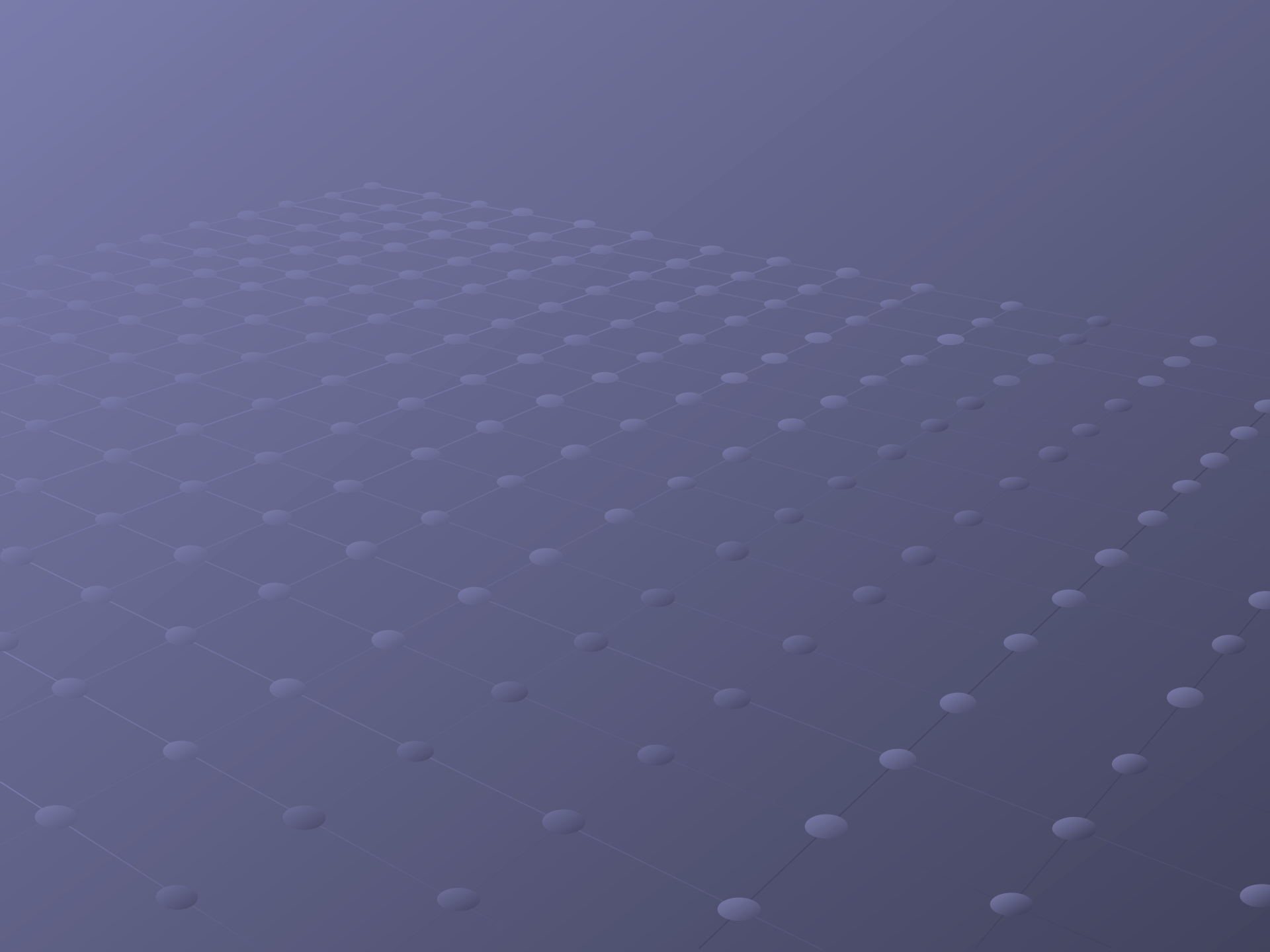

NETWORKS

Maths Club

February 2007

Su Doku

- Previous Maths Club talk
- The type of Su Doku that we do today is thought to have evolved from a simpler version
- This simpler version is Latin Squares

LEONHARD EULER

Portrait by Johann Georg Brucker

LEONHARD EULER

- On 15 April 1707, 300 years ago, Euler was born in Basel, Switzerland (He died in 1783)
- In 1727 he accepted the chair of mathematics at the new St. Petersburg Academy formed by Peter the Great.
- There where few students so the staff had time to delve into research.

LEONHARD EULER

- Euler is considered to be one of the most prolific of mathematical writers.
- He published a total of 886 books, averaging 800 printed pages a year.
- The editors of the St. Petersberg Academy Journal had so much material from him that it wasn't until 43 years after his death that they managed to publish it all.
- In 1736 Euler resolved a question then under discussion.

THE BRIDGES OF KÖNIGSBERG

QUESTION

- Is it possible to walk in the town of Königsberg in such a way that every bridge in the town would be crossed once and only once and the walker return to his starting point?

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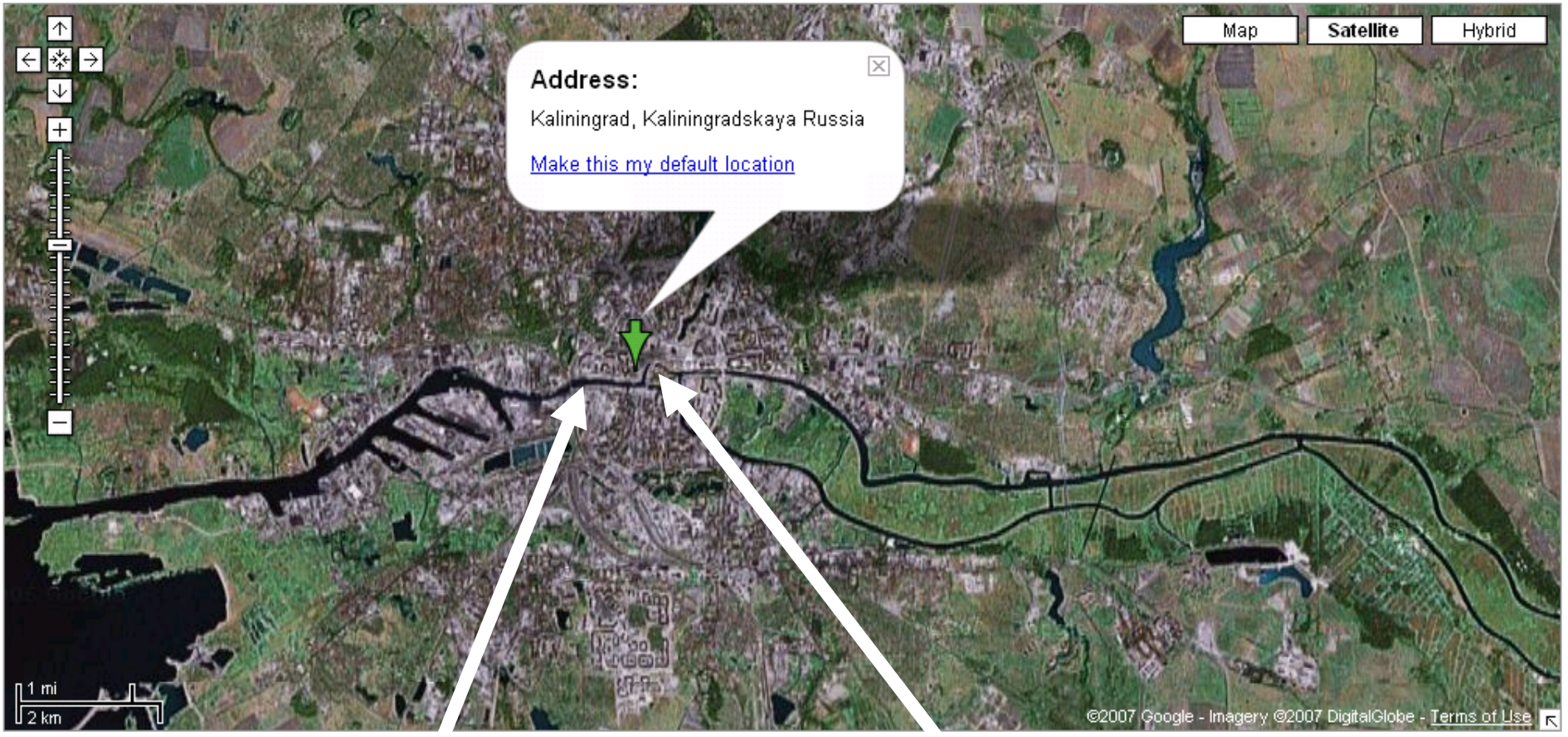
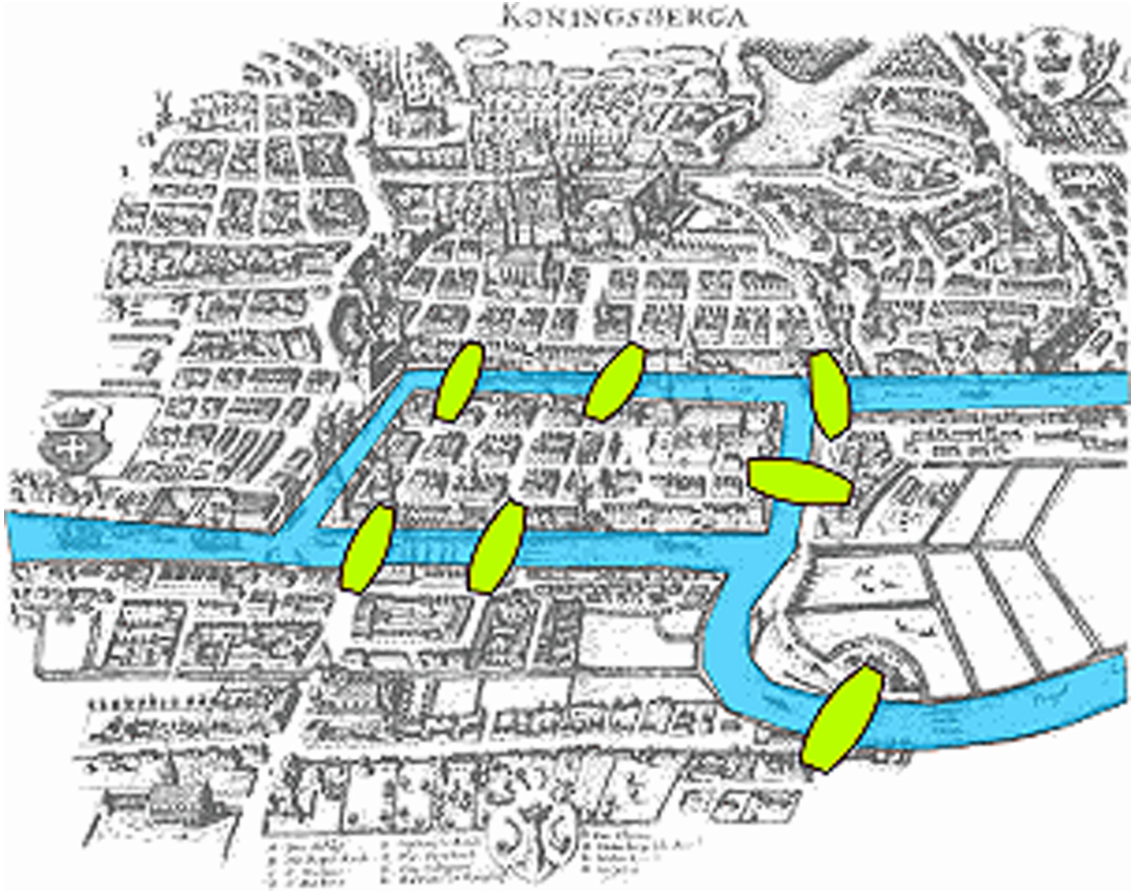


River Pregel

Kneiphof Island



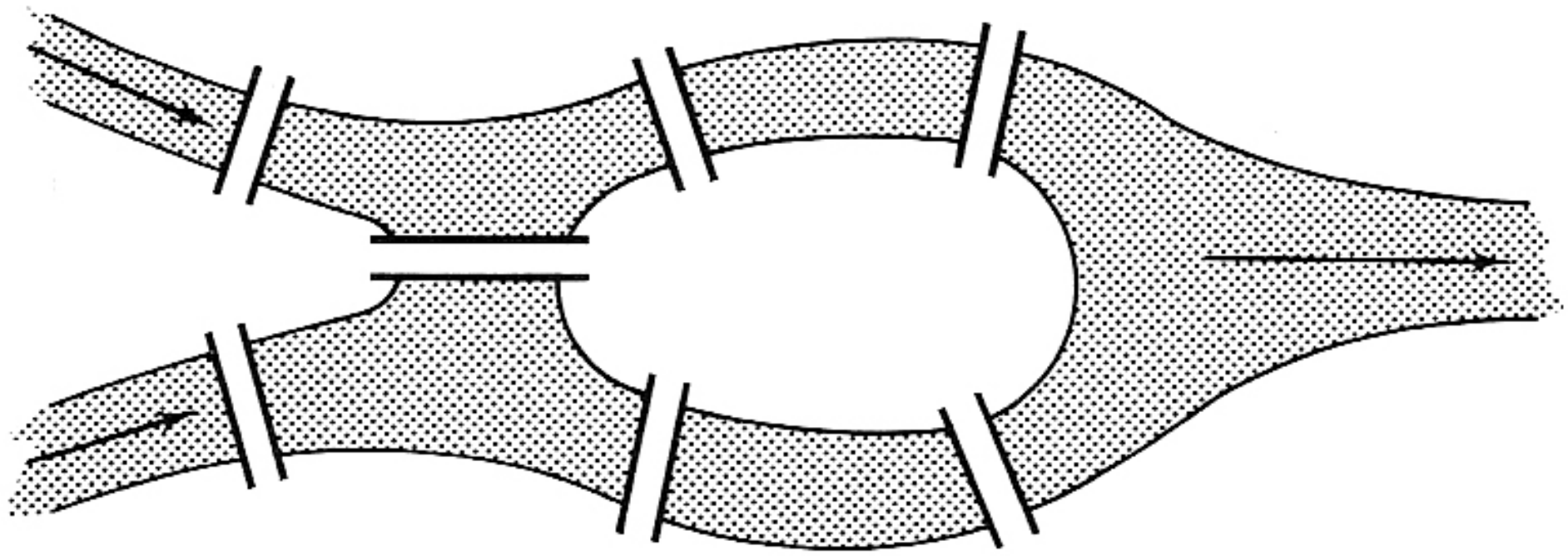
Two of the seven original bridges were destroyed by British bombing of Königsberg during World War II. Two others were later demolished by the Russians and replaced by a modern highway. The other three bridges remain, although only two of them are from Euler's time (one was rebuilt by the Germans in 1935).

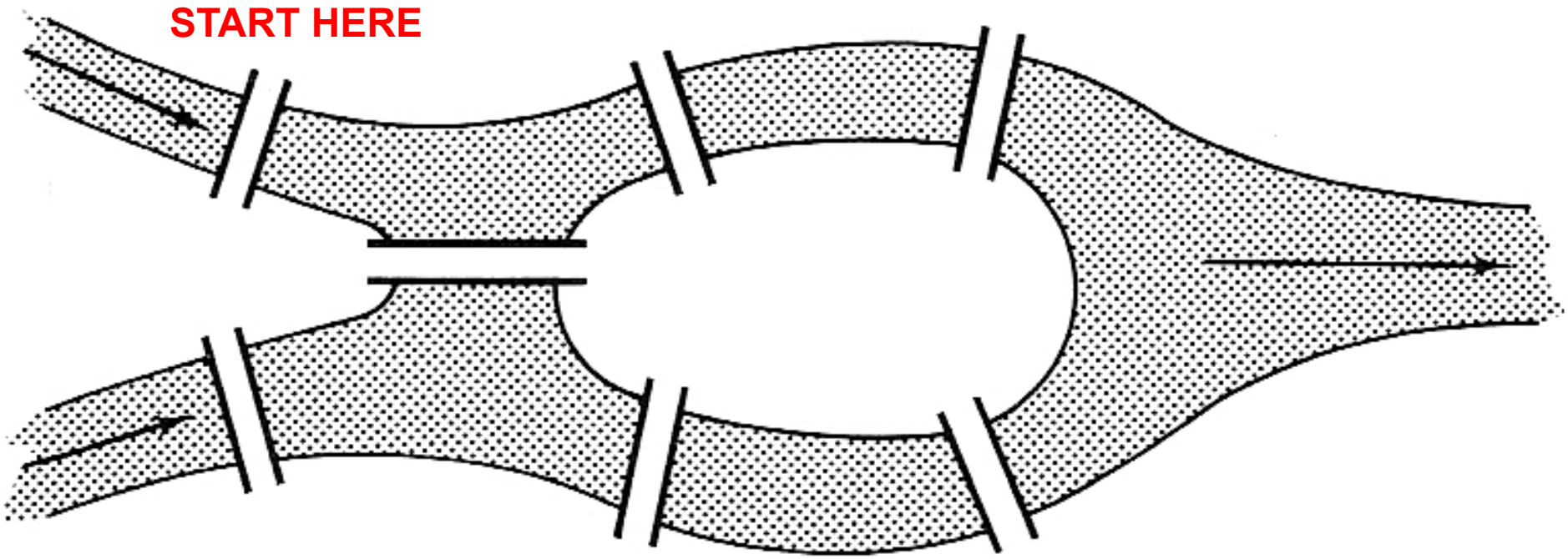
The name was changed by the Russians in 1945

THE BRIDGES OF KÖNIGSBERG

QUESTION

- Is it possible to walk in the town of Königsberg in such a way that every bridge in the town would be crossed once and only once and the walker return to his starting point?

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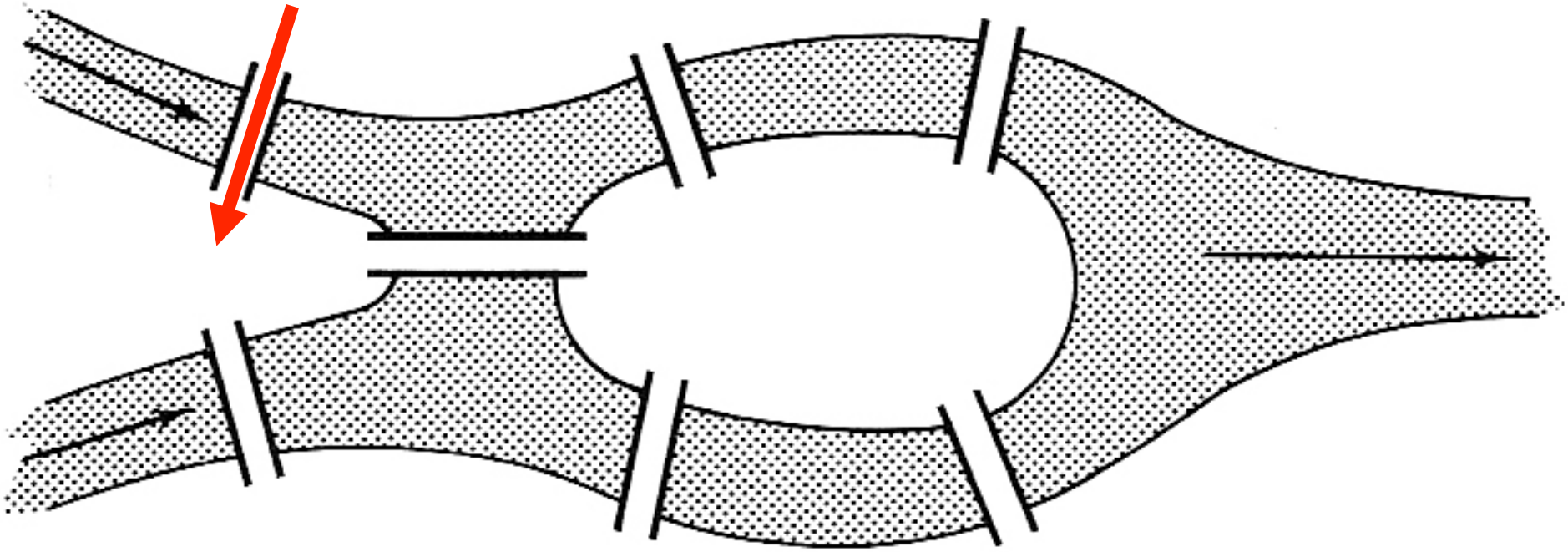

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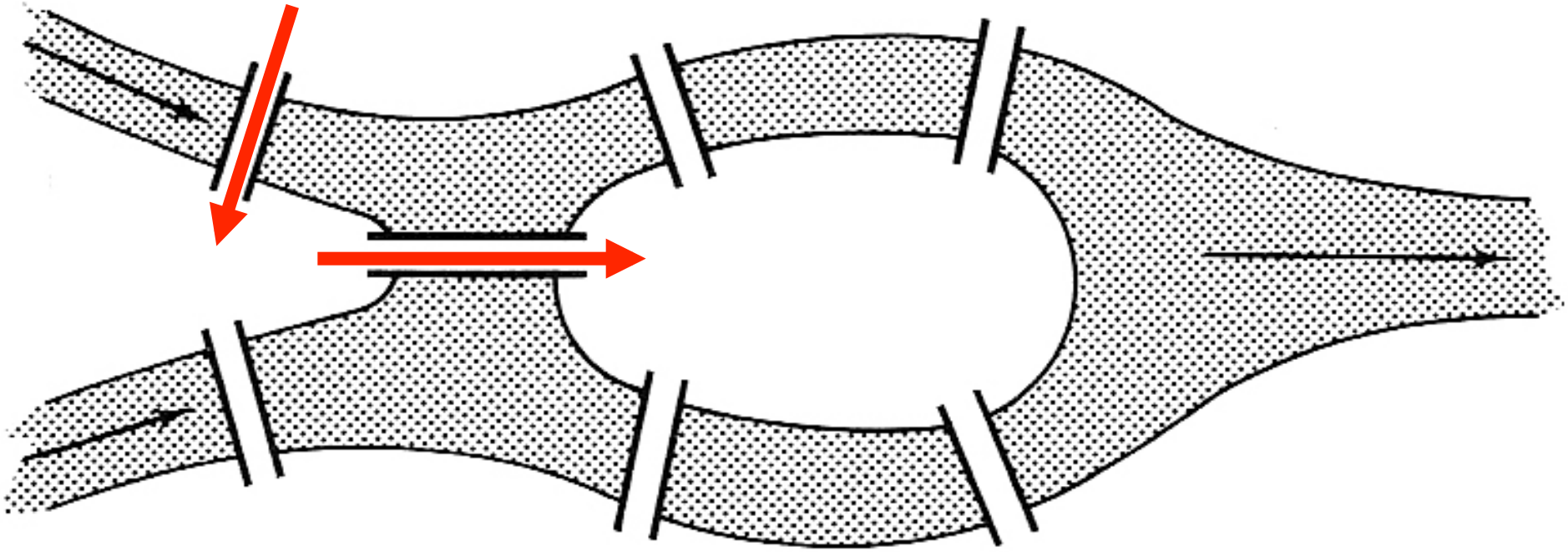

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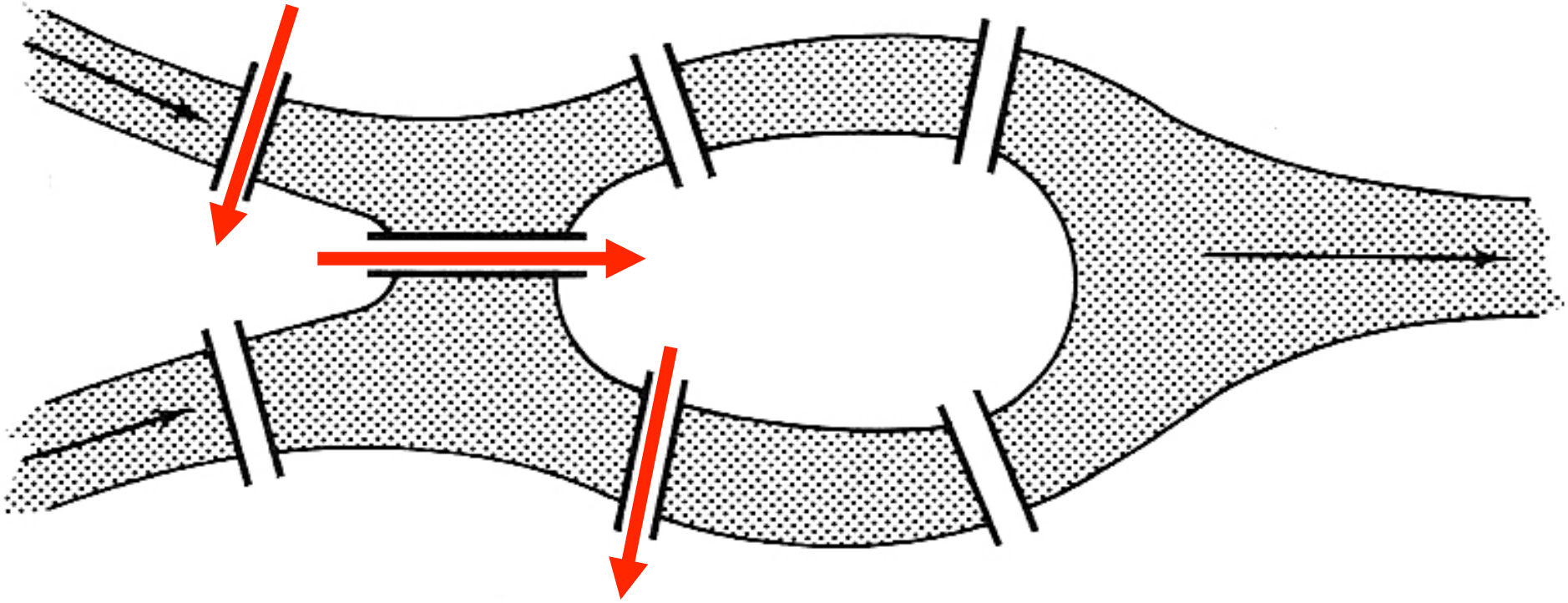

