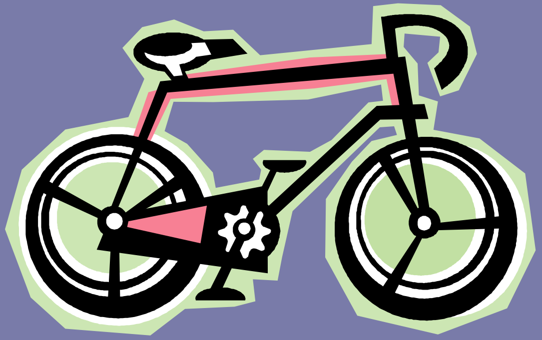
Round and Round We Go!

Universal Cycles



First Some Magic

Colours

- There are five cards in a row, each of which is either red or black. How many sequences of red and black are possible? (E.g. RRBRB)
- How can we tell what five cards were chosen?

It’s all about Universal Cycles!

- Suppose we have two binary digits, 0 or 1

Here are the four possibilities:

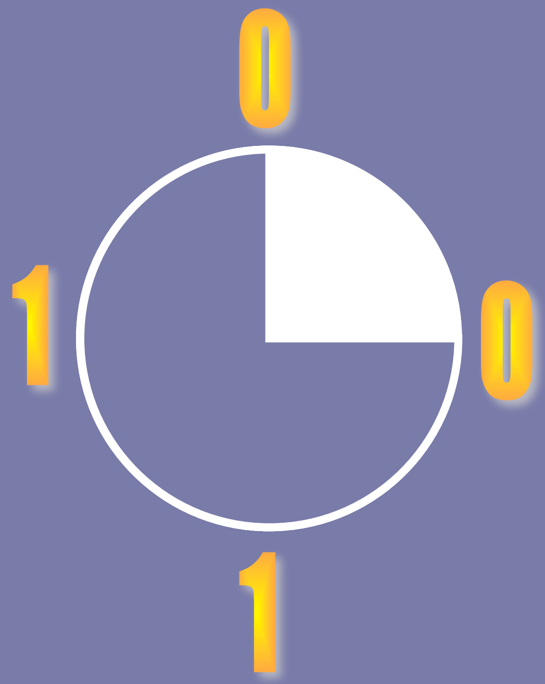
- 00, 01, 10, 11

Round the Circle

- By placing the digits around a circle, we can sweep out these combinations
- 00
- 01
- 11
- 10

All these are different!

- We can write the circle as 0011
- (or 0110 or 1100 or 1001)

