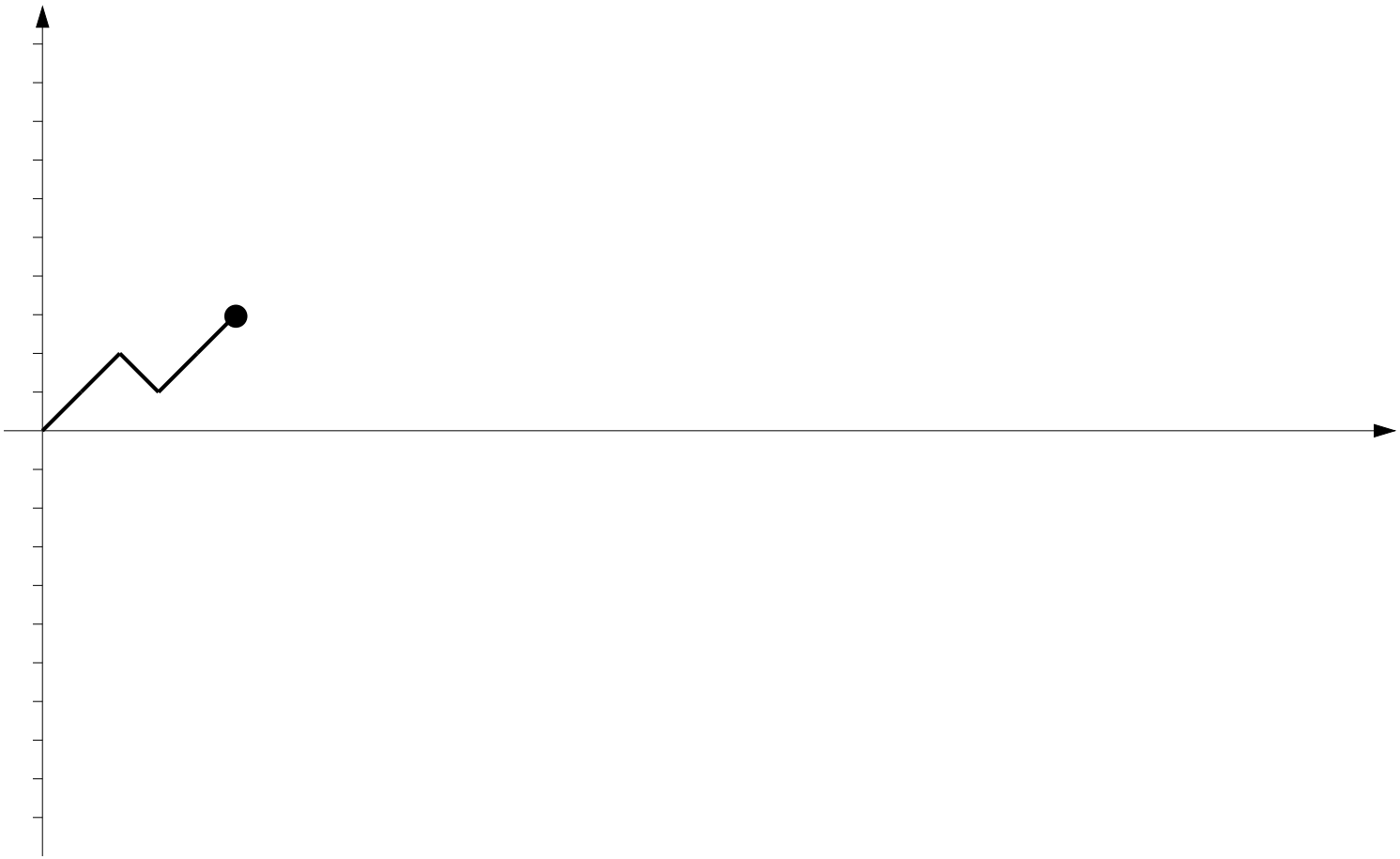

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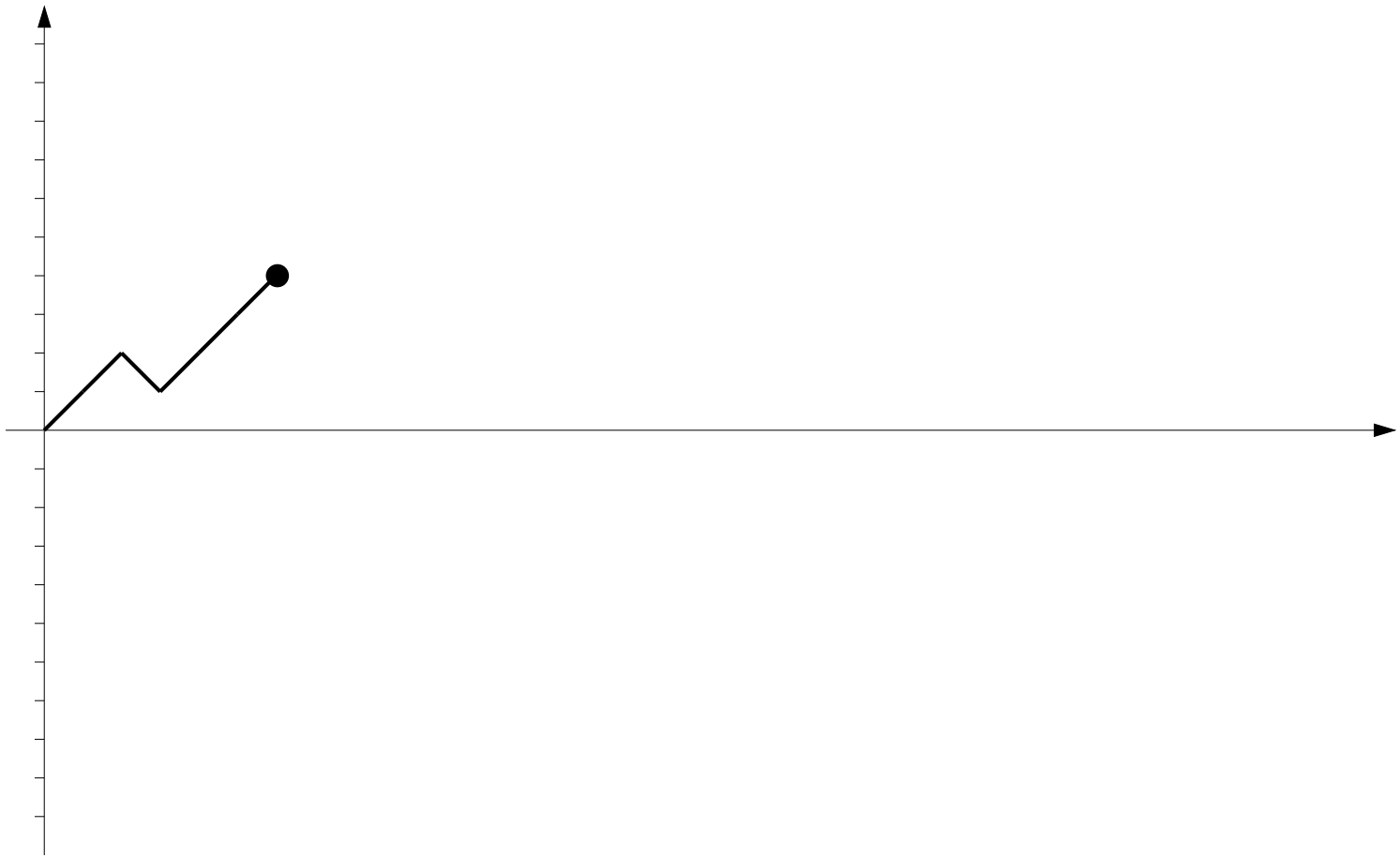

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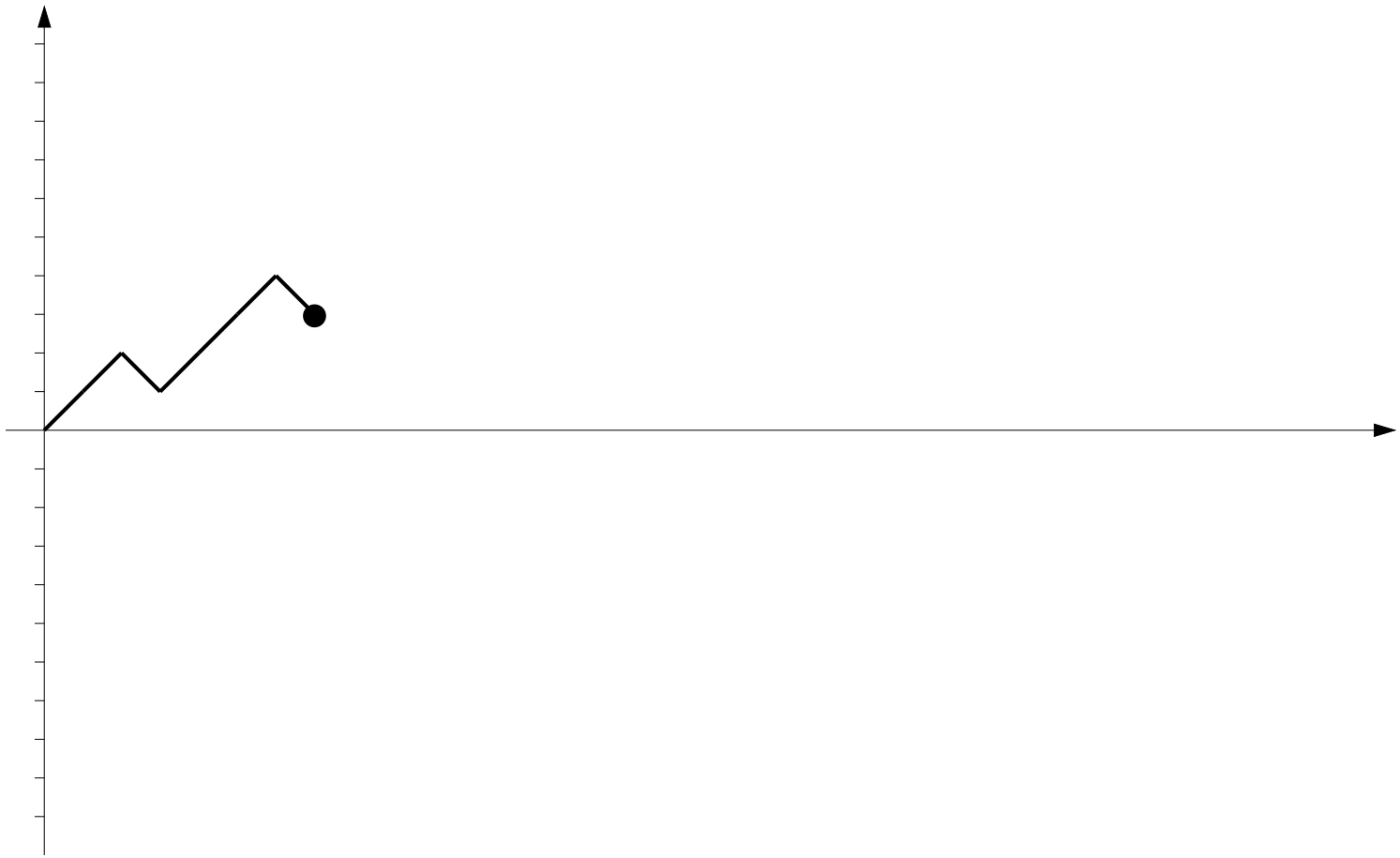

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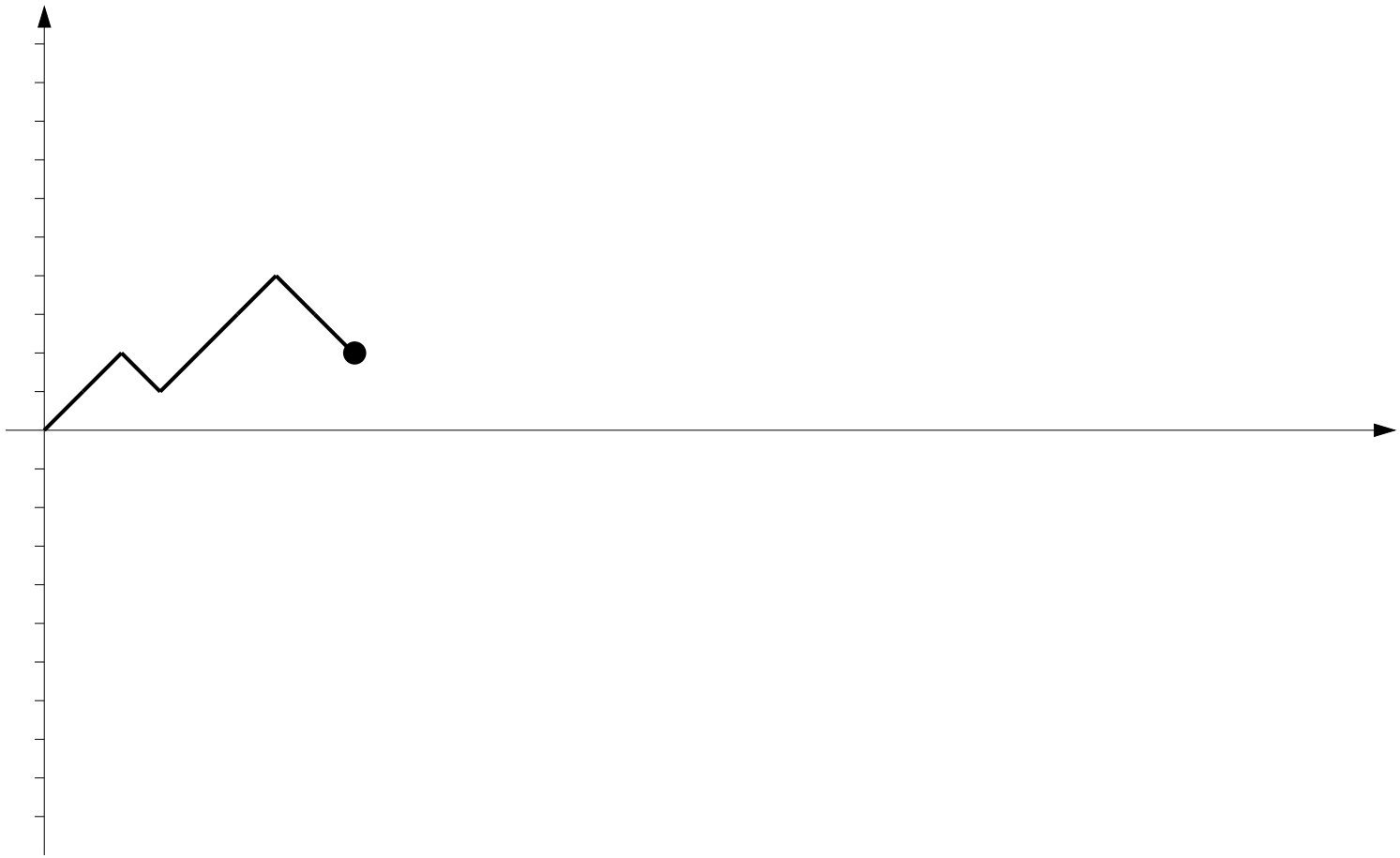

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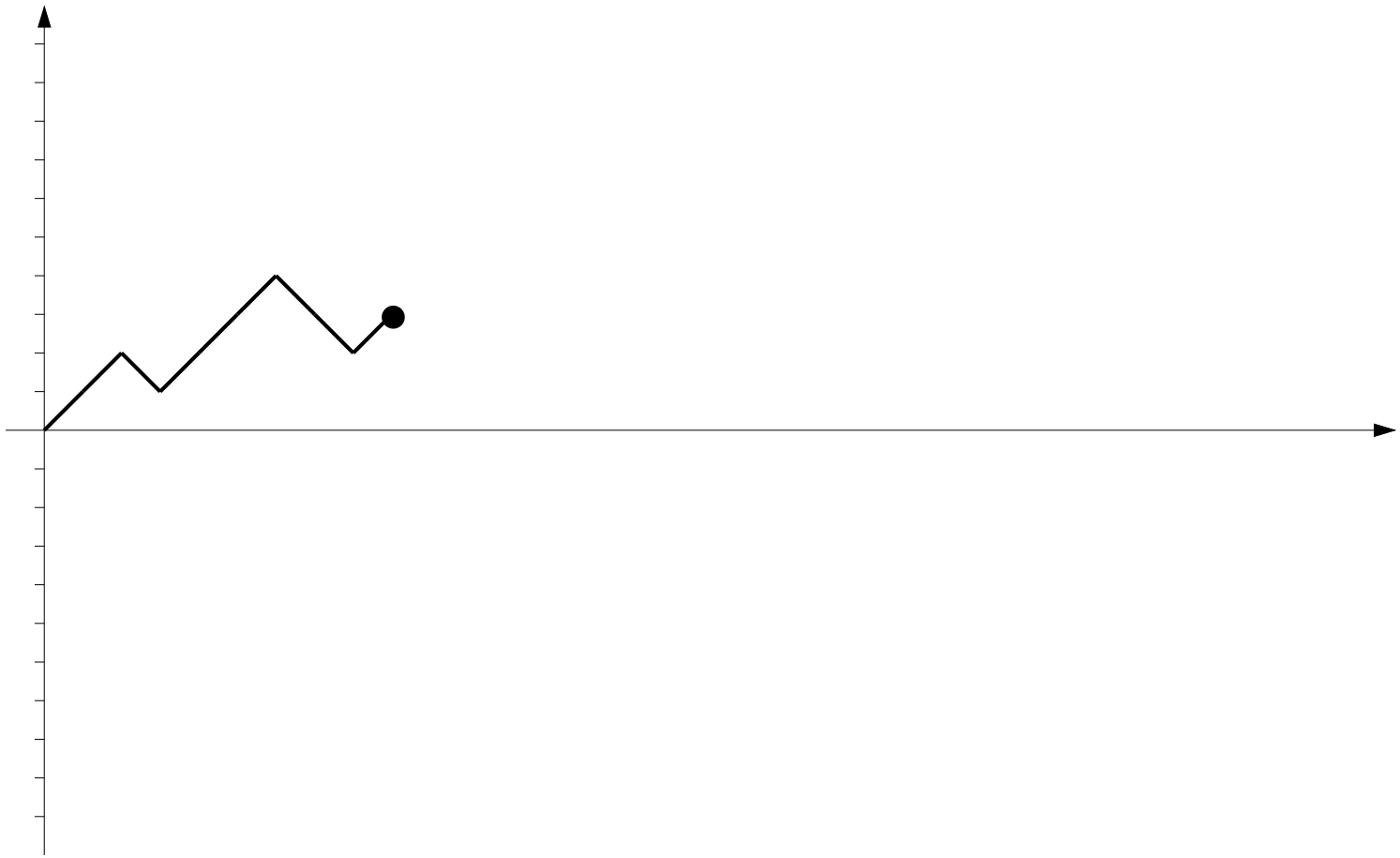

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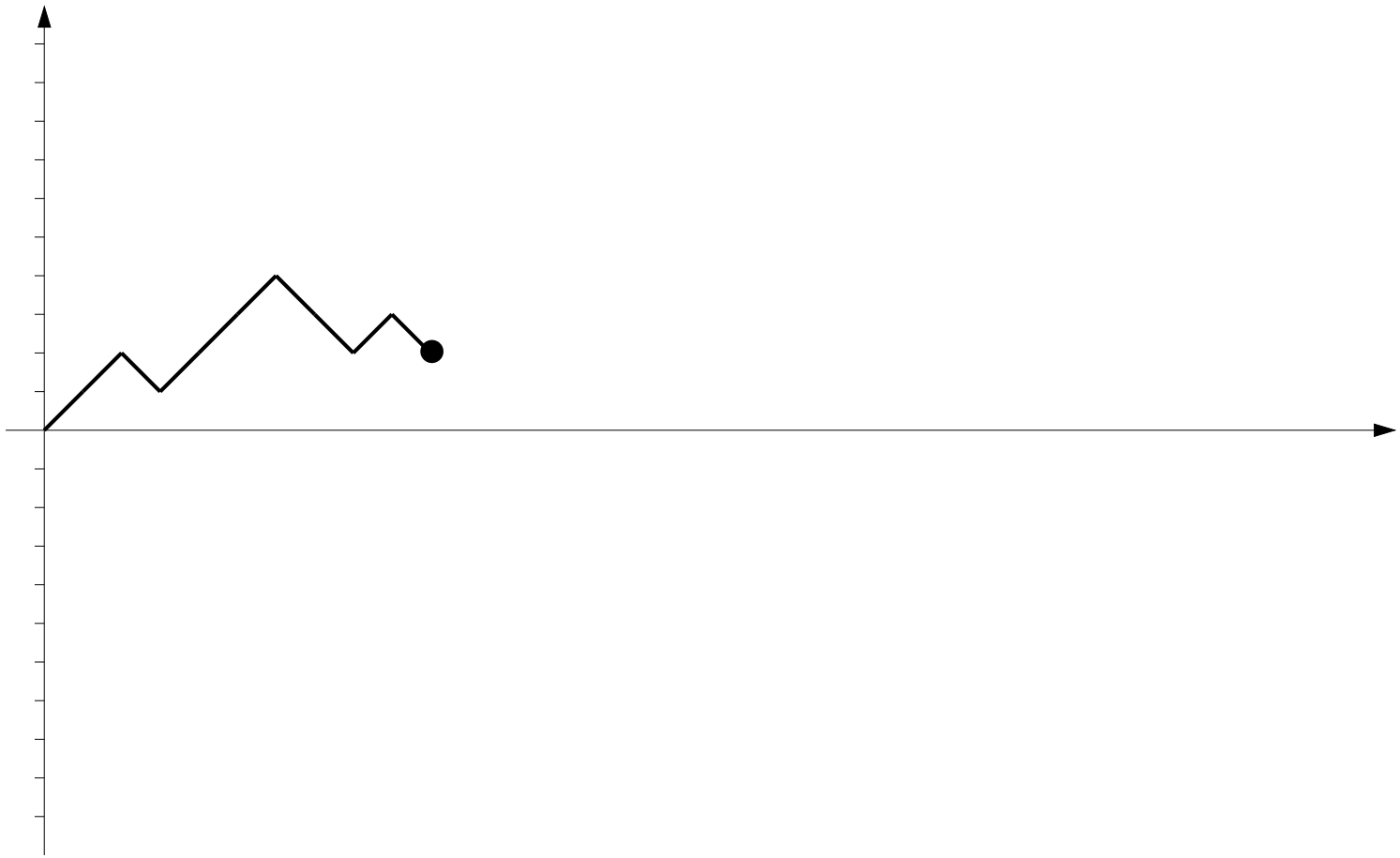

