Dungeon Master's dice
(and other probability problems)

Ian Thompson

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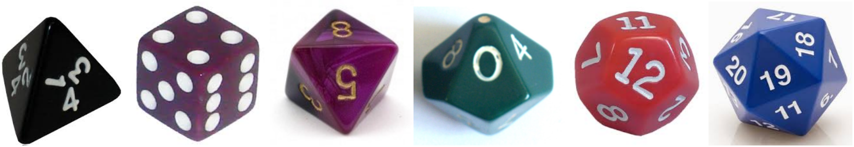
University of Liverpool



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10 :	⚅⚂⚀	⚅⚁⚁	9 :	⚅⚁⚀	⚄⚂⚀
	⚄⚃⚀	⚄⚂⚁		⚄⚁⚁	⚃⚃⚀
	⚃⚃⚁	⚃⚂⚂		⚃⚂⚁	⚂⚂⚂

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	⚃⚃⚁	⚃⚂⚂		⚃⚂⚁	⚂⚂⚂

- Six possible combinations in both cases: can you see the flaw in their reasoning?

Exercise 1

Complete the table showing the possible scores obtained by throwing two six-sided dice, and calculate their probabilities.

	⚀	⚁	⚂	⚃	⚄	⚅
⚀						
⚁						
⚂						
⚃						
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⚅						

Score	Probability
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

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  - Thus, there are 6 combinations with probability $1/36$ and 15 with probability $2/36$.
  - Some scores can be achieved by more than one combination, e.g. ⚀⚄, ⚁⚃ and ⚂⚂, but others (e.g. 11) can only be achieved by one.

Exercise 1 answers

Two dice:

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

Score	Probability
2	$1/36$
3	$2/36$
4	$3/36$
5	$4/36$
6	$5/36$
7	$6/36$
8	$5/36$
9	$4/36$
10	$3/36$
11	$2/36$
12	$1/36$

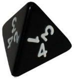
Three dice:

Score	3	4	5	6	7	8	9	10
Probability (/216)	1	3	6	10	15	21	25	27

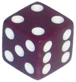
Score	11	12	13	14	15	16	17	18
Probability (/216)	27	25	21	15	10	6	3	1

Dungeons & Dragons

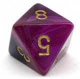
- Dungeons & Dragons is a game played with various types of dice.



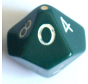
*D*4



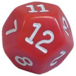
*D*6



*D*8



*D*10



*D*12

*D*20

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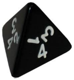

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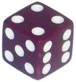

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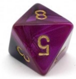

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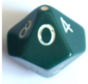
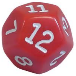


$D10$



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  - An ordinary long sword inflicts $1D8$ damage
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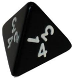

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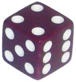

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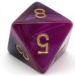

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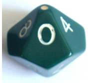

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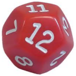

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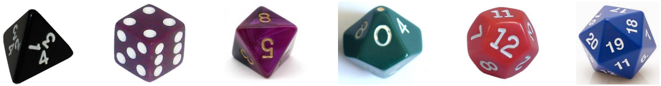

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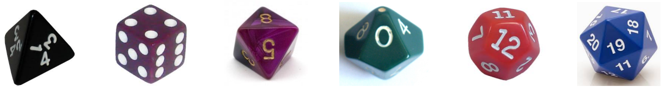

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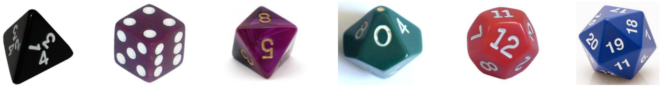

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- Can we simulate the other dice using multiple $D6$?
- We can use the scores, the combinations or (with different coloured dice) the permutations.

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> Upon an incorrect initial guess (goat), the host must open the other goat door, leaving the car as the only possible switch.

- Hence, the chance of winning is better if the contestant switches, but not everyone understands this, even after it has been explained.