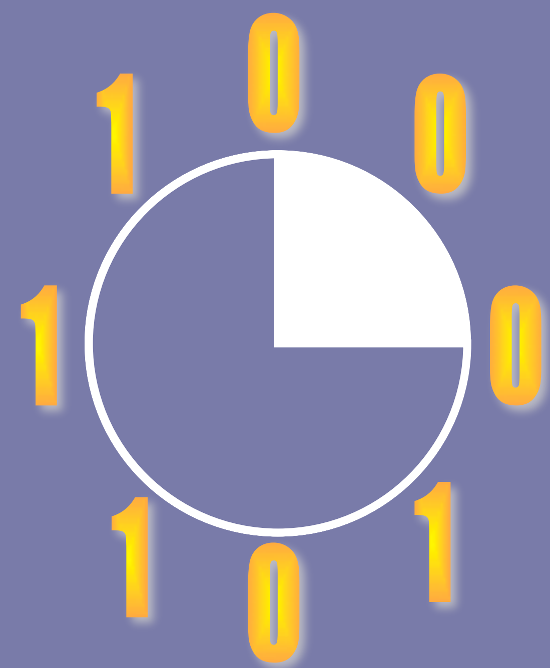


Universal Cycles

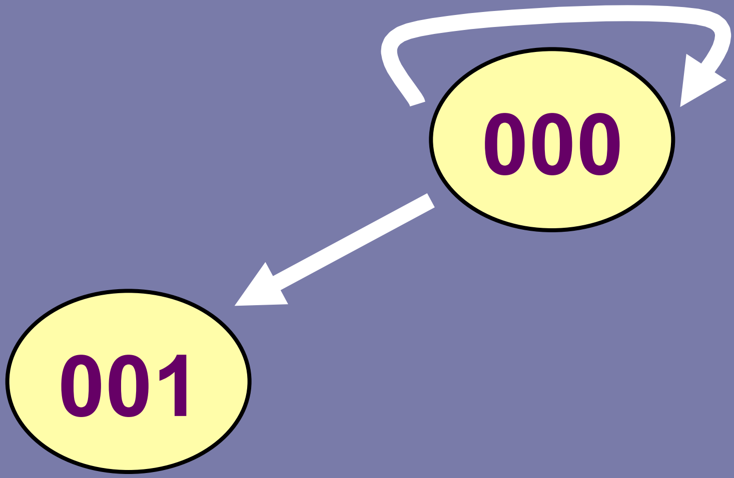
- Now we take three binary digits
- The possible combinations are:
- 000, 001, 010, 011,100, 101, 110, 111
- Can you put eight 0’s and 1’s round a circle so that each of these combinations occurs exactly once?
- Can you find more than one solution?

Round the Circle

- Again we can put 0's and 1's around a circle and sweep out the combinations
- 000
- 001
- 010
- 101
- 011
- 111
- 110
- 100



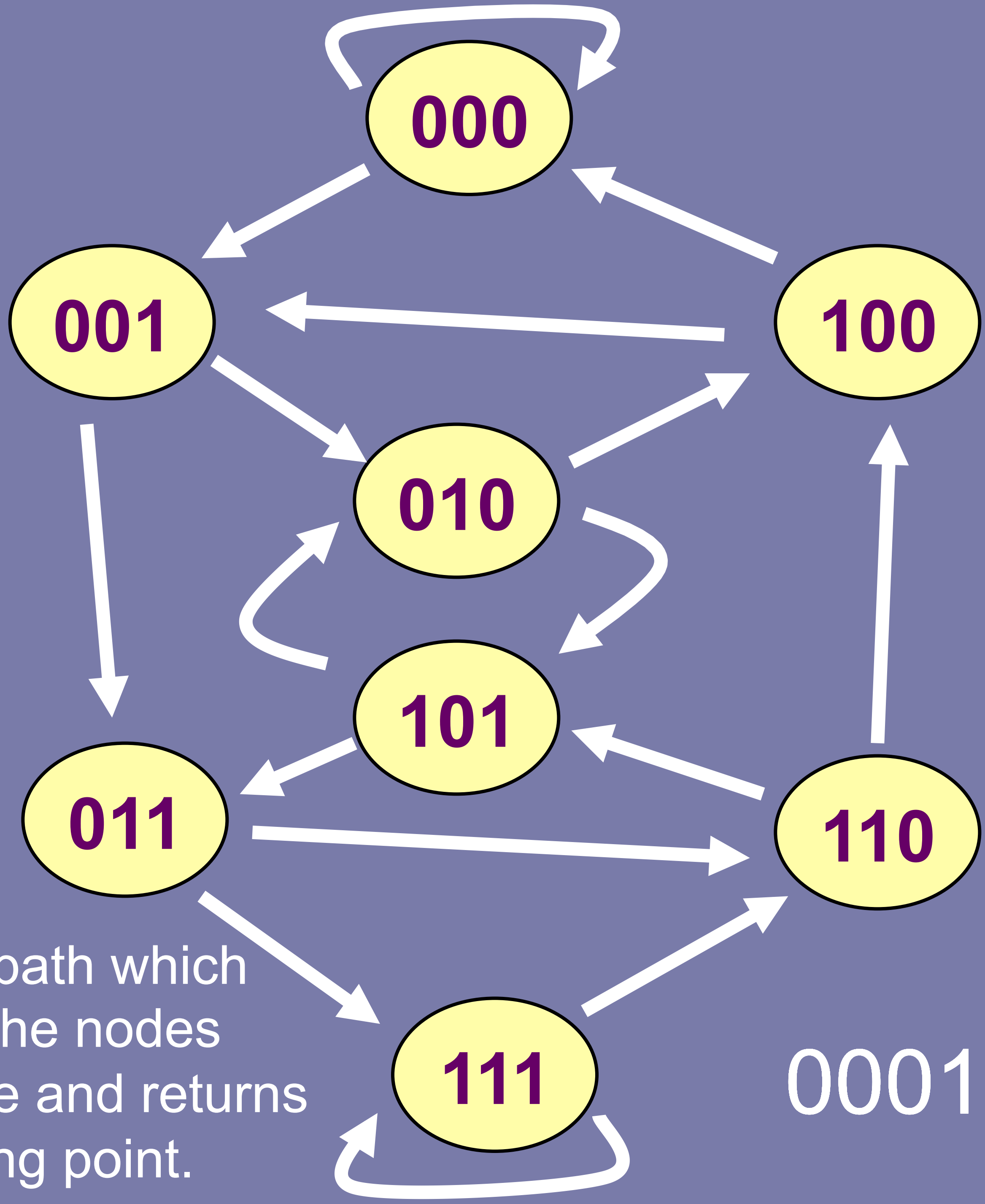
Again, we can write the circle as a string of digits, 00010111