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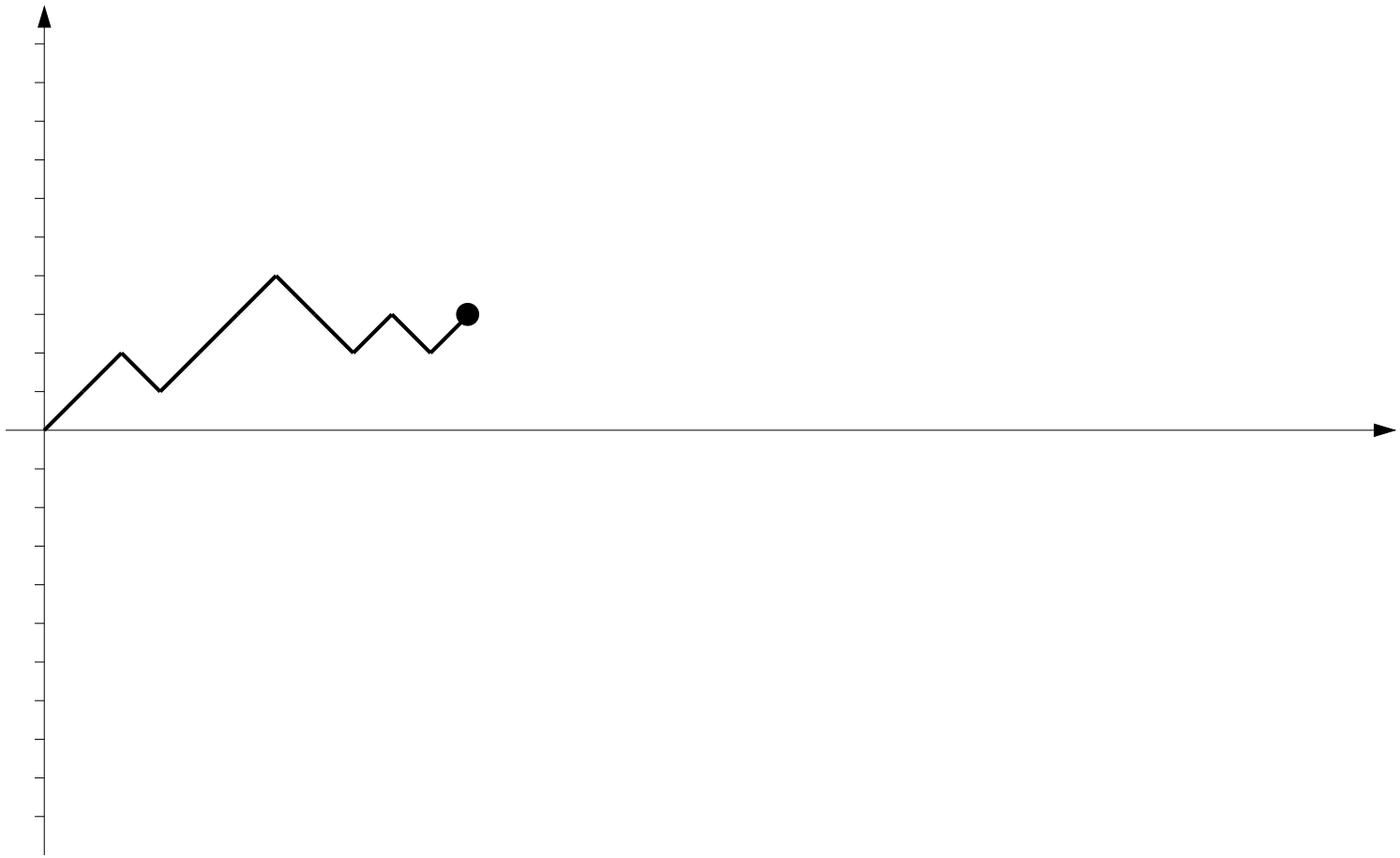

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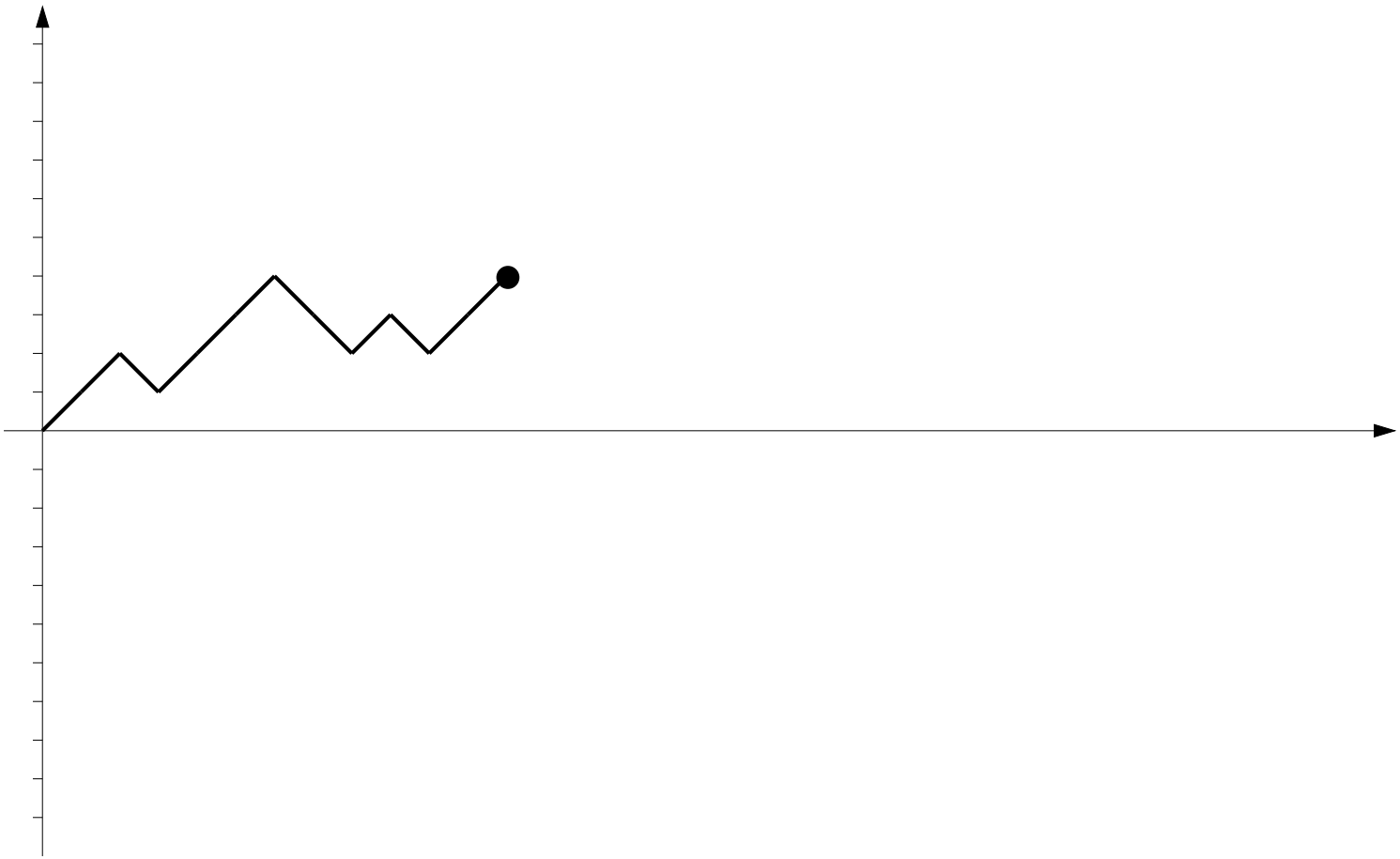

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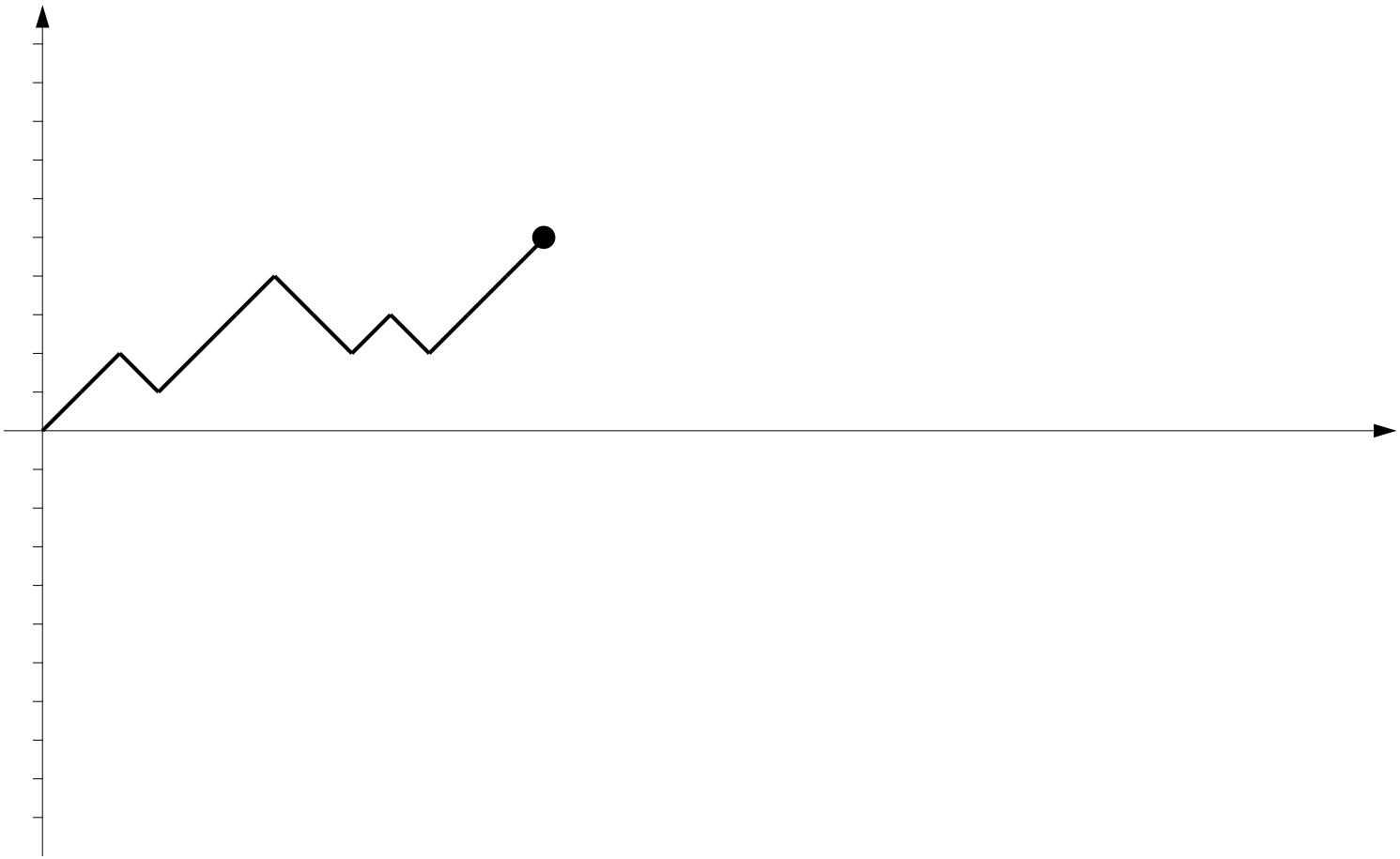

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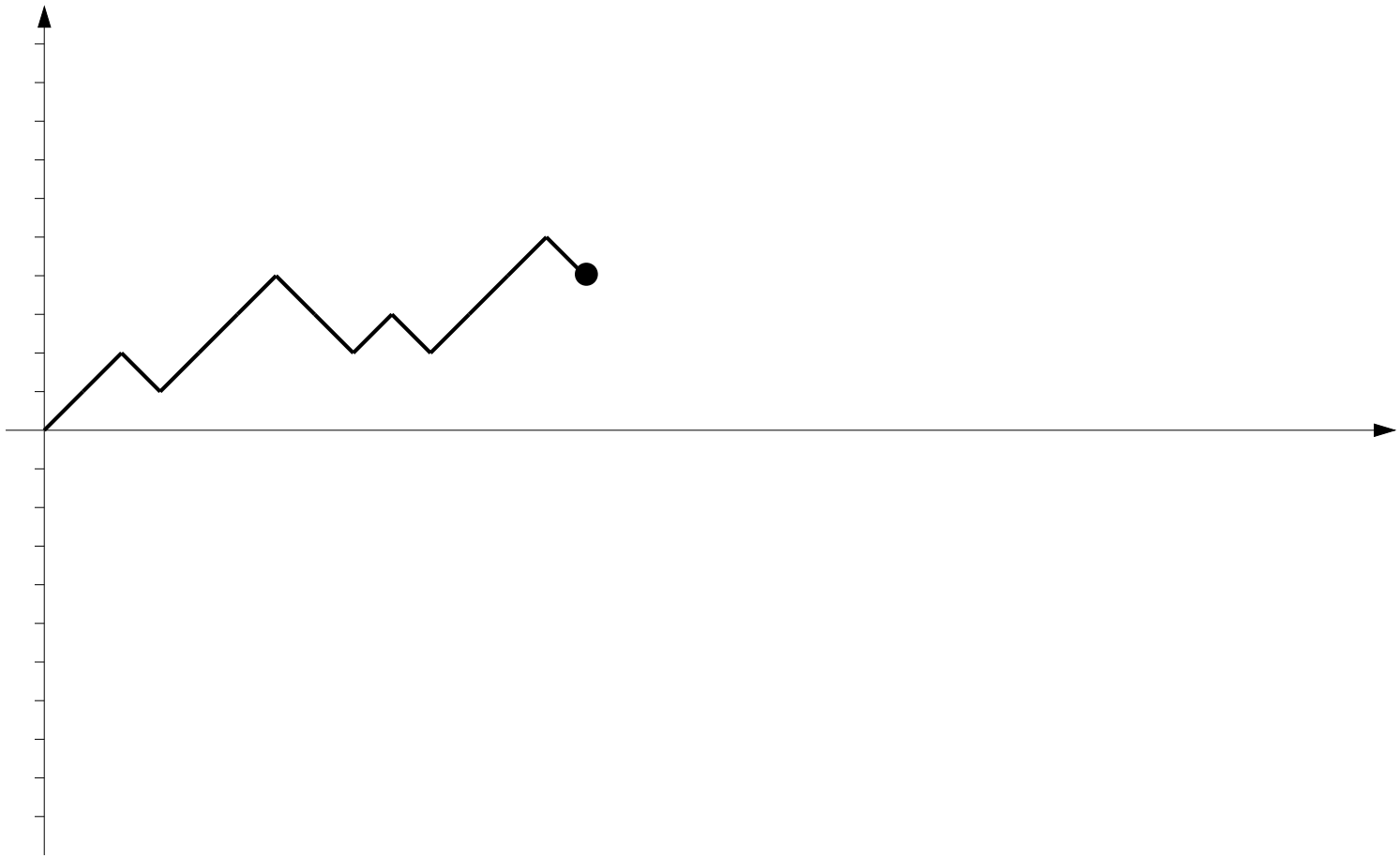

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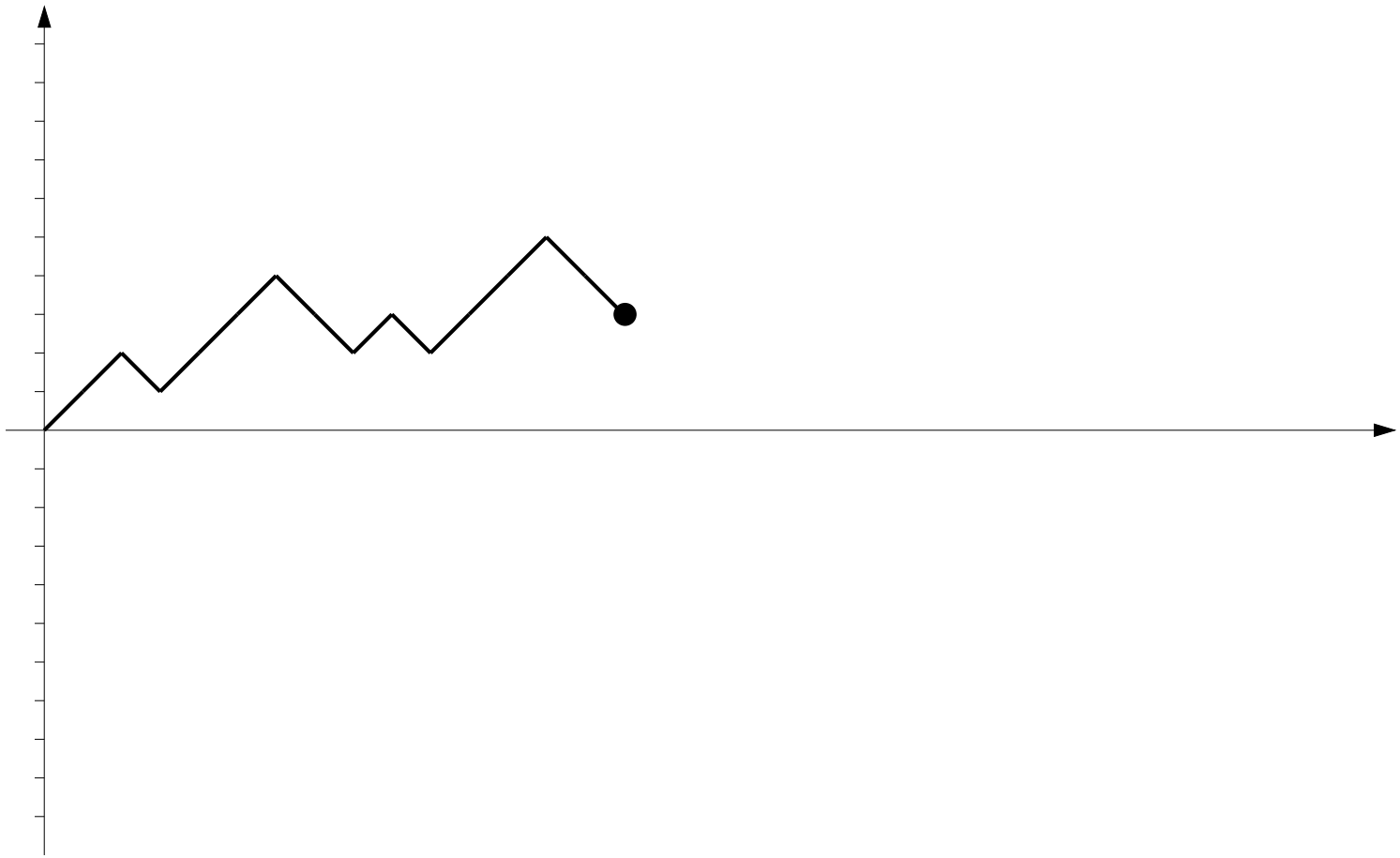

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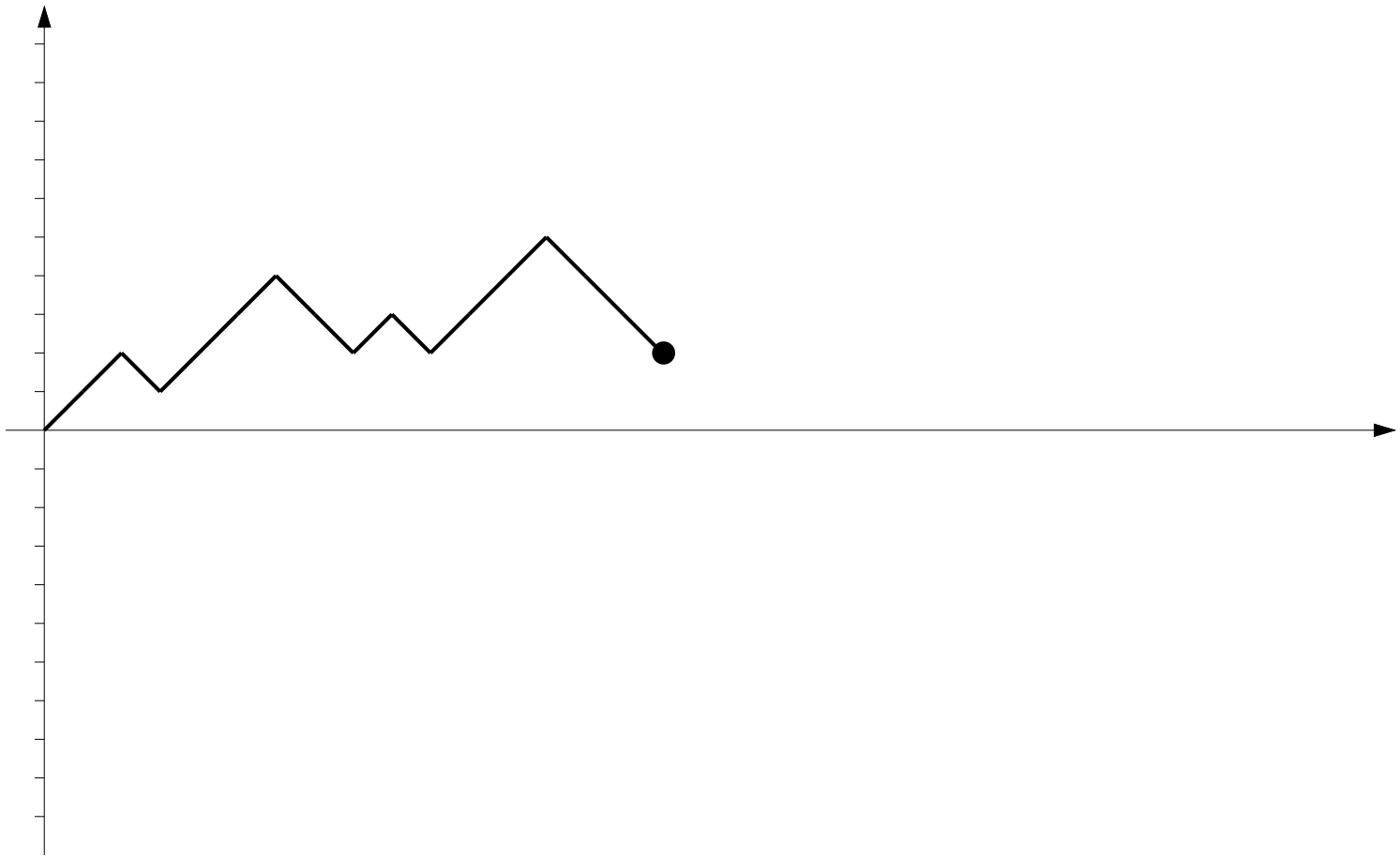

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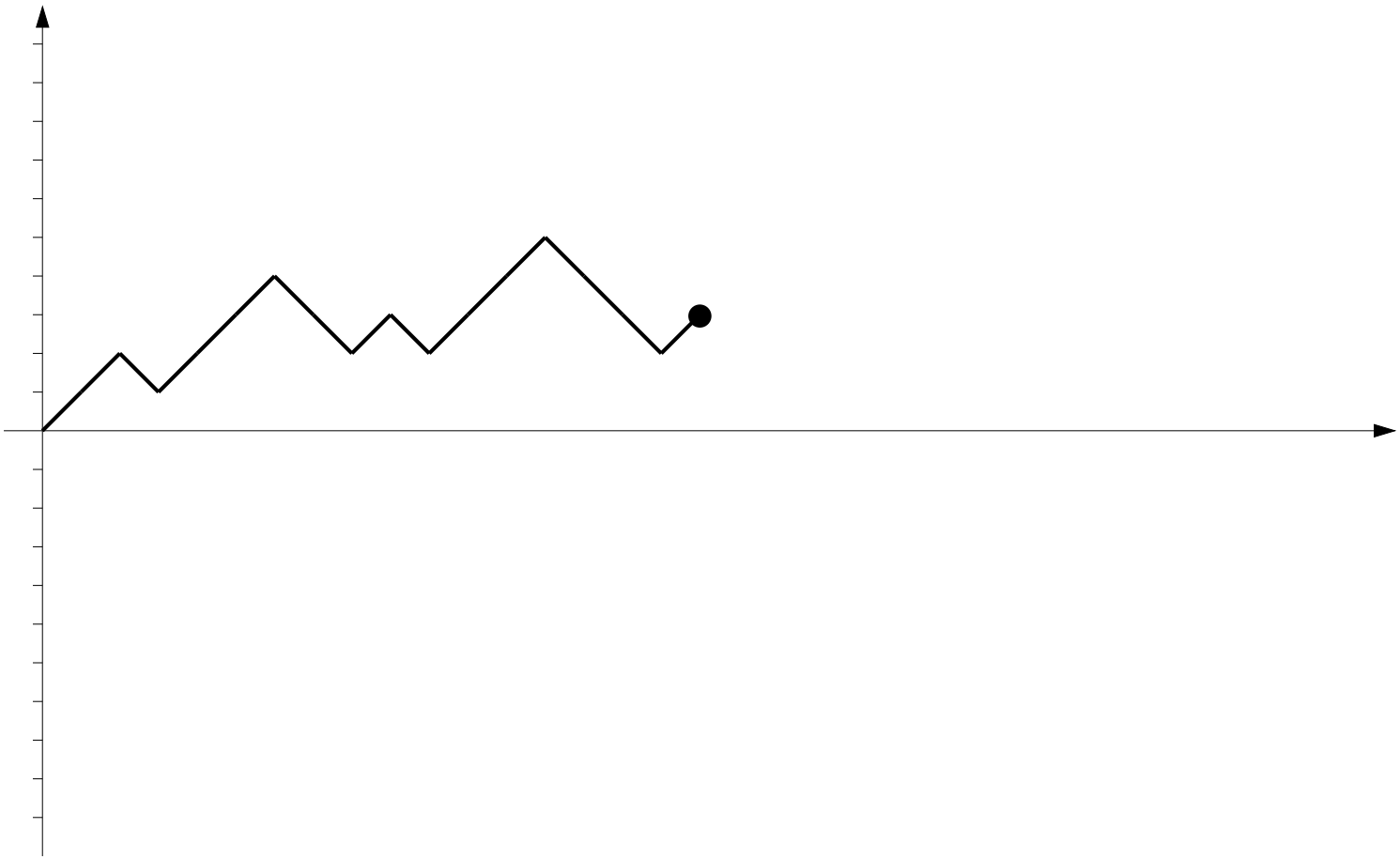

