## Astounding wonders of ancient Indian mathematics

Vedic mathematics deals mainly with various Vedic mathematical formulae and their applications for carrying out tedious and cumbersome arithmetical operations, and to a very large extent, executing them mentally. The current methods of arithmetical computations, for e.g. multiplying 87265 by 32117 , dividing 7031985 by 823 , finding reciprocal of 7246041 up to 11 decimal places etc are notoriously too long, tedious, cumbrous and clumsy and entails the expenditure of enormous time and toil. The Vedic formulae almost always give answers to these "easy" arithmetic operations in one-line. The sixteen simple mathematical formulae from the Vedas often always give answers to tedious arithmetic operations in one line.

We are going to look at some arithmetic operations viz., multiplication, division, and learn to find the product and quotients in matter of seconds. The following methods are detailed in the book "Vedic Mathematics" by "Tagadguru Swami Sri Bharati Krsna Tirthaji Maharaj". This book has dwelt on the almost incredible simplicity of the Vedic Mathematical Sutras (formulae) and the indescribable ease with which they can be understood, remembered and applied for the solution of the wrongly-believed-to-be "difficult" problems in the various branches of Mathematics.

## Multiplication

Multiplication in this section is performed by Urdhva-Tityagbhyam Sutra which is a general formula and is applicable to all cases of multiplication. The formula is very short and terse, and means "vertically and cross-wise".

Suppose we wish to multiply 23 by 21 . Here 23 is the multiplicand and 21 is the multiplier. We adopt the following modus operandi to compute the product.

| 2 3   <br>  2 1  <br>     | (i) | Multiply the left-hand-most digit 2 of the multiplicand vertically by the left-hand-most digit 2 of the multiplier, get their product 4 and set it down as the left-handmost part of the answer. <br> Then multiply 2 and 1 , and 2 and 3 cross-wise, add the two, get 8 as the sum and set it down as the middle part of the answer. <br> Multiply 3 and 1 vertically, get 3 as their product and put it down as the last, the right-hand-most part of the answer. |
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Note that when one of the results contains more than 1 digit, the right-hand-most digit is to be put down there and the preceding, i.e. left-hand-side digit or digits should be carried over to the left and placed under the previous digit or digits of the upper row. An example to demonstrate this method is given below:

Multiply $\mathbf{3 7}$ by 33 . In this case $\mathbf{3 7}$ is the multiplicand and 33 is the multiplier.


The algebraic principle involved in the multiplication performed by Urdhva-Tiryagbhyam Sutra is as follows:

Suppose we multiply $(\mathbf{a x}+\mathrm{b})$ by $(\mathbf{c x}+\mathrm{d})$. The product is $\mathrm{acx}^{2}+(\mathrm{ad}+\mathrm{bc}) \mathbf{x}+\mathrm{bd}$. This is equivalent to saying, the first term, i.e. the coefficient of $\mathbf{x}^{2}$ is got by vertical multiplication of $\mathbf{a}$ and $\mathbf{c}$; the middle term, i.e. the coefficient of $\mathbf{x}$ is obtained by the cross-wise multiplication of $\mathbf{a}$ and $\mathbf{d}$ and of $\mathbf{b}$ and $\mathbf{c}$ and the addition of the two products; and the independent term is arrived at by vertical multiplication of the absolute terms.

We now extend the algebraic principle applied above to three digit numbers. Consider the multiplication of 785 and 362 . It merely means that we are multiply $\left(\mathrm{ax}^{2}+\mathrm{bx}+\mathrm{c}\right)$ by $\left(d^{2}+e x+f\right)$ where $x=10$.
$a x^{2}+b x+c$
$\mathrm{dx}^{2}+\mathrm{ex}+\mathrm{f}$
$a d x^{4}+(a e+b d) x^{3}+(a f+b e+c d) x^{2}+(b f+c e) x+c d$

Let us multiply 785 by 362 by the above method.

## 785

## 362

## 216760 6741

## 284170

## Reciprocal

Here we shall consider the cases of the fractions whose denominator ends in 9 for e.g.: 1/19, $1 / 29,1 / 49$. Methods to find reciprocals of the fractions whose denominator does not end in 9 can be found in the chapter 25. Recurring Decimals in "Vedic Mathematics".

The modus operandi of the method is given below:
Consider the vulgar fraction $\mathbf{1 / 1 9}$. The first digit of the denominator is 1 . We add 1 to this and get our multiplicand 2 .

| $\begin{array}{\|cccccccccc} 1 / 19 & = & & & & & & & & \\ 0 . & 0 & 5 & 2 & 6 & 3 & 1 & 5 & 7 & 8 \\ 1 & 1 & & & & 1 & 1 & 1 & 1 \\ & & & & & & & & & \\ 9 & 4 & 7 & 3 & 6 & 8 & 4 & 2 & 1 & \\ 1 & & 1 & 1 & & & & \end{array}$ | (i) (ii) (iii) (iv) (v) | Put down 1 as the right-hand most digit. <br> Multiply that last digit $\mathbf{1}$ by 2 and put 2 down as the immediately preceding digit. <br> Multiply 2 by 2 and put 4 down as the next previous digit. <br> Multiply 4 by 2 and put 8 down as the next previous digit to 4 . Multiply 8 by 2 and get 16. Put 6 down immediately to the left of the 8 and keep the 1 on hand to be carried over to the left at the |
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|  | (vi)next step. <br> Multiply $\mathbf{6}$ by $\mathbf{2}$ and get $\mathbf{1 2}$ as the <br> product and keep the 1 on hand <br> as in previous case. |
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| (vii)Continue this until you get the <br> $18^{\text {th }}$ digit counting leftwards from <br> right. |  |

## Reference:

Jagadguru Swami Sri Bharati Krsna Tirthaji Maharaj, "Vedic Mathematics", Motilal Banarsidass Publishers PVT. LTD., Delhi - India, 2002.

Presented by:
Satish V. Malik
3P34 School of Mathematical Sciences, UWE Bristol
BS16 1QY
01173283178
Satish.Malik@uwe.ac.uk
www.cems.uwe.ac.uk/~svmalik