- This is part of the diagram on your sheet.
- Add an arrow from one node to another if the last two digits in the first node are the same as the first two in the second node.

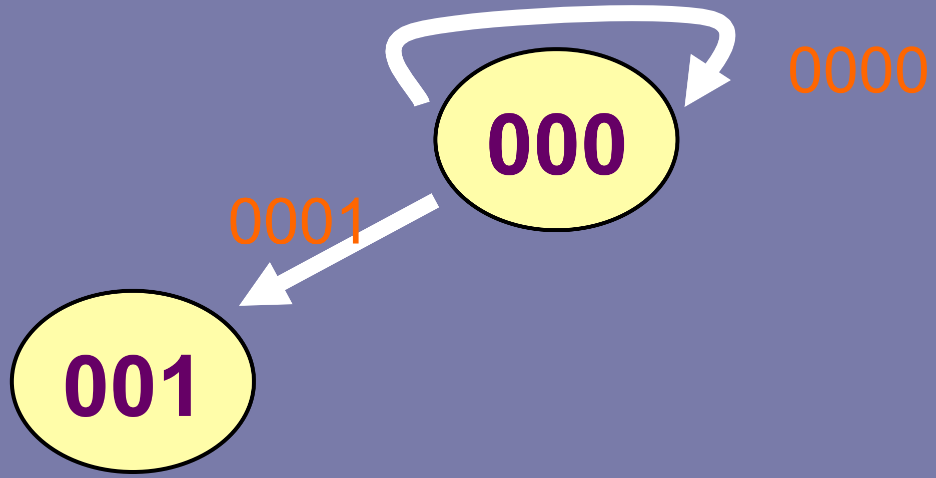
$$0\underline{00} \rightarrow \underline{00}1$$

$$0\underline{00} \rightarrow \underline{00}0$$



Now find a path which visits all of the nodes exactly once and returns to the starting point.