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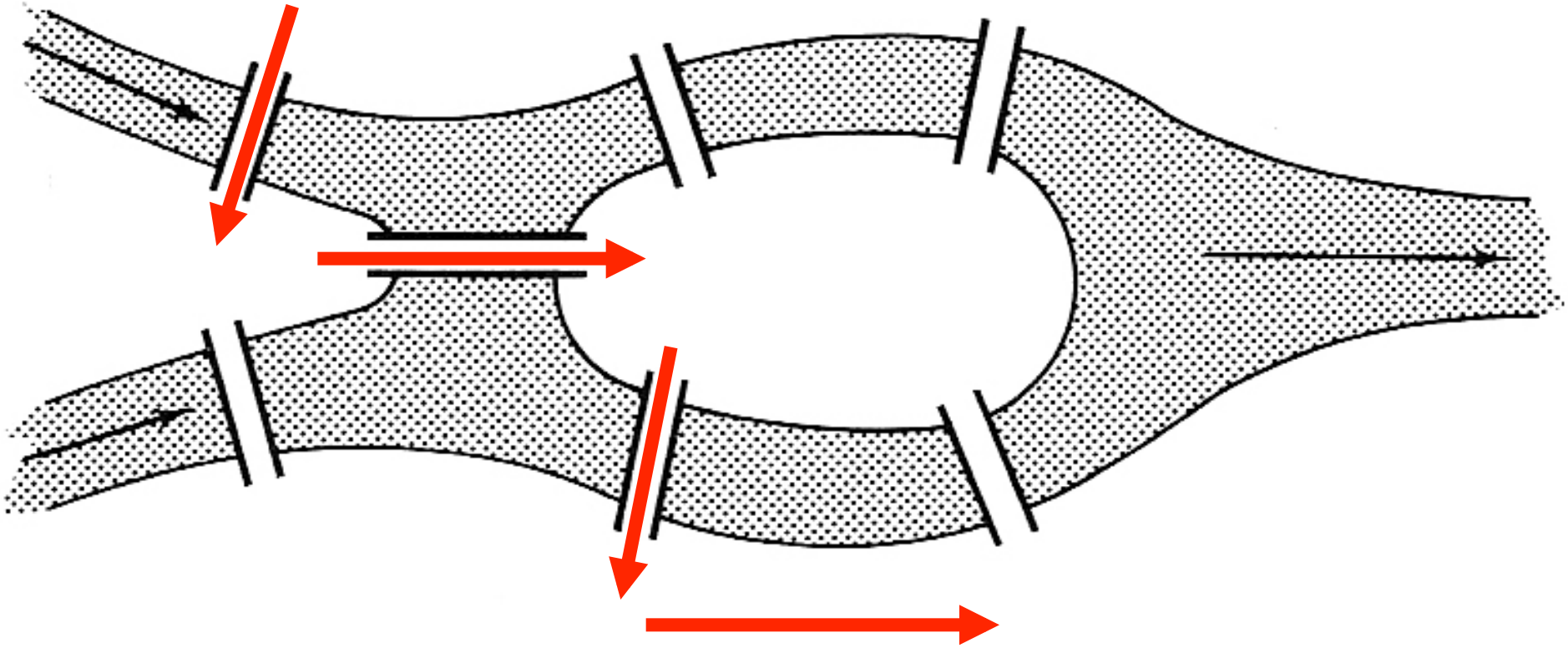

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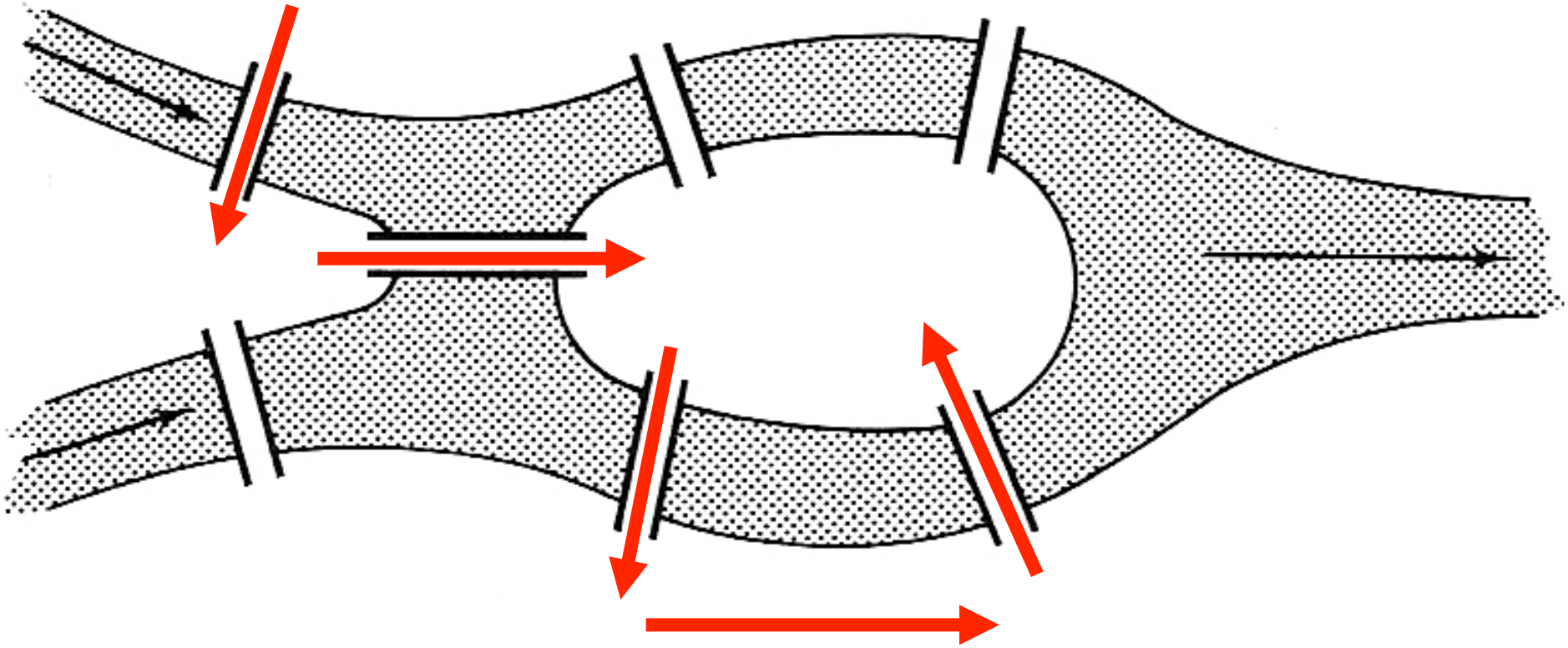

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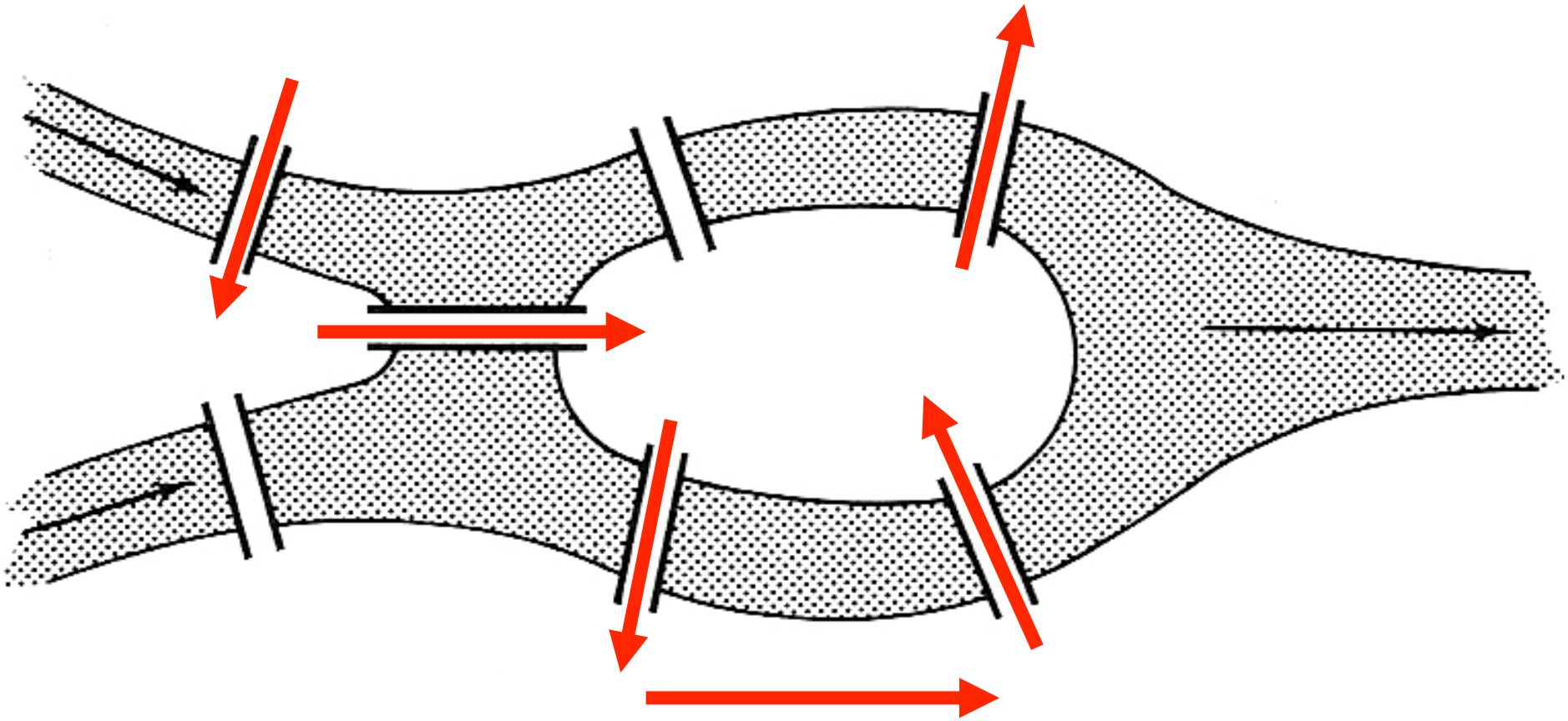

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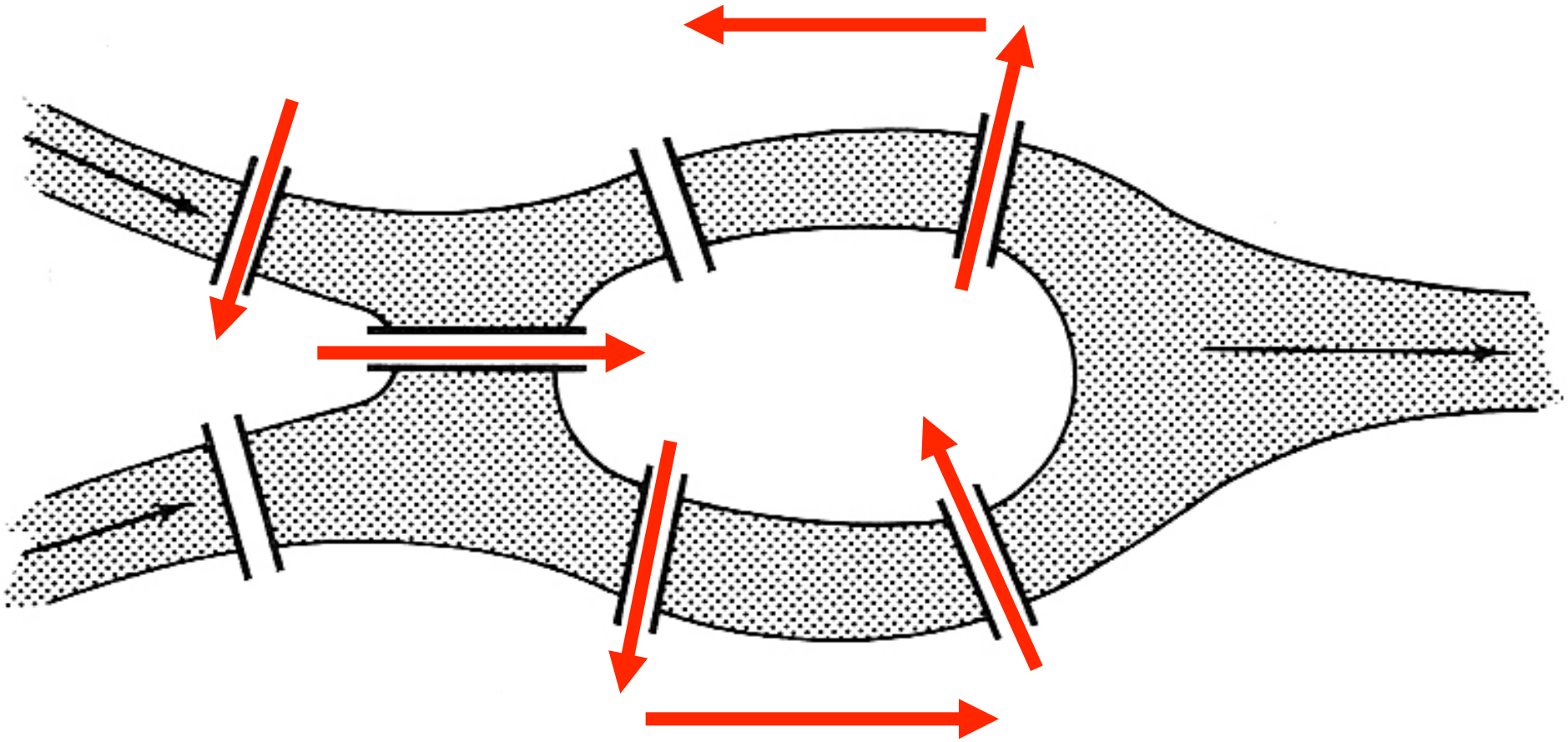

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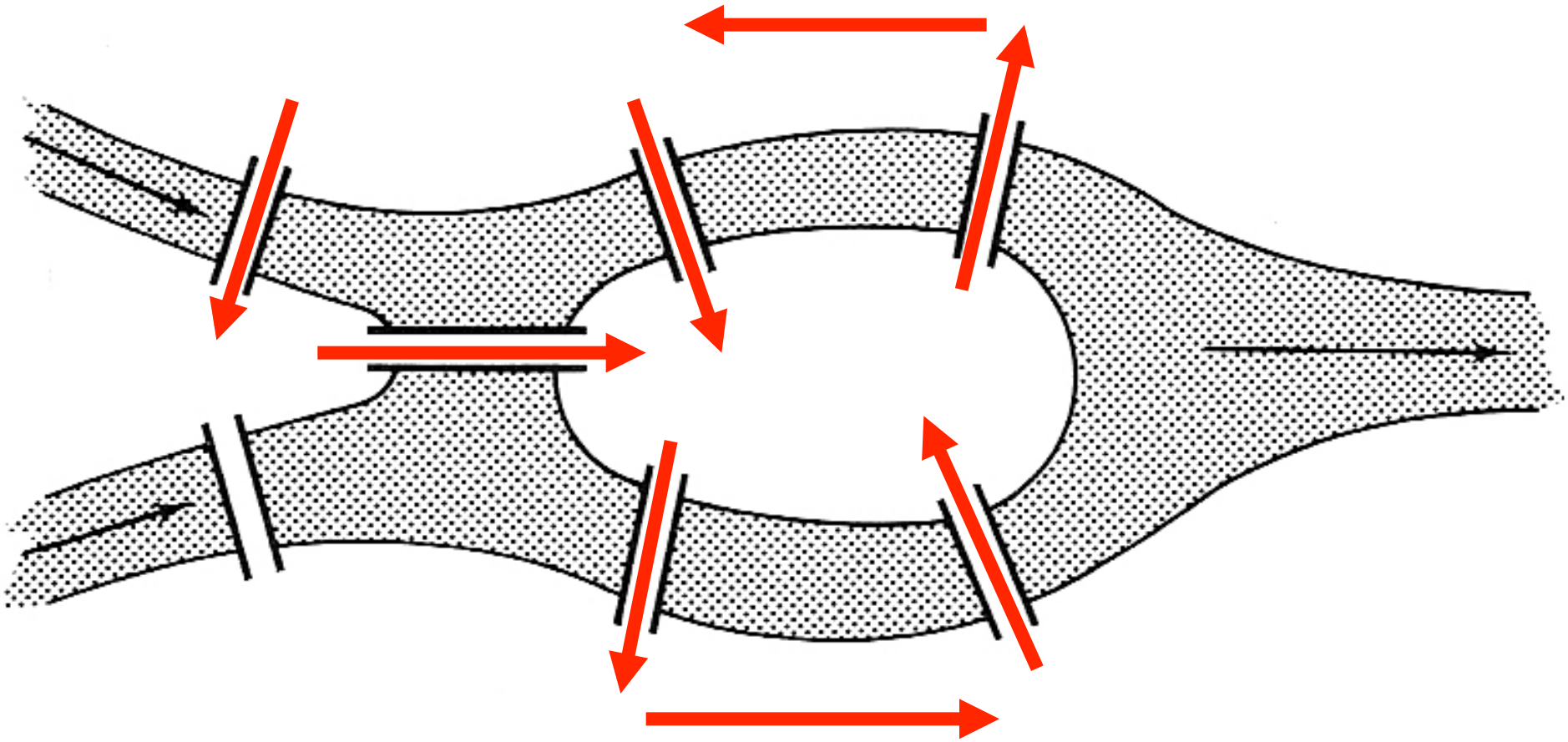

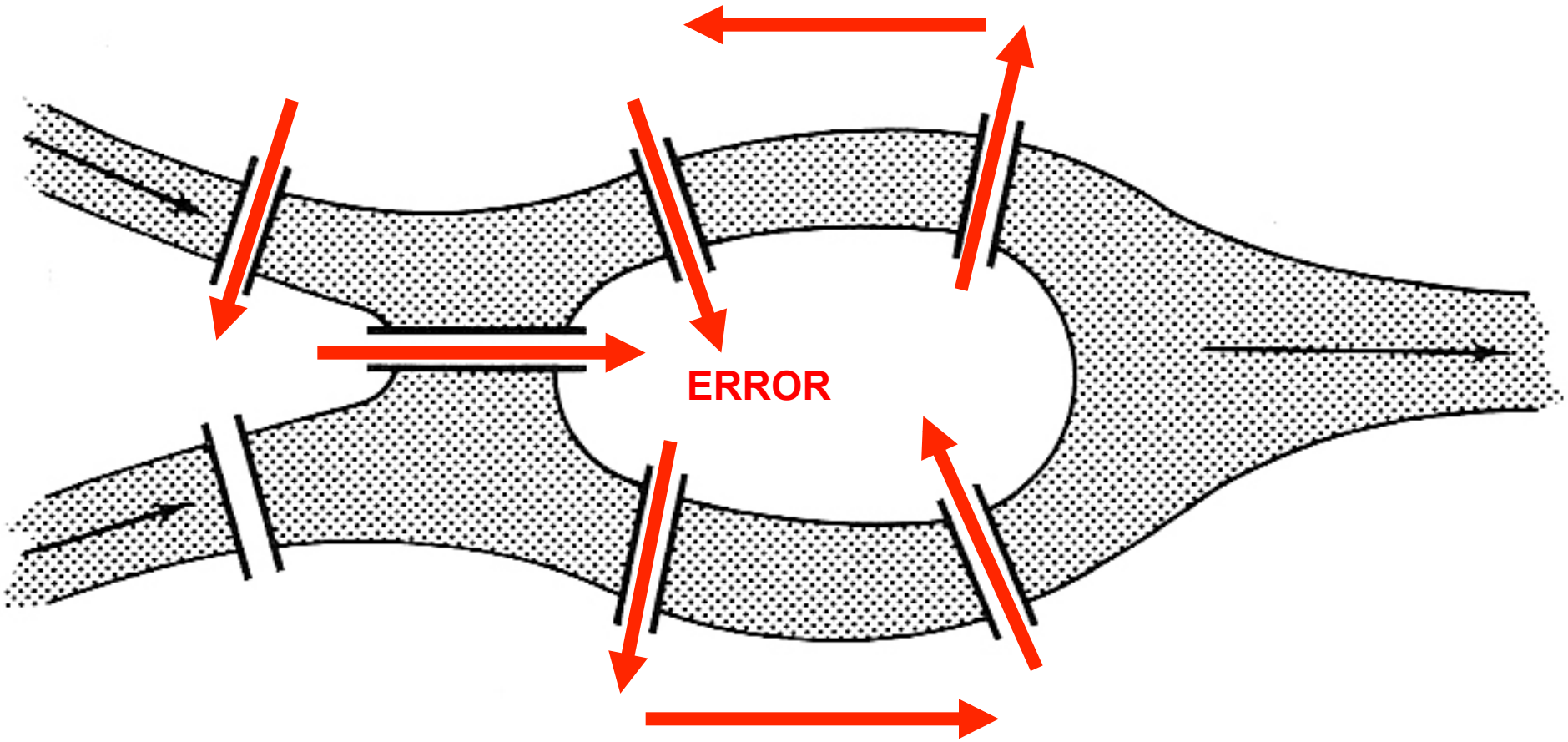
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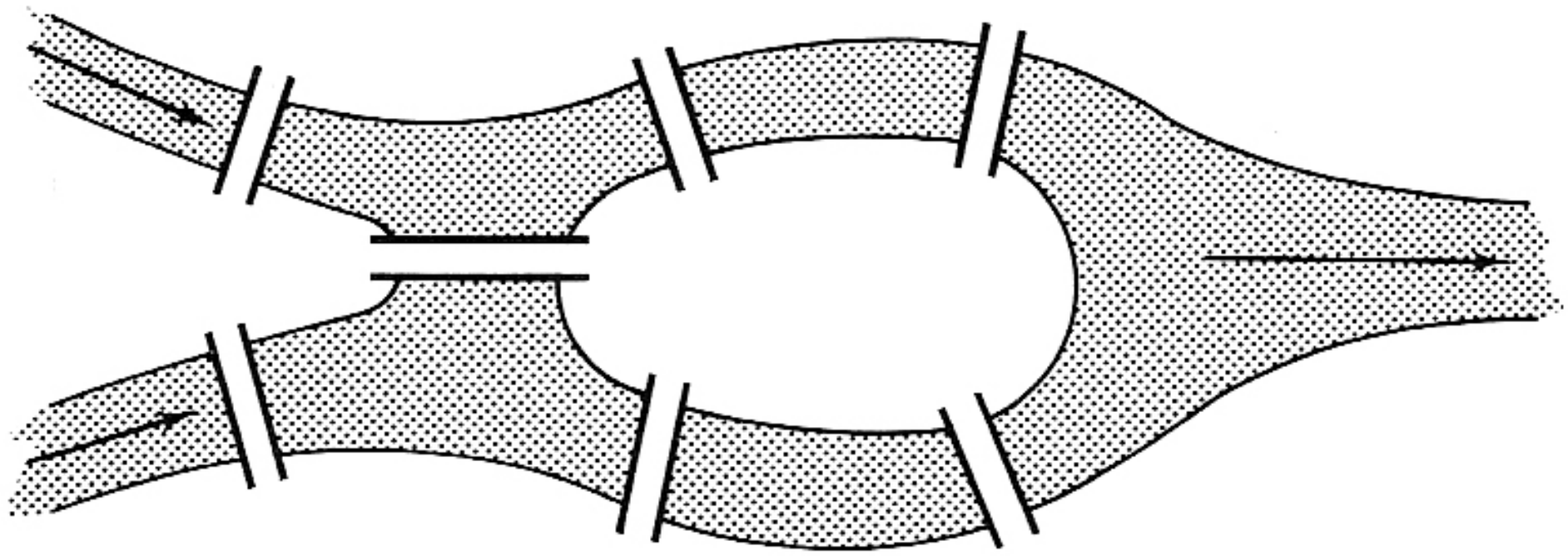

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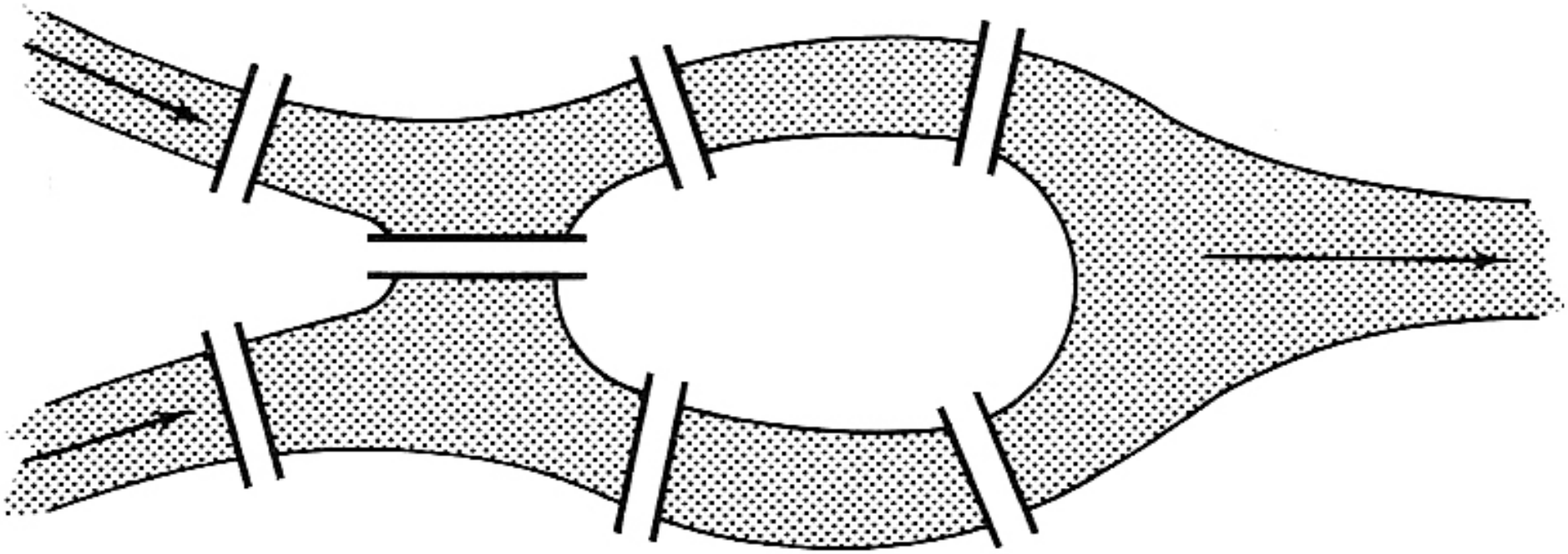