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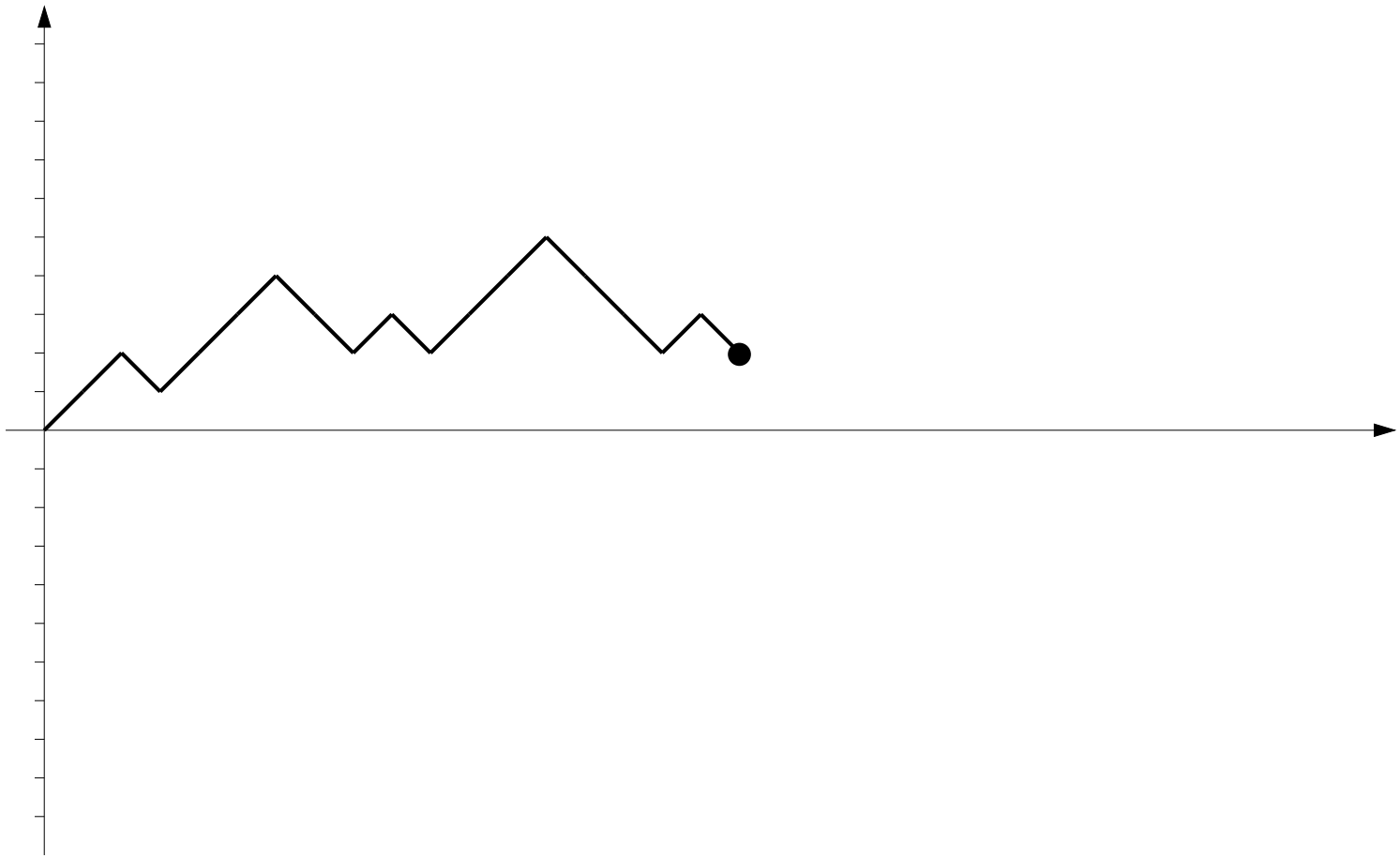

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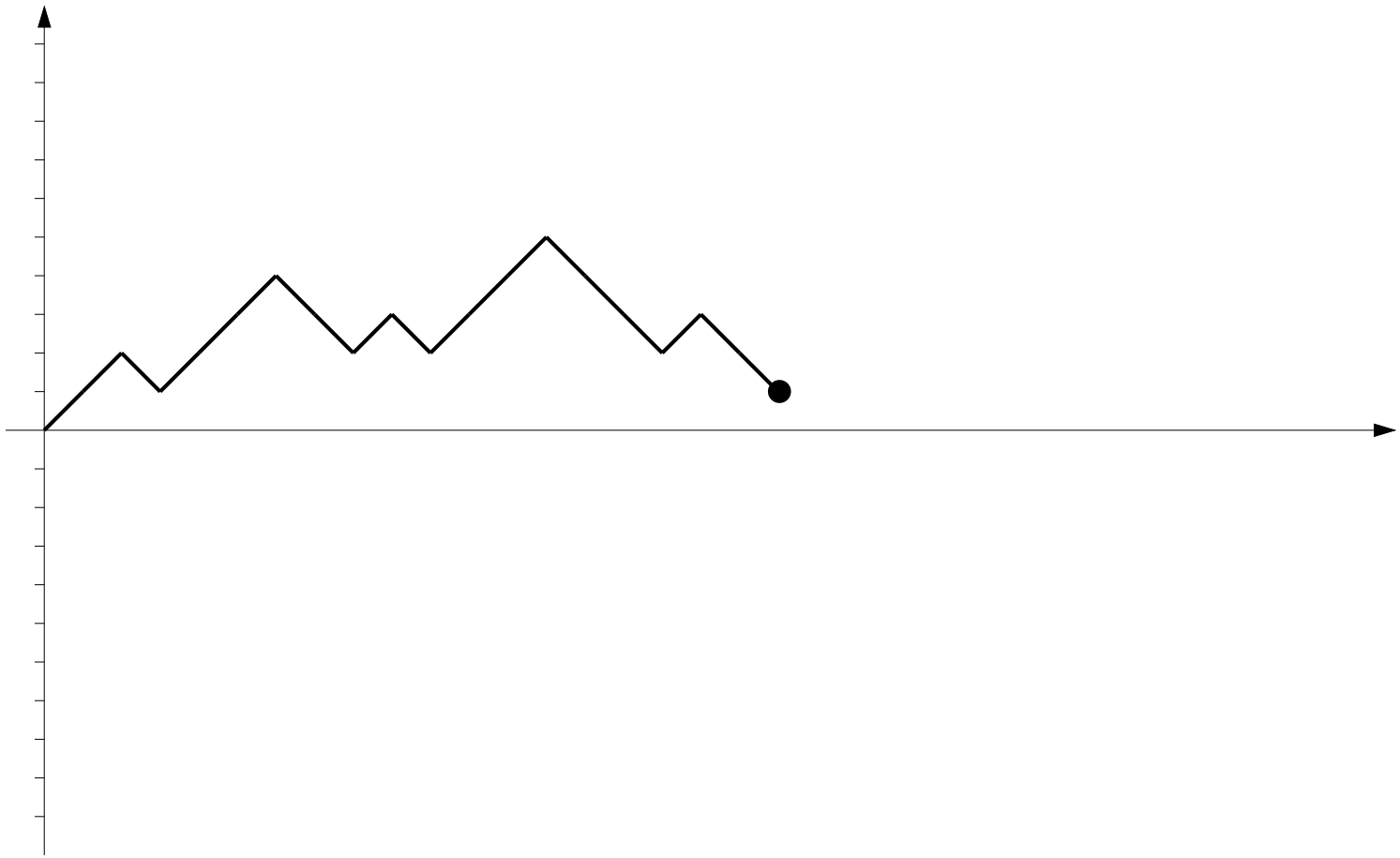

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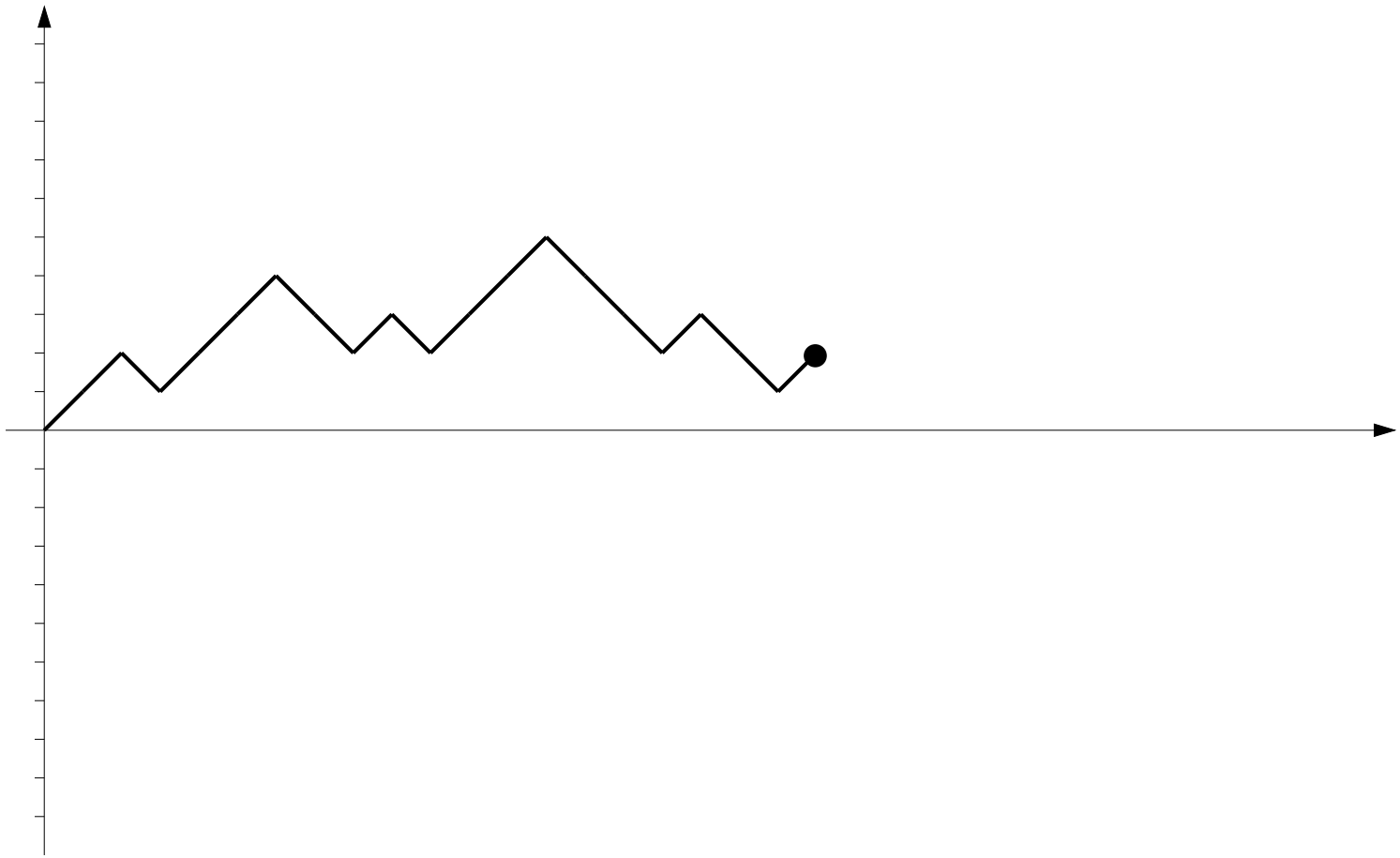

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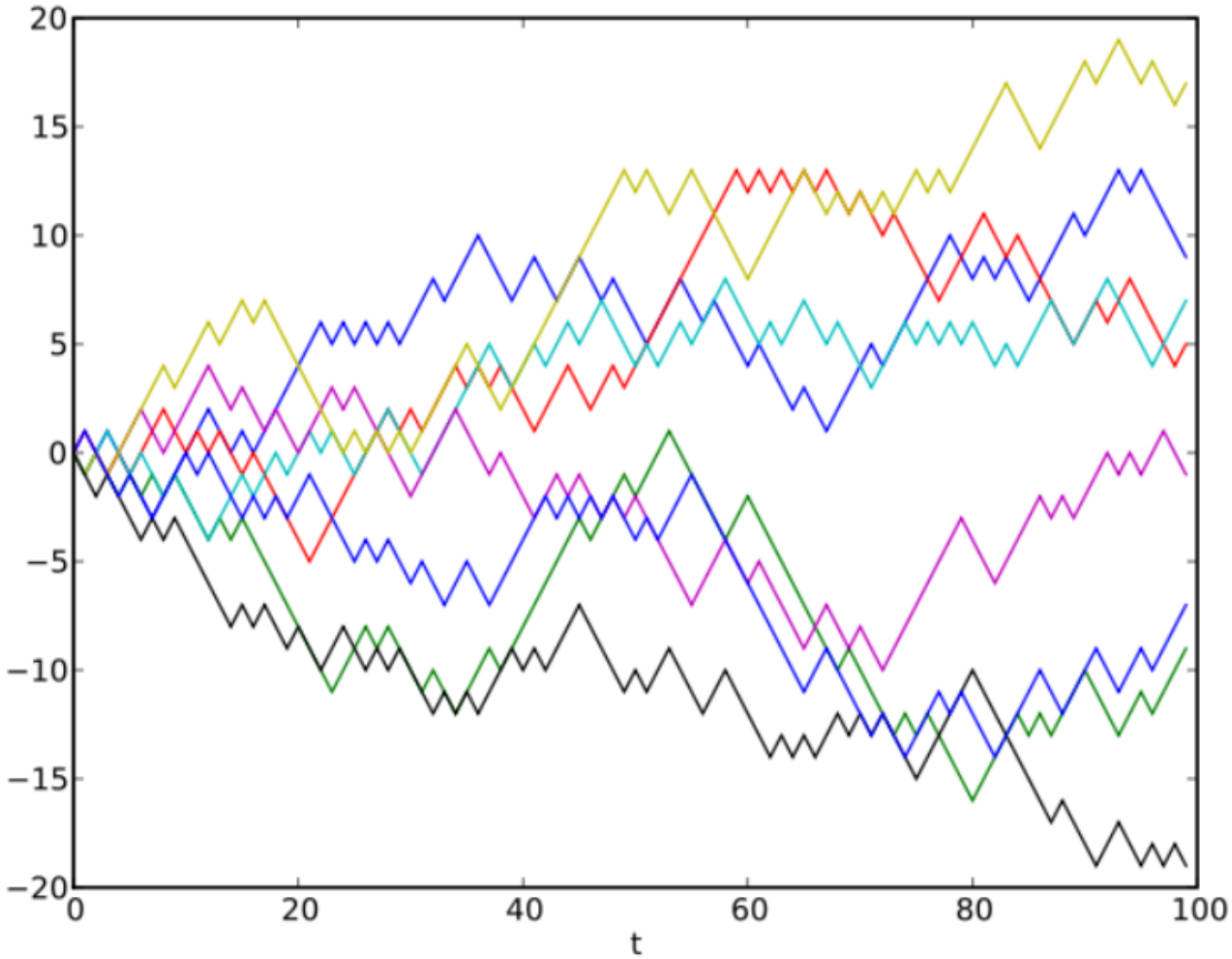

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From randomness to structure

H_n = fraction of Heads in n coin tosses
T_n = fraction of Tails in n coin tosses

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The Fundamental Theorem of Probability: Law (THEOREM) of Large Numbers

$$P(H_n - T_n \approx 0) = 1$$

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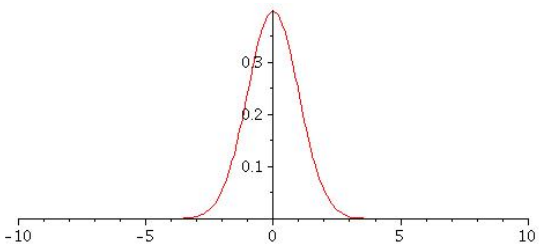
The Fundamental Theorem of Probability: Law (THEOREM) of Large Numbers

$$P(H_n - T_n \approx 0) = 1$$

The Fundamental Theorem of Statistics: Central Limit Theorem

$$P(x < \sqrt{n}(H_n - T_n) < x + h) \approx h\frac{\exp(-x^2/2)}{\sqrt{2\pi}}$$



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Structure

Here is then what I am busy with:

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By means of "Stochastic Mathematics".

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I prove limit theorems.

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An example follows.

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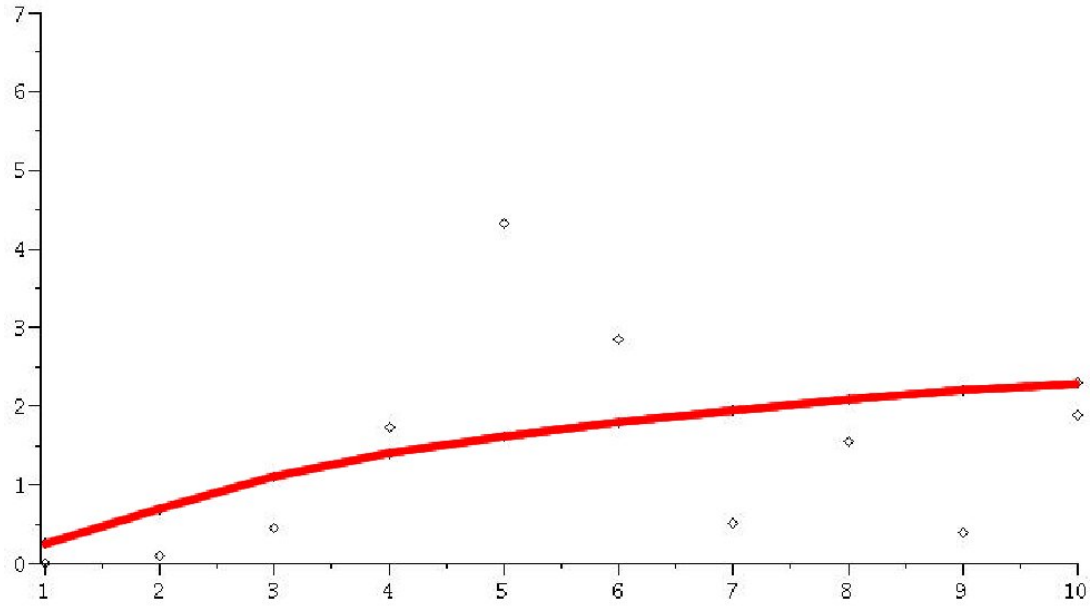
Structure: an example

red line $= 1.0001 \log n$



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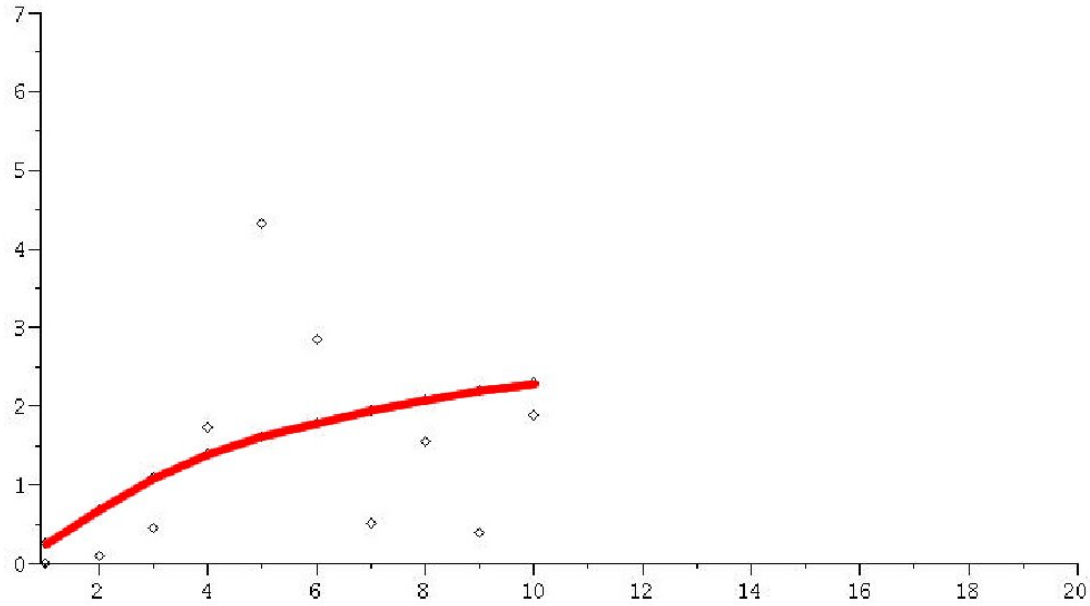

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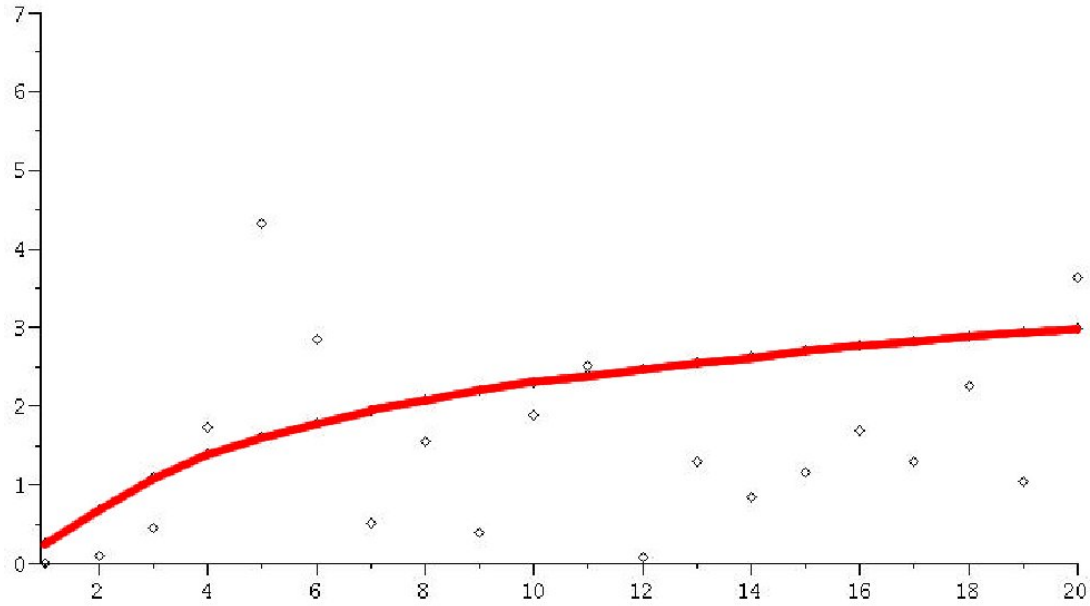

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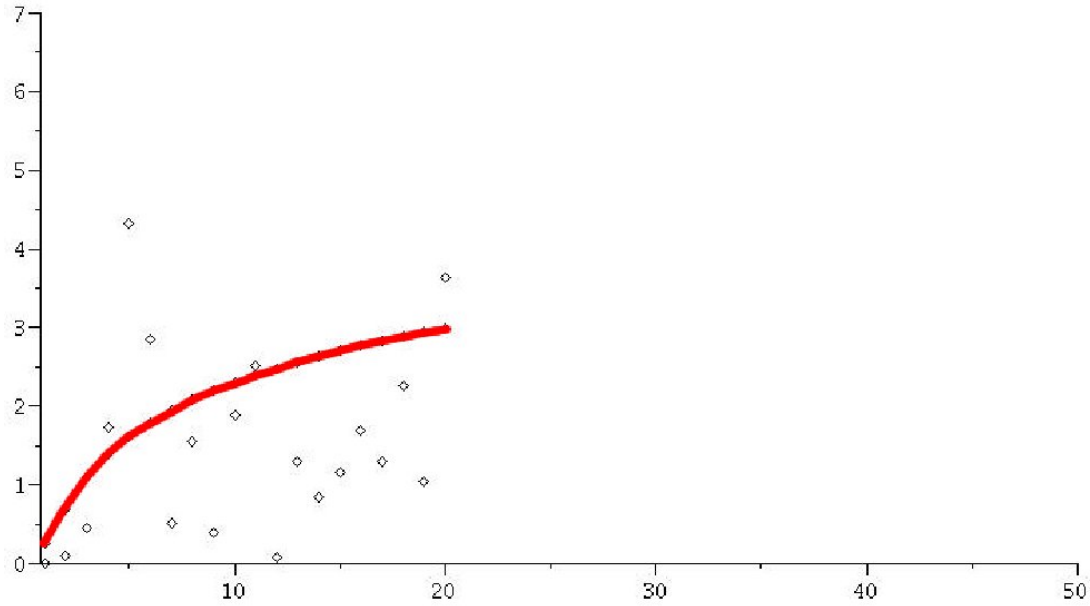