00010111

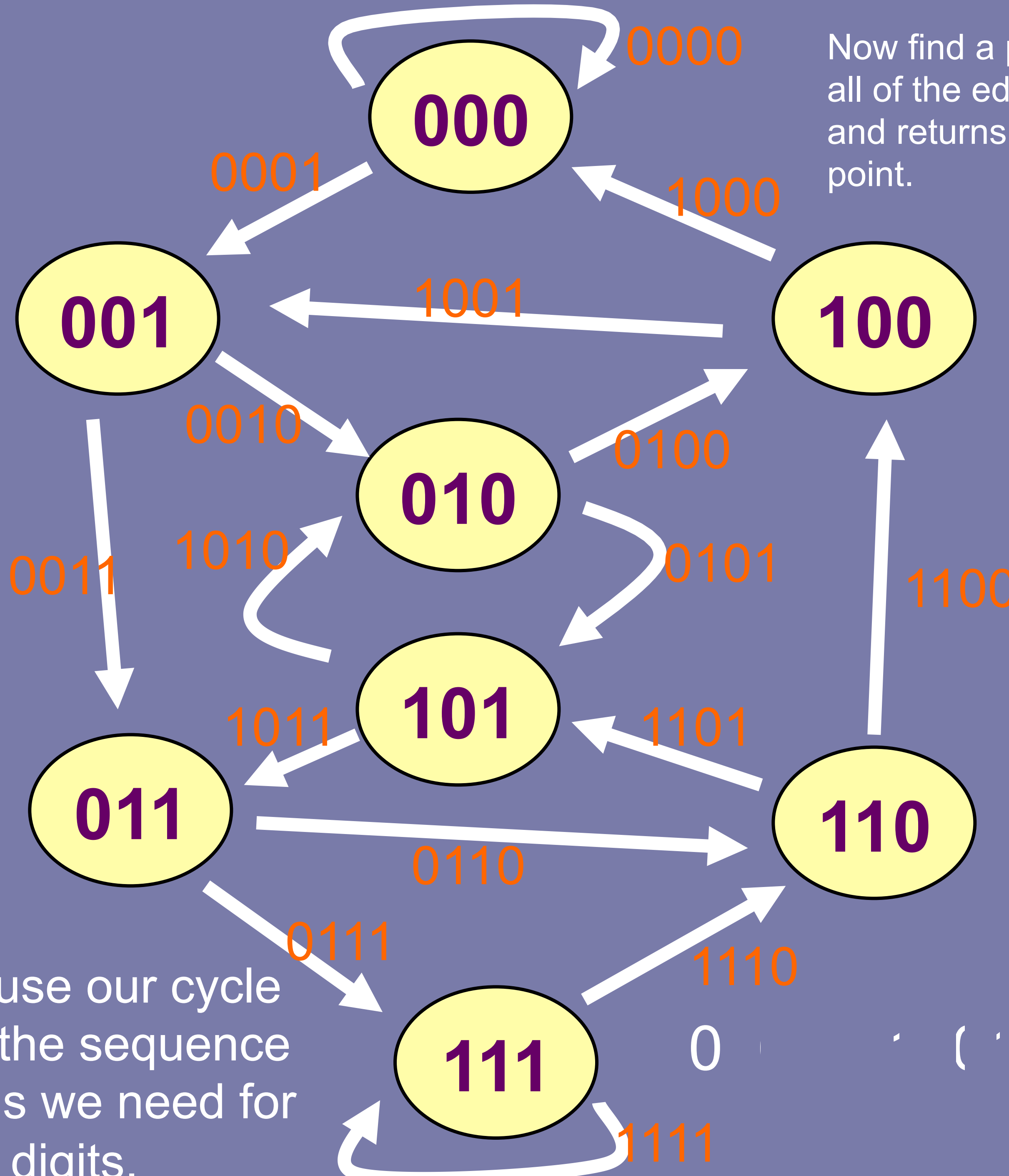


- This is part of the diagram on your sheet.
- Label each arrow with the three digits from the first node and the last digit from the second node.

$\underline{000} \rightarrow 00\underline{1}$ becomes 0001

$\underline{000} \rightarrow 00\underline{0}$ becomes 0000

Now find a path which uses all of the edges exactly once and returns to the starting point.



We can now use our cycle to determine the sequence of 0's and 1's we need for FOUR binary digits.

Paths which go over every arrow exactly once are called Euler paths.

These always exist provided there are the same number of arrows going into every node as there are arrows coming out of it. (Two in, two out in our example.)

But this is a hard thing to prove. Try typing Euler path or Eulerian path or Eulerian circuit into Google and see what you come up with.

0000111101100101

contains all the sequences of four binary digits exactly once, remembering to go ‘round the corner’ at the end

so if we have 16 cards, say the ace,2,3,4 of each suit, arranged so that the red = 0 and black = 1 cards are in the above order