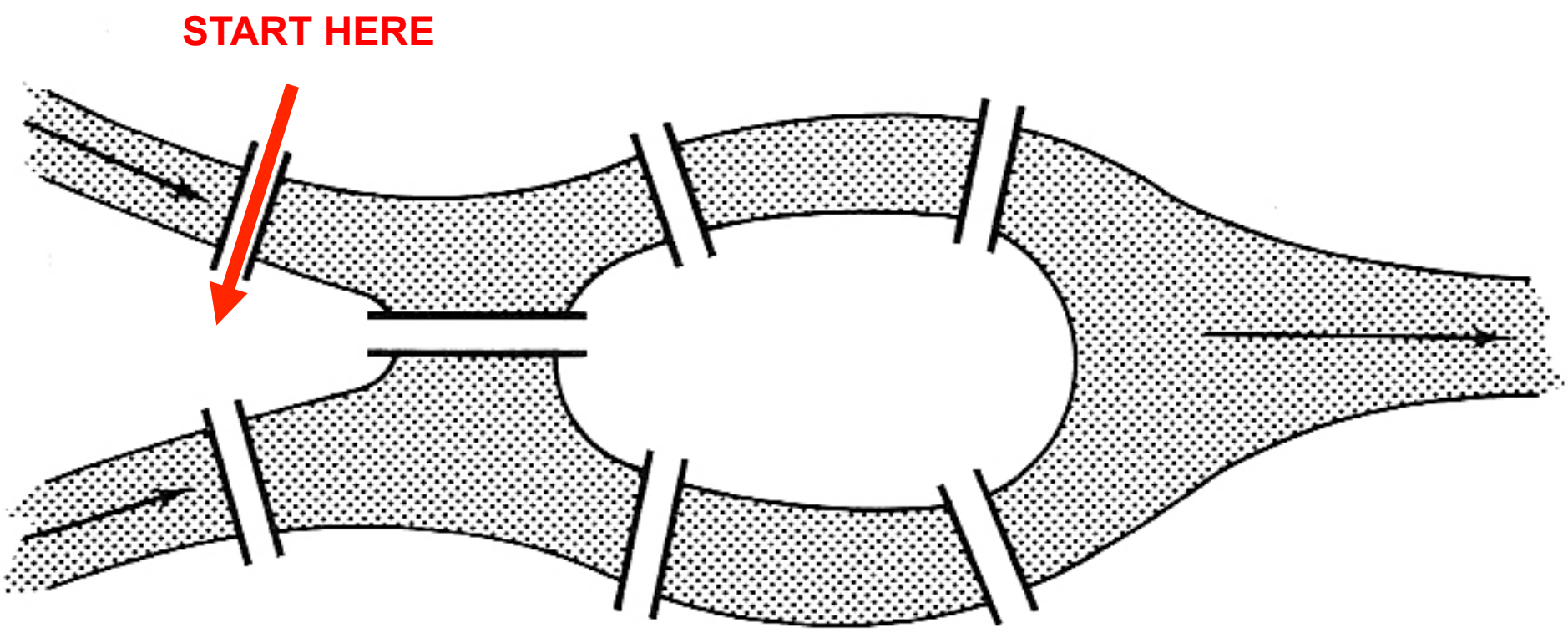
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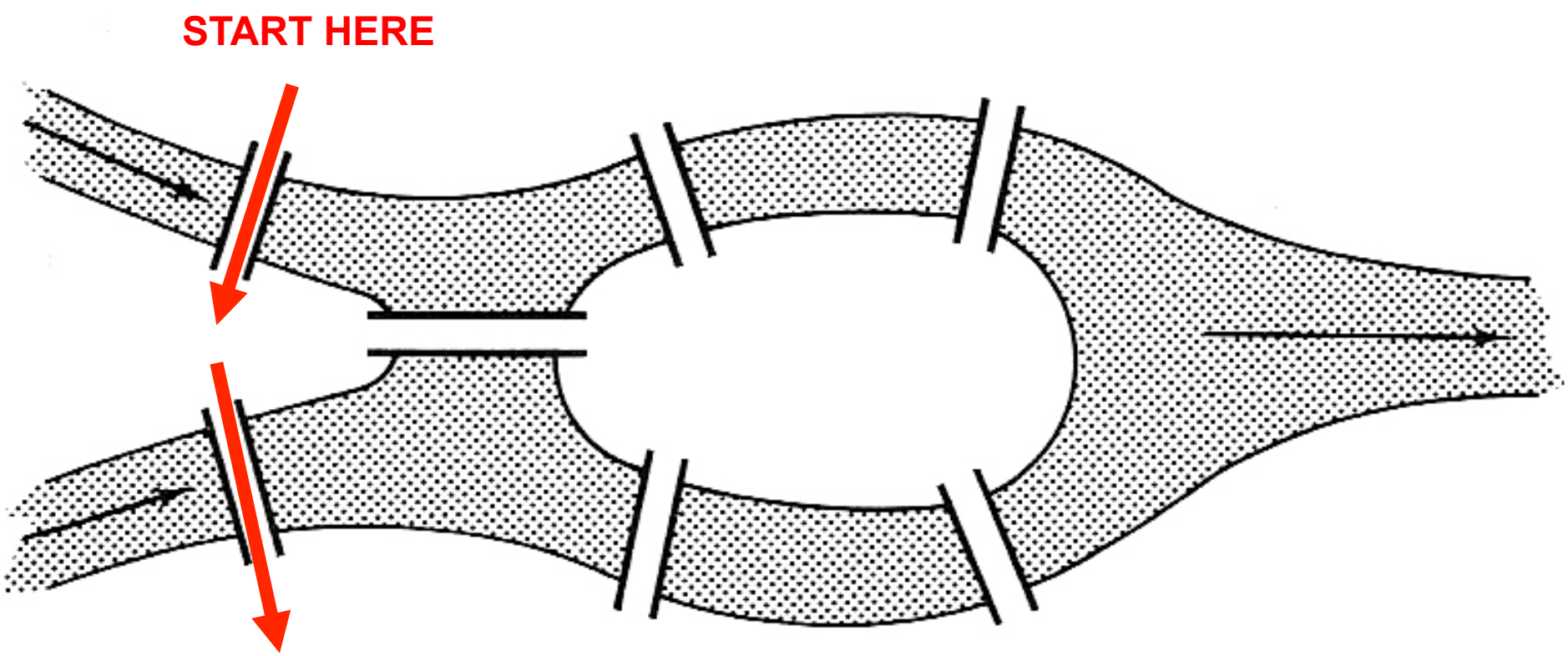
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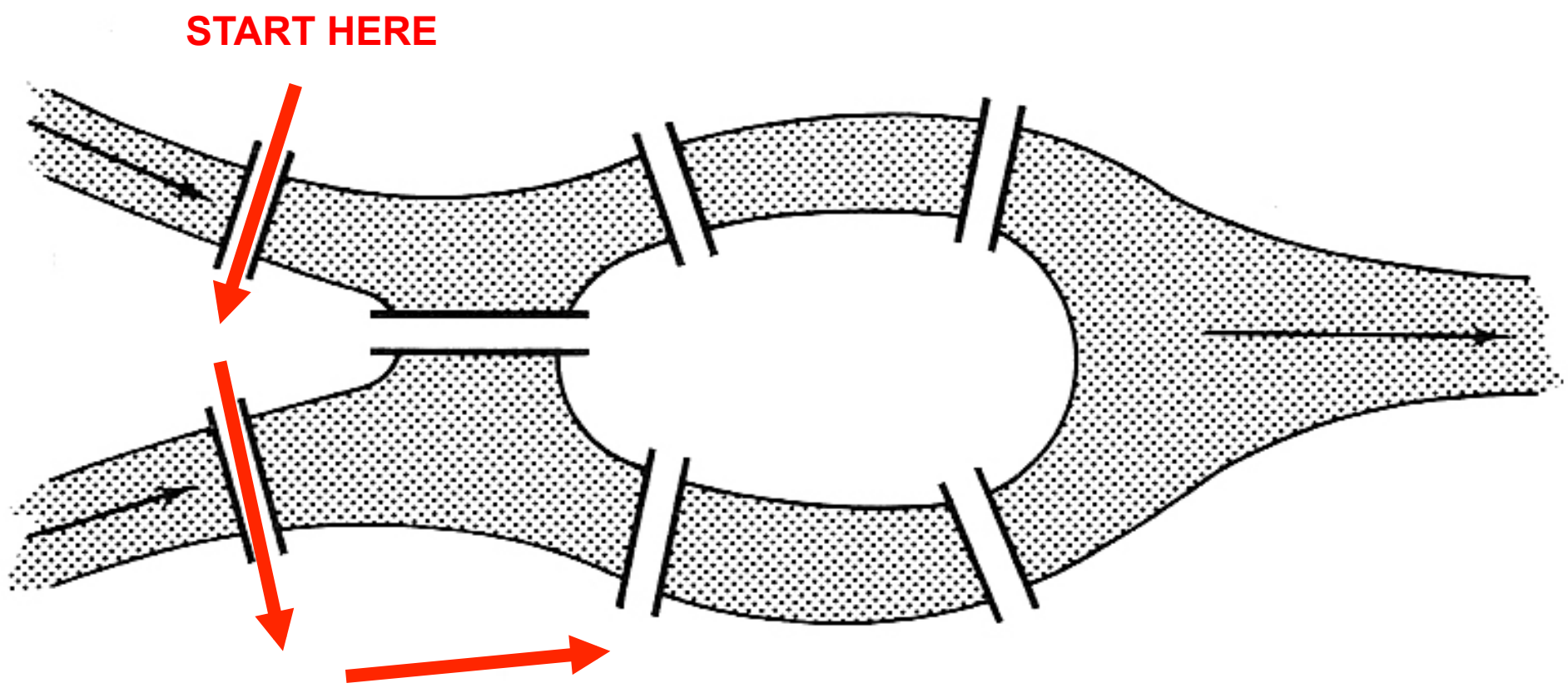
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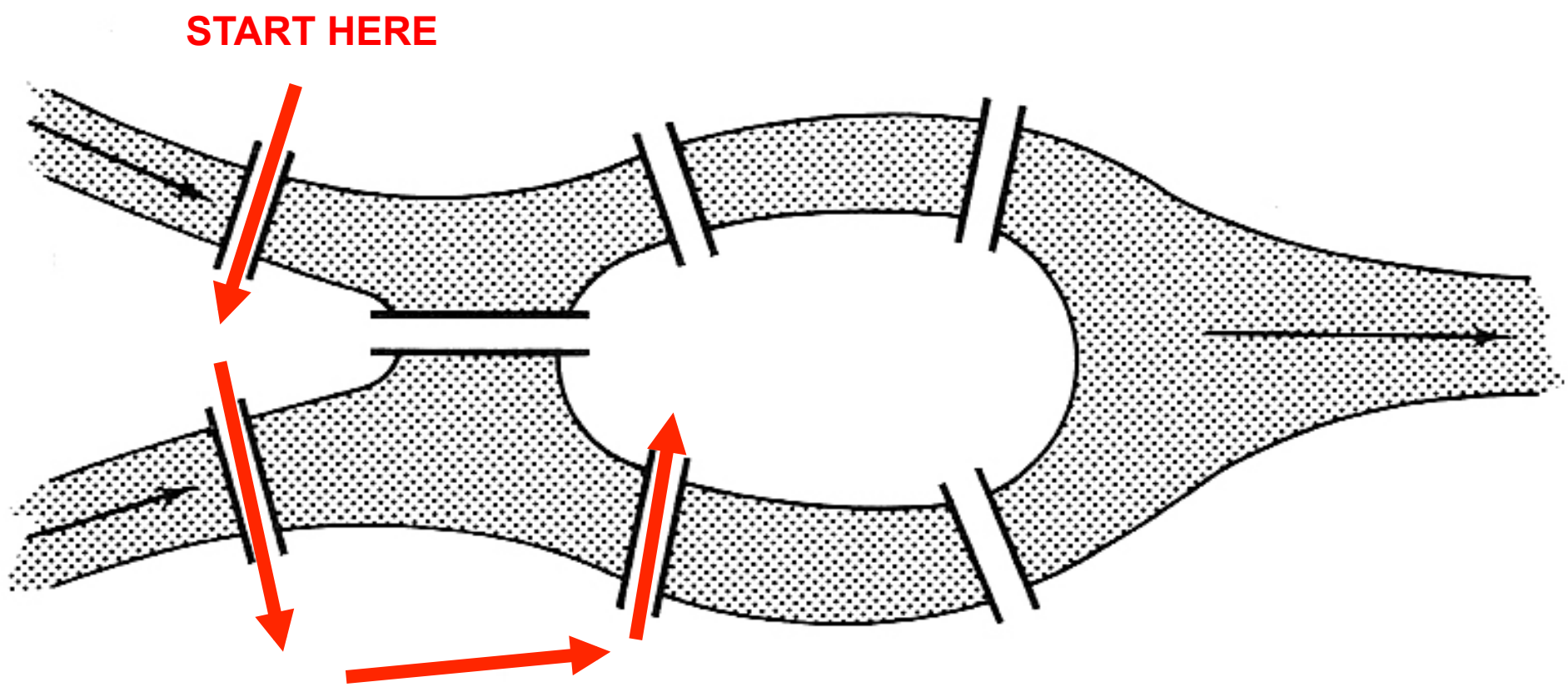
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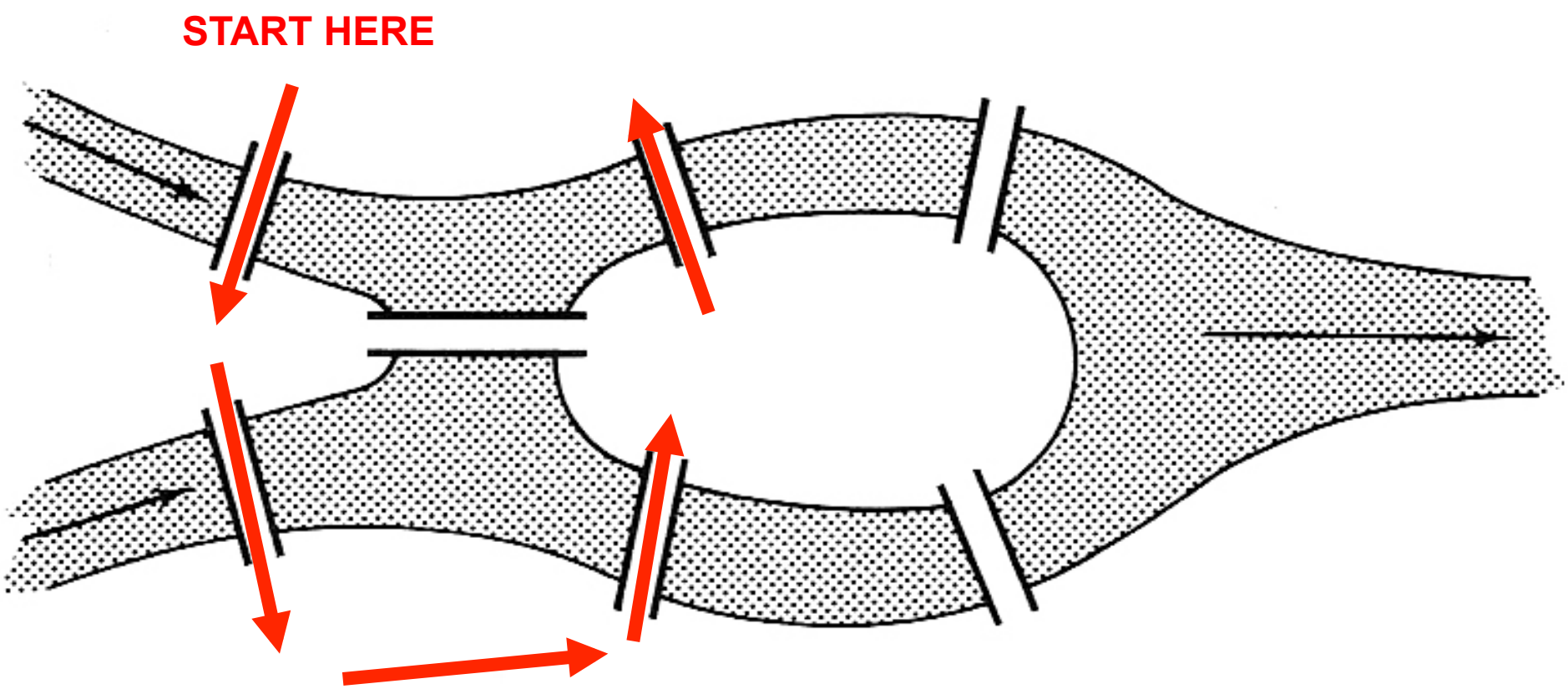
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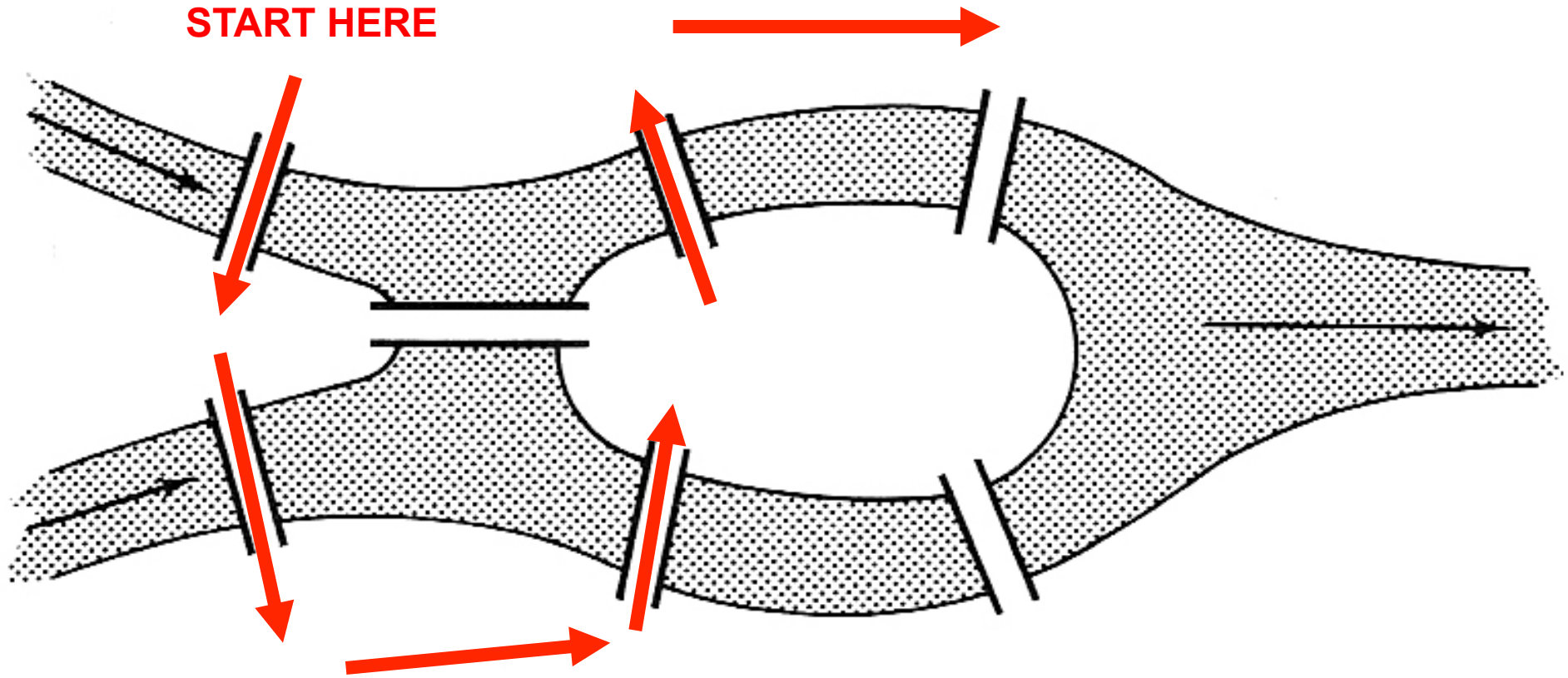
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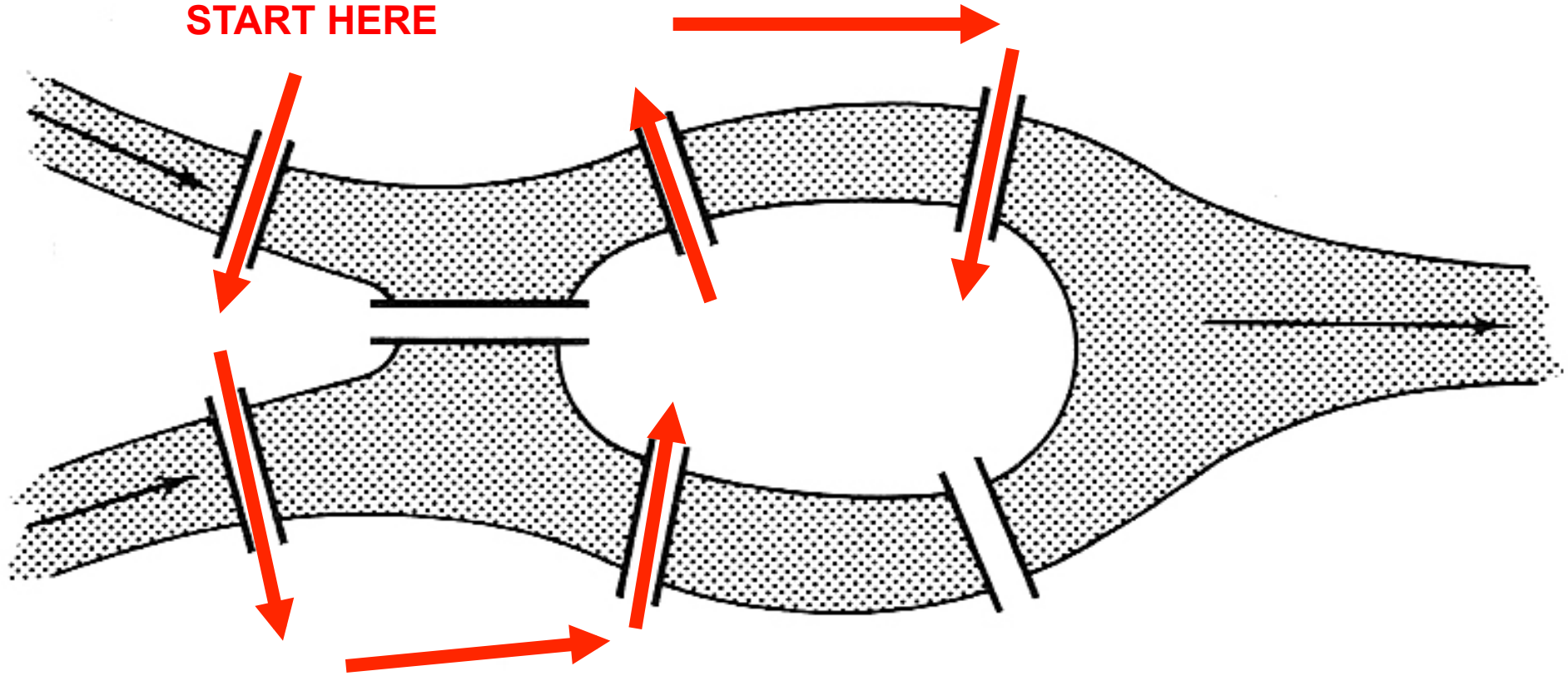
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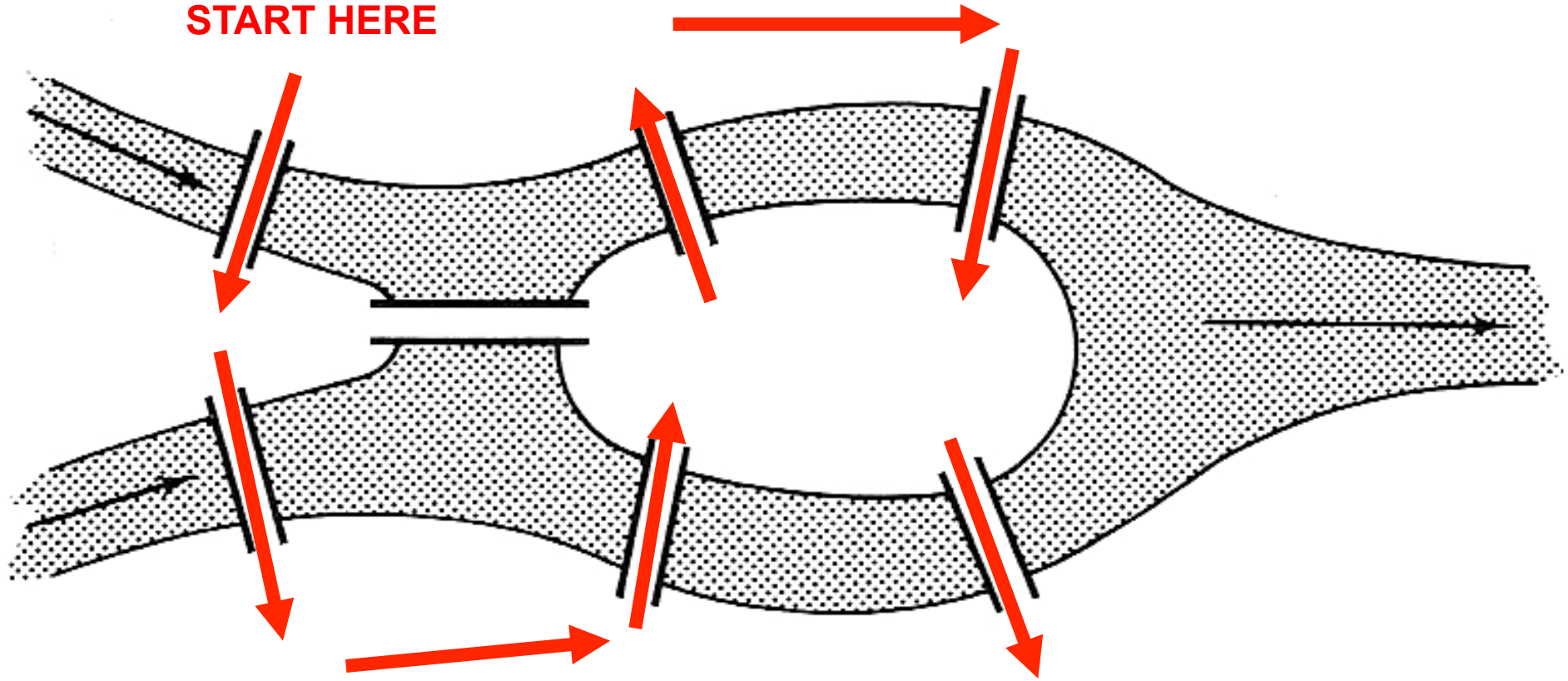
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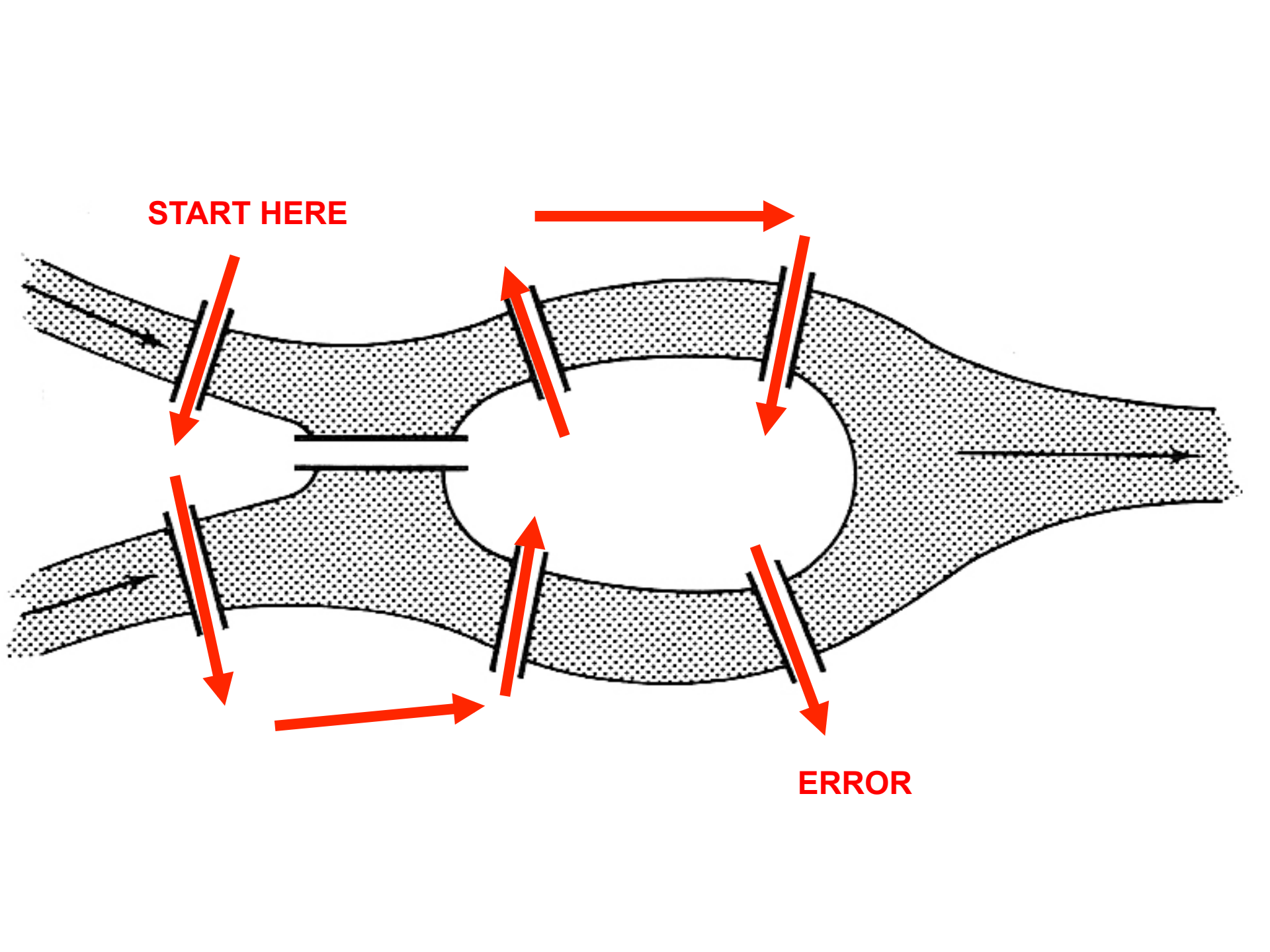
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THE BRIDGES OF KÖNIGSBERG

QUESTION

- Is it possible to walk in the town of Königsberg in such a way that every bridge in the town would be crossed once and only once and the walker return to his starting point?