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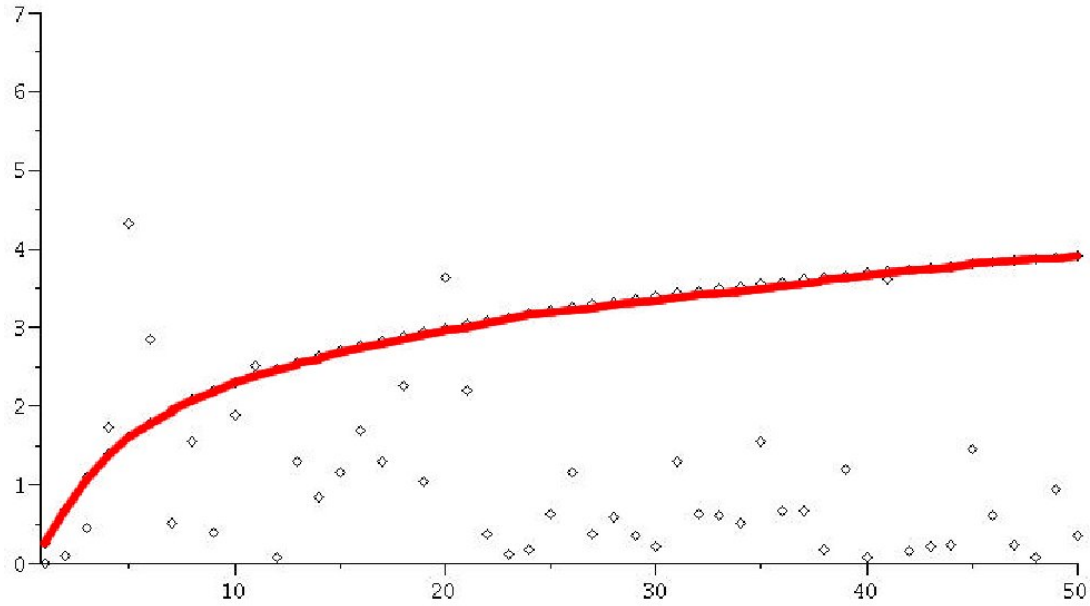

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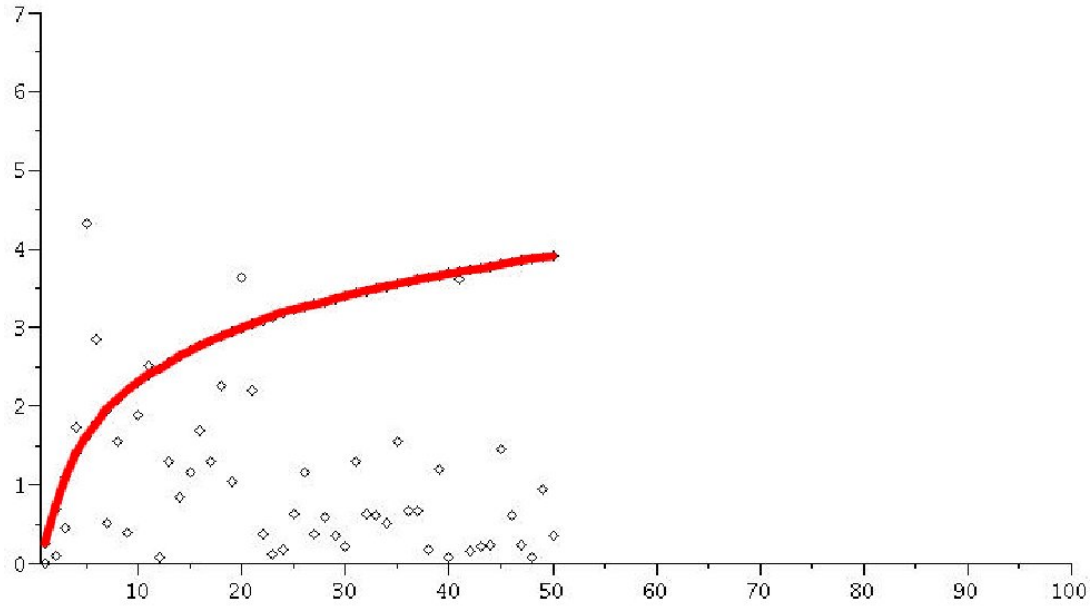

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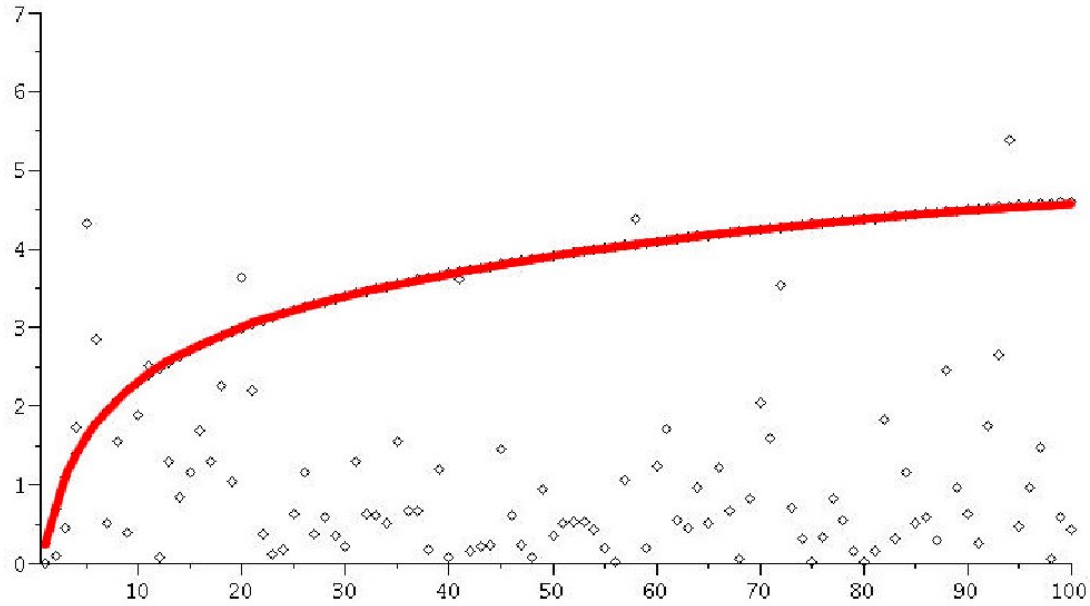

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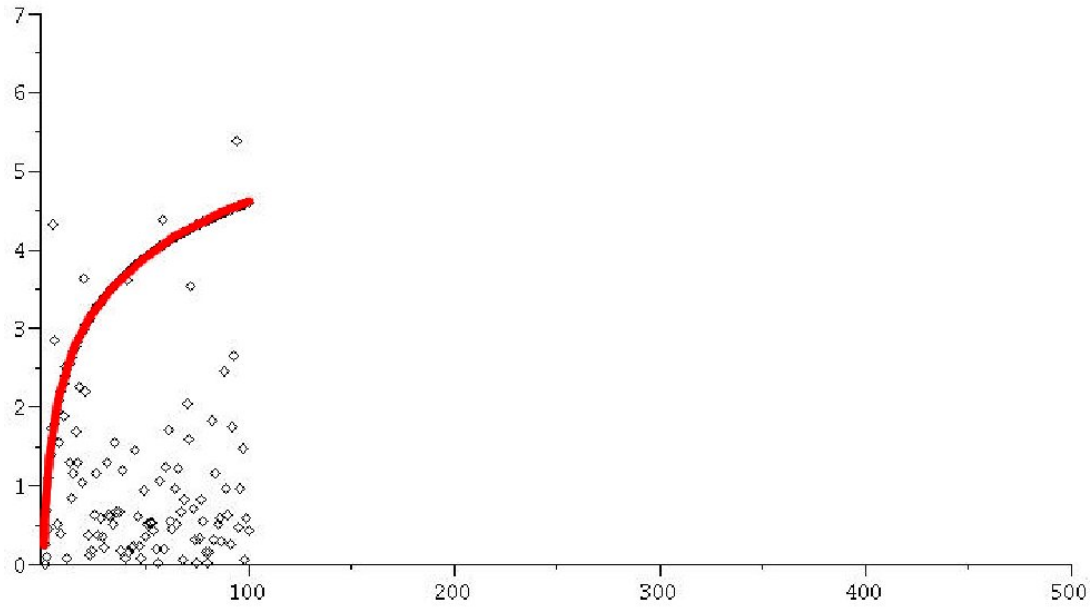

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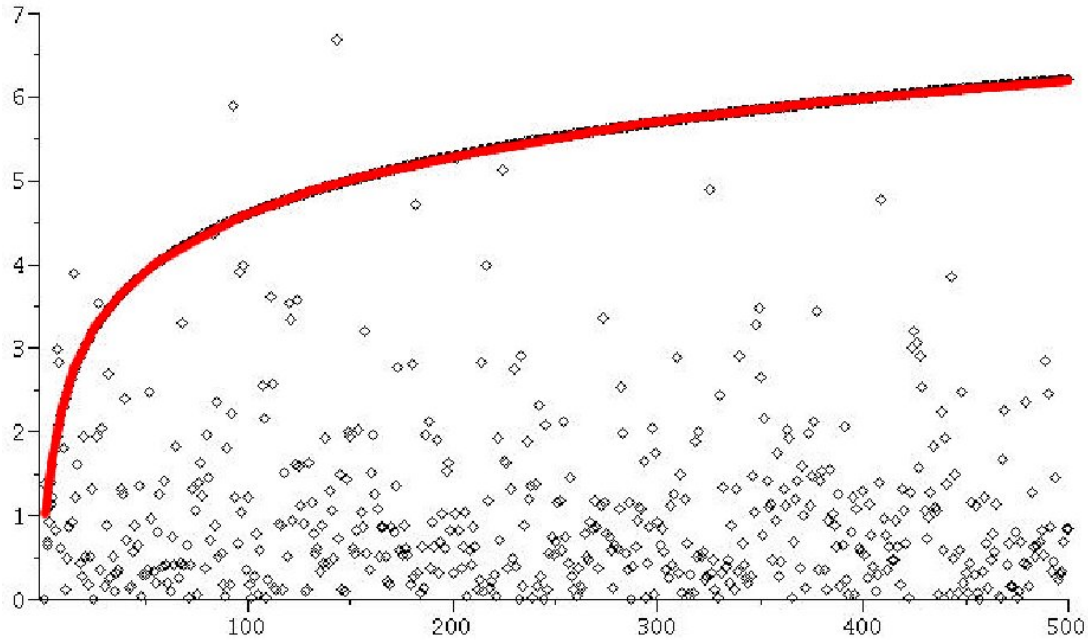

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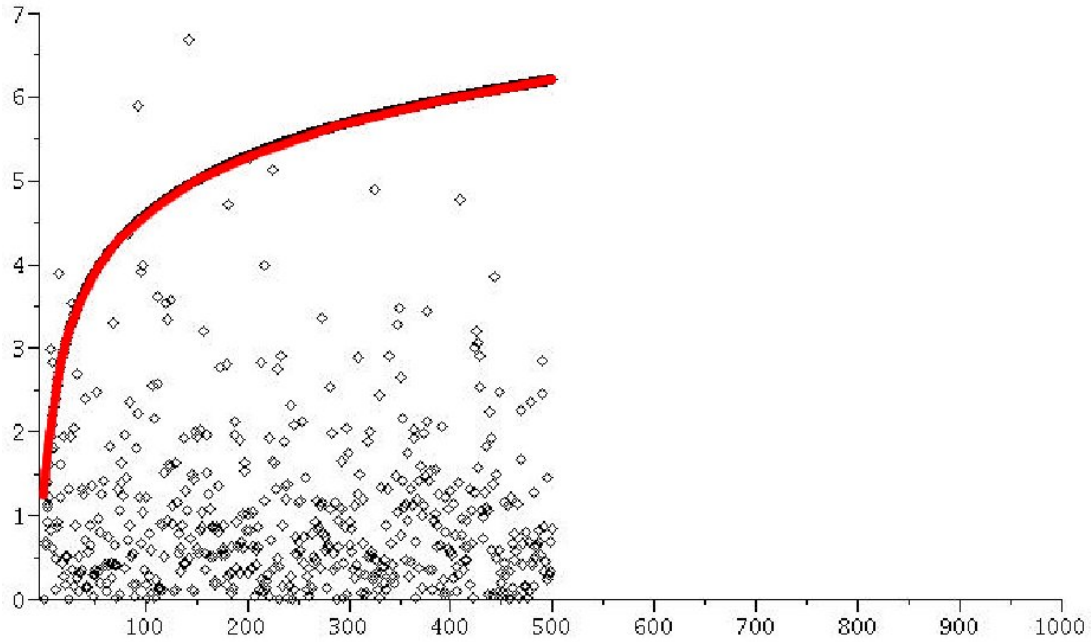

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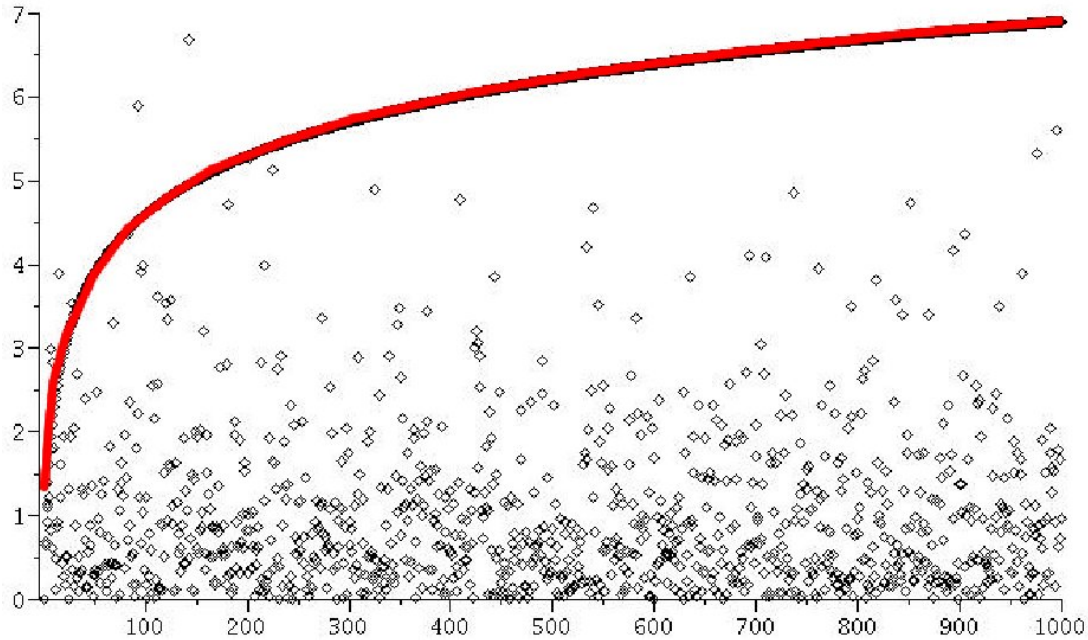

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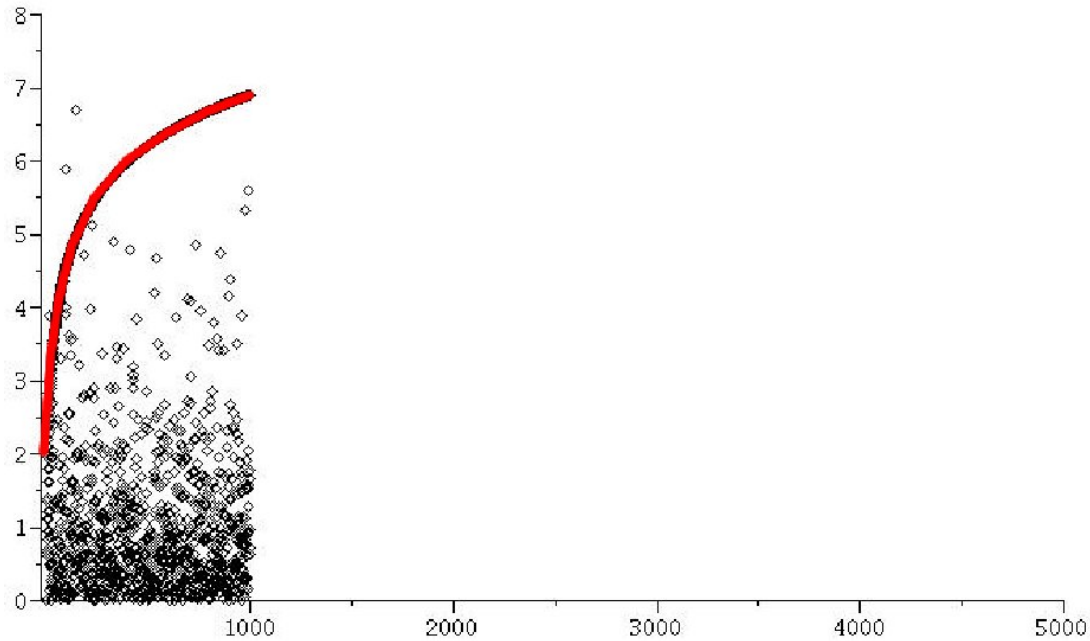

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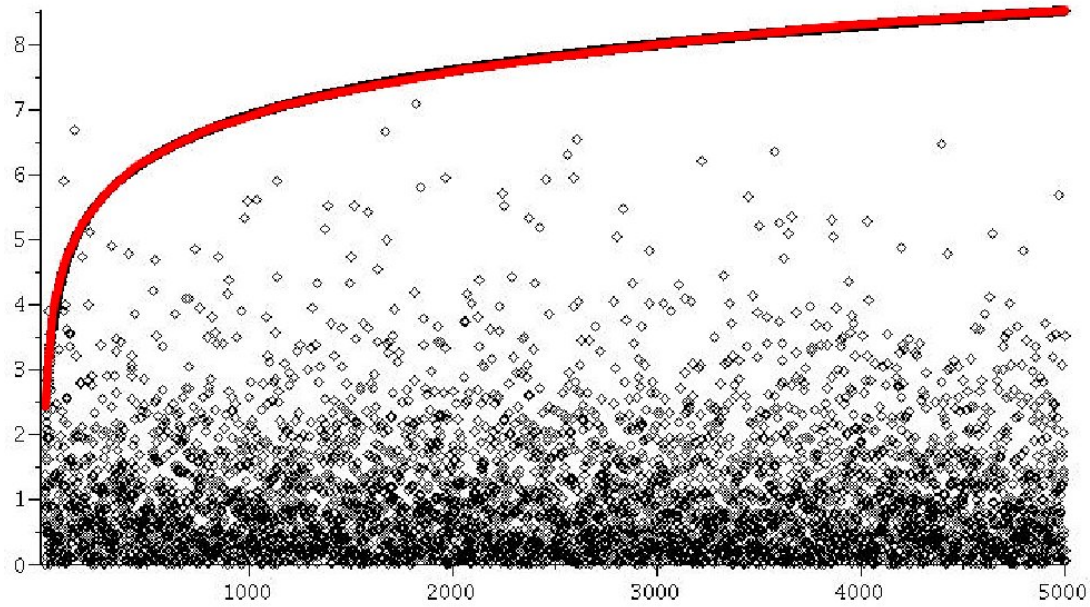

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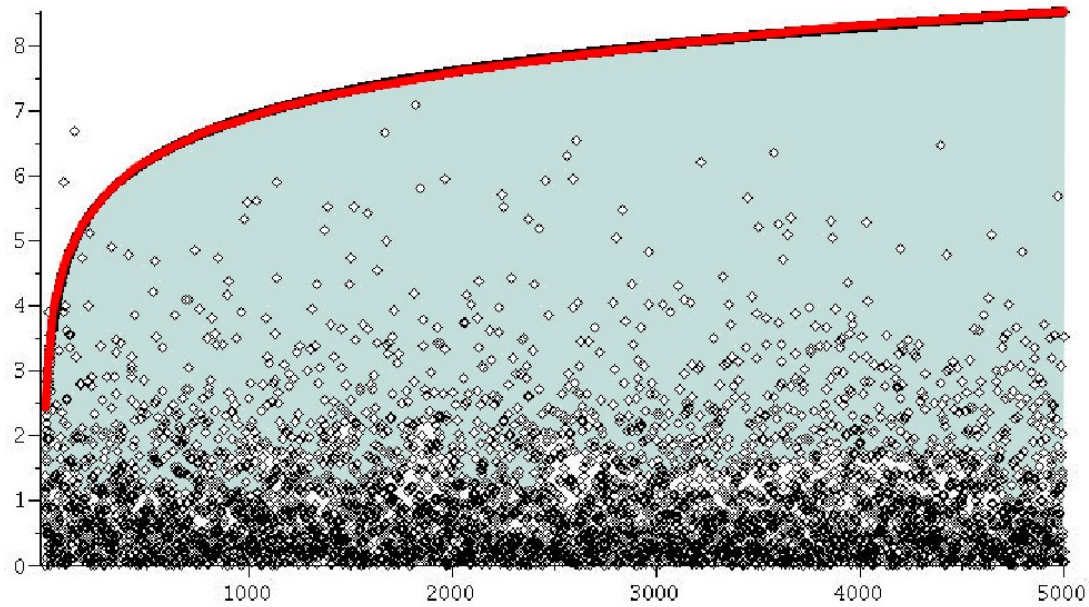

5000 points

Despite randomness, all points from now on lie under the red curve!

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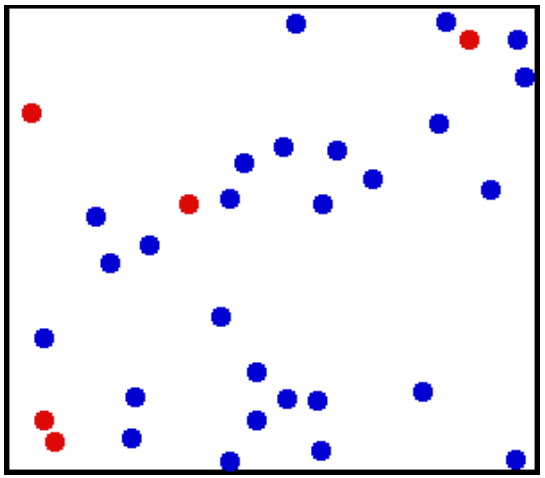
Structure and universality

We say that a class of dynamical systems exhibits universality with respect to a certain property if this property does not depend on the details of the systems.



Systems display universality in a scaling limit, when a large number of interacting parts come together.