"You blew it, and you blew it big! Since you seem to have difficulty grasping the basic principle at work here, I'll explain. After the host reveals a goat, you now have a one-in-two chance of being correct. Whether you change your selection or not, the odds are the same. There is enough mathematical illiteracy in this country, and we don't need the world's highest IQ propagating more. Shame!"

Scott Smith, Ph.D. University of Florida

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In making the calculation, the numbers themselves are irrelevant! Each ball is either in the player's choice of six (•) or not (•).

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- In fact this is the probability of any arrangement of 3 ● and 3 ●. Changing the order just swaps around the numbers in the numerator.

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$$\text{Probability of } \bullet\bullet\bullet\bullet\bullet\bullet \times 20 = \frac{11,713}{22,528,737} \times 20 \approx \frac{1}{96}.$$

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- Sally Clark was convicted, and sentenced to life in prison.

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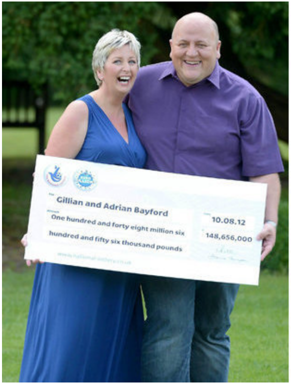

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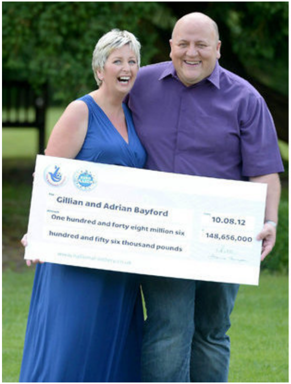

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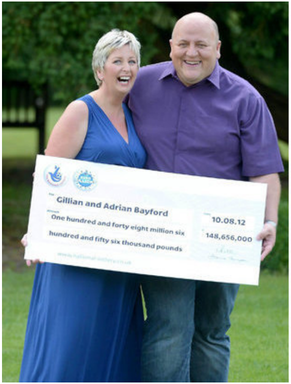

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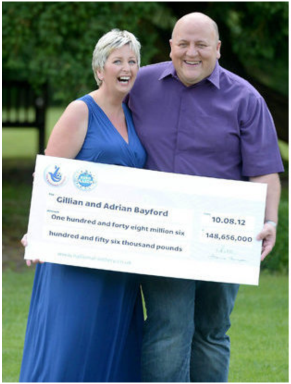

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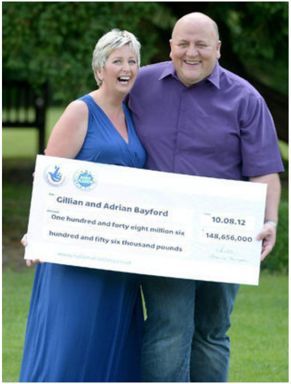

- Wrong. Unlikely things do happen to some people, especially in large populations!
- The only useful way to view the probability of winning after the event is to compare it to the probability that the Bayfords somehow defrauded the lottery.

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2. The probability of two instances is calculated by squaring the probability of one. The Royal Statistical Society wrote that this

   "would only be valid if SIDS cases arose independently within families, an assumption that would need to be justified empirically. Not only was no such empirical justification provided in the case, but there are very strong a priori reasons for supposing that the assumption will be false. There may well be unknown genetic or environmental factors that predispose families to SIDS, so that a second case within the family becomes much more likely.

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- After a lengthy legal battle, Sally Clark's conviction was finally overturned by the Court of Appeal in 2003. The judges wrote that

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- In 2004, Ray Hill published an article in which he tried to calculate the probability more carefully, noting several places where the available data was insufficient, or difficult to interpret. He noted that

  "One wonders whether the Clark jury would have convicted if, instead of being given the 'once in a hundred years figure', they had been told that second cot deaths occur around four or five times a year and indeed happen rather more frequently than second infant murders in the same family."

Think carefully about probability!

Thanks for your attention.