ANSWER

- No!

THE BRIDGES OF KÖNIGSBERG

WHY?

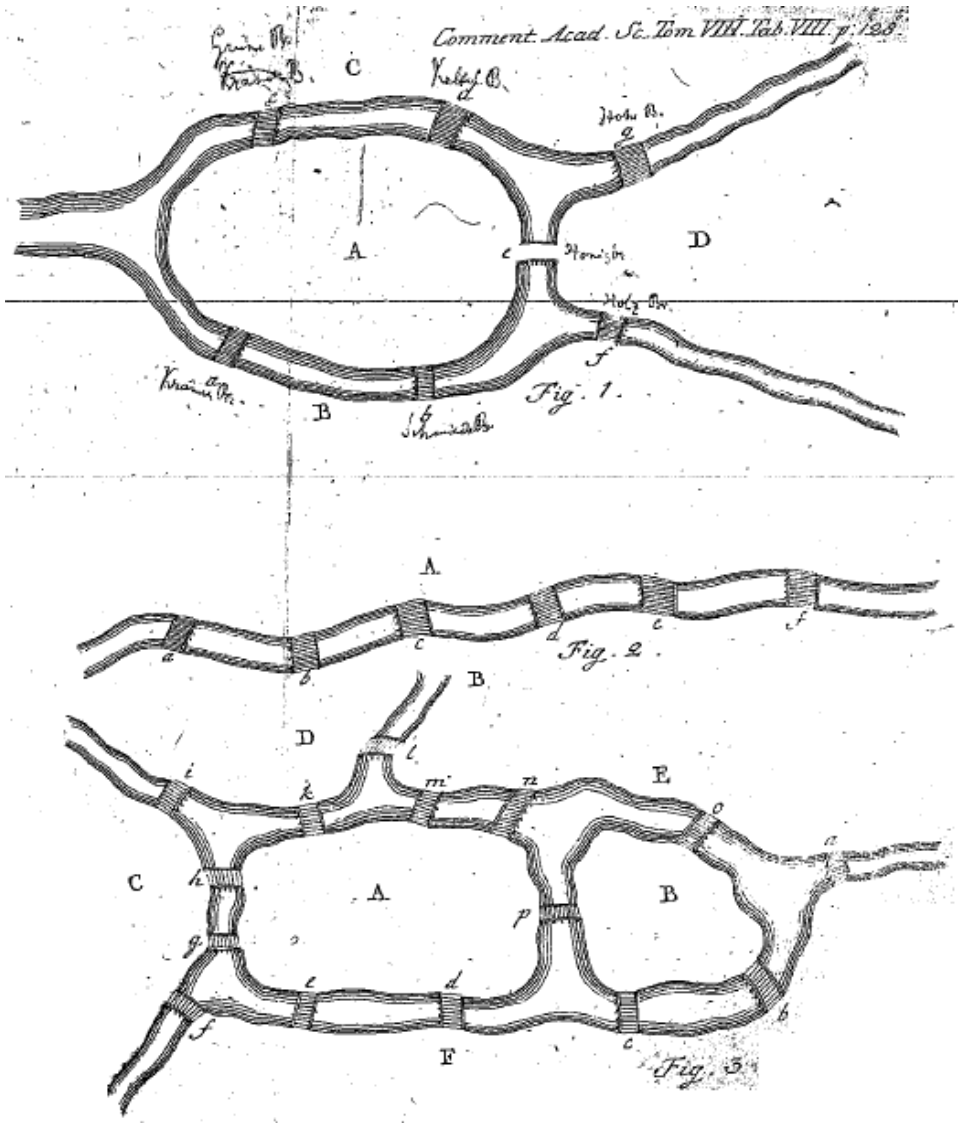
SOLVTIO PROBLEMATIS AD GEOMETRIAM SITVS PERTINENTIS.

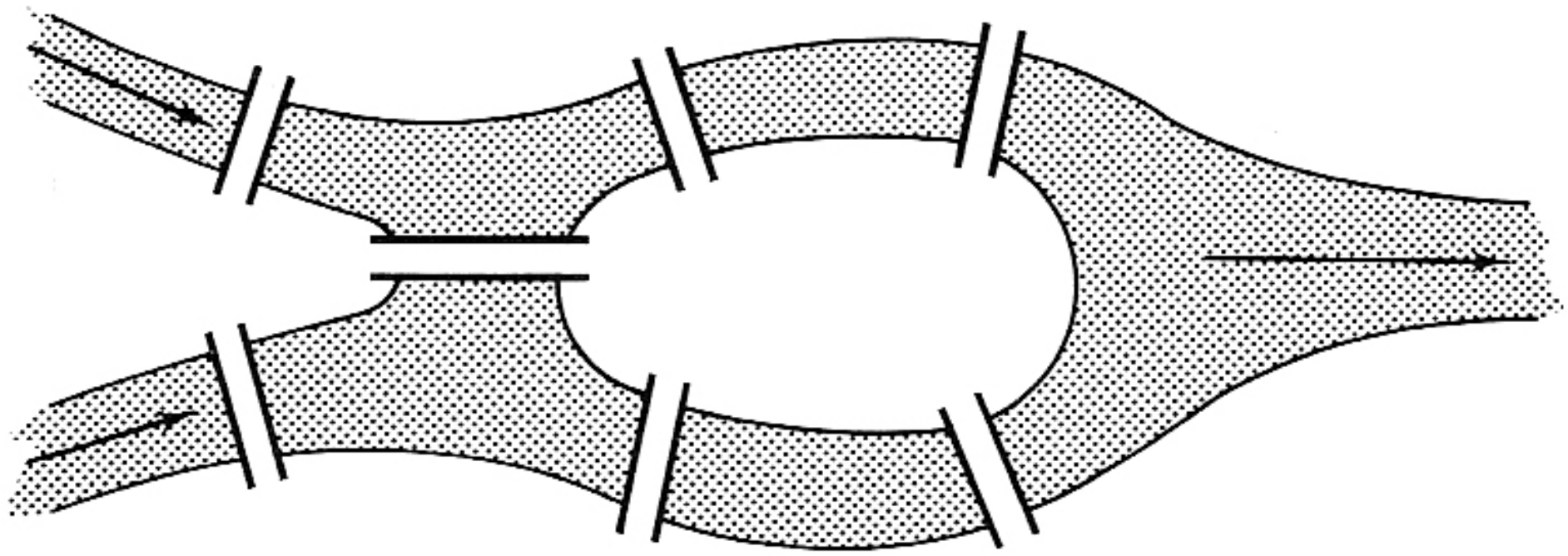
AVCTORE
Leonh. Eulero.

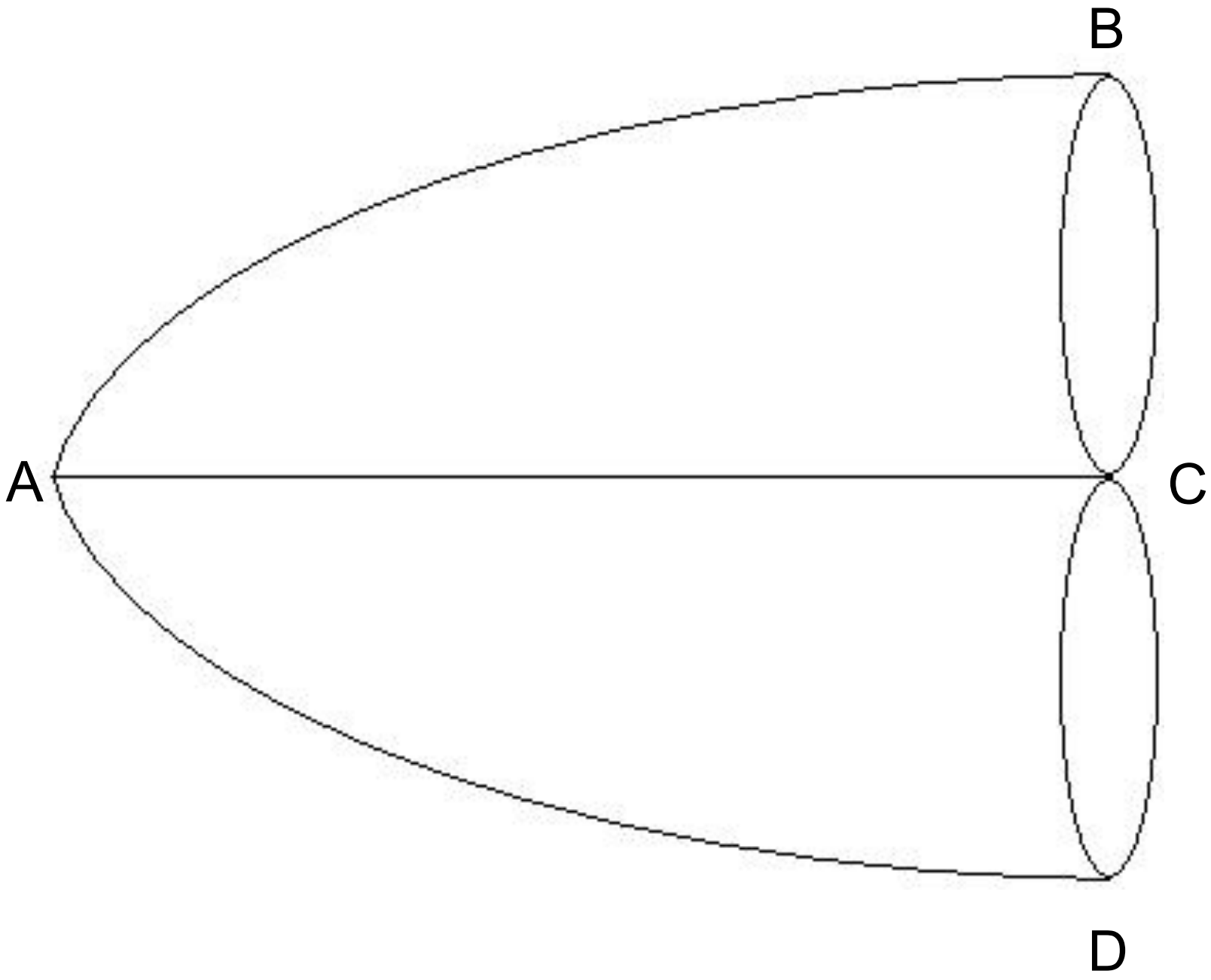
§. 1.

Tabula VIII.

PRaeter illam Geometriae partem, quae circa quantitates verſatur, et omni tempore ſummo ſtudio eſt exculta, alterius partis etiamnum admodum ignotae primus mentionem fecit *Leibnitzius*, quam Geometriam ſitus vocauit. Iſta pars ab ipſo in ſolo ſitu determinando, ſitusque proprietatibus eruendis occupata eſſe ſtatuitur; in quo negotio neque ad quantitates reſpiciendum, neque calculo quantitatum vtendum ſit. Cuiusmodi autem problemata ad hanc ſitus Geometriam pertineant, et quali methodo in iis reſoluendis vti oporteat, non ſatis eſt definitum. Quamobrem, cum nuper problematis cuiusdam mentio eſſet facta, quod quidem ad geometriam pertinere videbatur, at ita erat comparatum, vt neque determinationem quantitatum requireret, neque ſolutionem calculi quantitatum ope admitteret, id ad geometriam ſitus referre haud dubitaui: praeſertim quod in eius ſolutione ſolus ſitus in conſiderationem veniat, calculus vero nullius prorſus ſit vſus. Methodum ergo meam quam ad huius generis problemata



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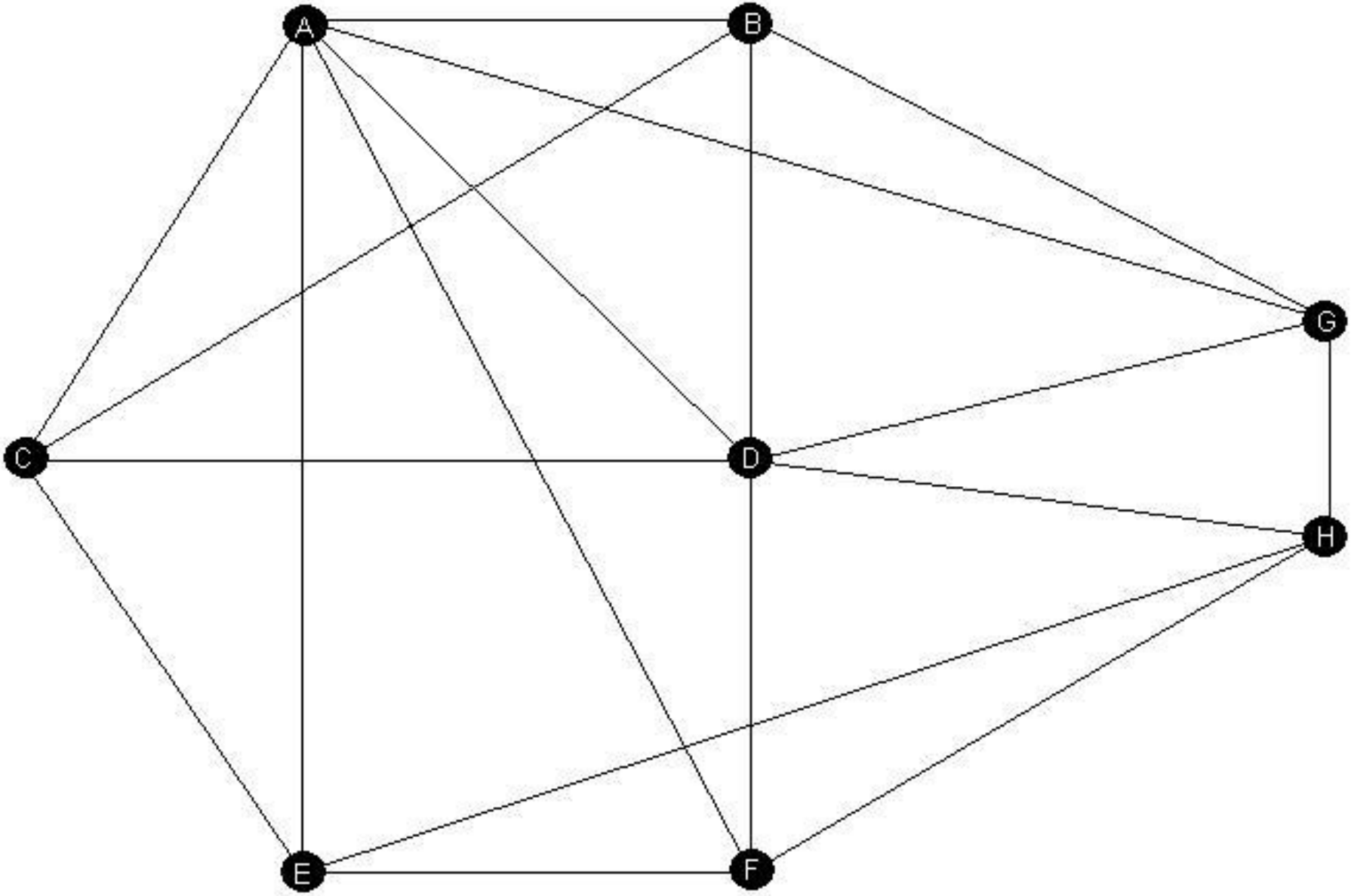
DEFINITIONS

1. A Euler Walk is possible in any graph if the number of nodes is even.
2. A graph with no odd nodes can be traced in one route that starts and ends at the same point. This is called a Euler Tour
3. A graph with exactly two odd nodes can be traced in one route starting at one of the odd nodes and finishing at the other.
4. A graph with more than two odd nodes can't be traced in one route.

ODD OR EVEN?

- What do we mean by ODD and EVEN nodes.
- An Odd Node is said to be a node with an odd number of edges (lines) meeting at that node.

Sheet_2 and Sheet_3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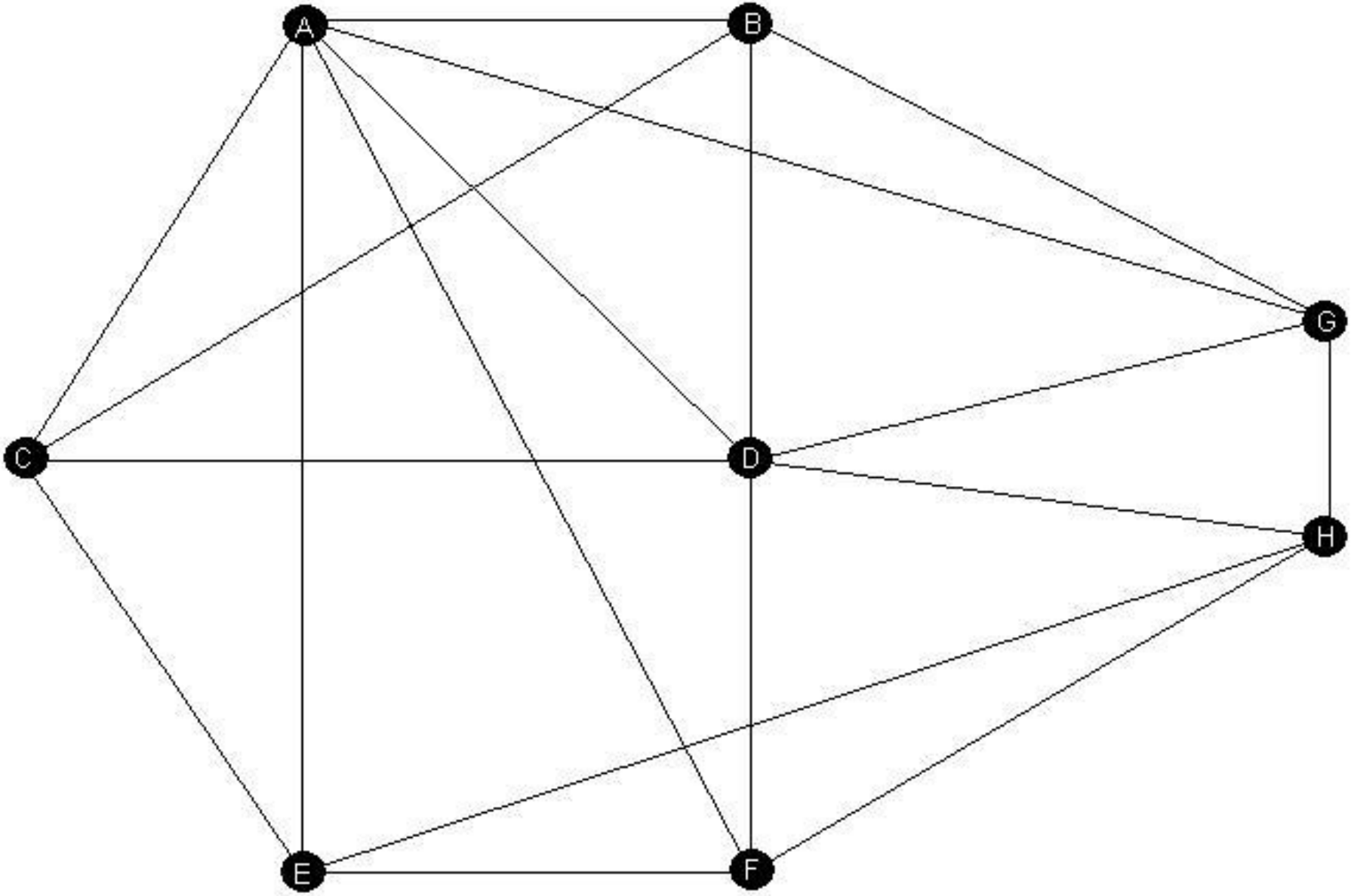