for example

1♦ 2♥ 3♦ 4♥ 1♣ 2♠ 3♣ 4♠ 2♦ 2♣ 1♠ 1♥ 4♦ 4♣ 3♥ 3♠

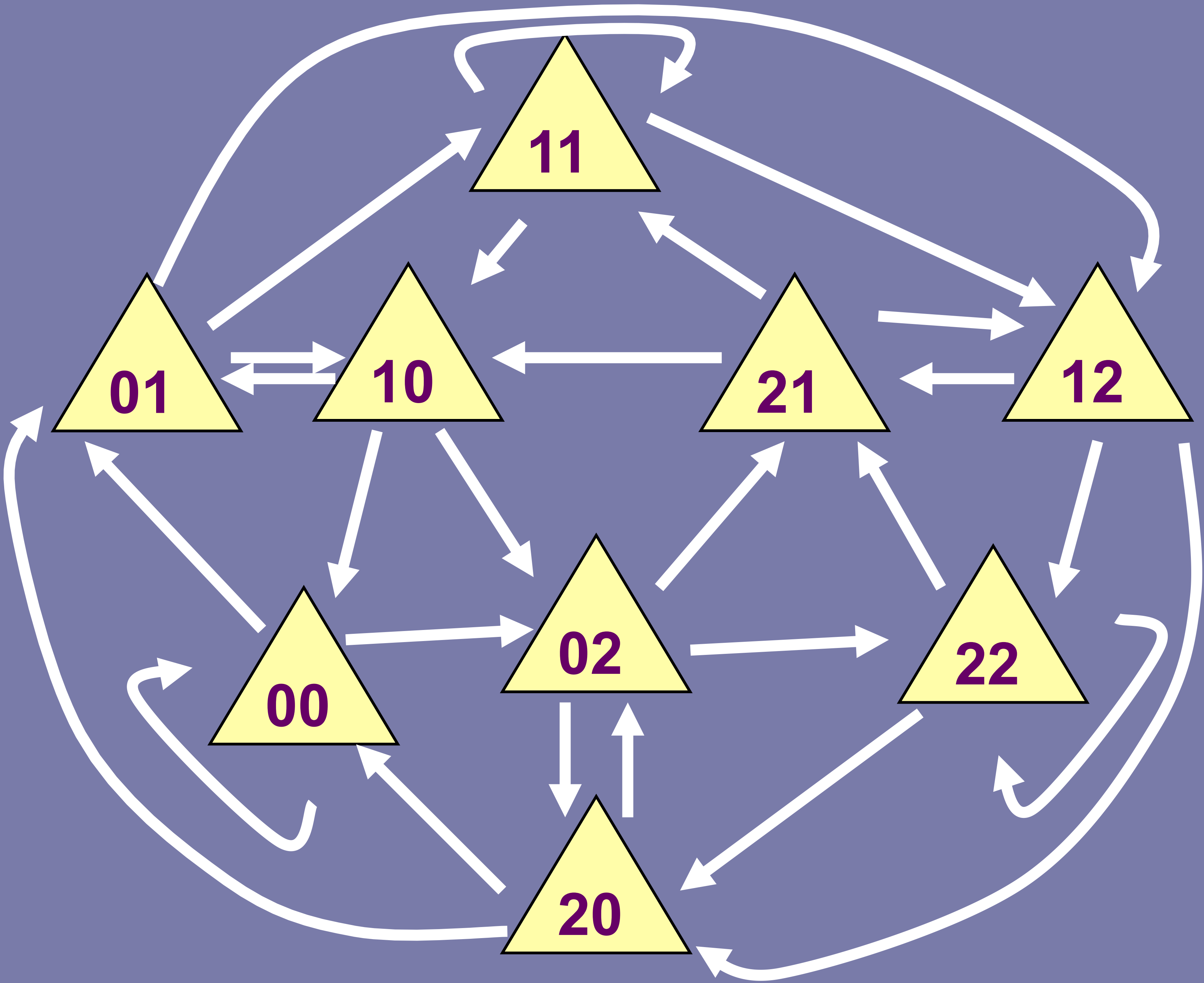
then even after cutting we can tell which four cards in a row are chosen if we know the sequence of red, black

Now maybe you can use the same method (NOT guesswork!!!) to find the universal cycle for sequences of FIVE binary digits, or 32 cards

This time there are 4096 different solutions!

Ternary

- Suppose now we have a system with three possible values: 0, 1 and 2.
- If k=2
- The possible combinations are:
- 00, 01, 02, 10, 11, 12, 20, 21, 22
- Using the network provided, can you find a cycle of 27 digits to cover all the combinations for k=3 in a ternary system?
- Email your solutions to c.j.marchant@liv.ac.uk or pjgiblin@liv.ac.uk


11
01
10
21
12
00
02
22
20