"Probability and the Law", by Grahame Settle
A presentation to Mathematics Club, 24th September 2005

Introduction

Sally Clark case: Sally Clark, a Cheshire solicitor, lost her first baby in 1996. The death was recorded as natural, a "cot death". In 1998, her second child died at 8 weeks old. Mrs Clark was charged with the murder of both babies. A key witness at her trial in November 1999 was a distinguished British consultant specialising in the treatment of babies, Professor Sir Roy Meadow. He had previously said that 'one cot death is a tragedy, two cot deaths is suspicious, and, until the contrary is proved, three cot deaths is murder.' This statement became known as "Meadow's Law".

Mrs Clark appealed unsuccessfully in October 2000, despite challenge to the statistical arguments used in the 1999 trial. A second appeal in January 2003 succeeded, but this was on the basis of new medical evidence, so new statistical evidence was not heard. Nevertheless, the judgment stated that if the new statistical evidence had been needed, it would probably have also been sufficient. Meadow's evidence at the original trial was described by this new judgment as "manifestly wrong" and "grossly misleading".

BBC News (Health) website 15 July 2005: "The General Medical Council has struck off … Professor Sir Roy Meadow after his "misleading" evidence in the Sally Clark case. The GMC announced on Friday that Meadow had been found guilty of serious professional misconduct. Meadow had stood by his evidence, but admitted his use of statistics at Mrs Clark's 1999 trial was "insensitive".... During the trial, Meadow said the probability of two natural unexplained cot deaths in the family was 73 million to one. The figure was later disputed by the Royal Statistical Society…" [Meadow actually said something equivalent to: "In a family like Sally Clark's, the chance of two babies both dying a cot death was I in 73 million", he wasn't claiming a *probability of 73 million to one* as the BBC News (Health) website stated, as probability cannot exceed 1.]

So, why did the RSS take an interest? Was Meadow incorrect in his calculation? Let's find out…

First, here are some warm-up exercises on probability. Remember that, conventionally, a probability must lie in the range 0 to 1, with 0 representing impossibility and 1 representing certainty. Incidentally, one of these exercises is a 'catch question'.

1. A fair coin is tossed once. What is the probability of "heads"? [I shall call this Pr (heads)]………………………………………..
2. A fair die is rolled once. What is Pr(6)? ………………………..
3. A fair die is rolled once. What is the probability that the result is *not* a 6? …………………………………………………………..
4. A fair die is rolled twice. What is Pr (6,6)? ……………………..
5. A fair coin is tossed once and a fair die is rolled once. I look at the coin, and tell you that it shows "heads". *Given that extra information*, what is Pr(6|heads)? [The vertical line **|** is used to separate what's not known from what is already known.] ………
6. A fair die is rolled once. I look at it, and tell you that the number I see is *not* a prime number. What is Pr(6|not prime)? ……………

Next, let's look at two older applications of probability in law courts:

- ***First, a 1962 trial of a parking charge in Sweden.*** *When I've explained what happened, chat with the person next to you (or with two other people near you): do you think the probability calculation was reasonable? If you do, would you have convicted? If you don't, would you have convicted anyway?*
- ***Second, a 1968 robbery trial in California, known as 'the Collins case'.***
  When I've explained this one, have another chat with the same person or group, and consider whether or not you accept the argument that
  Pr (randomly-chosen couple has these characteristics) = 1/12000000.
  If not, why not? Even if you disagree with the calculation, I expect you would agree that the probability is a very small one. Is this the probability that the two people charged are innocent?

Some more probability calculations now:

7. A *biased* die might look like a fair one, but actually have the probability of one of the numbers increased and the others decreased. Suppose Pr(6)=1/2, Pr(any other value)=1/10.
   [Note that the total probability is still
   (1/2) + (1/10) + (1/10) + (1/10) + (1/10) + (1/10) =1 as required.].
   What is Pr (6,6|biased)? ………………………………………
8. Now suppose that we don't know whether we are using a fair die or the biased die. We get two sixes in two throws. *Given this result,* what is the probability that this is the biased die? [Work this out as Pr (6,6|biased) / { Pr (6,6|biased) + Pr (6,6|fair)}].
   …………………………………………………………………
   There is an implicit assumption in making the calculation by the method I've given, can you see what it is? ….
9. In the calculation you have just carried out, suppose that the die had been chosen at random from a drawer containing 4 fair dice and a biased one, but the result is still two sixes. Chat with someone nearby about whether it would change your view about Pr (biased|result). If so, how would you suggest modifying the calculation?

Finally, back to the Clark case:

Meadow used figures from a study of cot deaths, which estimated 1/8543 as the probability of a randomly-chosen baby dying a cot death in an affluent, non-smoking family with the mother over 26. He squared this figure to yield about 1/73000000. Although he didn't claim that this was the probability that Sally Clark was innocent, there were fears that the jury might have interpreted it that way, and that even if they had accepted that his calculation was flawed (it was challenged later in the trial) they might still have inferred that there was a very, very small probability of innocence. Compare with the Swedish parking trial and the Collins case. Discuss with someone near you what might have been the errors in the Clark case.
If you'd like to read more about the Clark case and other similar ones, you can do so in "Significance", Vol 2 Issue 1, March 2005.

One to take home:

If time, I'll leave you with "The Case of the Guilty Beaker". N.B. You need to be able to work out Binomial probabilities to solve it.

The “Guilty Beaker” problem

There are two identical beakers, the ‘innocent’ beaker and the ‘guilty’ beaker. The innocent beaker has two red marbles for every blue one, the guilty beaker has two blue marbles for every red one. Each beaker may be assumed to contain so large a number of marbles that the number may be treated as infinite. One of the two beakers is chosen at random, and a handful of marbles drawn from it. The handful proves to contain 20 blue marbles and 13 red ones. What are the odds in favour of this being the guilty beaker? (If you obtain a probability of m/n, then the odds in favour are ‘m to n-m’). Incidentally, before you do any calculation, try writing down what approximate answer you expect, as it’s quite interesting to compare this with the result).