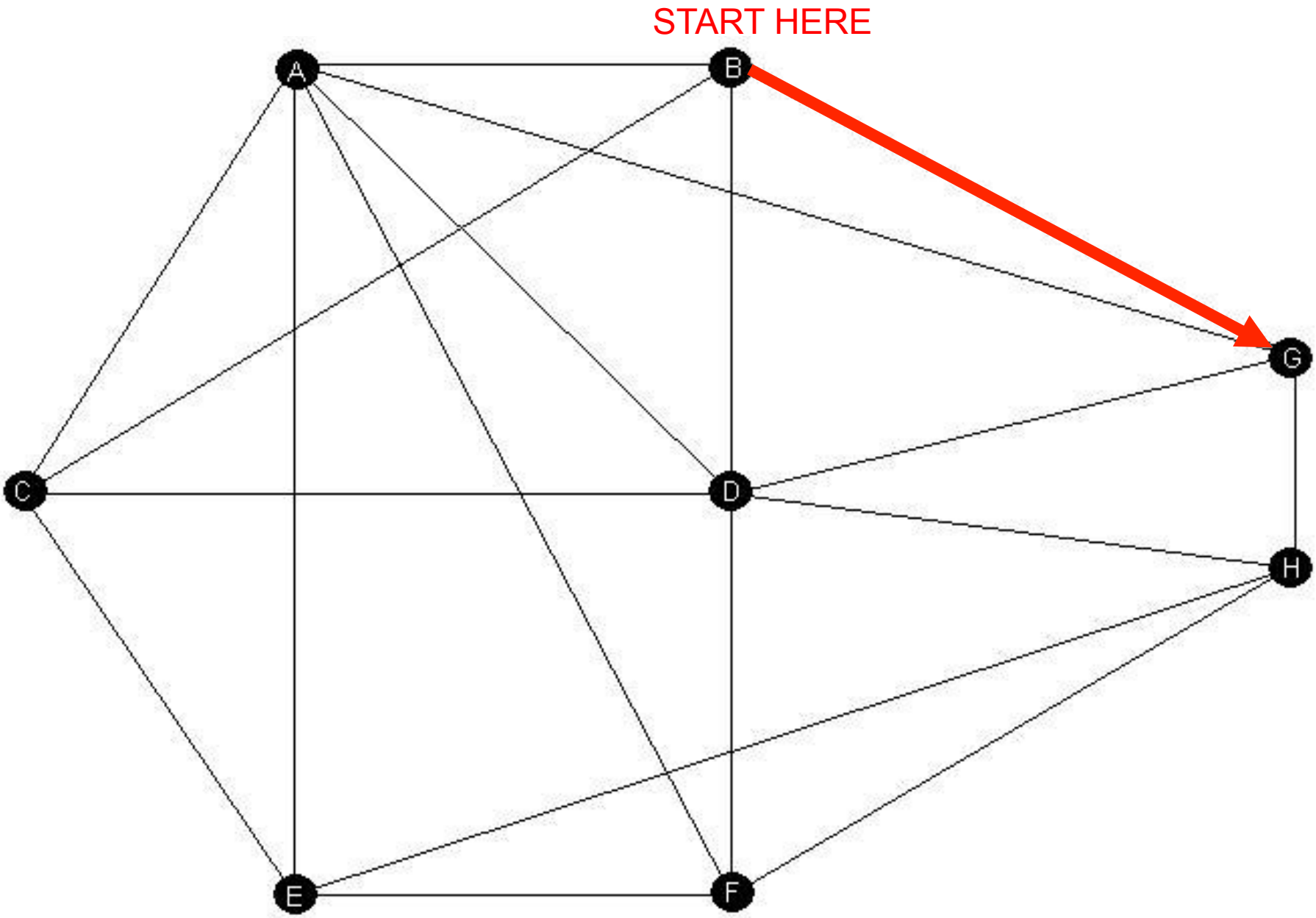
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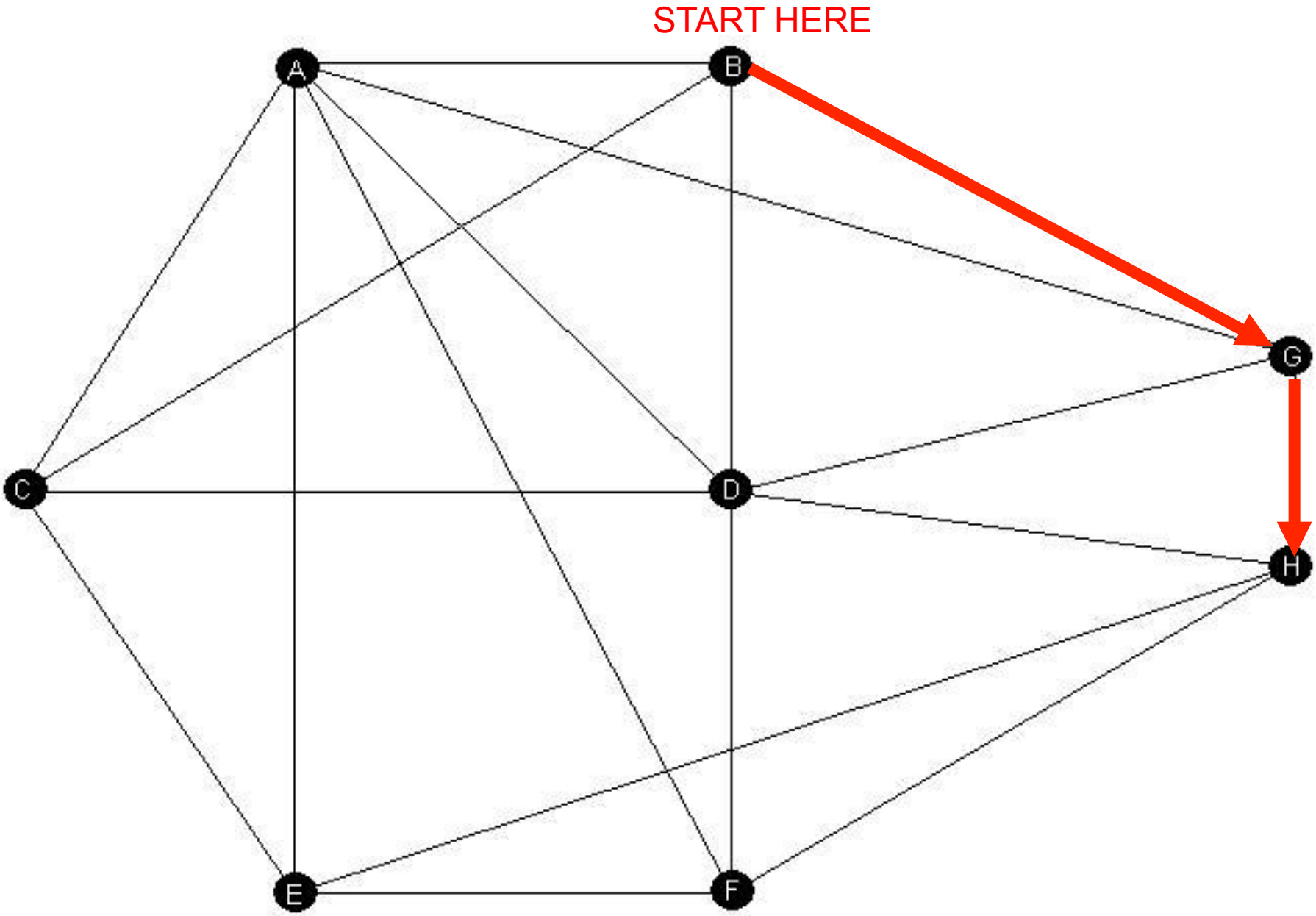
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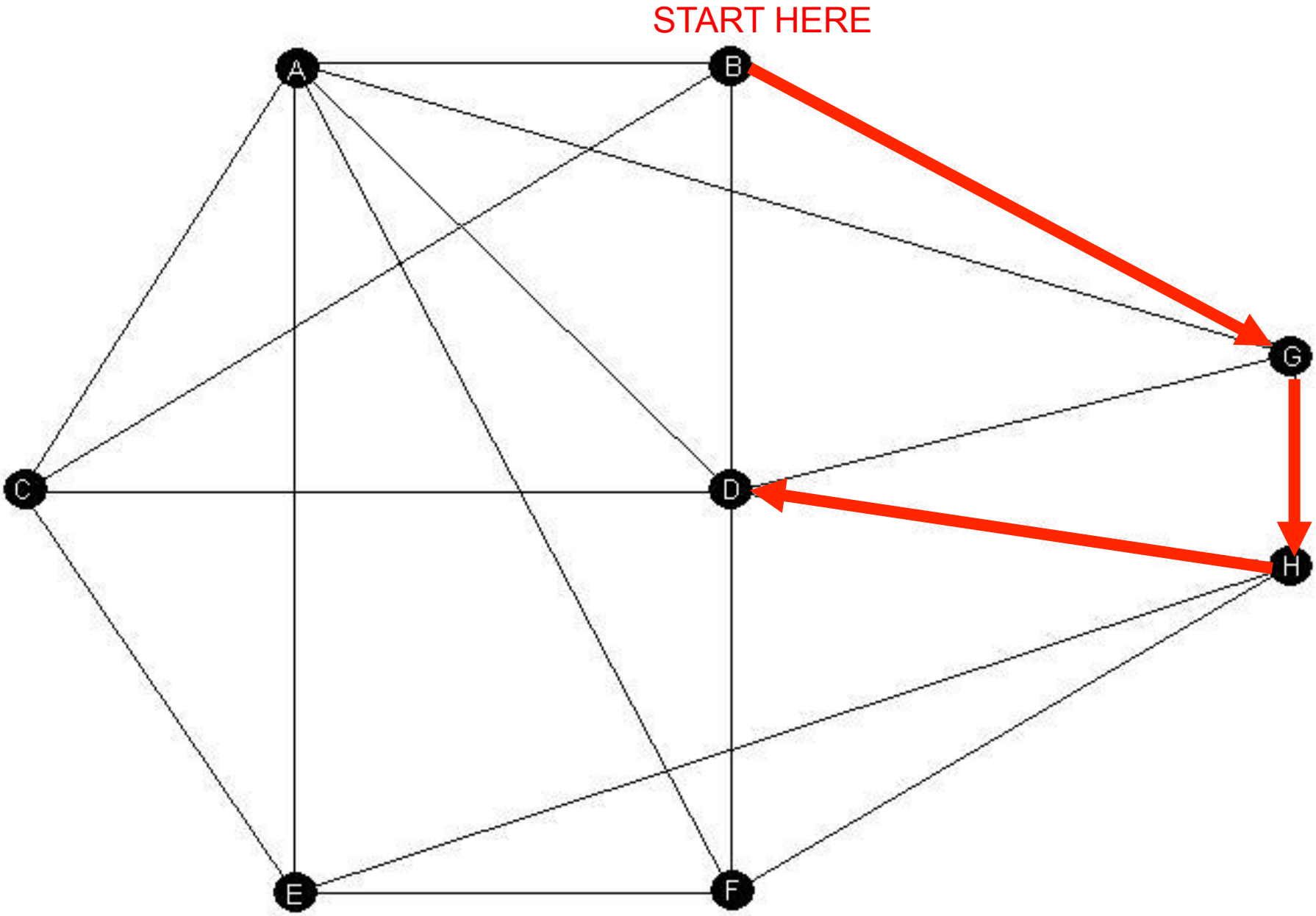
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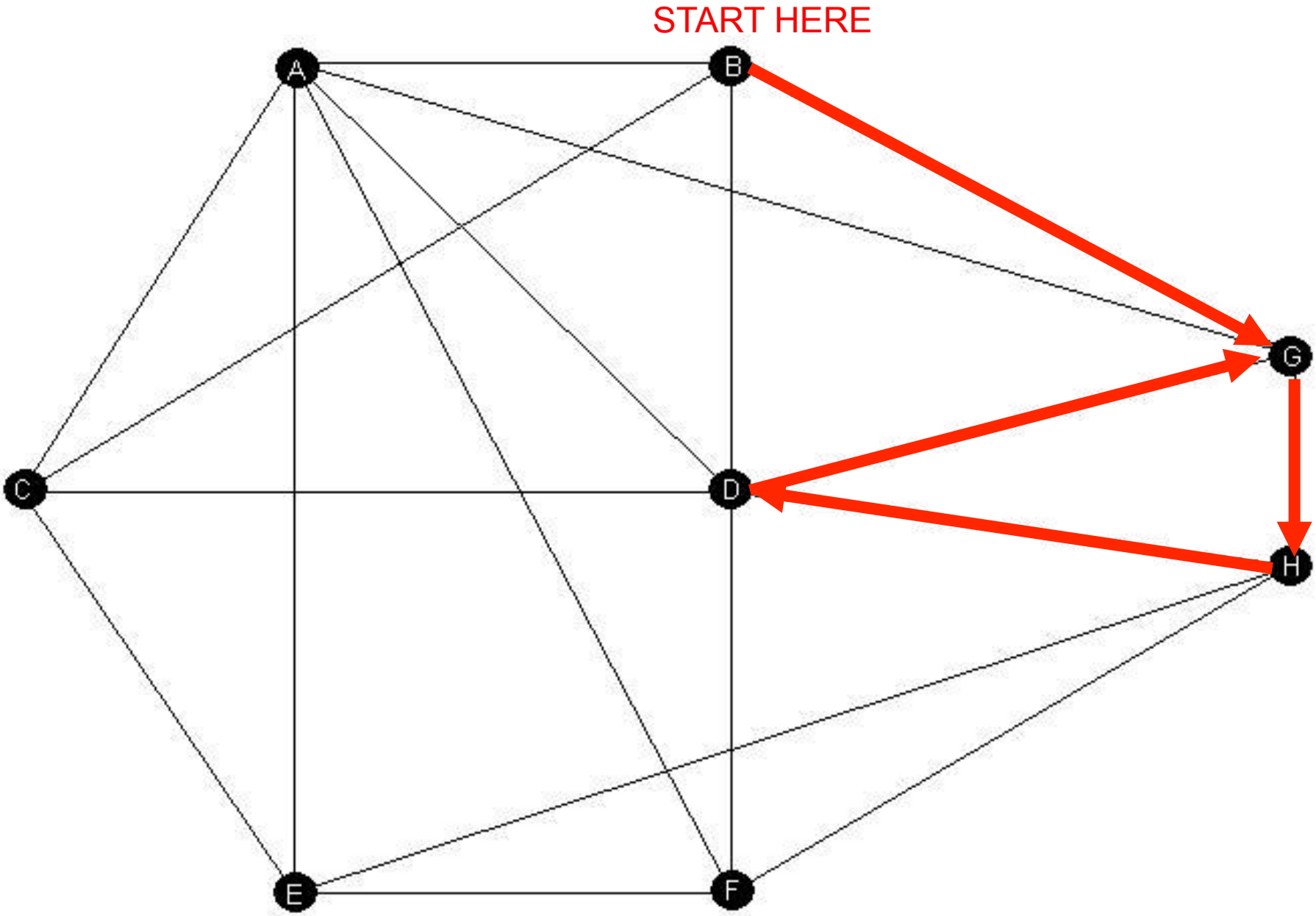
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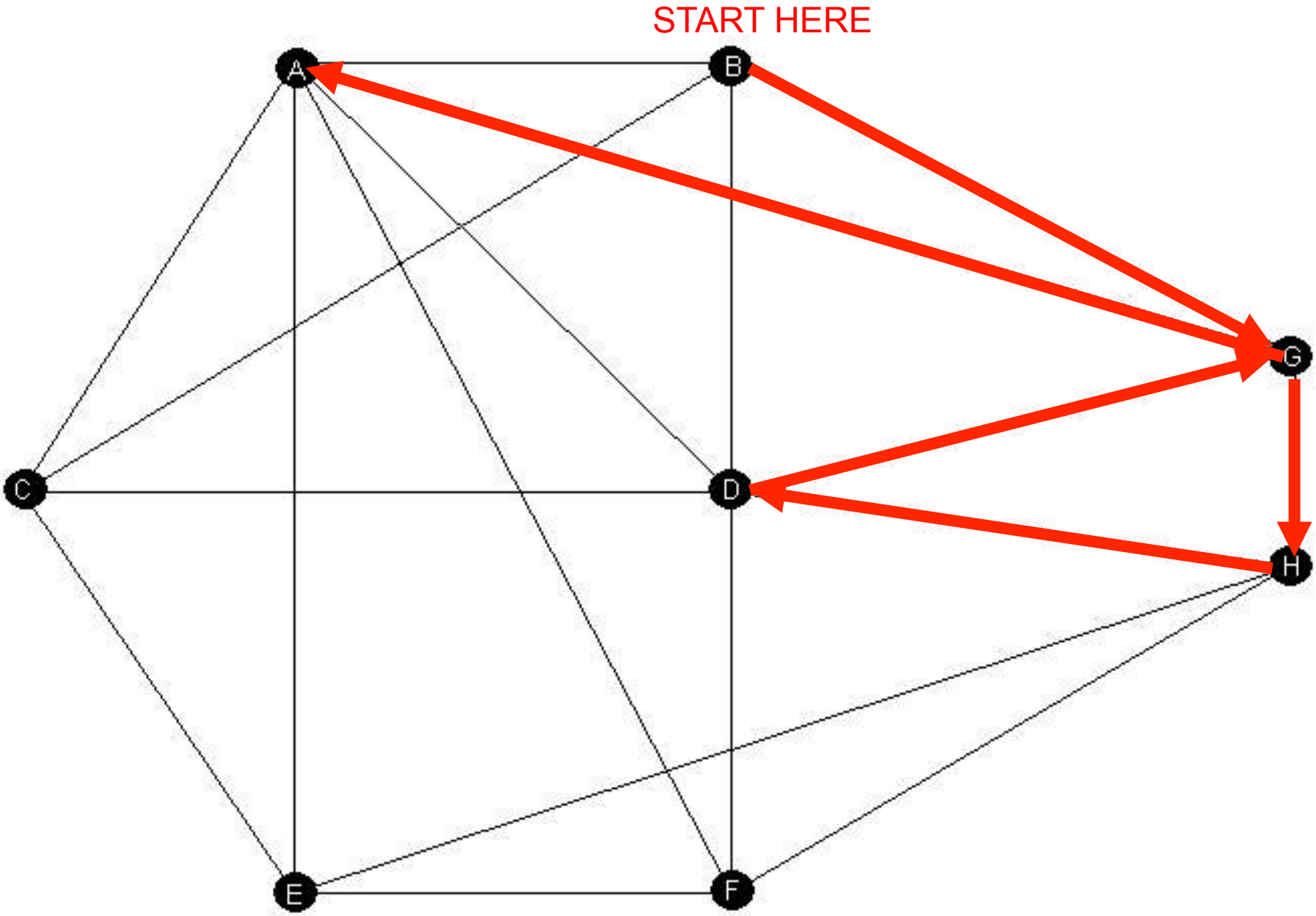
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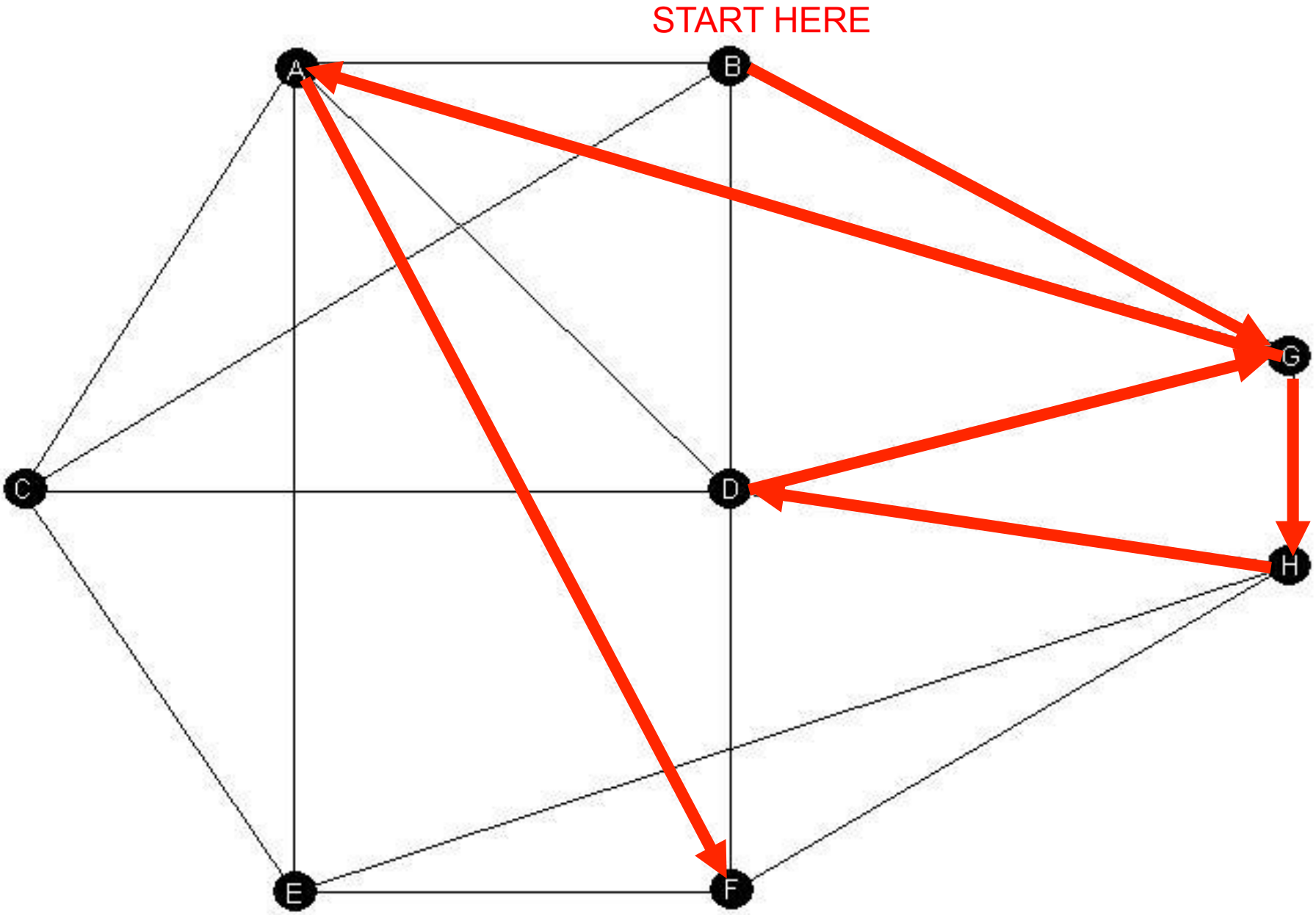
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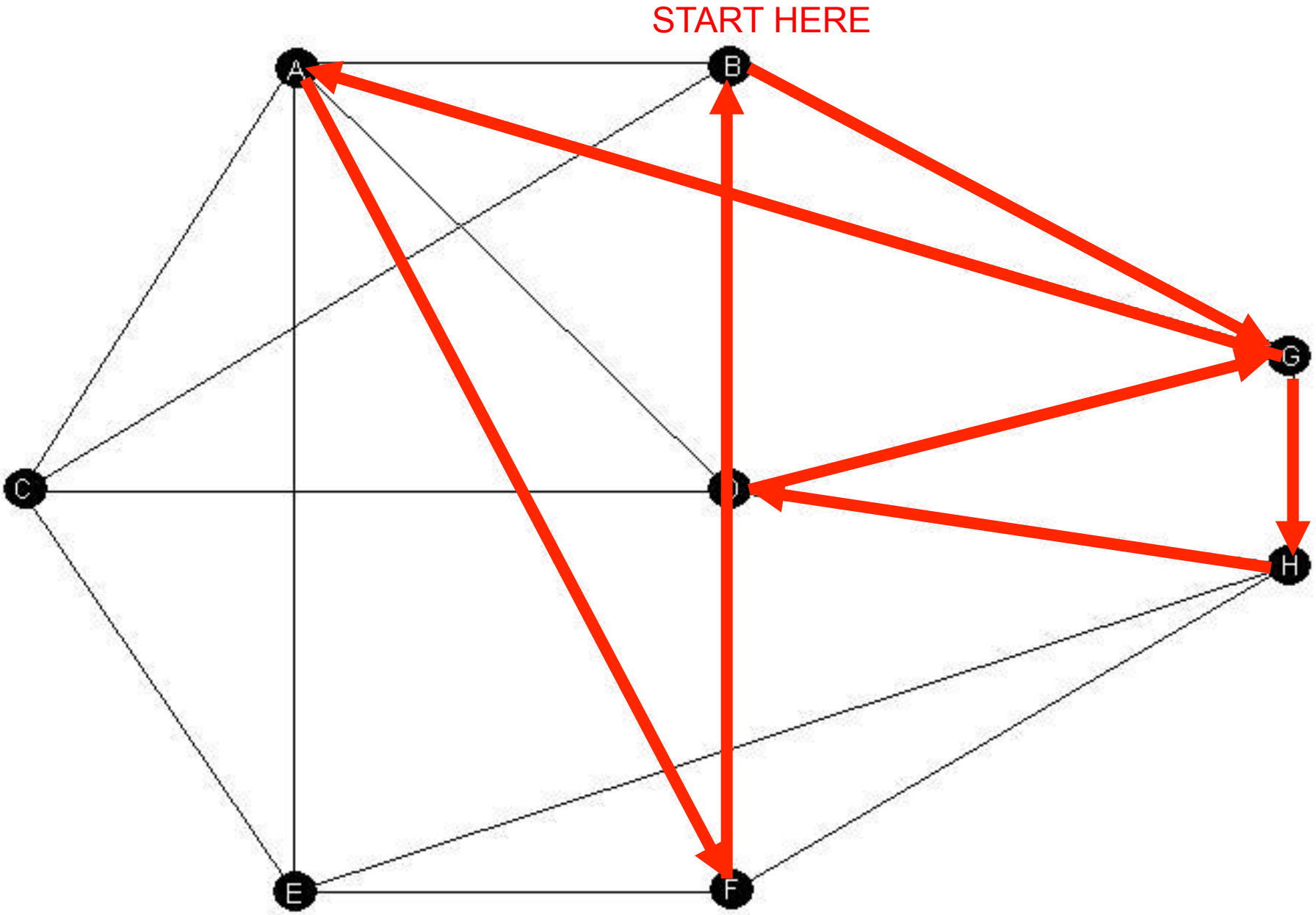
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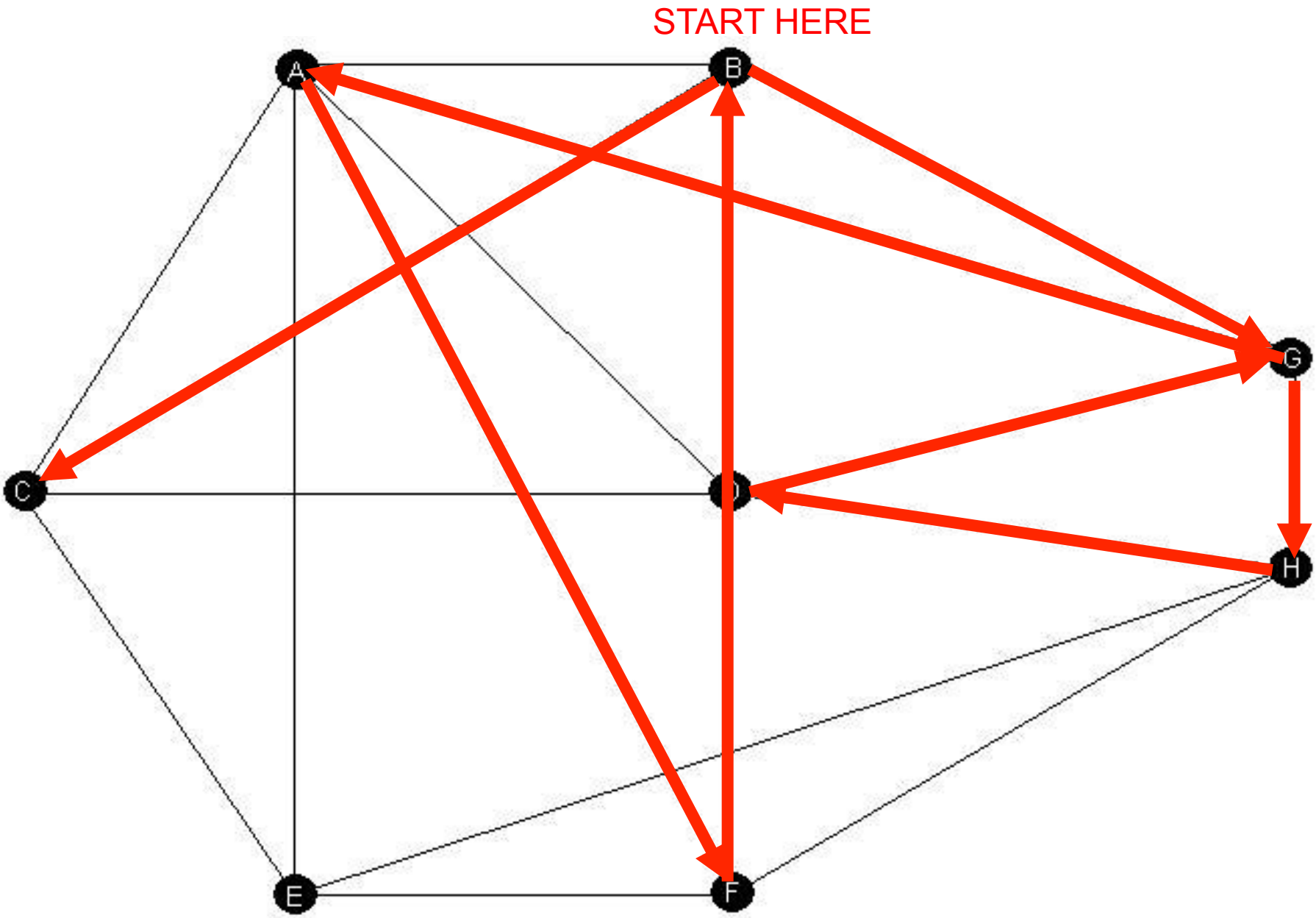
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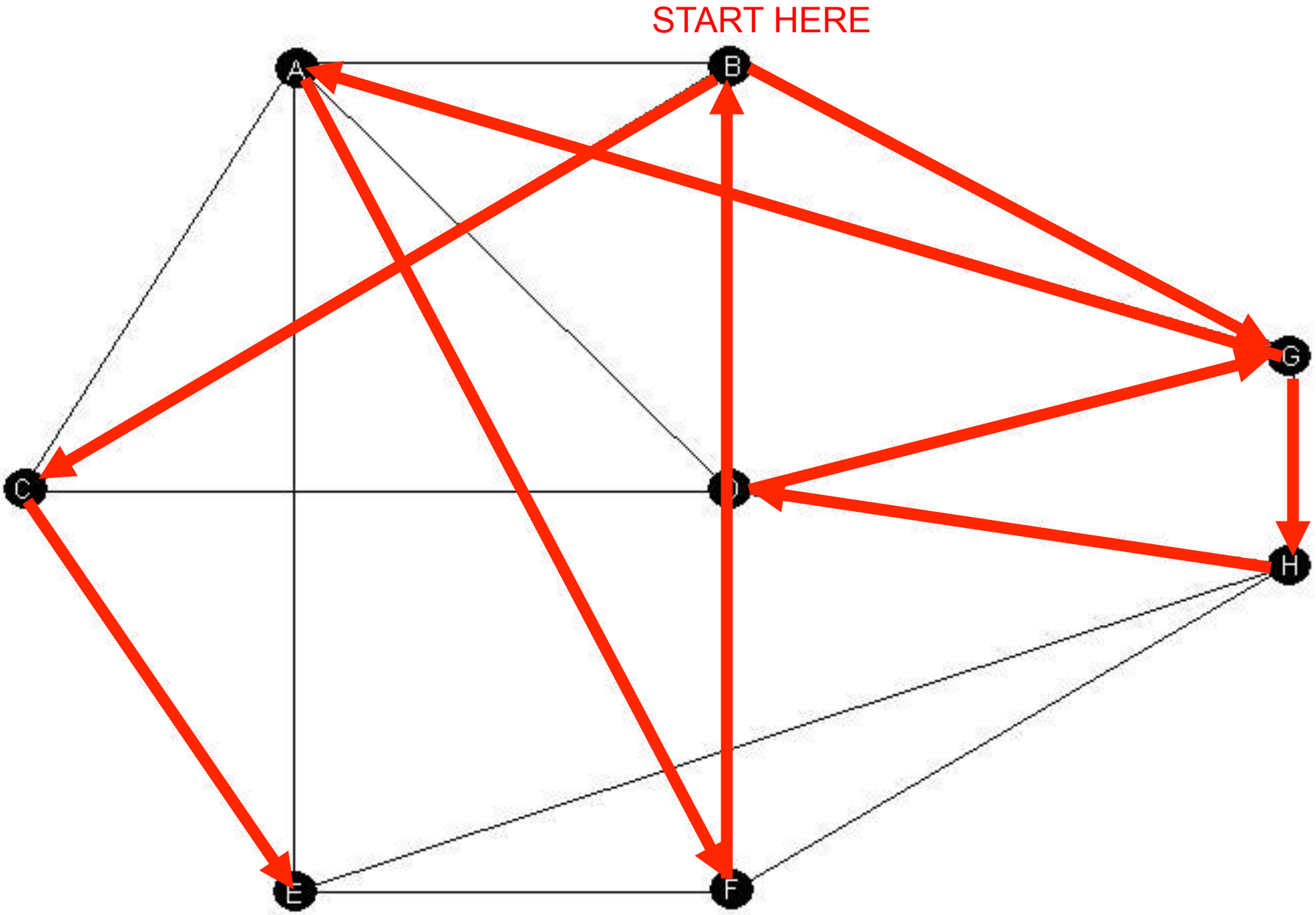