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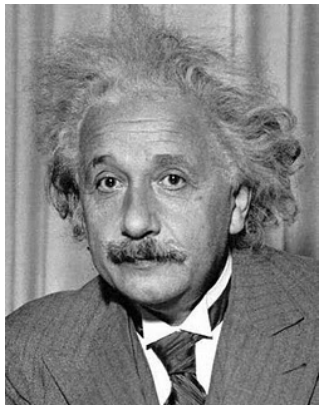
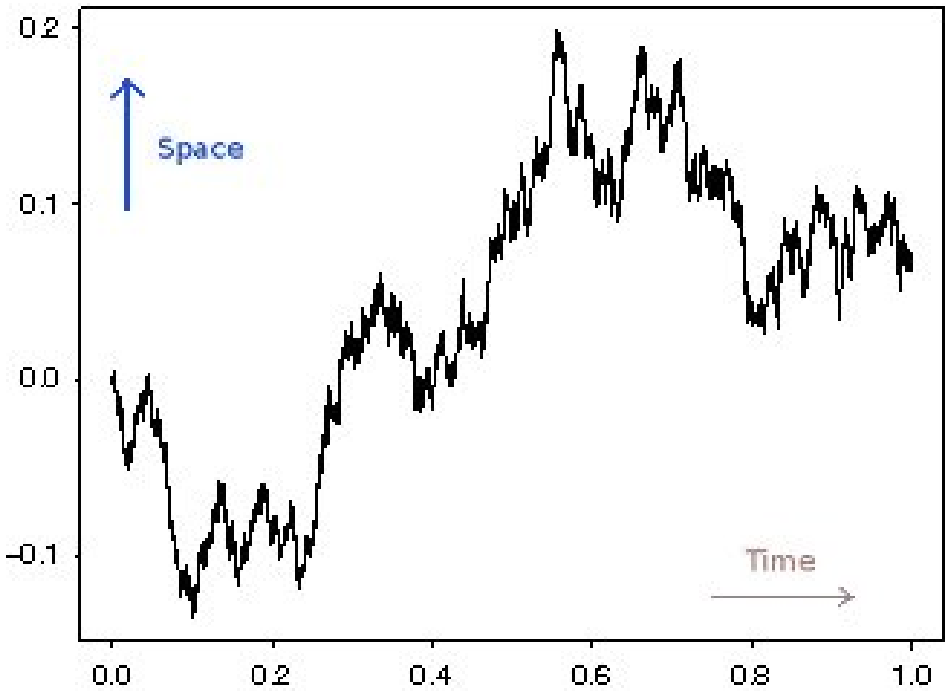
Scaling limit of Random Walk = Brownian Motion





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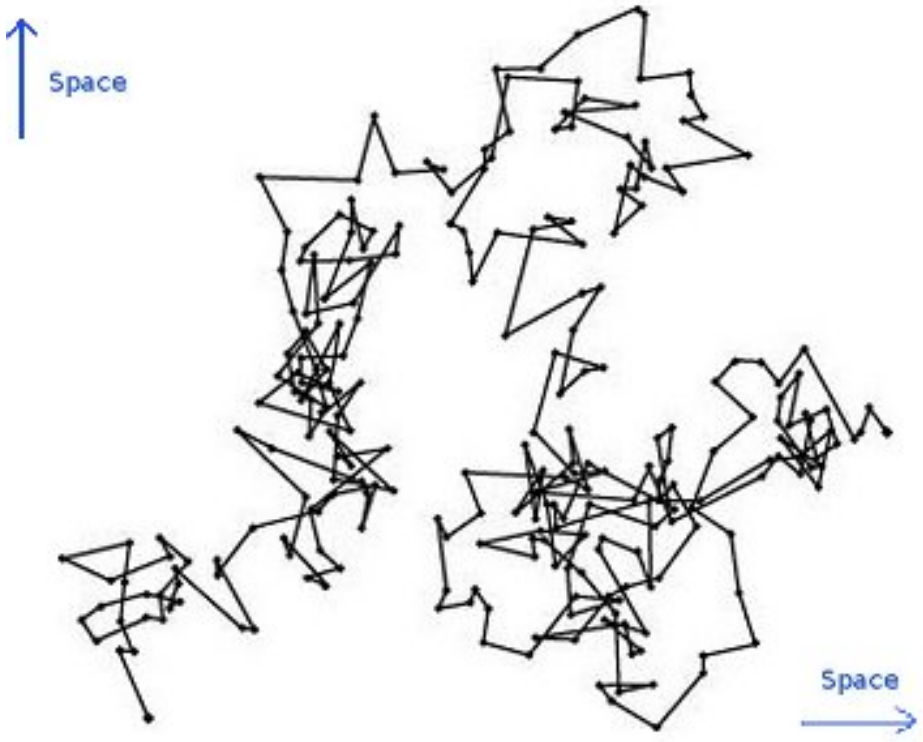
Random walk in 2 dimensions



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Rescaling to get 2 dimensional Brownian motion

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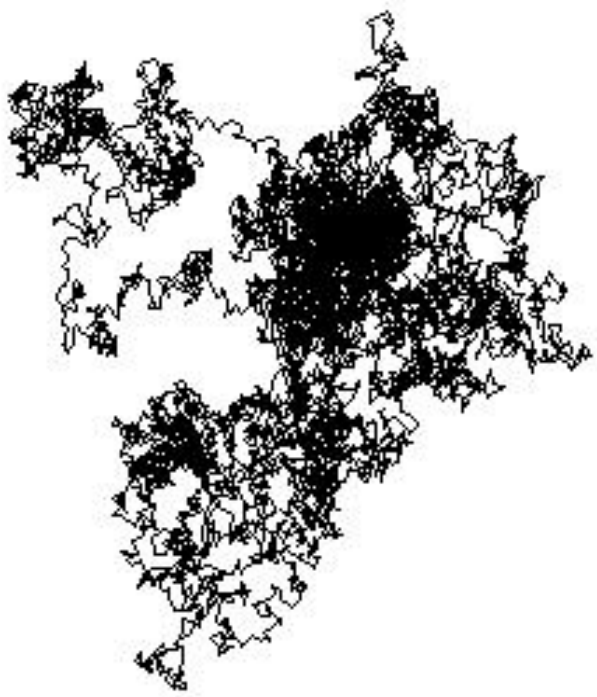

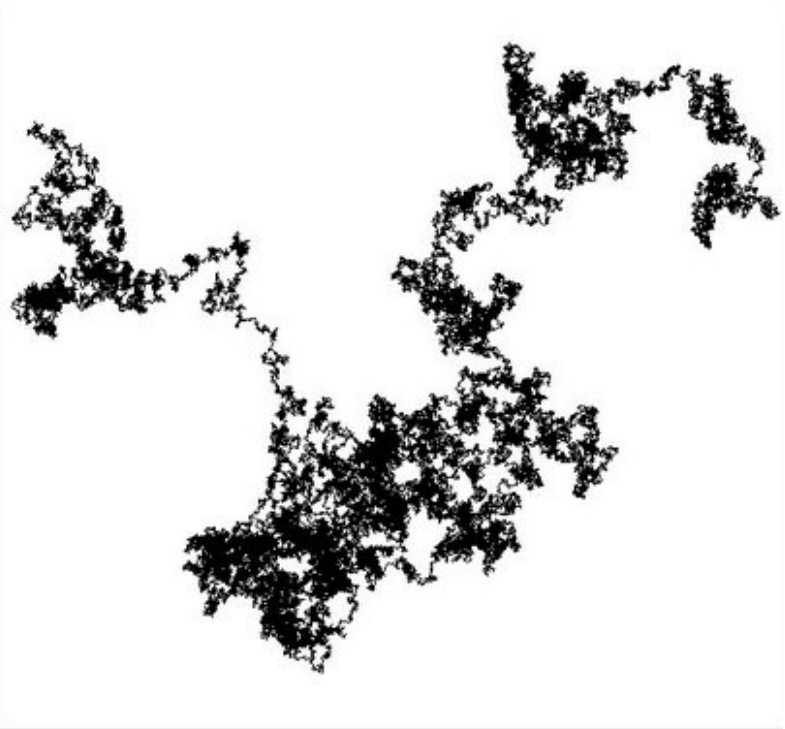

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$S(1), \quad S(S(1))$

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$S(3), \quad S(S(3))$

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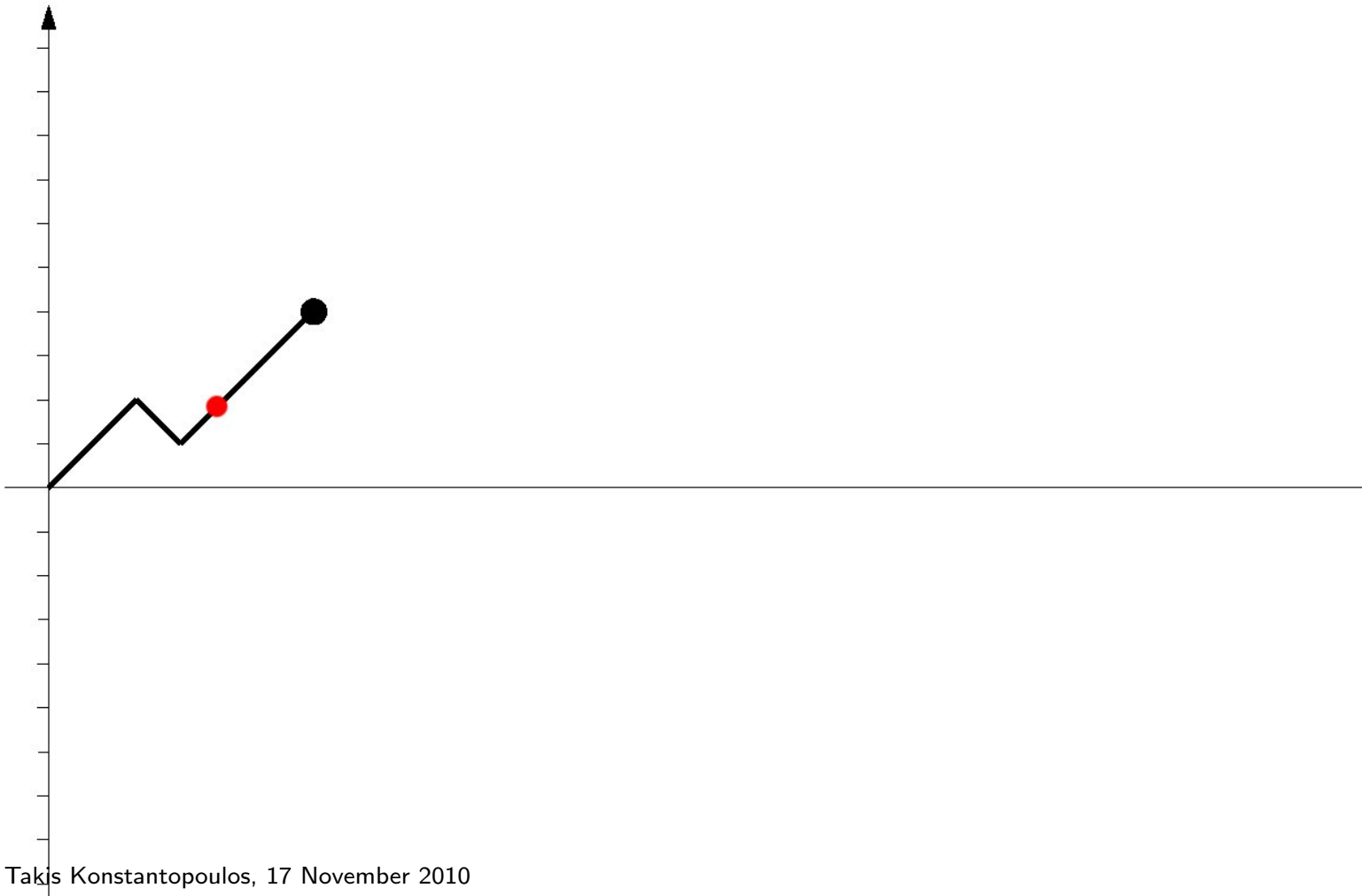

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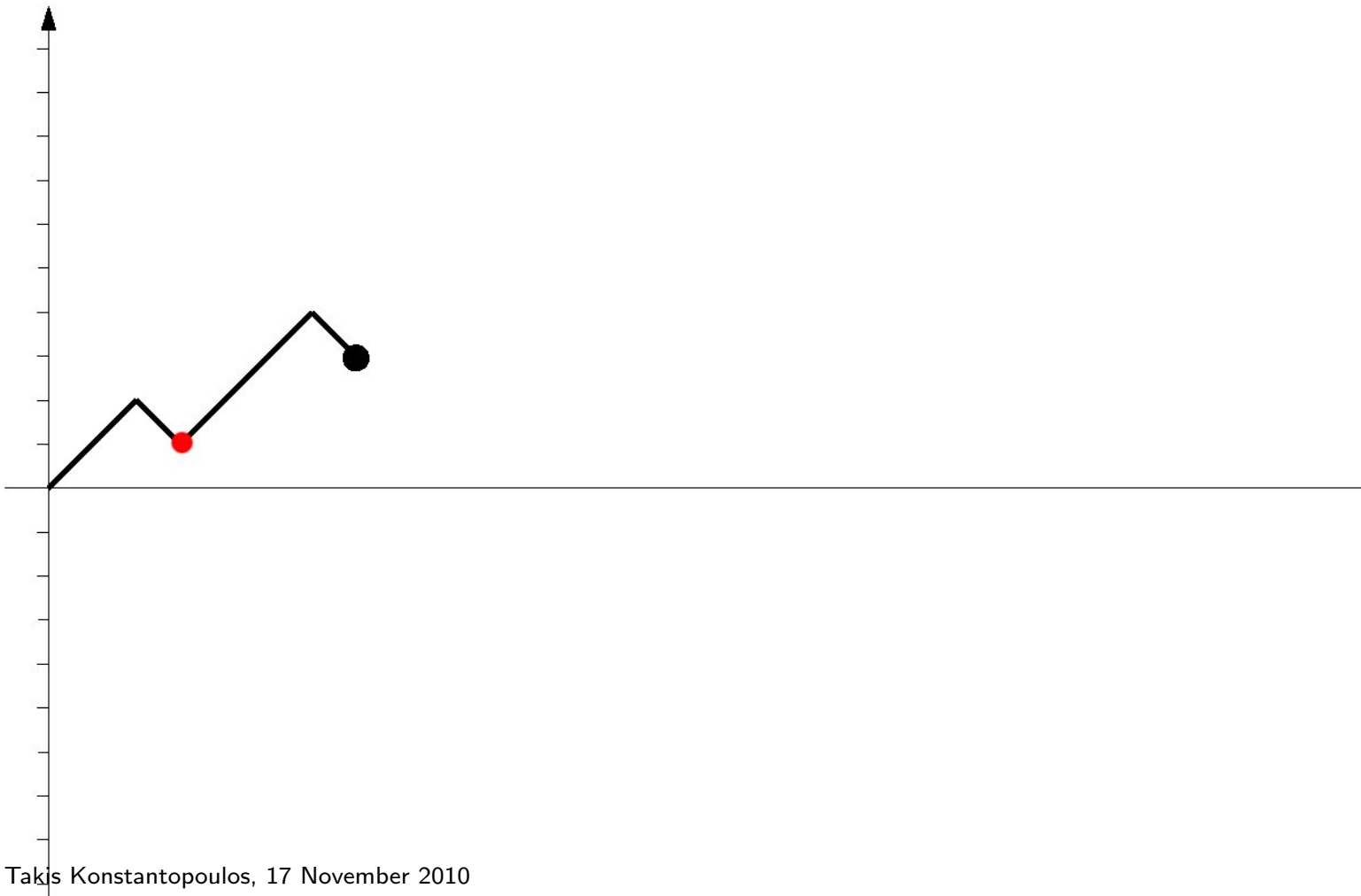

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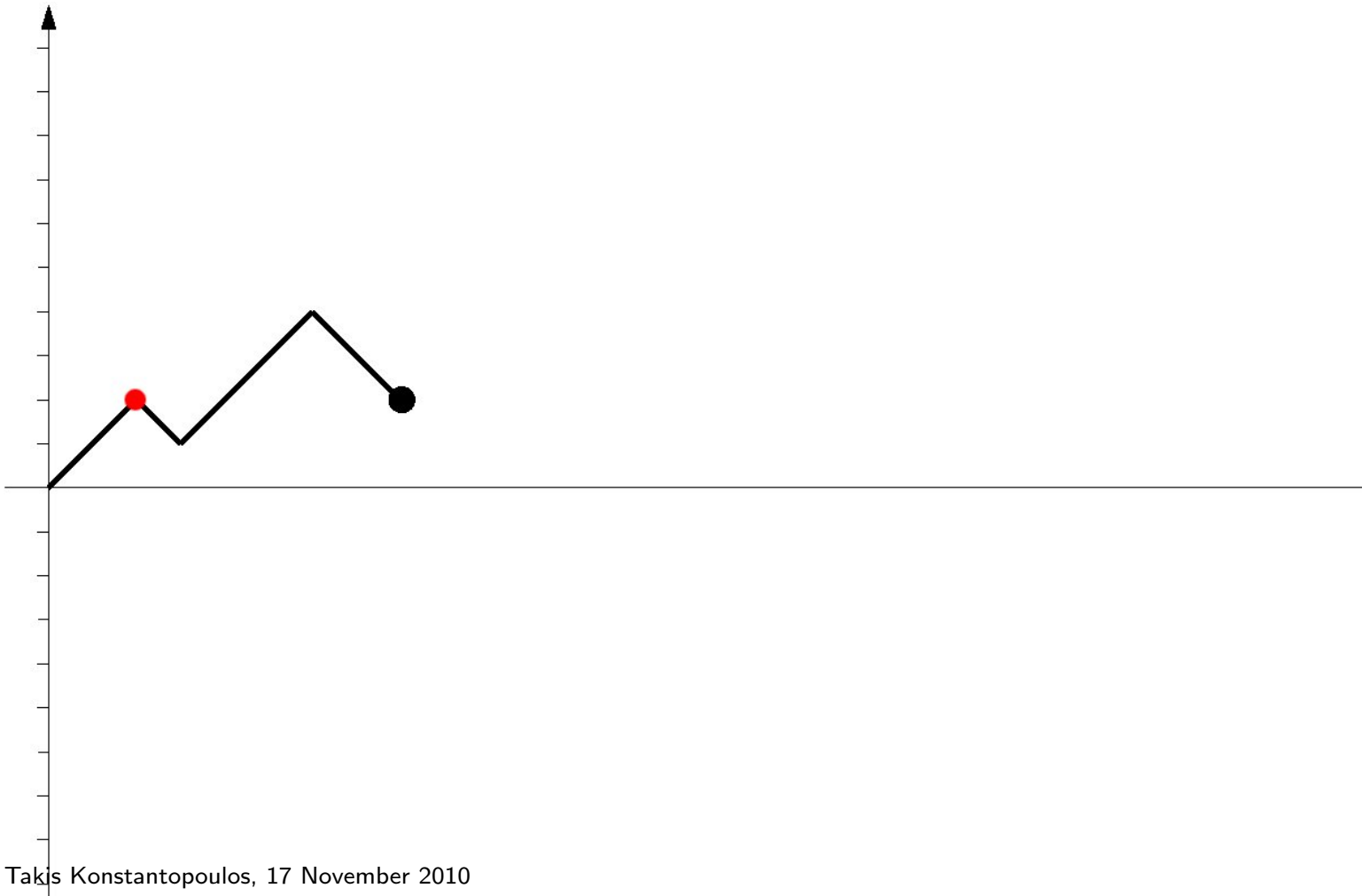
$S(8), \quad S(S(8))$



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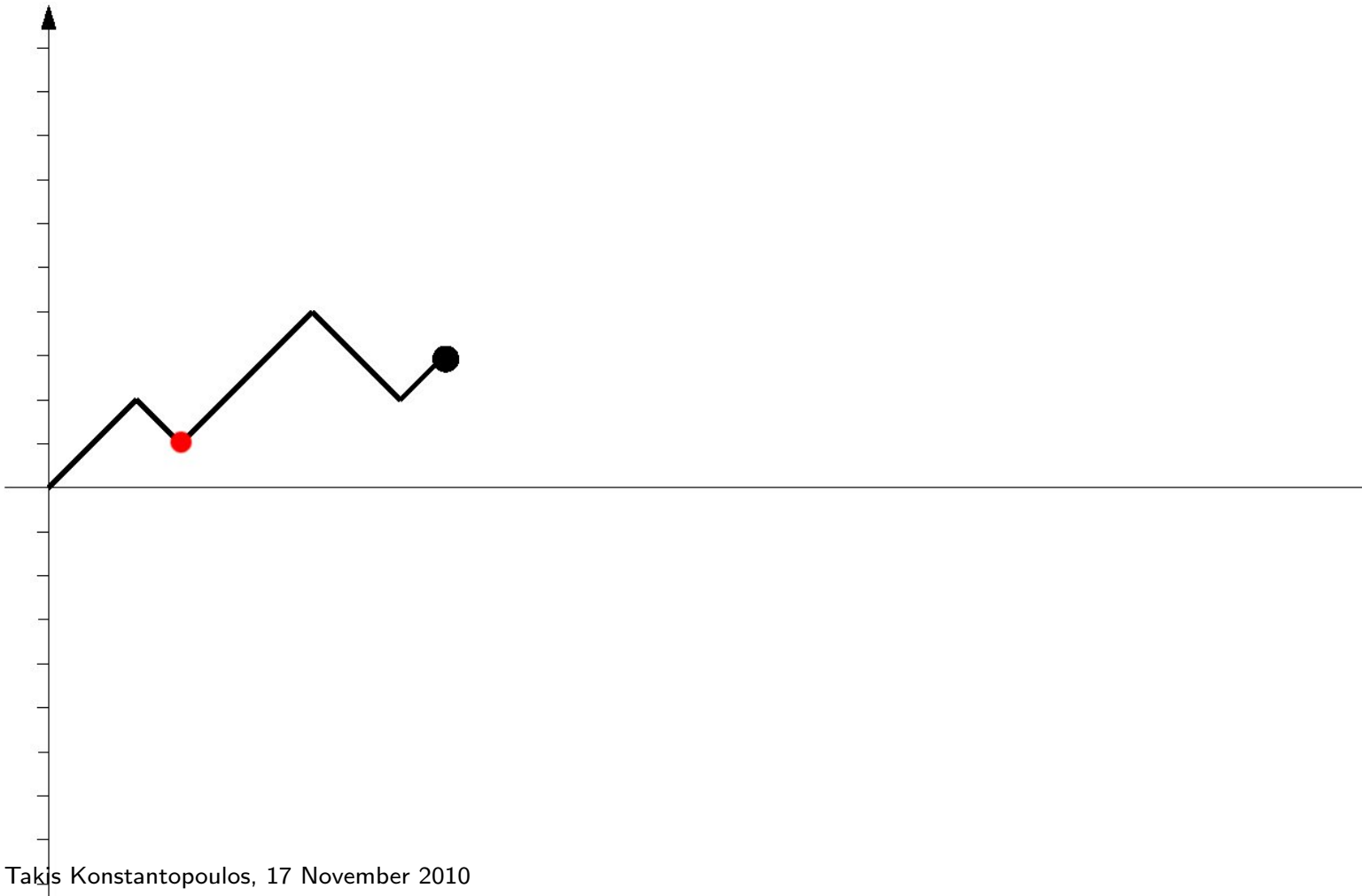
$S(9), \quad S(S(9))$



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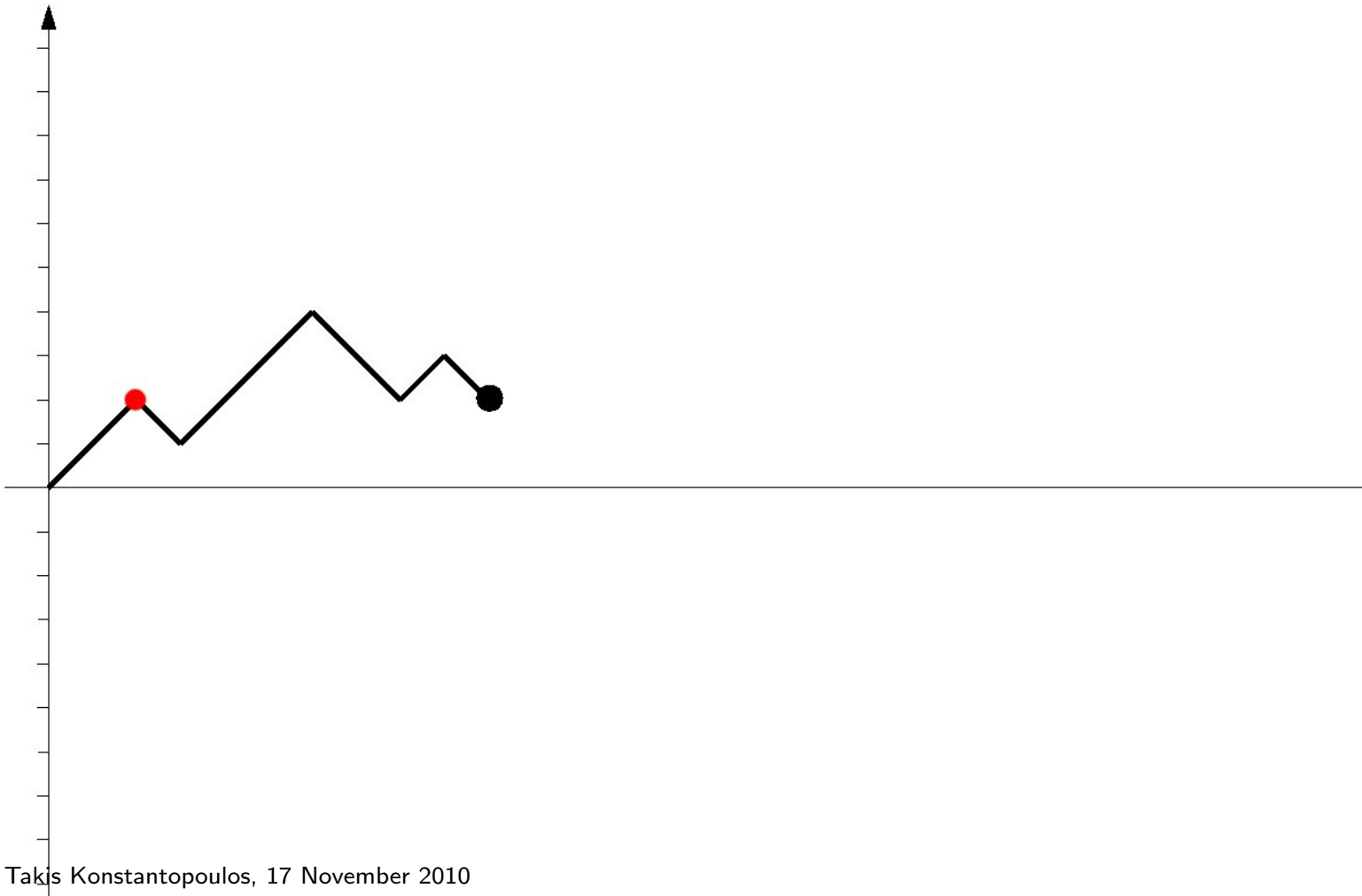

