Maths Club 26th February 2005

EXPECTED VALUES

Session Commentary

Copies of the documentation as presented to the participants are reproduced here.

At the outset, each participant was provided with a set of session notes and a Data Record sheet.

Participants, working in pairs or singly, generated their personal data sets by spinning a 2p coin 50 times and recording the outcomes, H or T, on their own Data Record sheets. A partly completed Data Record sheet was displayed on a room screen for their guidance.

On their Data Record sheets the participants noted a sequence of proportions of cumulative heads obtained and the lengths of runs (of heads or tails) observed. A completed Data Record sheet was displayed on a room screen for their guidance.

The intuitive concept of an expected value was established in the context of the sequence of proportions of cumulative heads obtained, here ½. Participants calculated their average run lengths and speculated that the expected value of a run length would be 2.

A brief introduction to the notion of discrete probability distributions was followed by the formal definition of an expected value.

A set of exercises served to consolidate participants' understanding of this formal definition.

The calculation of the expected value of the run lengths was flagged as the motivation for the derivation of some standard series summation results. Aspects of binomial and negative binomial expansions were reviewed and the relevant interpretations of Pascal's triangle highlighted.

The appropriate probability distribution for run length was derived. The sum of the probabilities was verified as 1 and the corresponding expected value was confirmed to be 2.

As a light-hearted finale. Participants were invited to spin repeatedly their 2p coin, scoring +1 for a head and -1 for a tail and, at each spin, and at each spin, calculating their cumulative sum. The number of spins taken to the cumulative sum's first return to zero was to be recorded.

After a very few minutes, several returns to zero were noted but several participants had cumulative sums that were drifting away from zero.

At this point the presenter suggested that it might make a party game for the participants friends over the forthcoming Christmas holiday period, revealing that the expected number of spins required to the first return to zero was "infinite". The participants appeared to appreciate, and enjoy, the implications of this statement in the given context.