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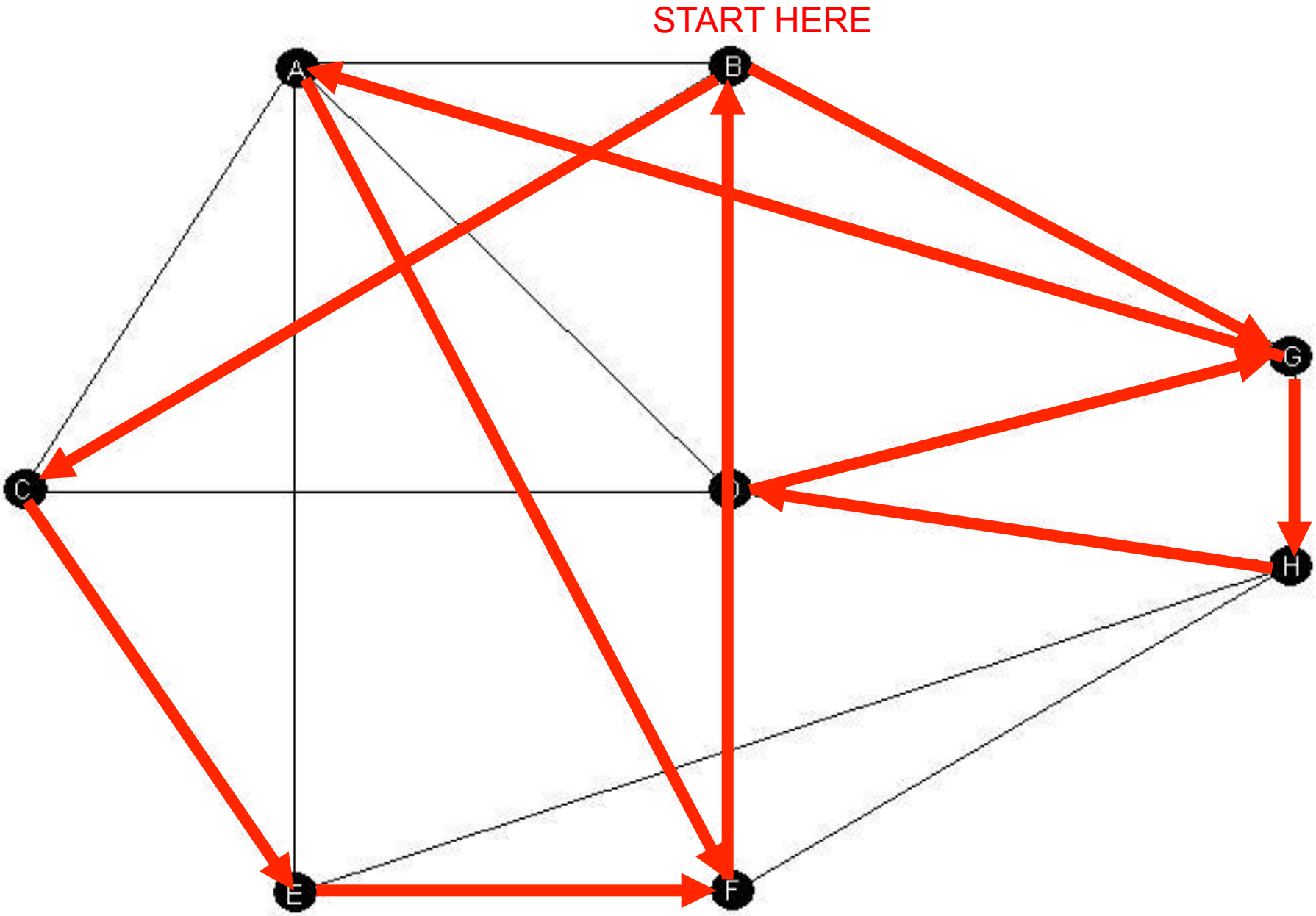
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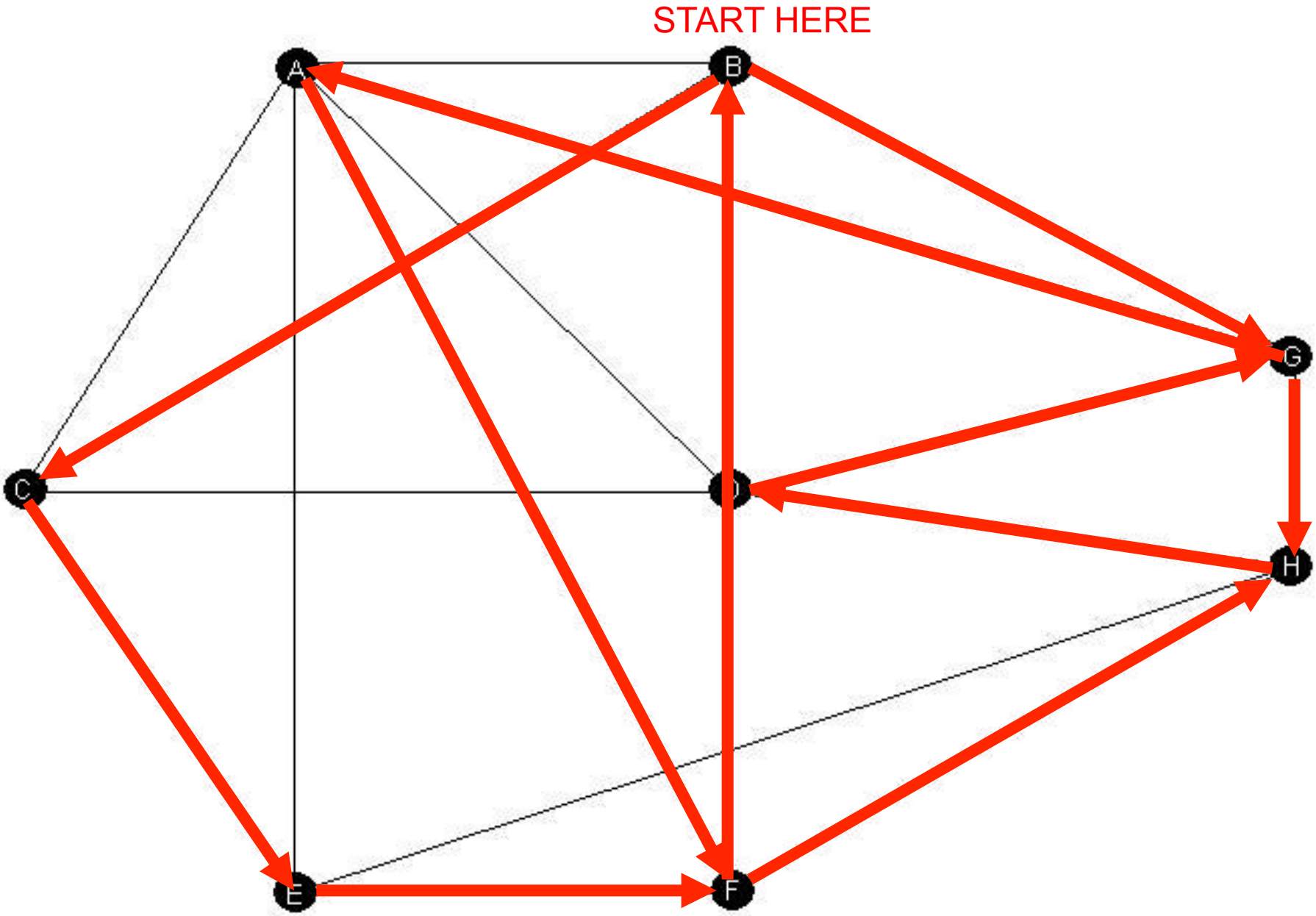
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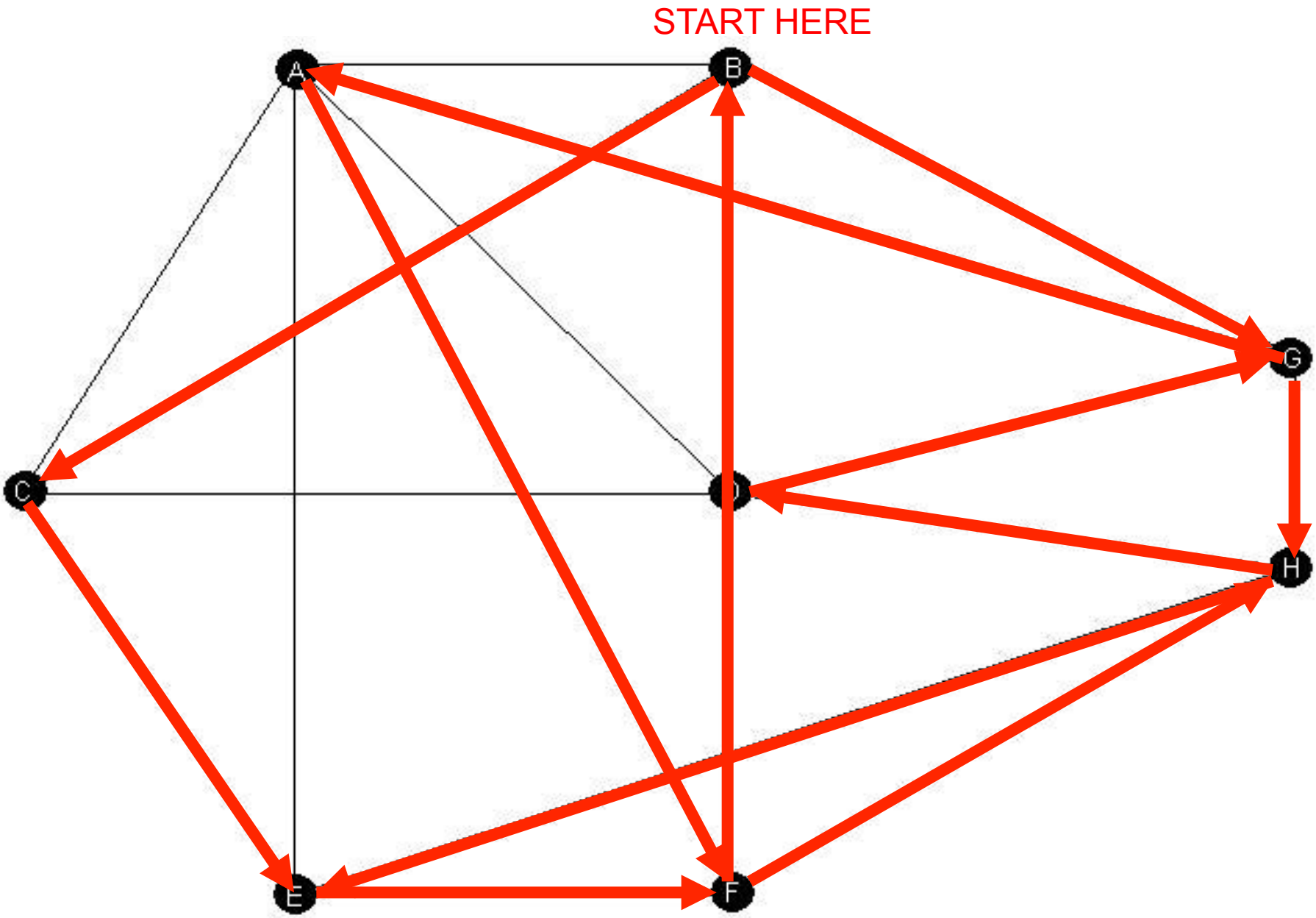
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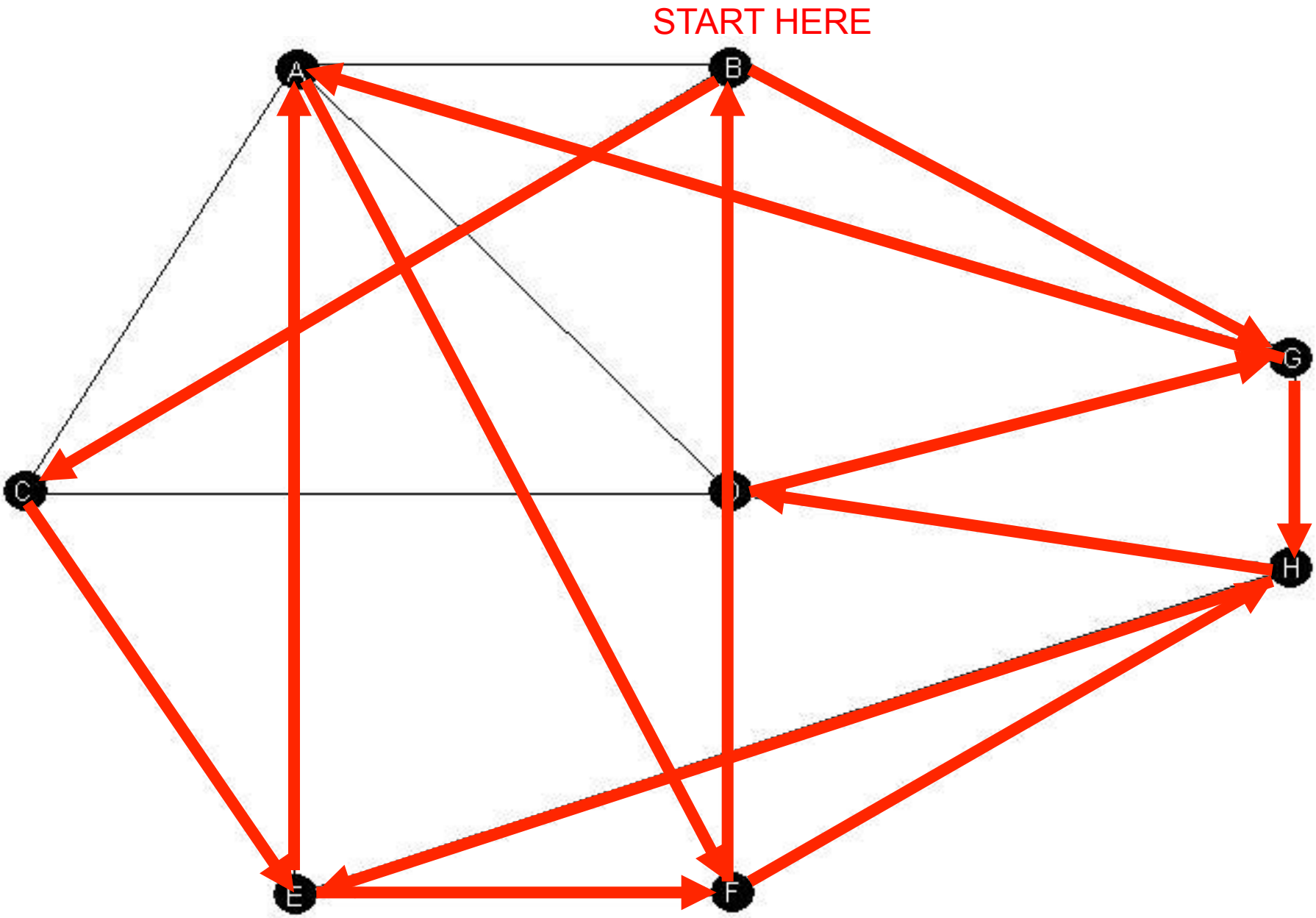
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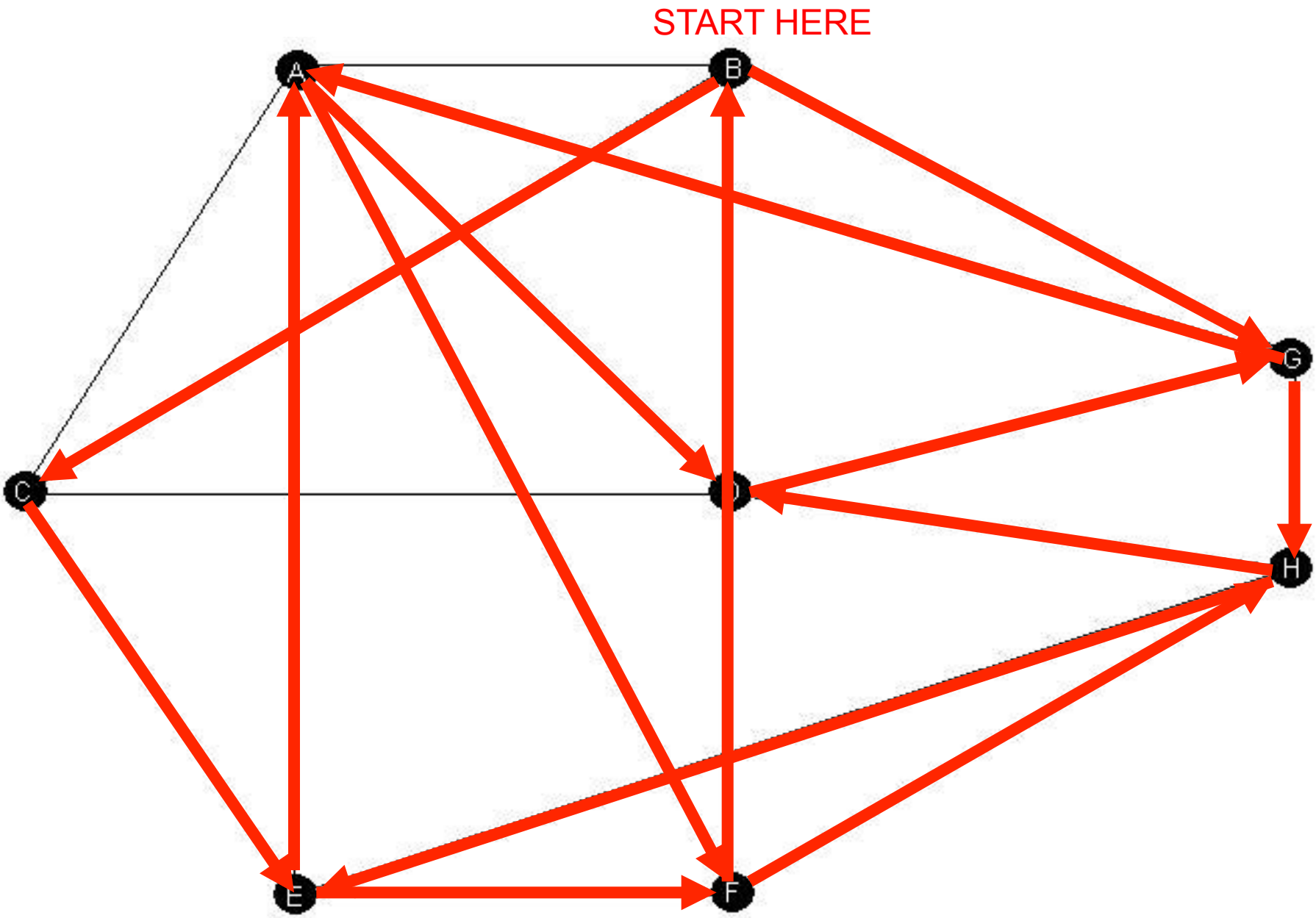
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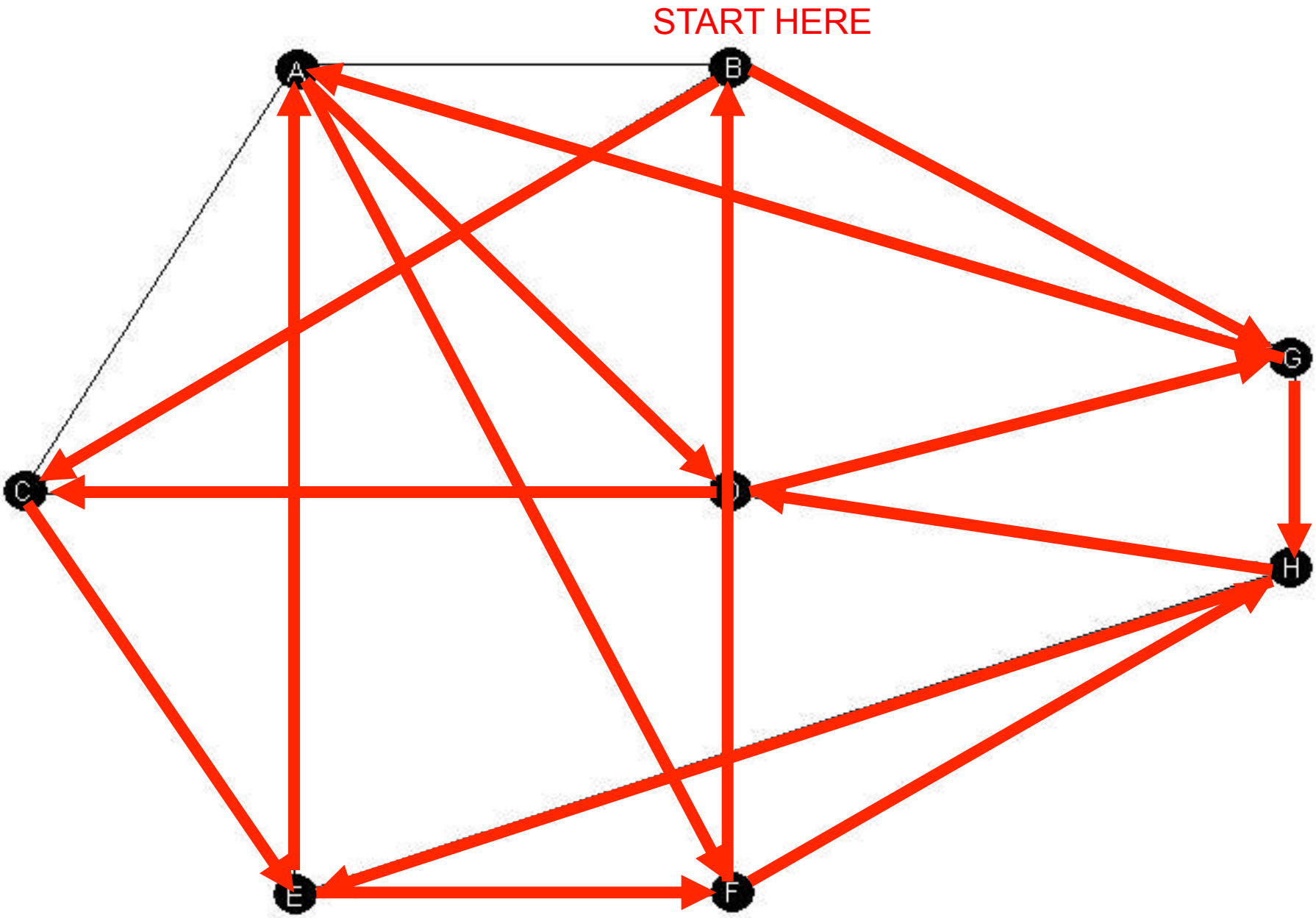
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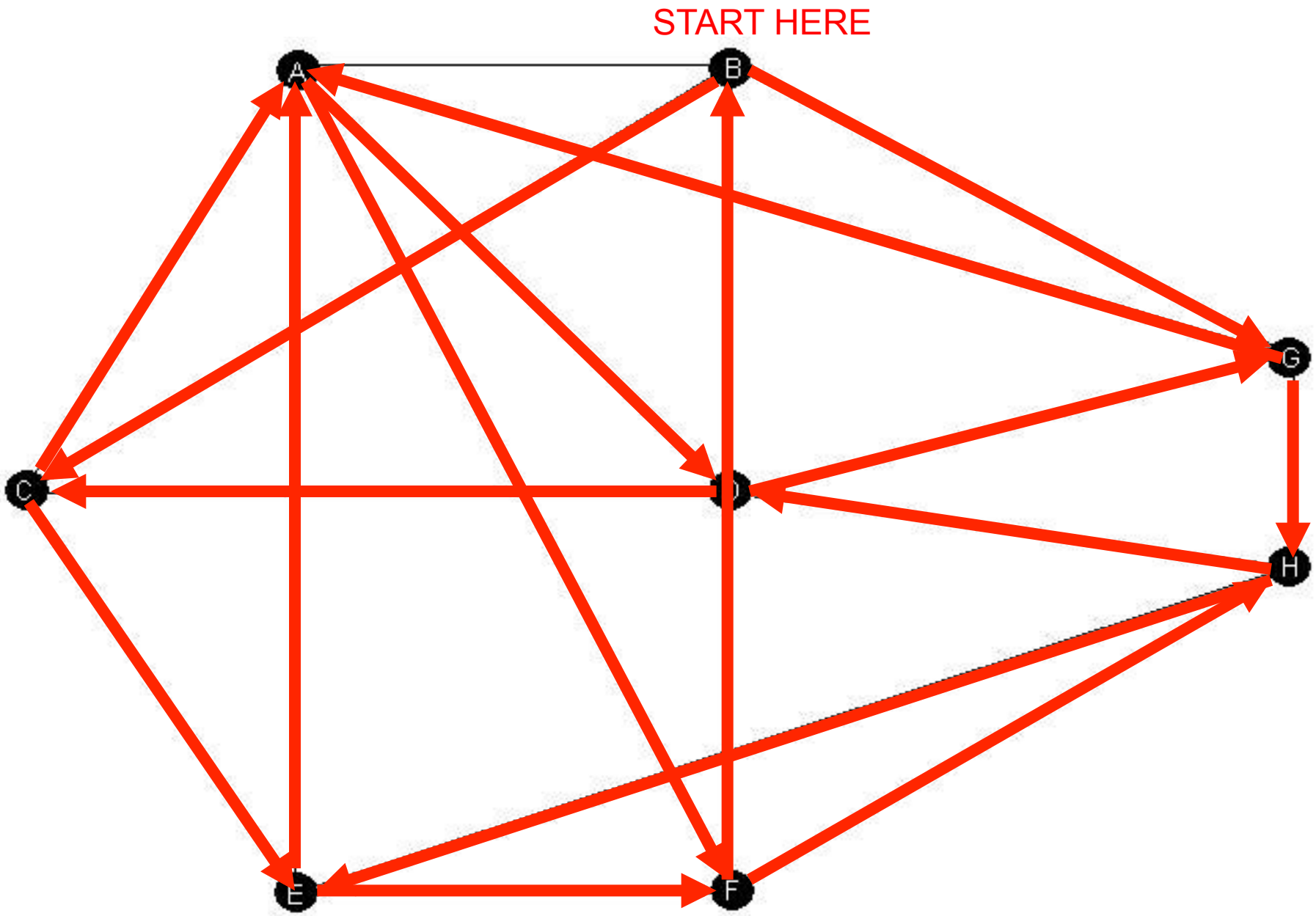
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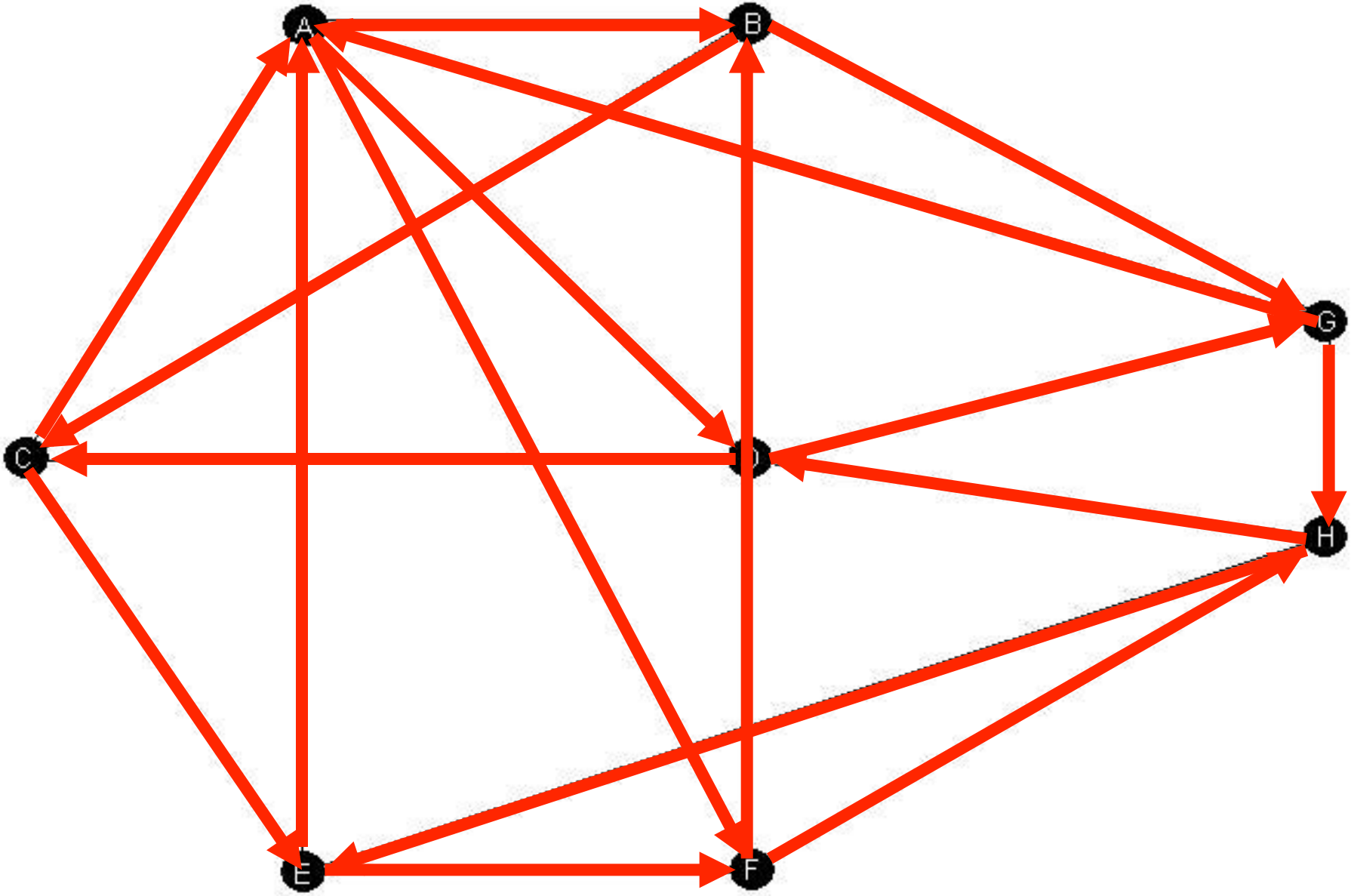