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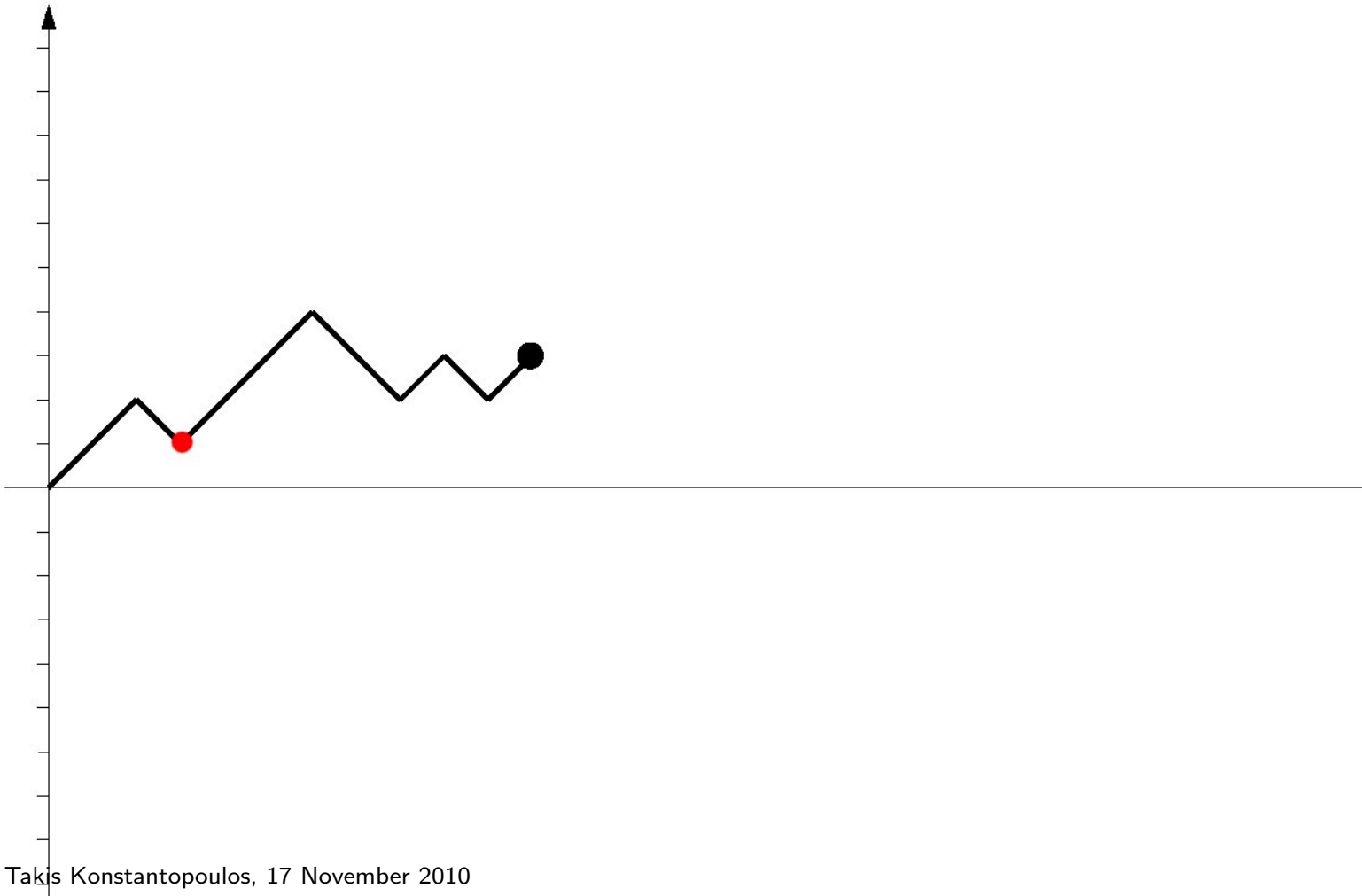
$S(11), \quad S(S(11))$



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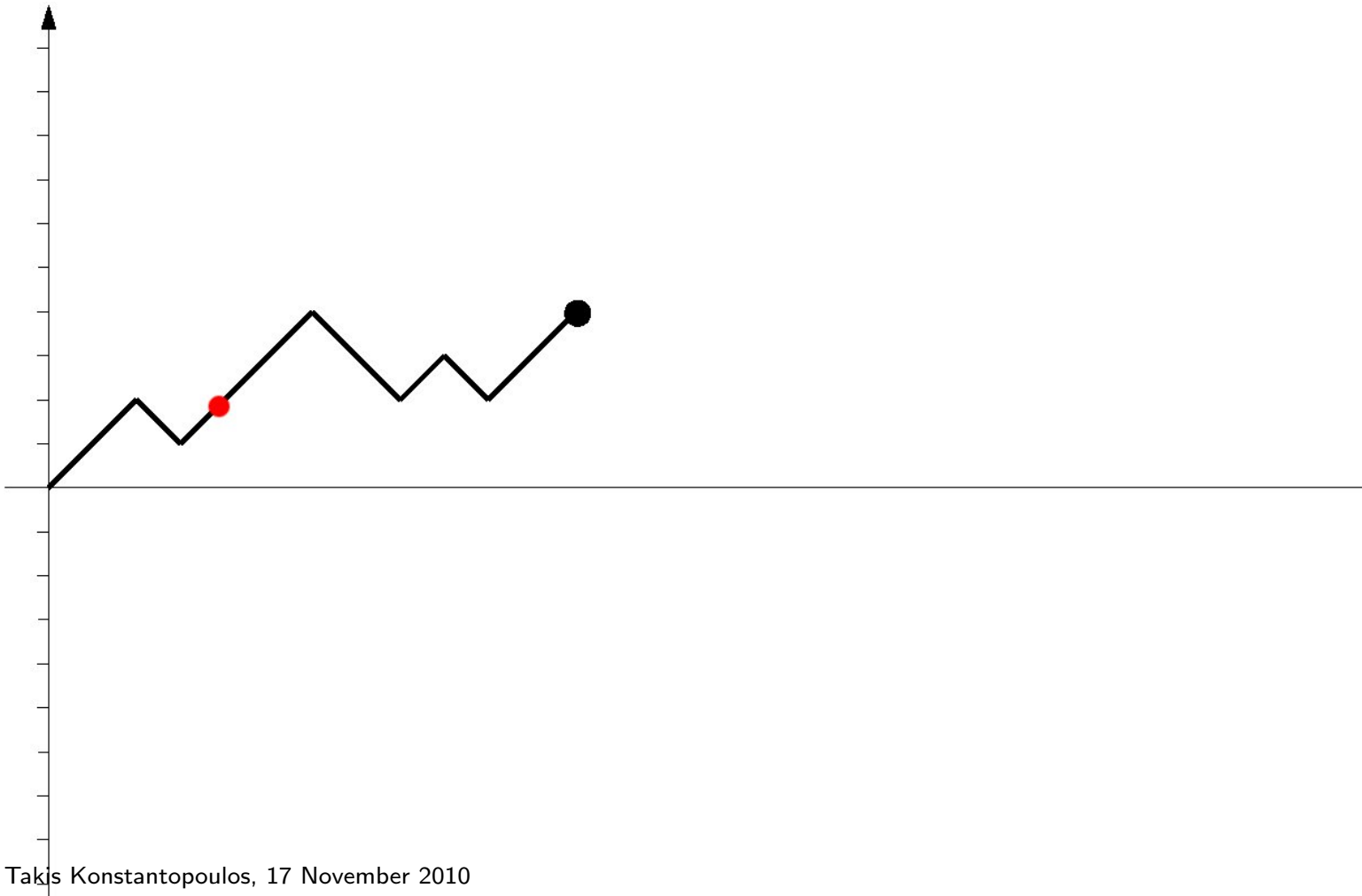

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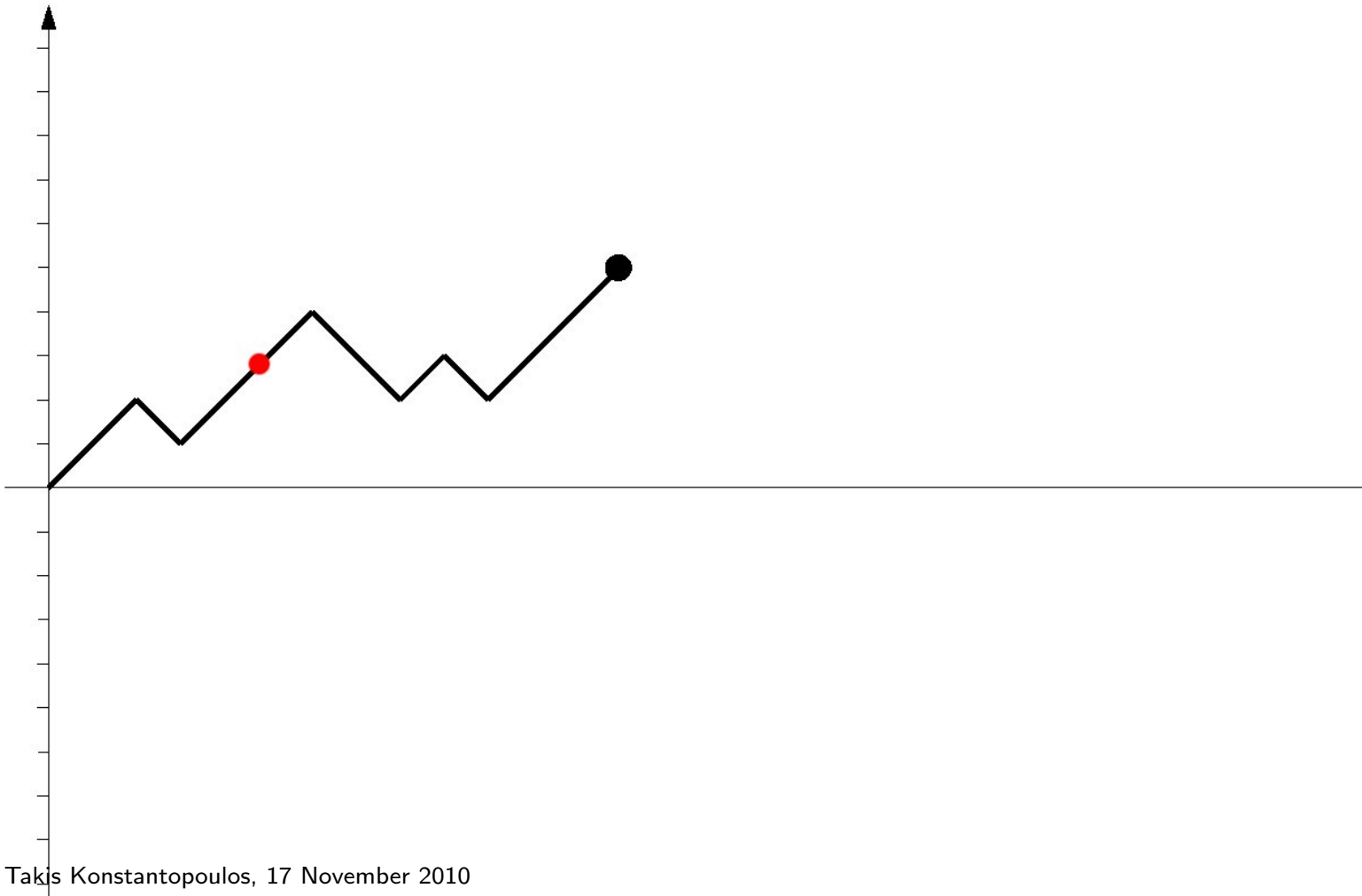

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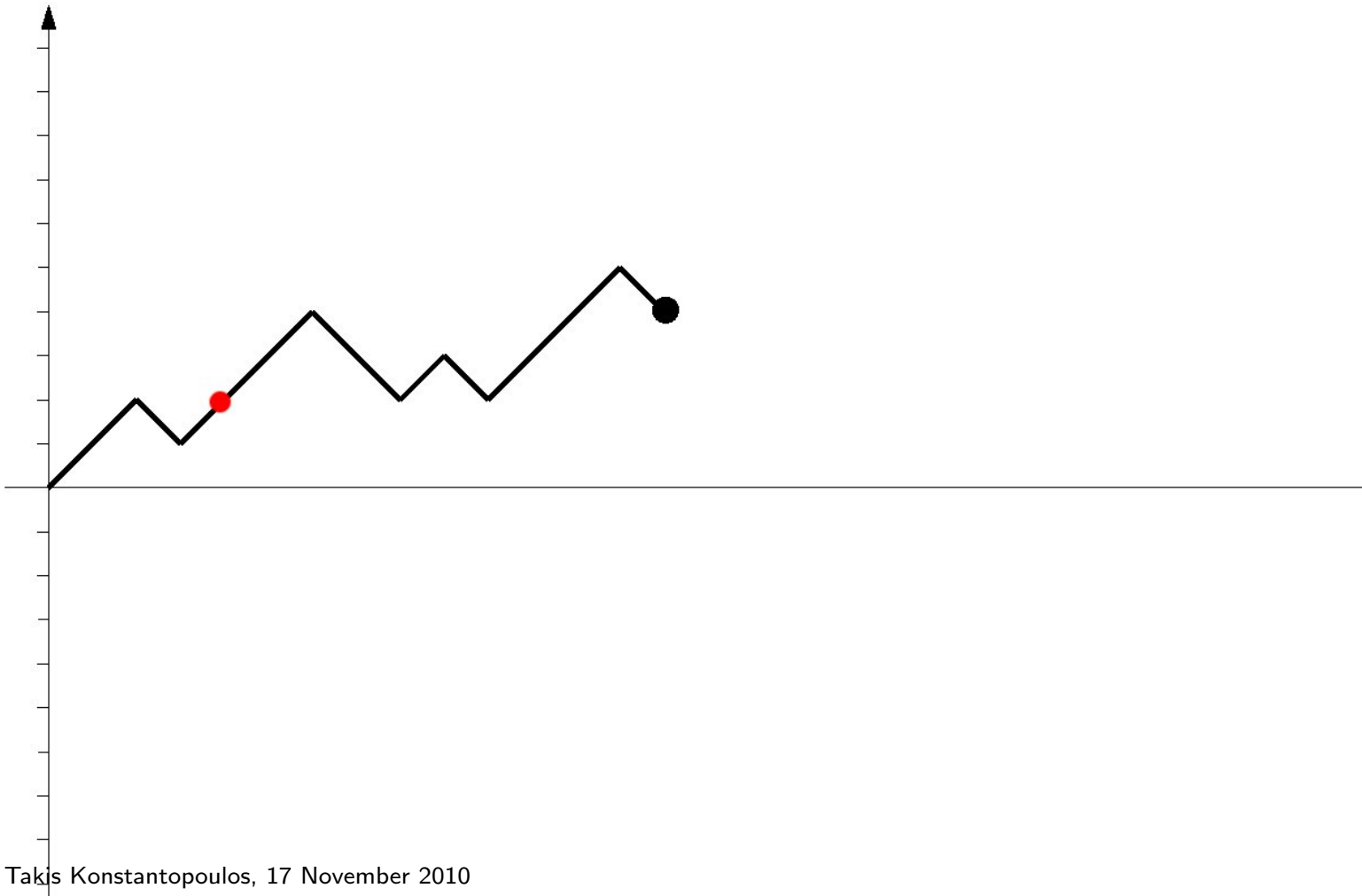

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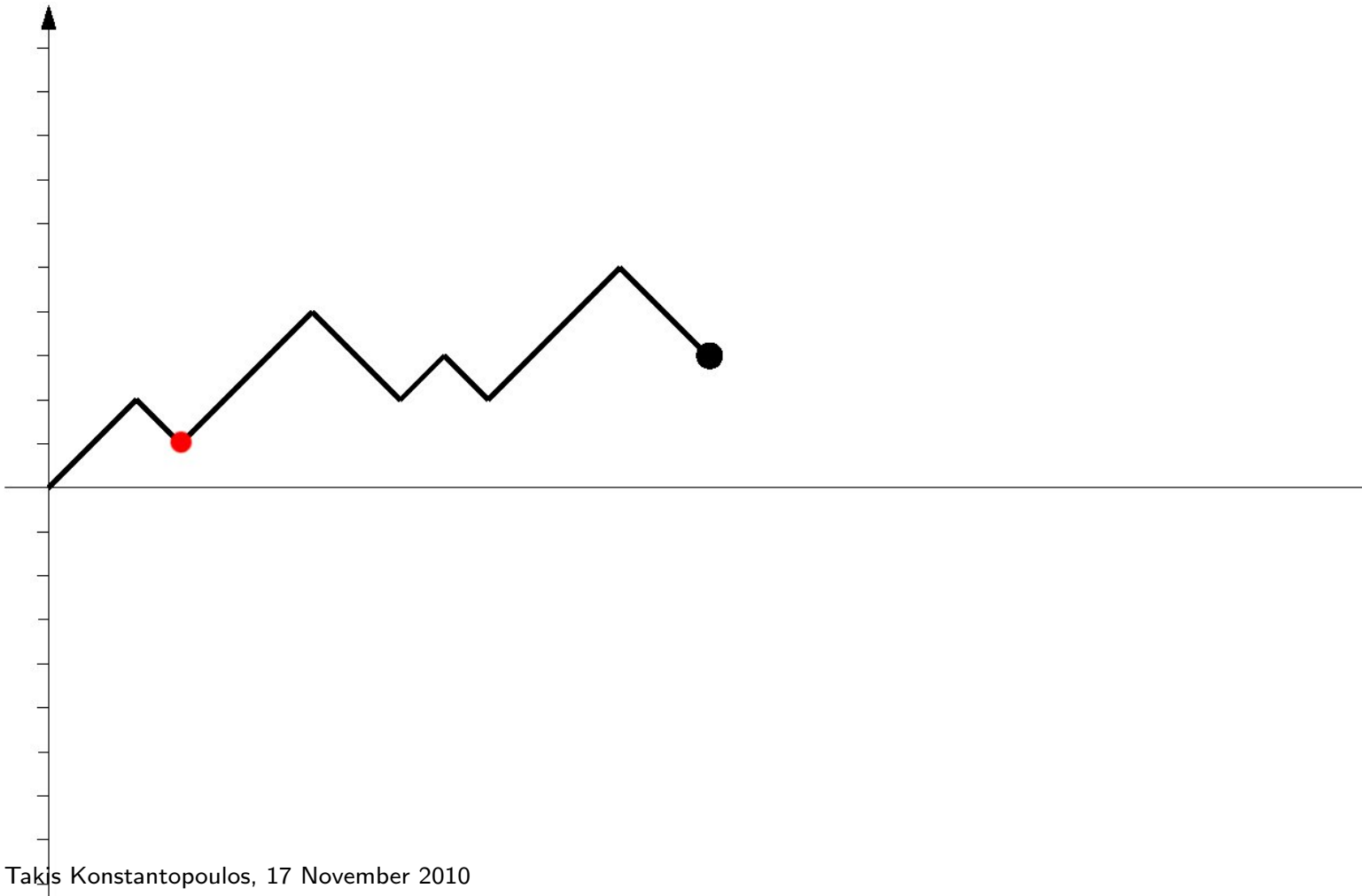

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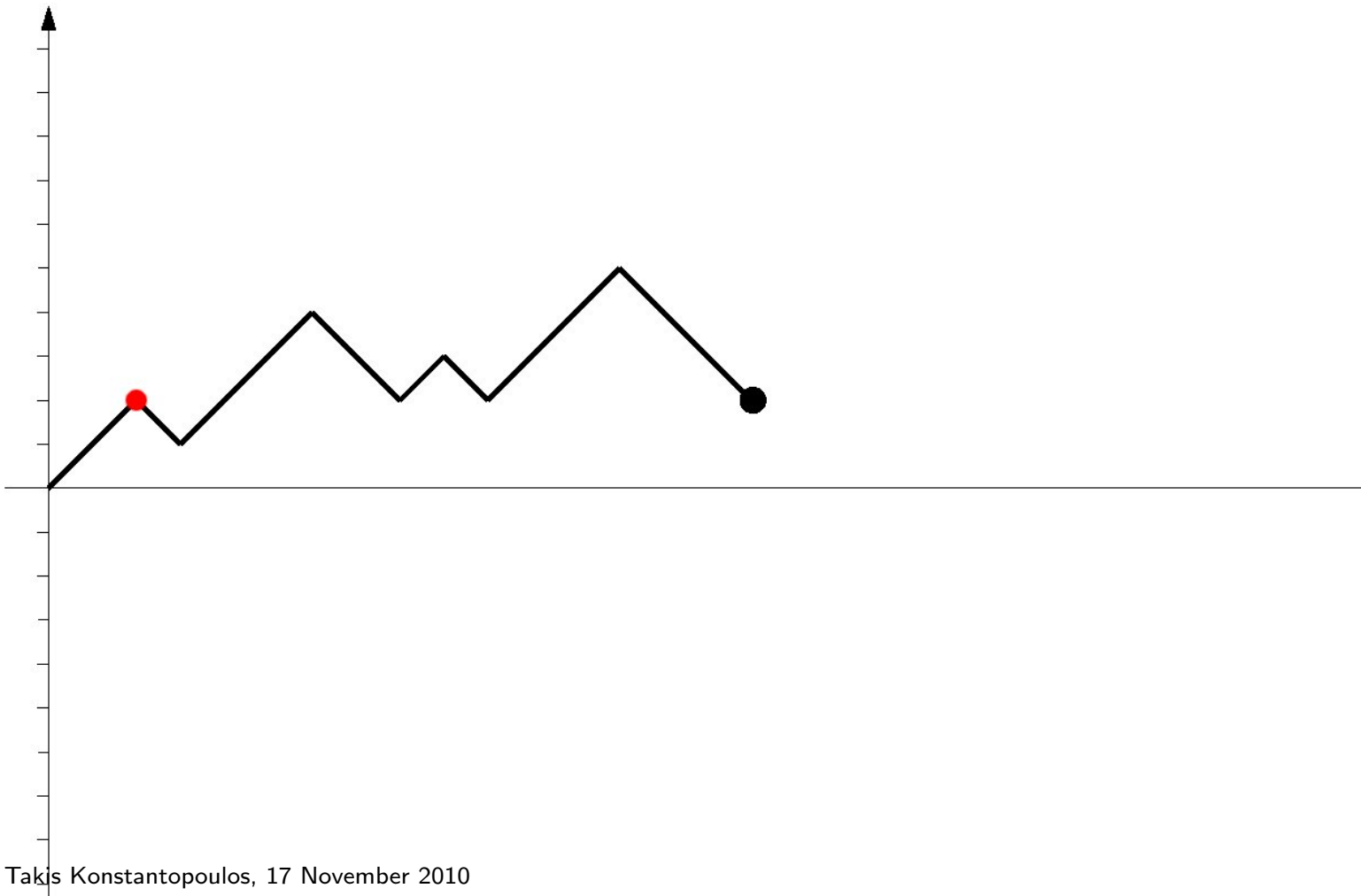
Self-iterated random walk $S(S(n))$