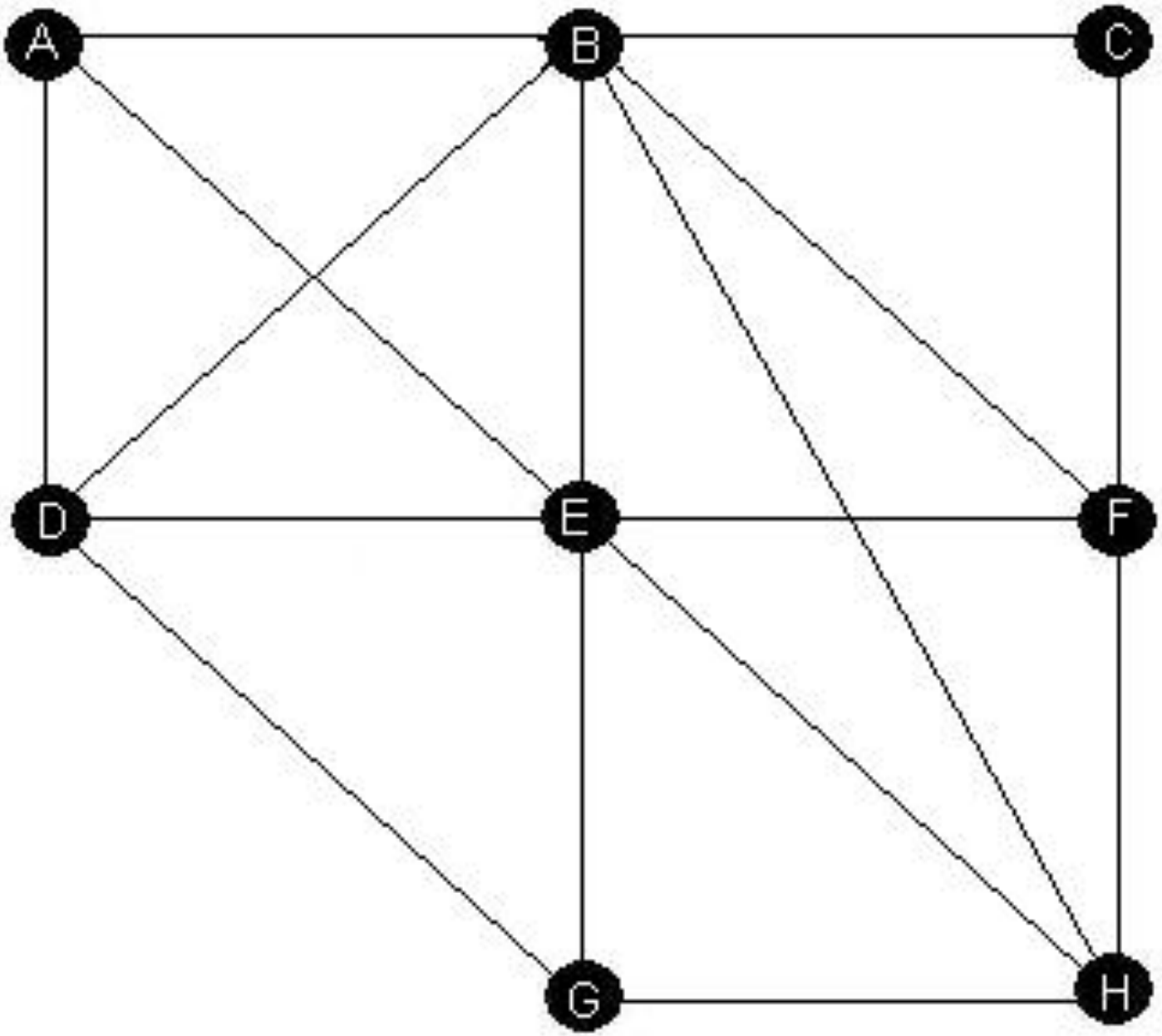
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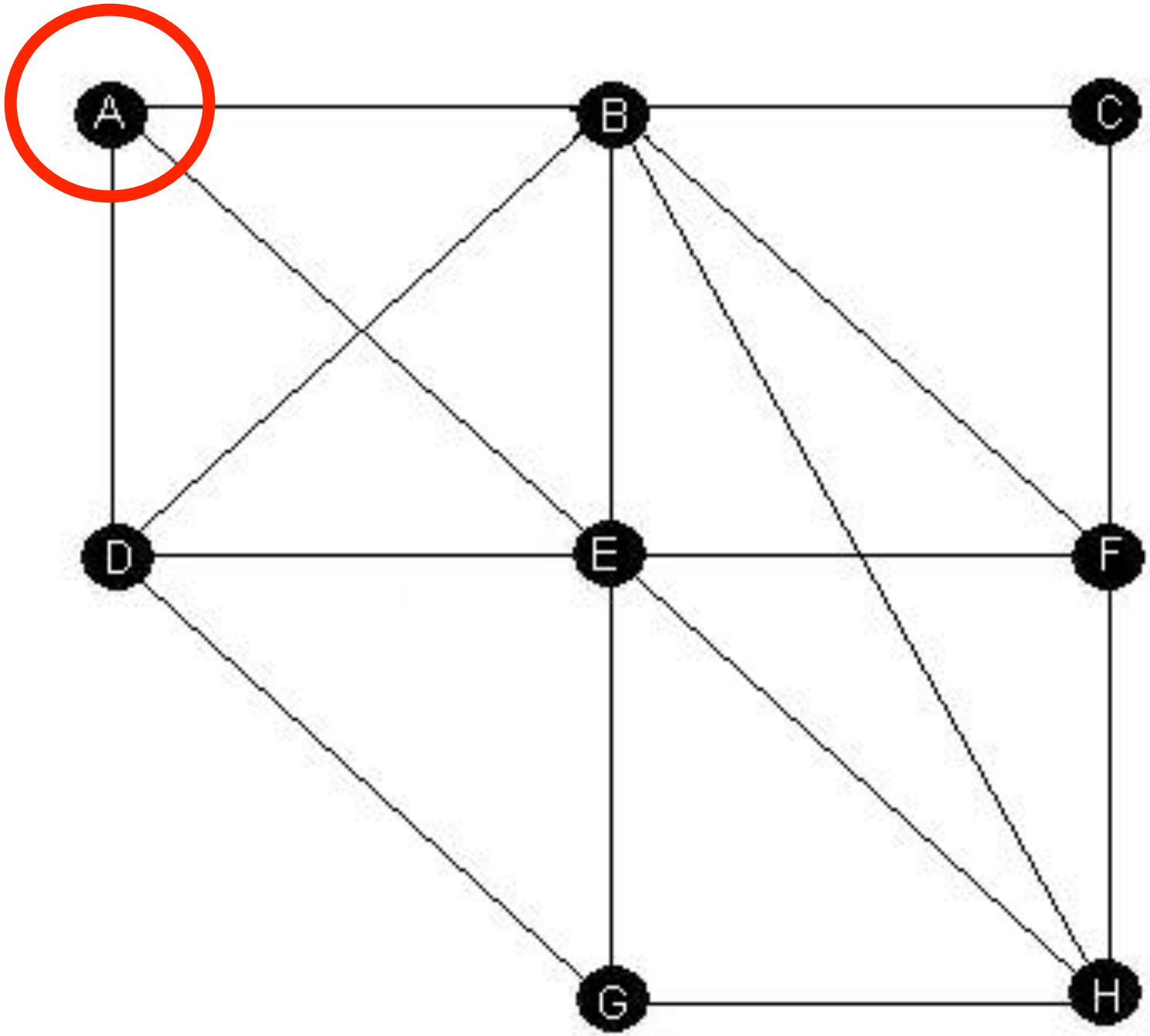
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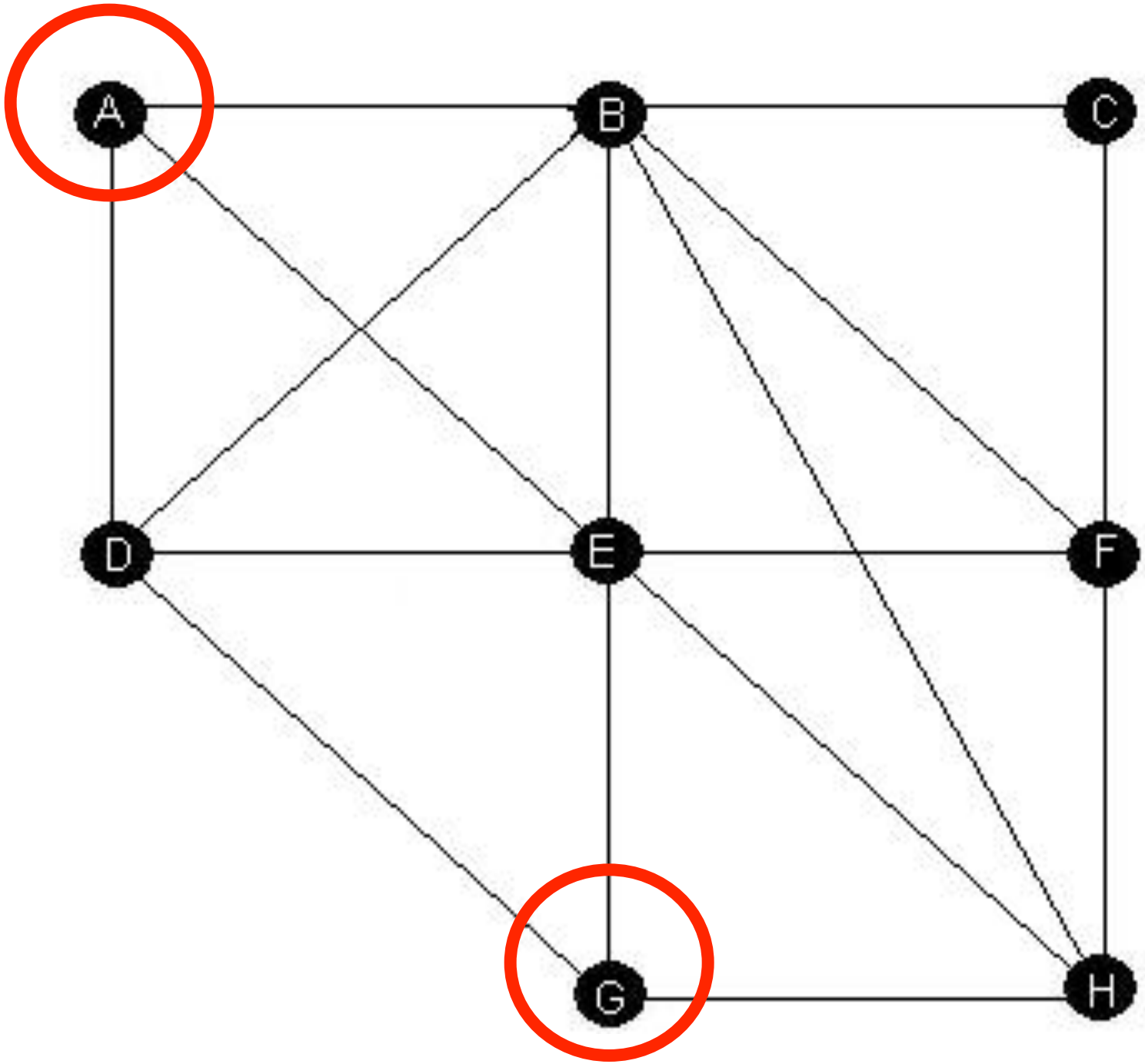
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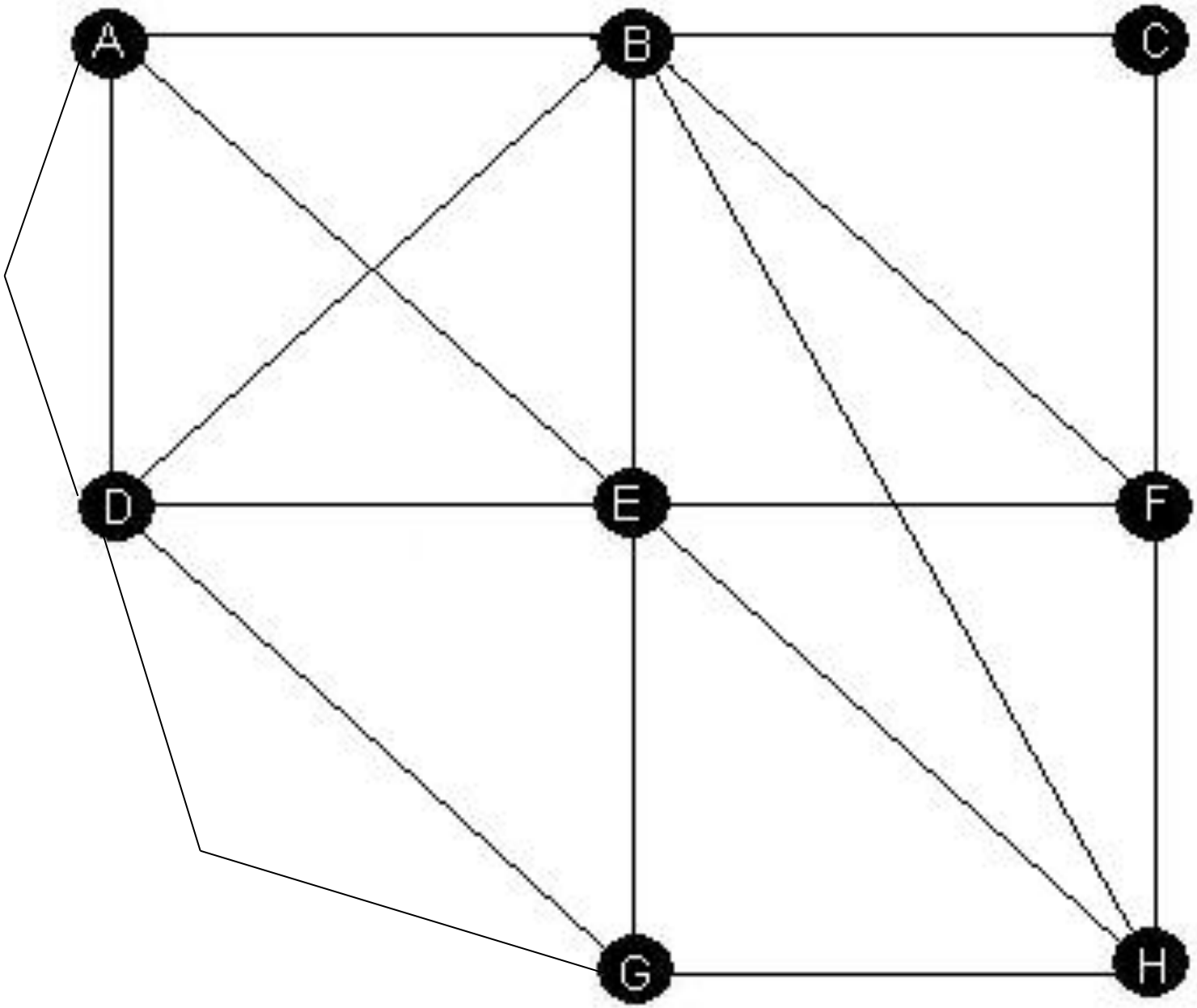
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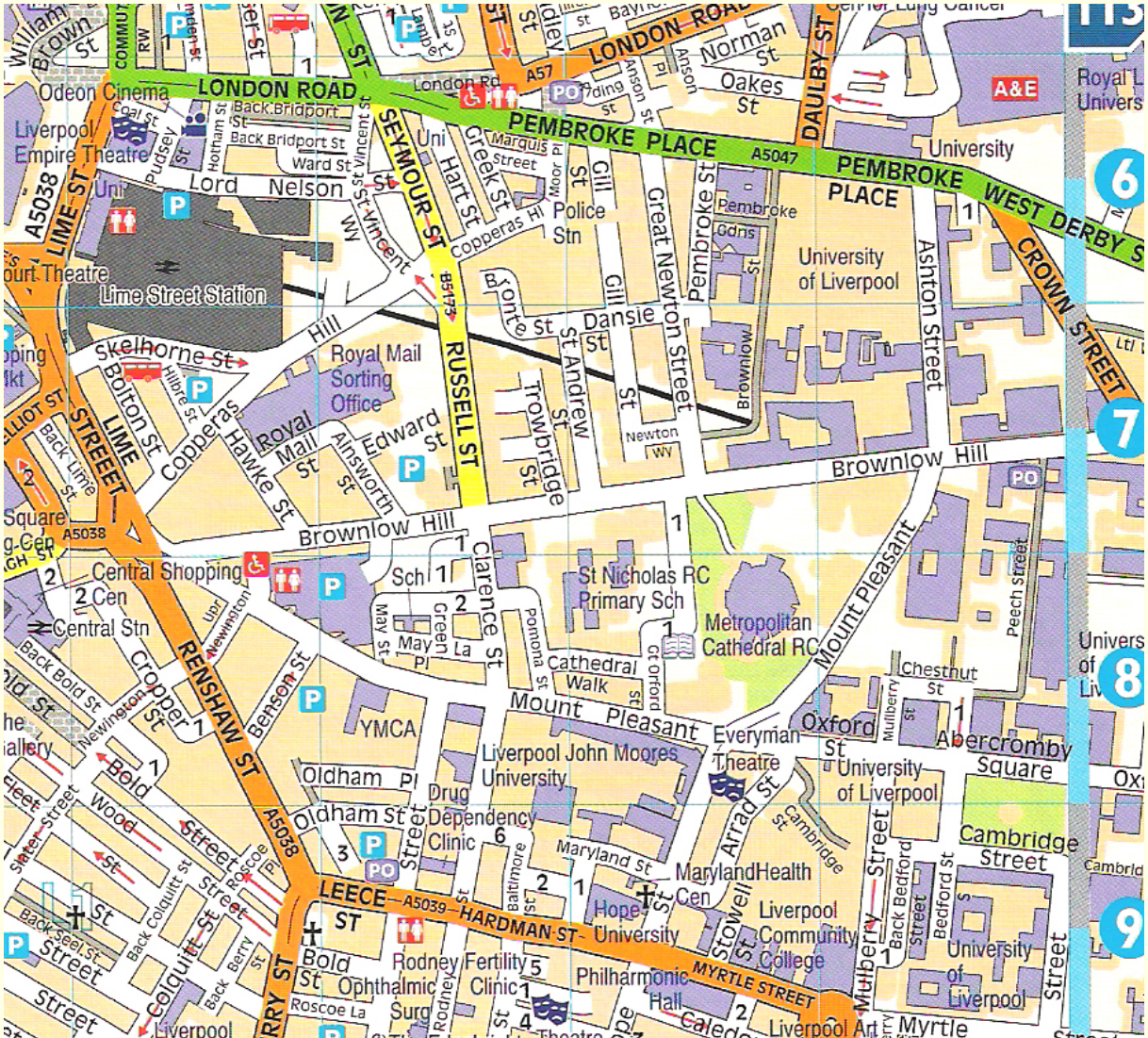
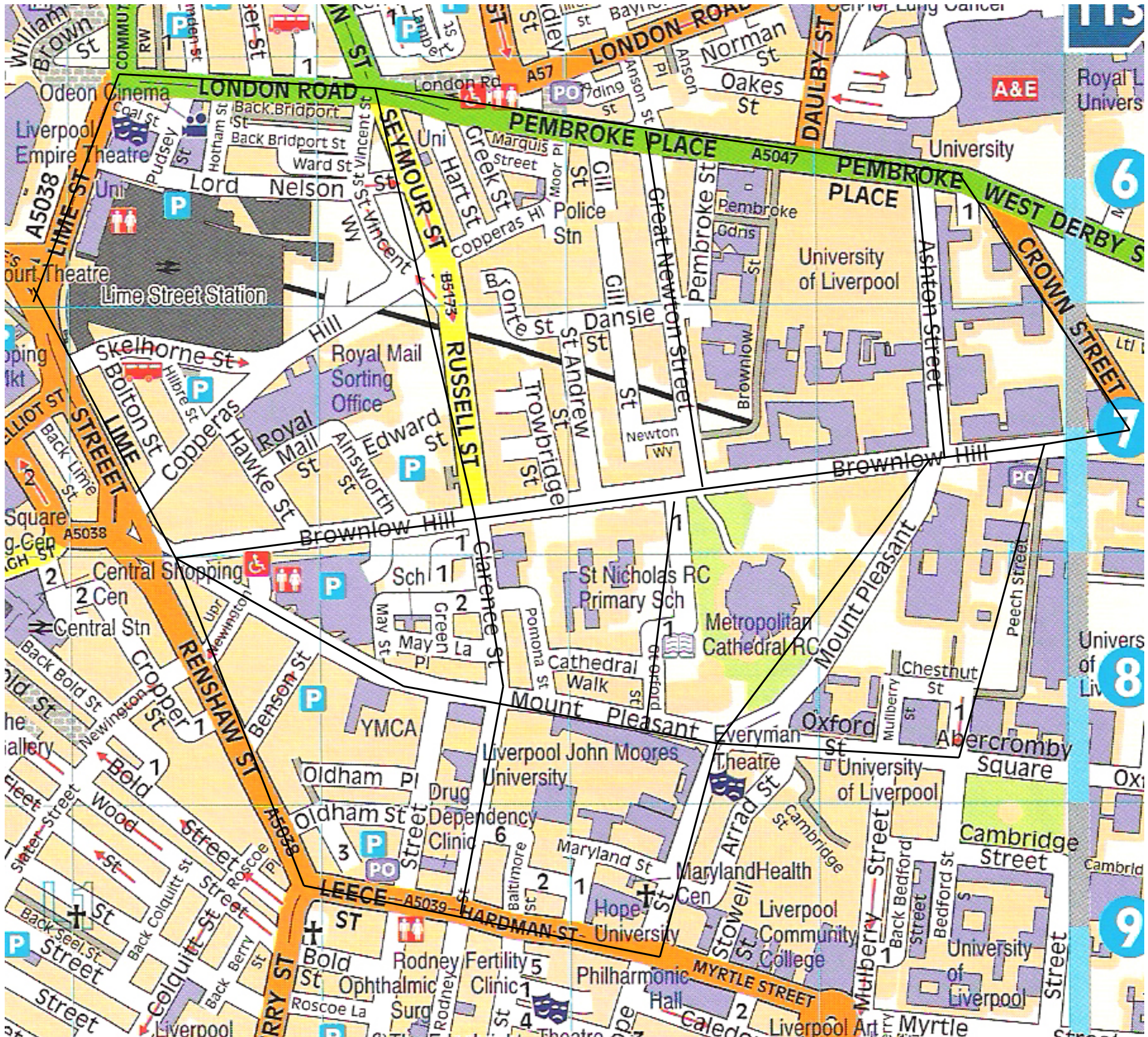
Any Practical Use?

- Honestly, there is!

In The Real World




LONDON ROAD
PEMBROKE PLACE
A5047
PEMBROKE PLACE
WEST DERBY ST
CROWN STREET
DAULBY ST
Norman St
Oakes St
A57
Odeon Cinema
Liverpool Empire Theatre
Lord Nelson St
Back Bridport St
Ward St
Coal St
Pudsey St
Hotham St
St Vincent St
St Vincent Wy
SEYMOUR ST
RUSSELL ST
B5173
Hart St
Greek St
Marquis Street
Copperas Hl
Police Stn
Gill St
Great Newton Street
Pembroke St
Pembroke Gdns
University of Liverpool
University
Ashton Street
Brownlow St
A&E
Royal Liverpool University
LIME ST
A5038
Lime Street Station
Court Theatre
Skelhorne St
Bolton St
Hilbre St
Copperas Hill
Hawke St
Royal Mail Sorting Office
Royal Mail St
Ainsworth St
Edward St
Bronte St
Dansie St
St Andrew St
Trowbridge St
Newton Wy
Brownlow Hill
LIME STREET
Back Lime St
ELLIOT ST
Square
A5038
Central Shopping Cen
Central Stn
RENSHAW ST
Cropper St
Newington
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Benson St
Sch
May St
May Pl
Green La
Clarence St
Pomona St
St Nicholas RC Primary Sch
Metropolitan Cathedral RC
Cathedral Walk
Mount Pleasant
Mount Pleasant
Oxford St
Chestnut St
Mulberry St
Peech Street
Abercromby Square
Everyman Theatre
YMCA
Oldham Pl
Oldham St
Drug Dependency Clinic
Liverpool John Moores University
Maryland St
Maryland Health Cen
Arrad St
Cambridge St
University of Liverpool
Cambridge Street
Hope University
Stowell St
Liverpool Community College
Mulberry Street
Back Bedford Street
Bedford St
University of Liverpool
LEECE ST
A5039
HARDMAN ST
MYRTLE STREET
Bold St
Back Bold St
Wood St
Bold Street
Colquitt St
Back Colquitt St
Roscoe St
Back Seel St
Seel Street
Slater Street
Roscoe La
Ophthalmic Surg
Rodney Fertility Clinic
Philharmonic Hall
Myrtle
Liverpool Art
Baltimore St
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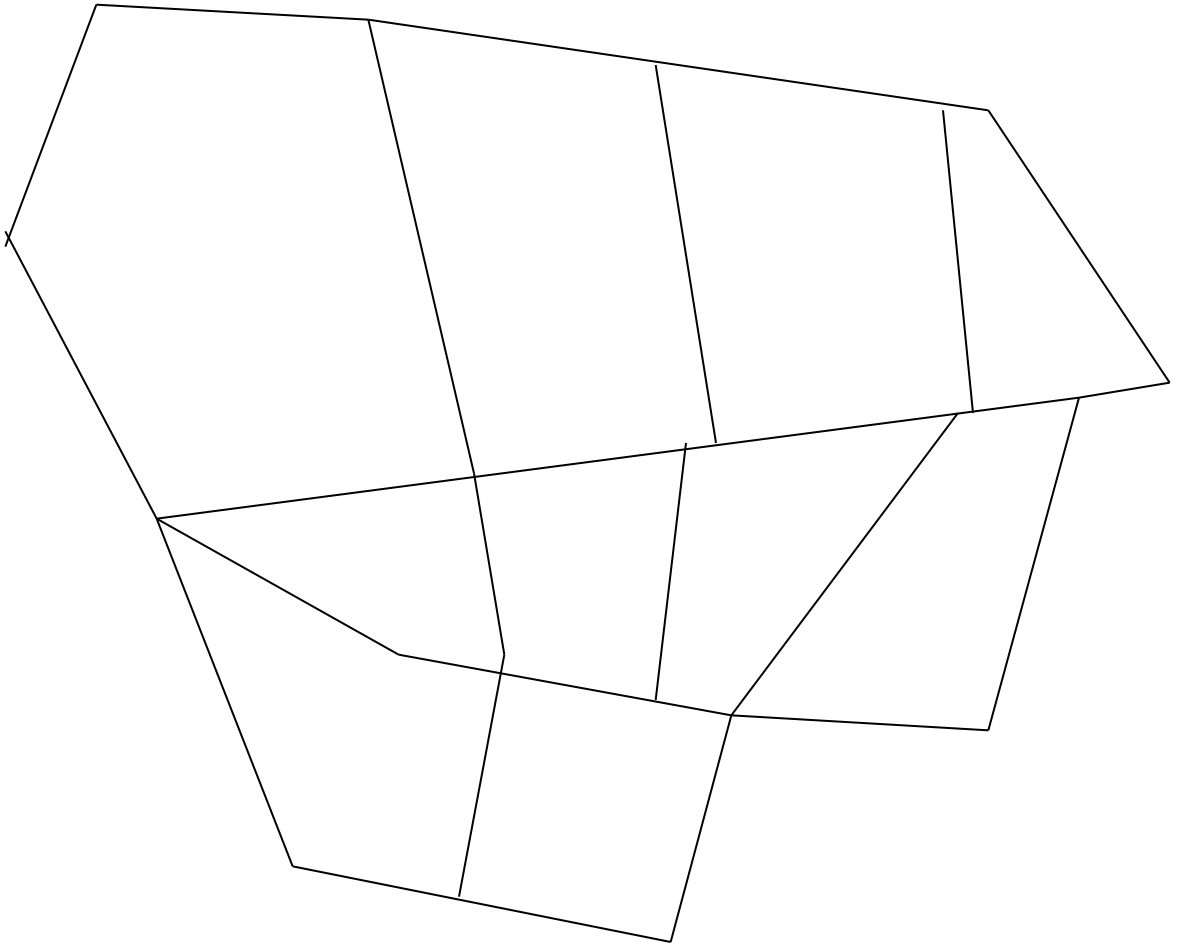

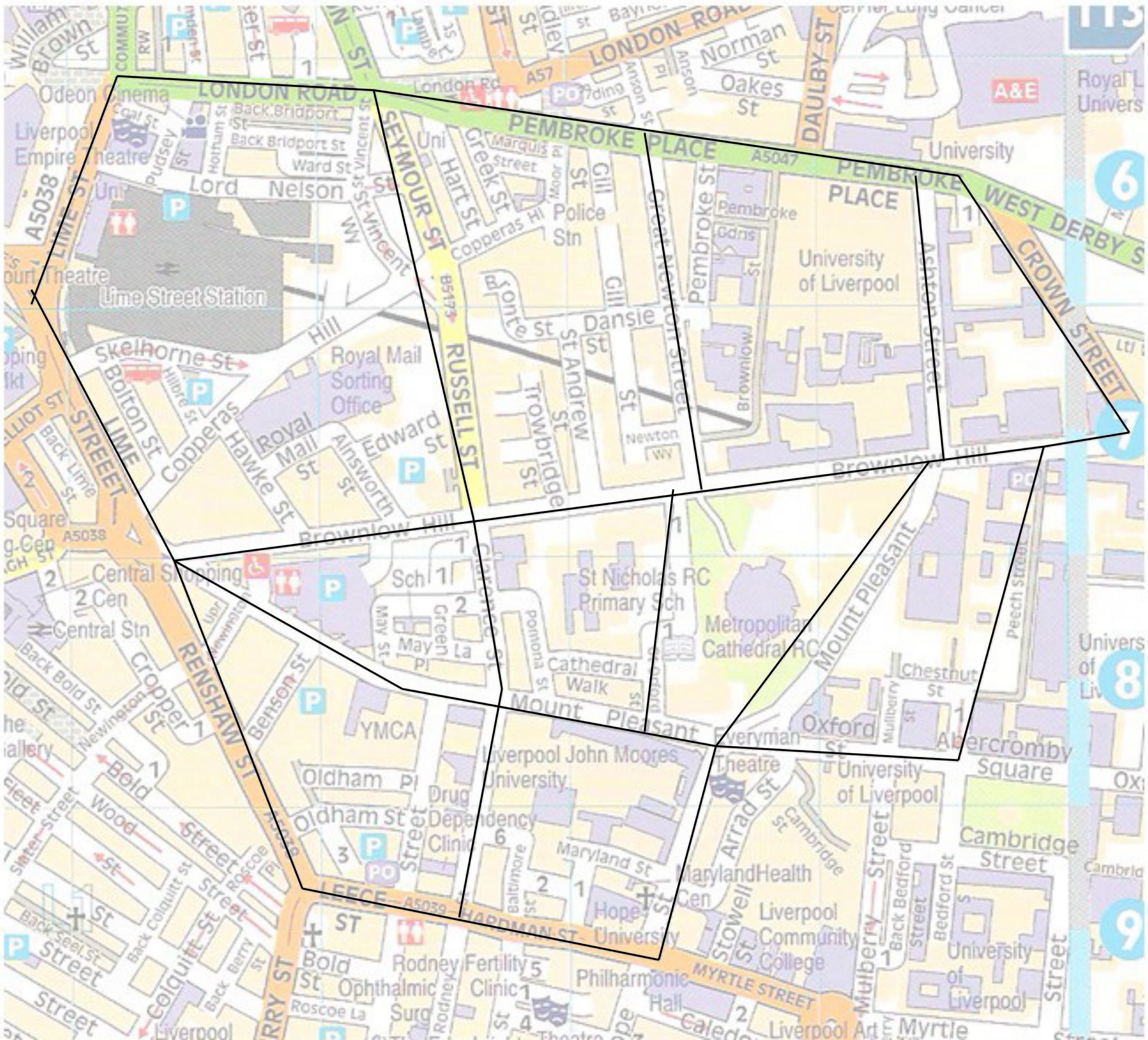
LONDON ROAD
PEMBROKE PLACE
PEMBROKE PLACE
WEST DERBY ST
CROWN STREET
DAULBY ST
Norman St
Oakes St
Odeon Cinema
Liverpool Empire Theatre
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SEYMOUR ST
RUSSELL ST
Hart St
Greek St
Police Stn
Bronte St
Dansie St
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Pembroke St
Pembroke Gdns
Brownlow St
University of Liverpool
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Central Stn
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Cropper St
Benson St
YMCA
St Nicholas RC Primary Sch
Metropolitan Cathedral RC
Mount Pleasant
Mount Pleasant
Cathedral Walk
Clarence St
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Everyman Theatre
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MYRTLE STREET
Philharmonic Hall
Rodney Fertility Clinic
Bold St
Ophthalmic Surg
University of Liverpool
Mulberry Street
Bedford St
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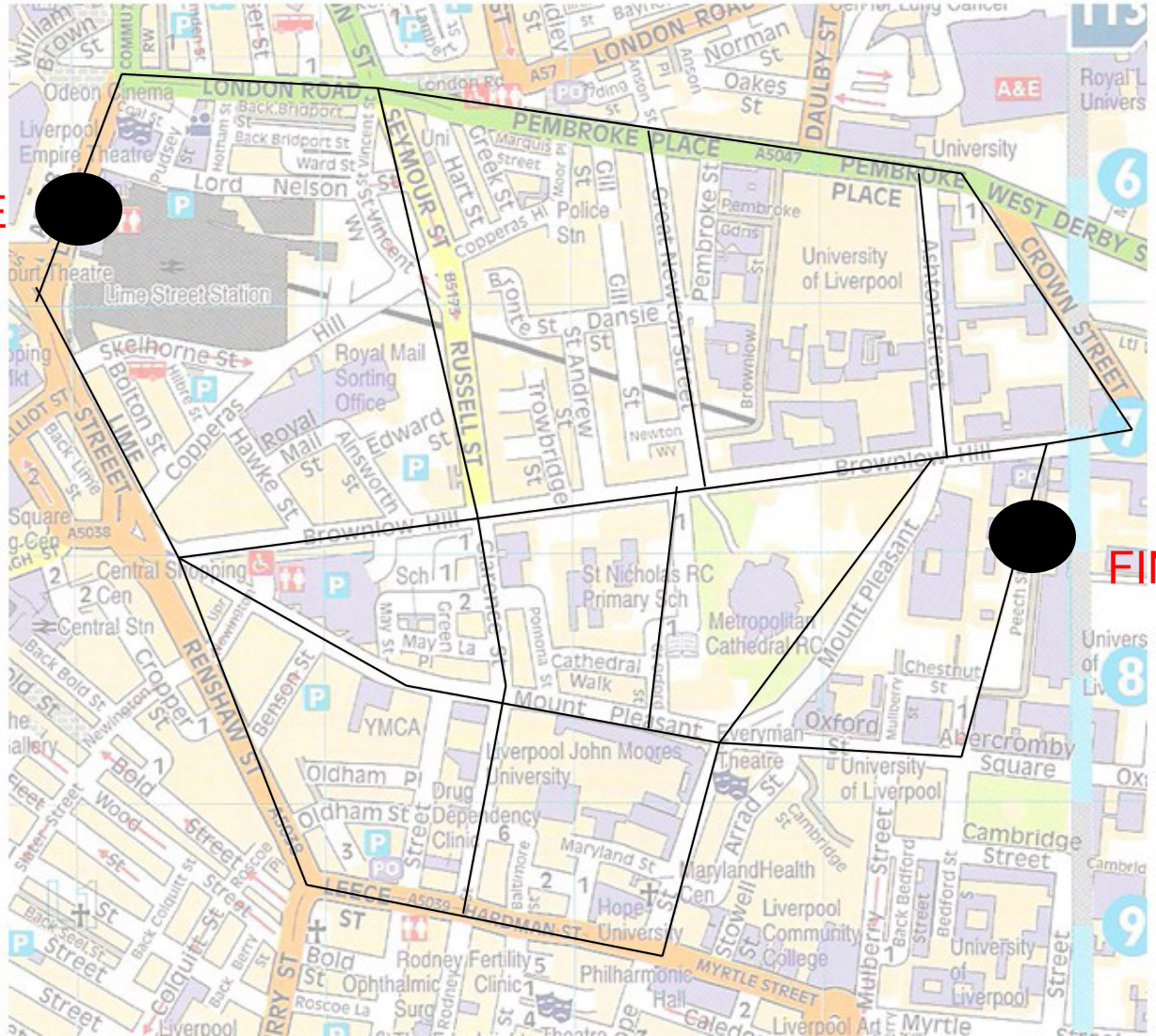
START HERE



FINISH HERE

Shortest Paths

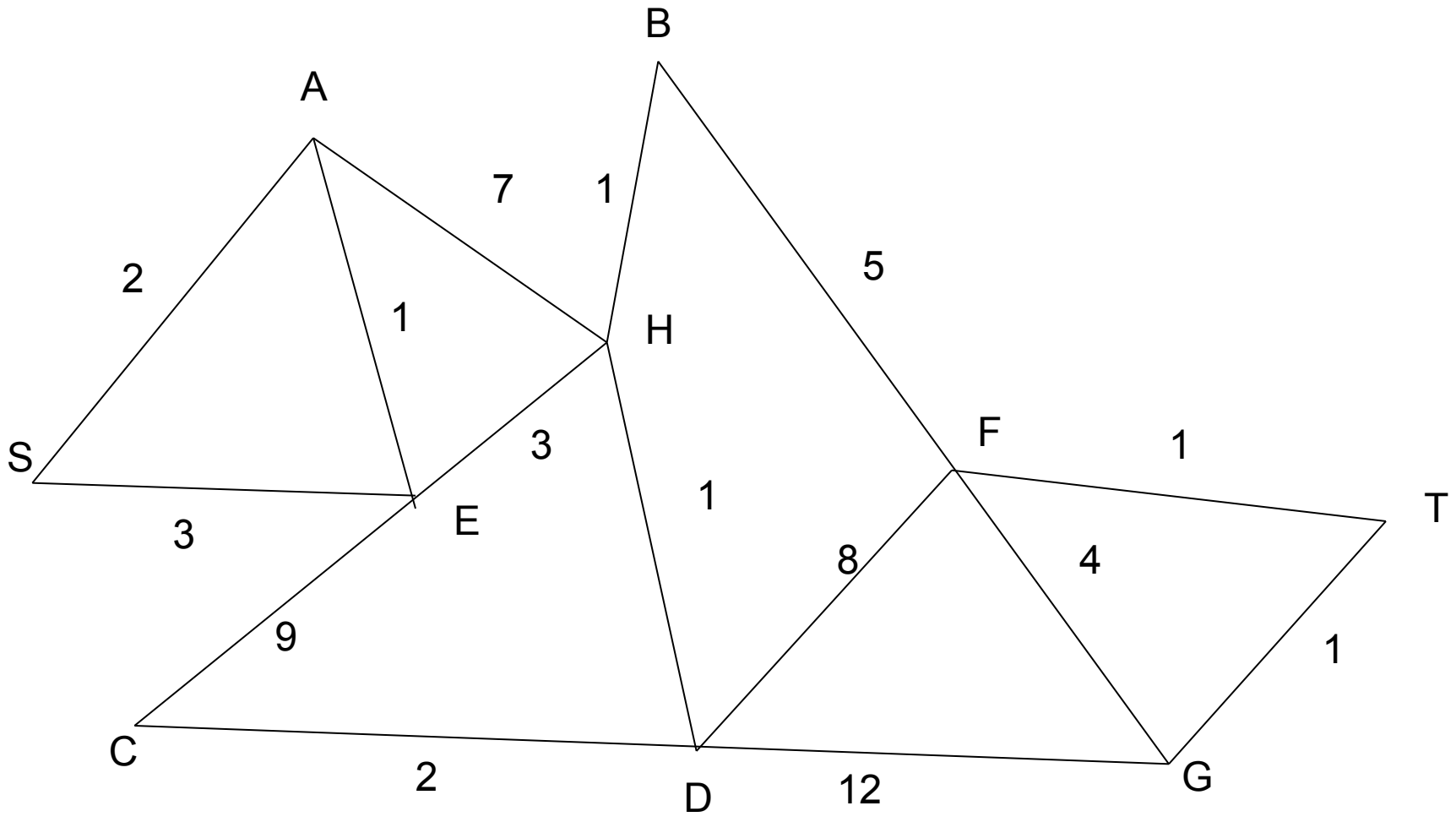
- Firstly, we need to assign some values to each edge.
- This will be the travel time, say, between the nodes i and j on the network.
- So an edge looking like this

A

Shortest Paths

- Firstly, we need to assign some values to each edge.
- This will be the travel time, say, between the nodes i and j on the network.
- So an edge looking like this

- Means that it takes 3 minutes (or 3 measures of time if you like) to move between A and B in either direction.

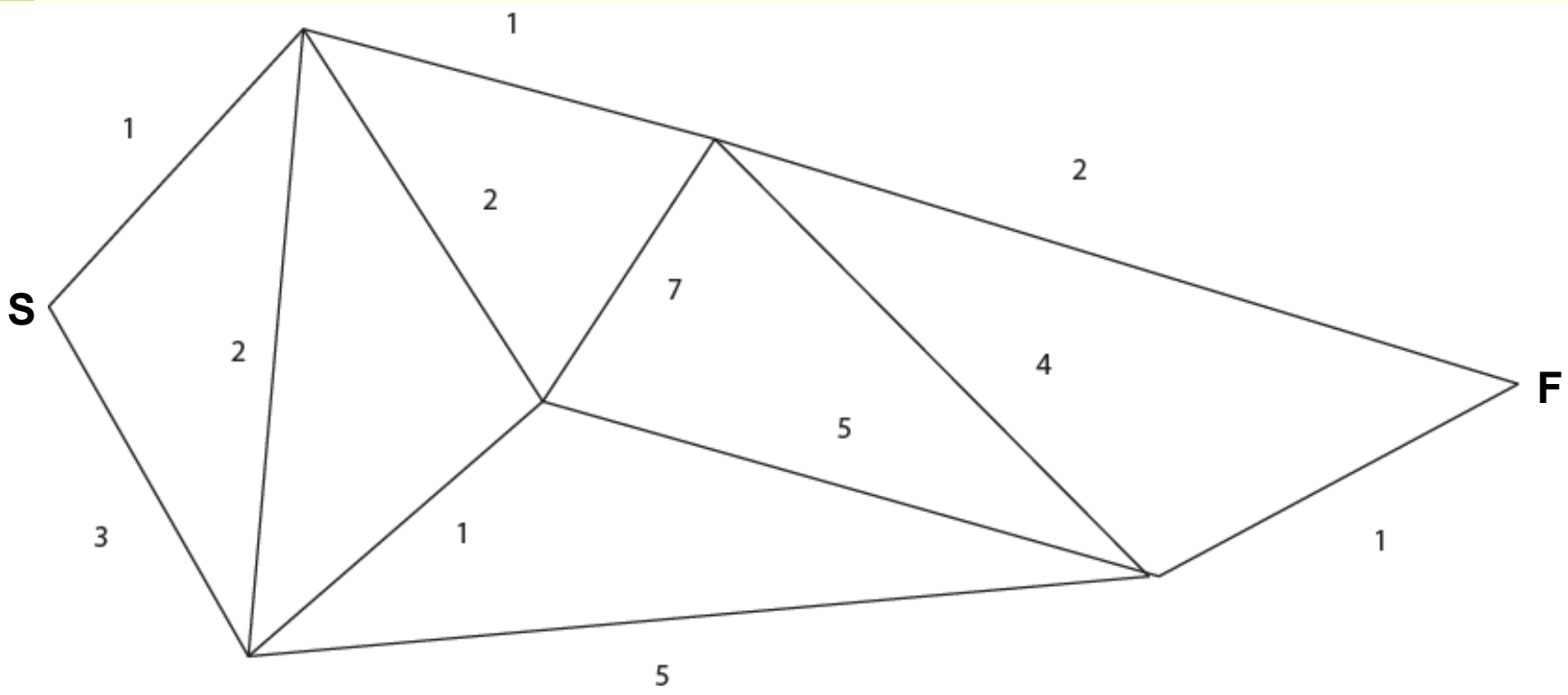

B
A
7
1
2
5
1
H
S
3
F
1
3
1
T
E
8
4
9
1
C
2
D
12
G

Shortest Paths

- How do we know that we found the shortest path?
- Dijkstra's Method

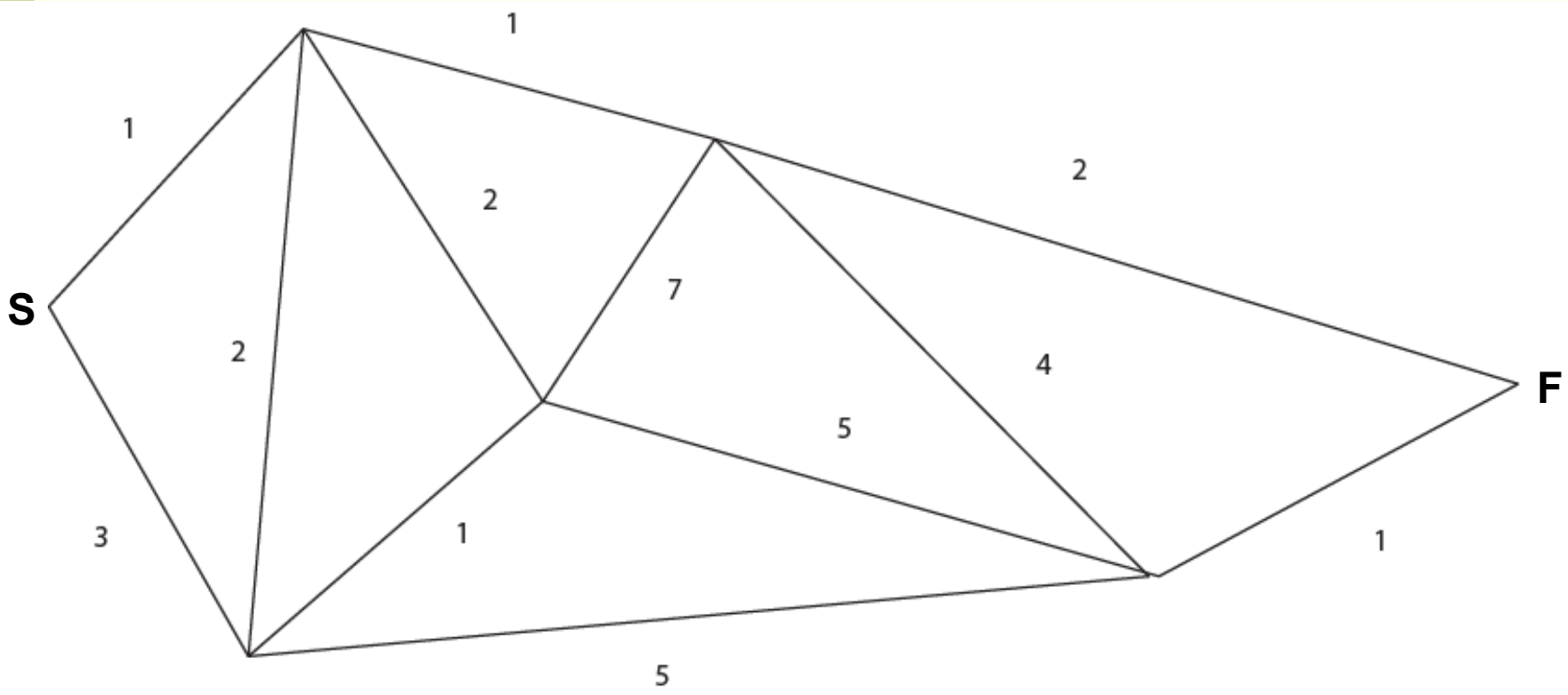
Dijkstra

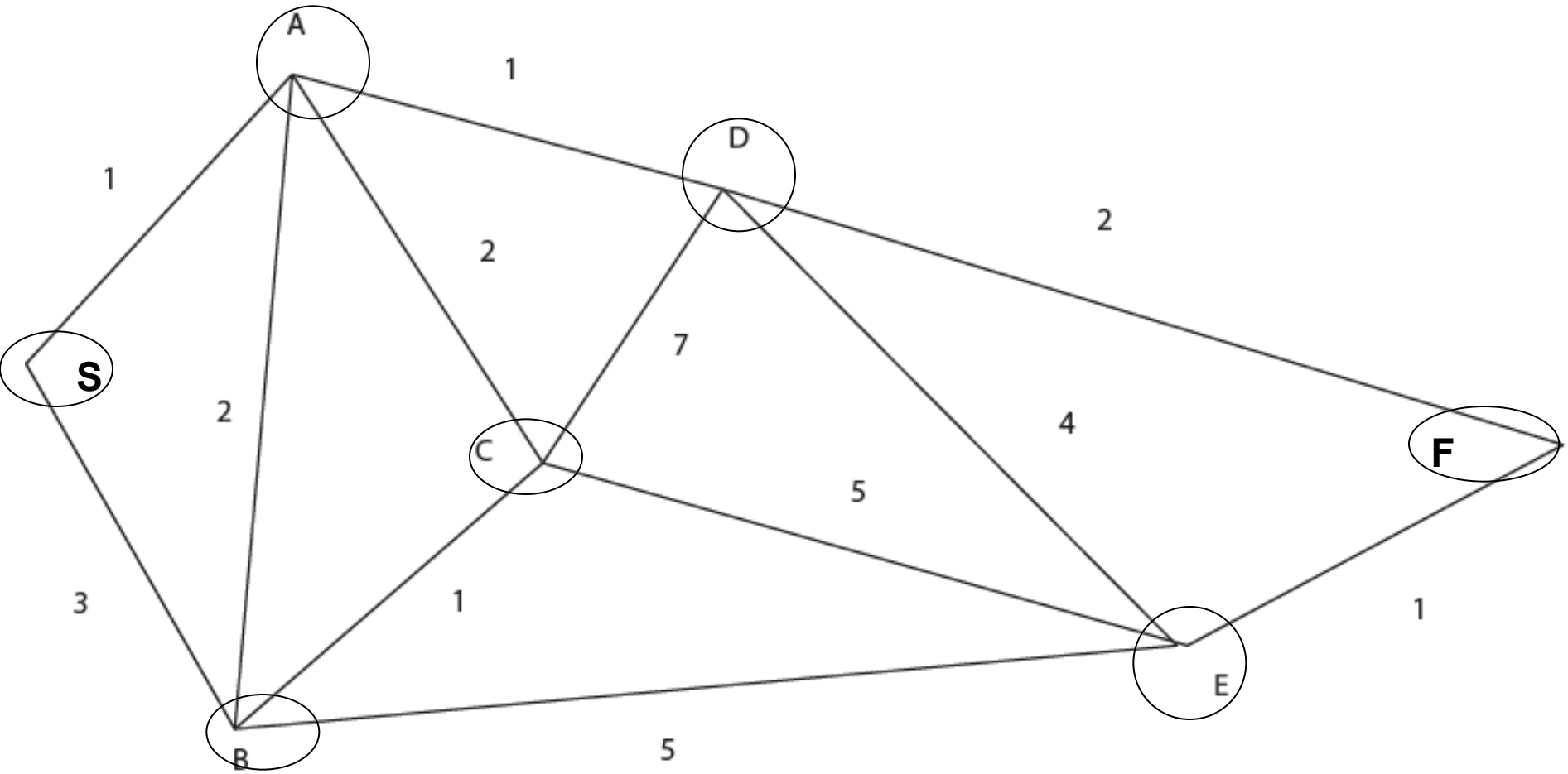
1. Label the starting node with ‘S’
2. Label the finishing node with ‘F’



Dijkstra

3. Label the remaining nodes.
4. Fill in the table





Sheet_4 (Remind error)

And Finally…

- Networks all around us.
- Maps are Networks, which we use every day.