$S(16), \quad S(S(16))$



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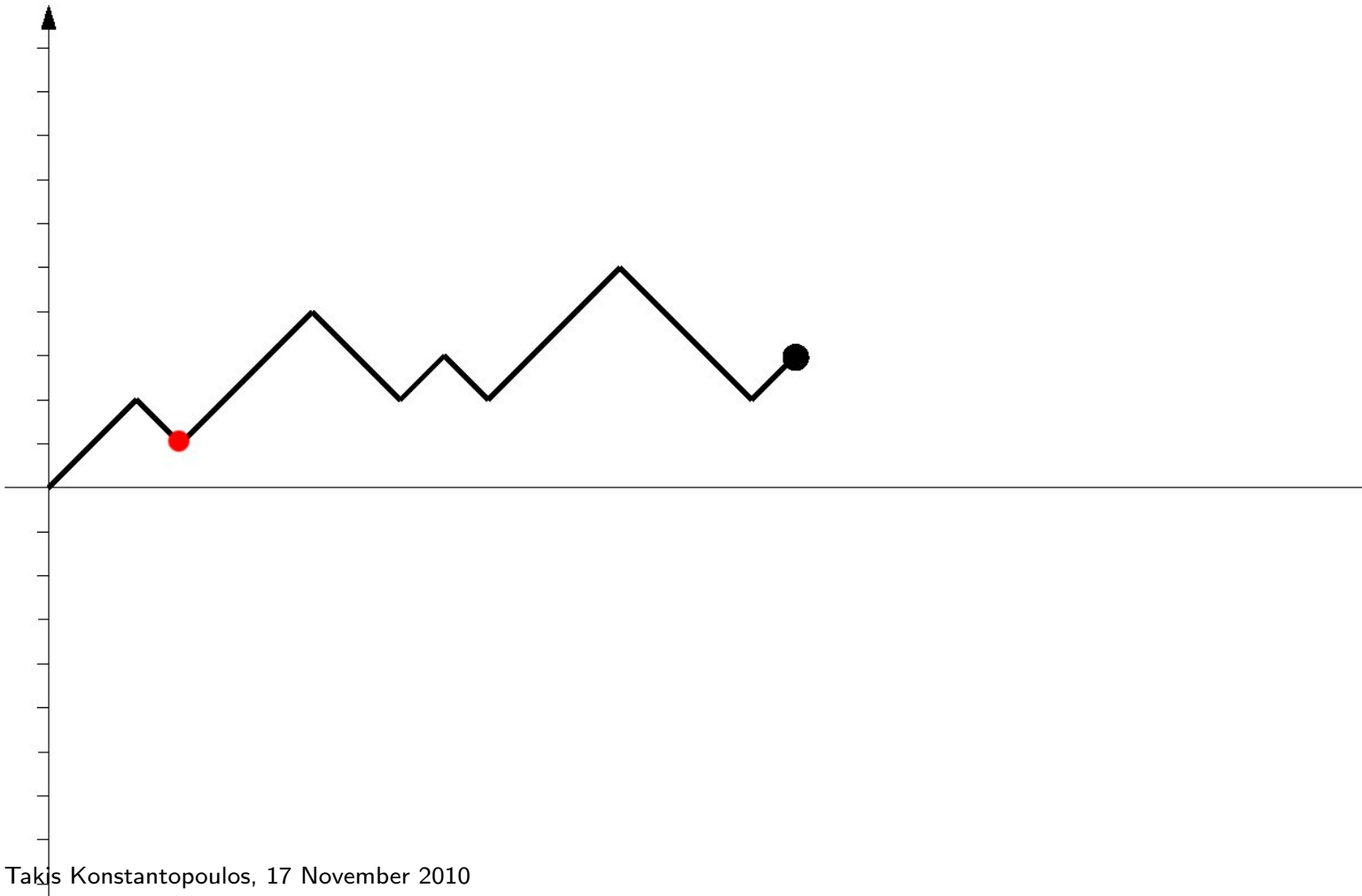
Self-iterated random walk $S(S(n))$

$S(17), \quad S(S(17))$



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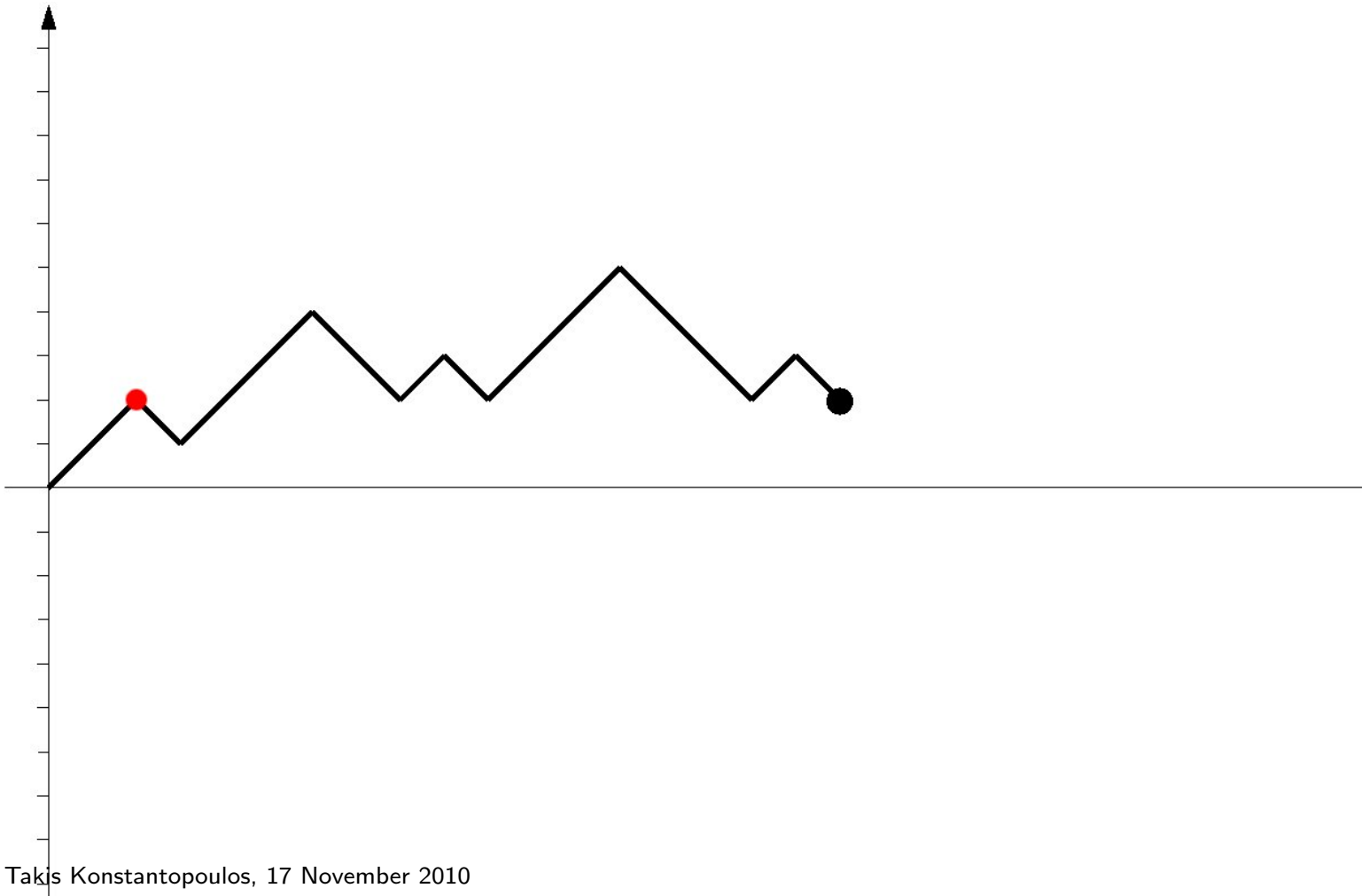
Self-iterated random walk $S(S(n))$

$S(18)$, $S(S(18))$



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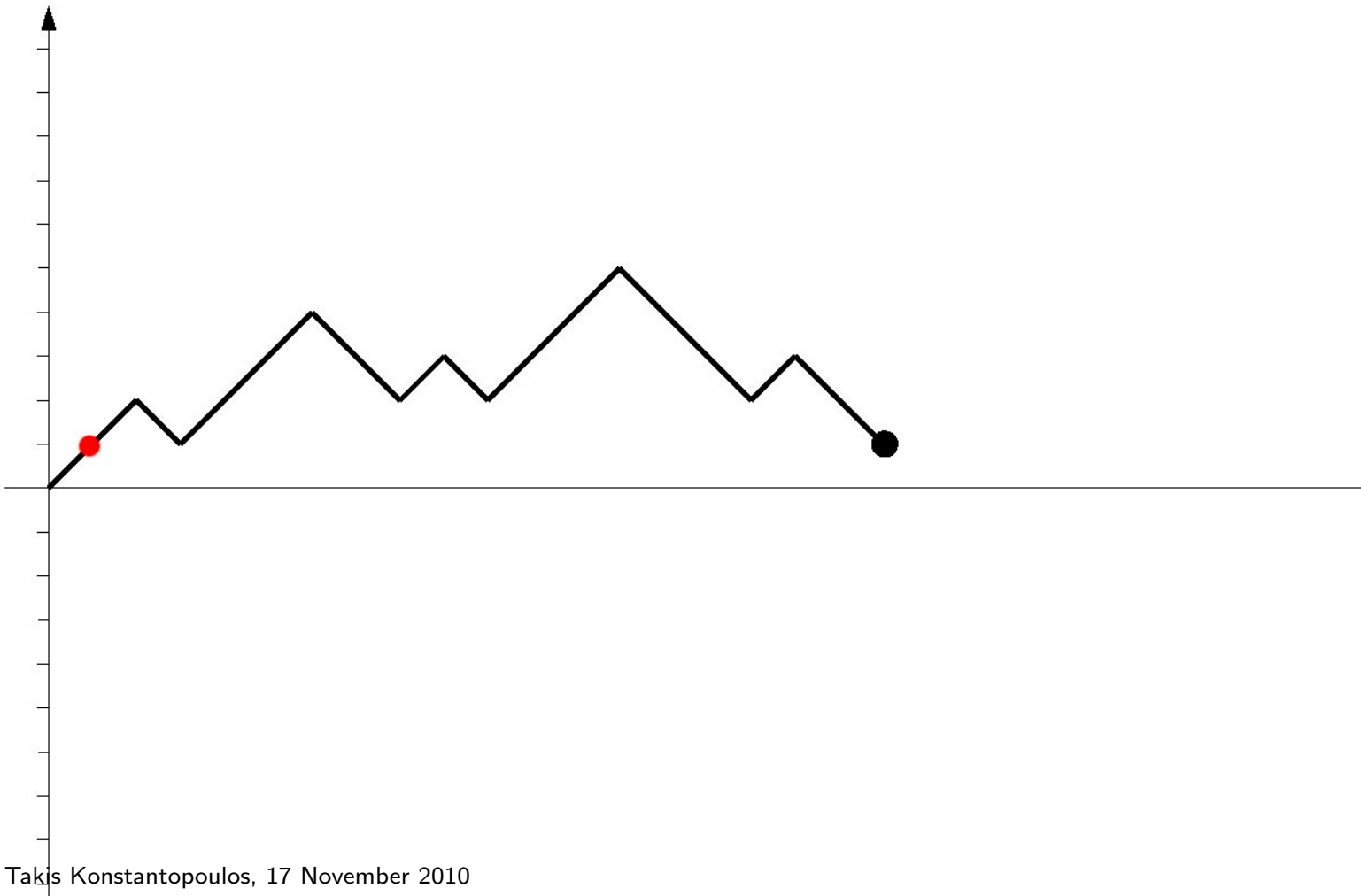
Self-iterated random walk $S(S(n))$

$S(19), \quad S(S(19))$



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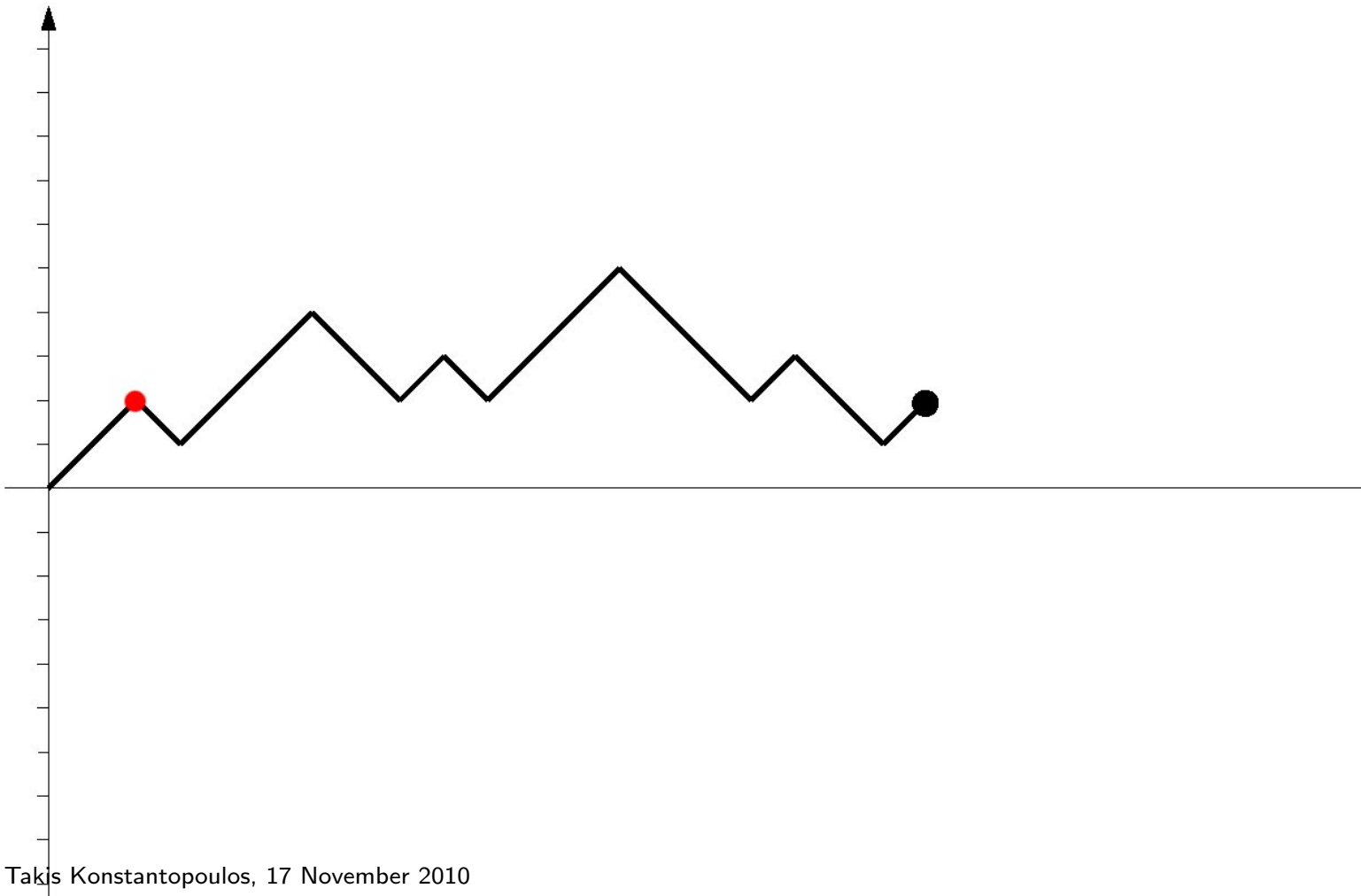
Self-iterated random walk $S(S(n))$

$S(20)$, $S(S(20))$



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Self-iterated random walk -many times

$$S(n)$$
$$S(S(n))$$
$$S(S(S(n)))$$
$$S(S(S(S(n))))$$
$$S(S(S(S(S(n)))))$$
$$S(S(S(S(S(S(n))))))$$
$$\ldots\ldots\ldots$$

What happens when we do this infinitely many times?
What is the scaling limit?

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... as a huge random monster

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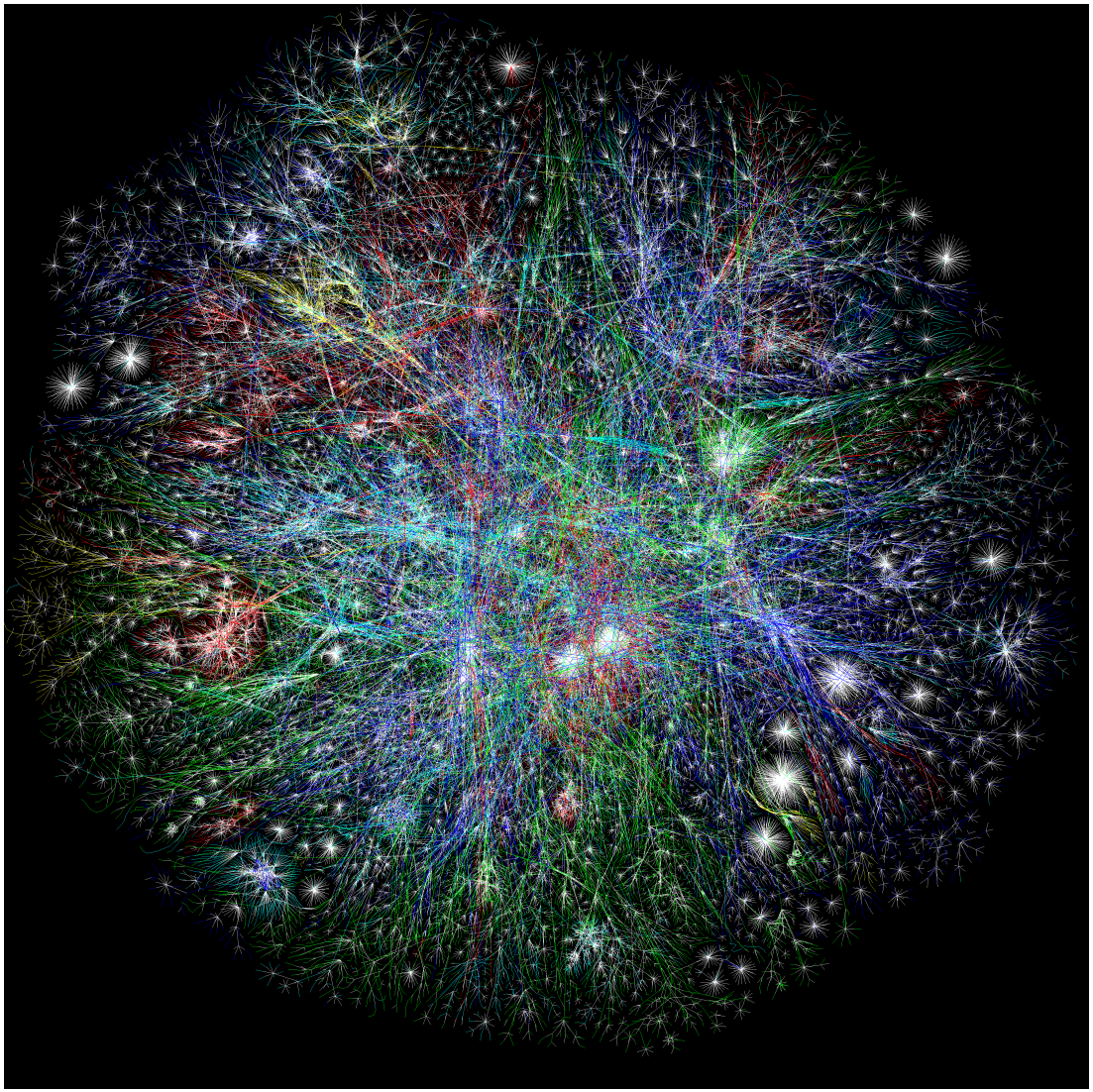

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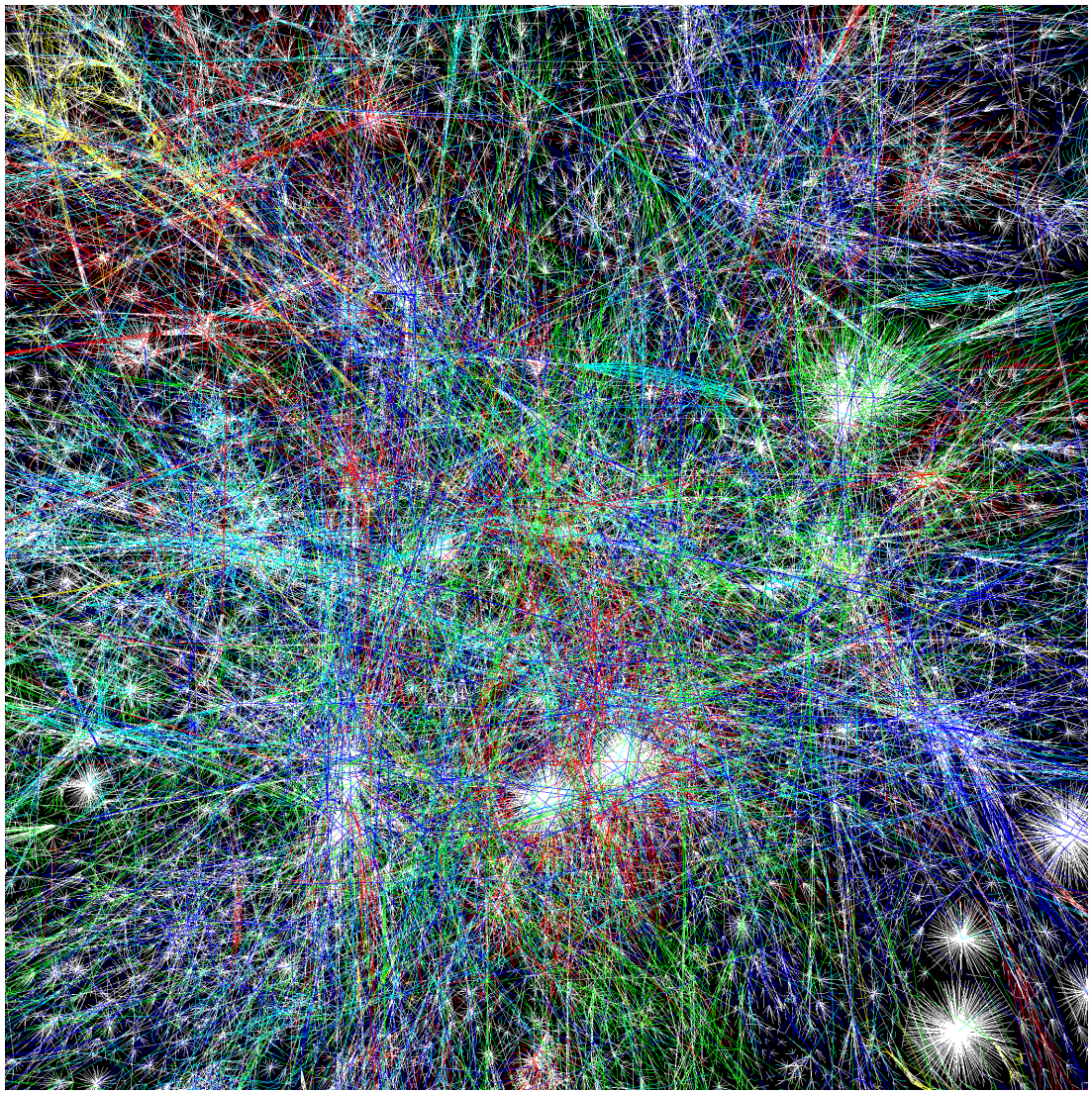

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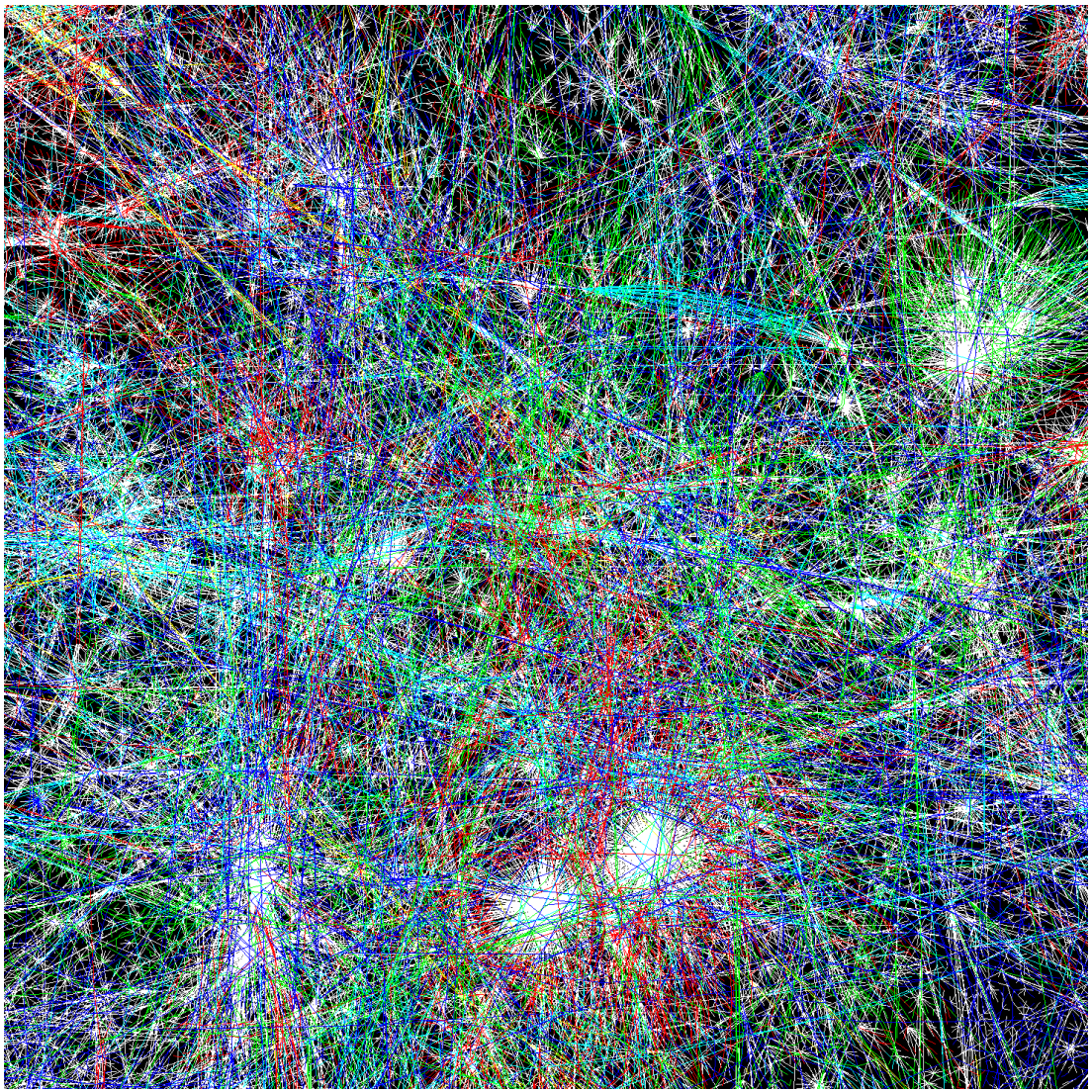

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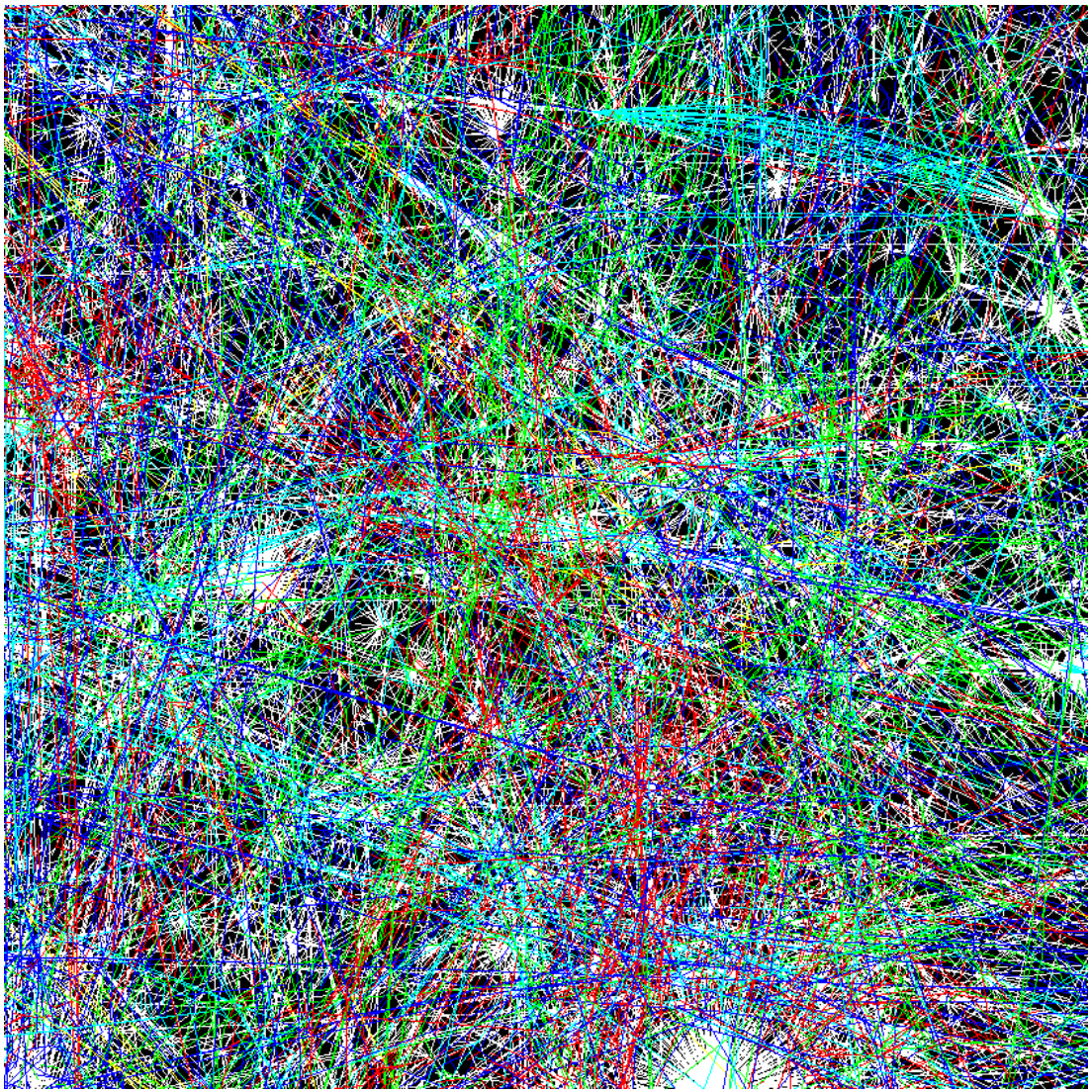

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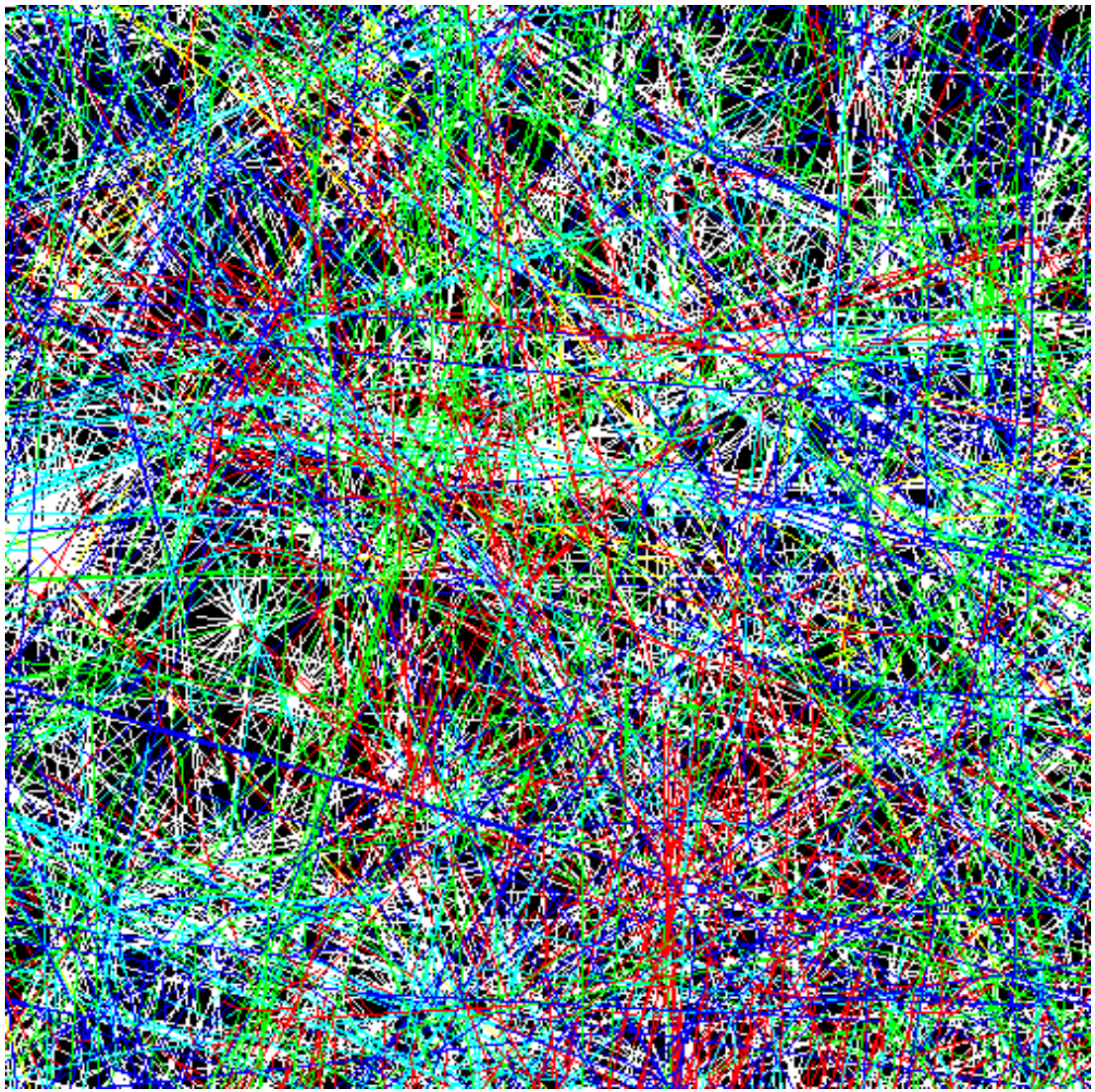

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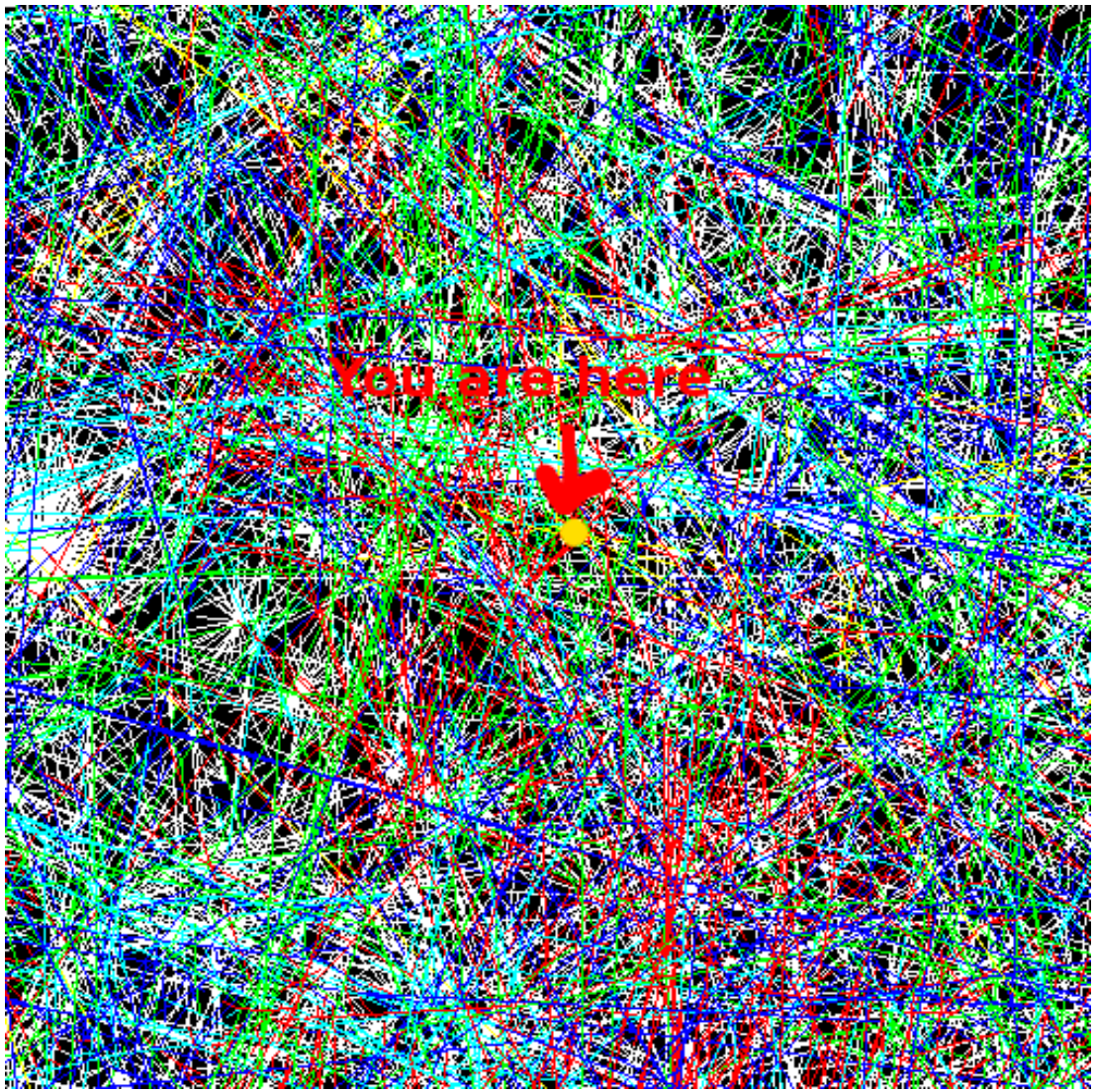

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Internet: dynamical behavior

Looked from very far in space,

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Internet: dynamical behavior

Looked from very far in space,

and very far in time (in an appropriate time scale),

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Internet: dynamical behavior

Looked from very far in space,

and very far in time (in an appropriate time scale),

we can reveal structure in the Internet,

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Internet: dynamical behavior

Looked from very far in space,

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we can reveal structure in the Internet,

we can describe it by Mathematics,

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Internet: dynamical behavior

Looked from very far in space,

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we can reveal structure in the Internet,

we can describe it by Mathematics,

control it

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Internet: dynamical behavior

Looked from very far in space,

and very far in time (in an appropriate time scale),

we can reveal structure in the Internet,

we can describe it by Mathematics,

control it

design it,

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Internet: dynamical behavior

Looked from very far in space,

and very far in time (in an appropriate time scale),

we can reveal structure in the Internet,

we can describe it by Mathematics,

control it

design it,

and understand its behavior

In fact, when a system (random or not) becomes large, the proper language for describing it is Probability.

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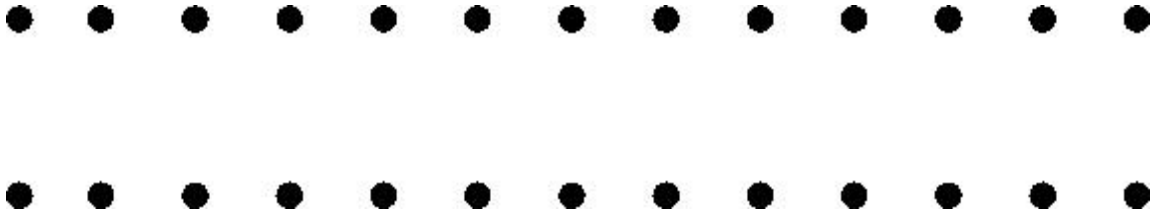
A random graph



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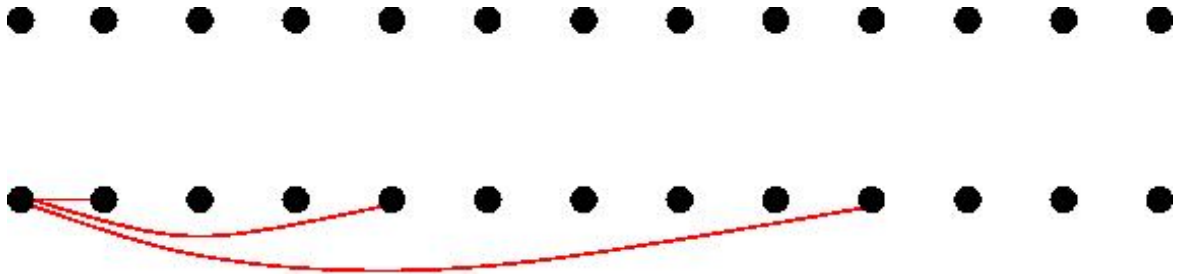

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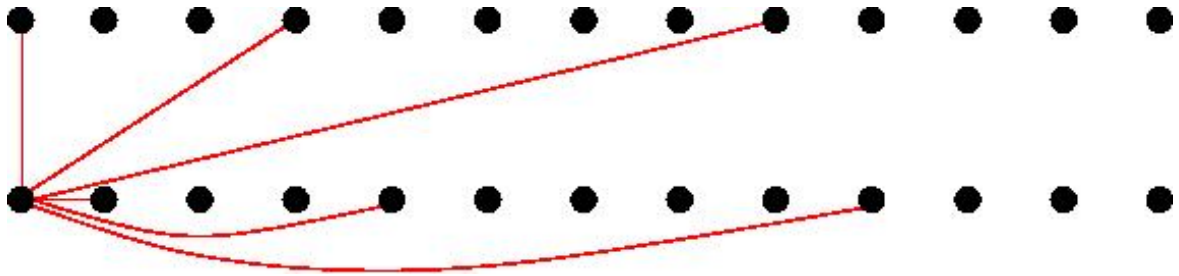

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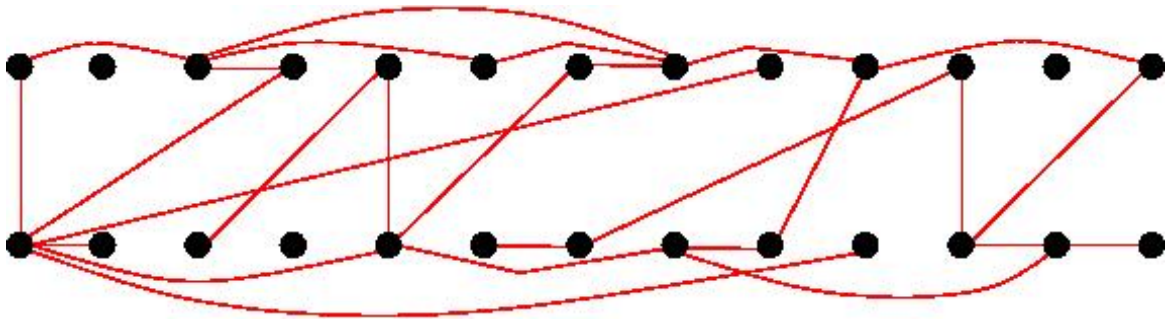

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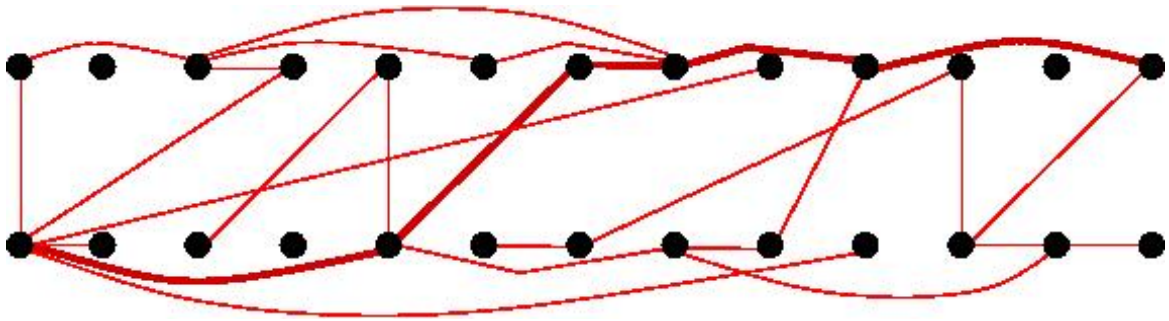

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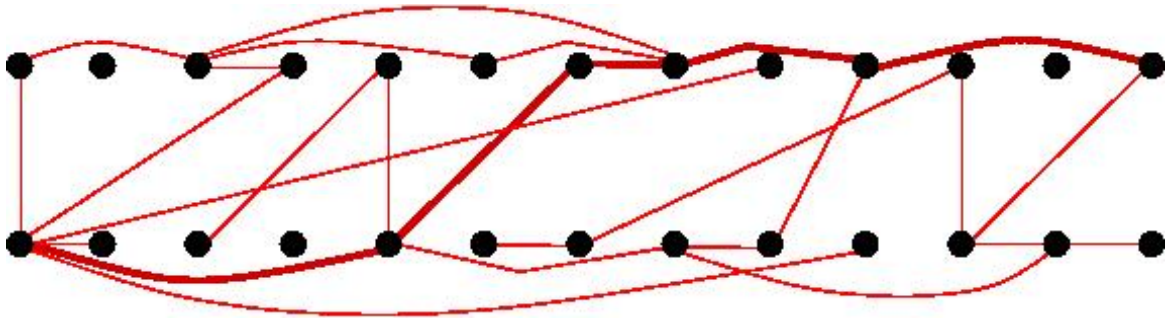

The length of the longest path in this graph is a random variable with distribution similar to the distribution of a certain random matrix.

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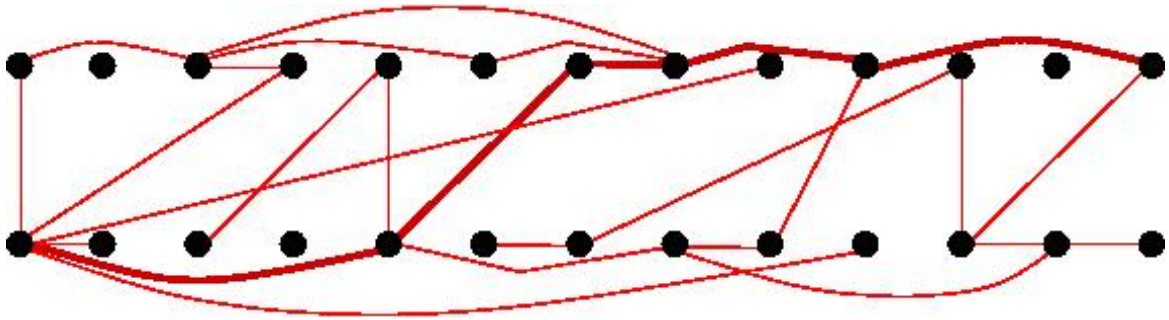

The length of the longest path in this graph is a random variable with distribution similar to the distribution of a certain random matrix.
The specific details are unimportant.

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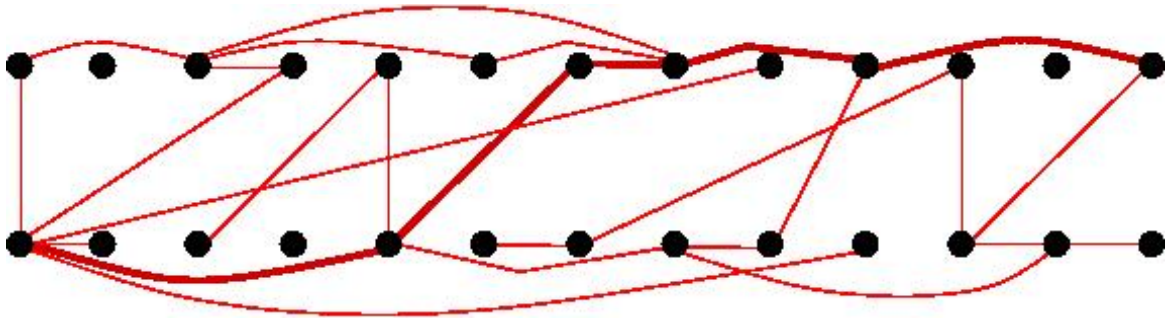

The length of the longest path in this graph is a random variable with distribution similar to the distribution of a certain random matrix.
The specific details are unimportant.
This is another example of universality.

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Random matrices

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Random matrices

They originate in Physics.

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Random matrices

They originate in Physics.

They have applications in Physics, Engineering, and even in Pure Mathematics.