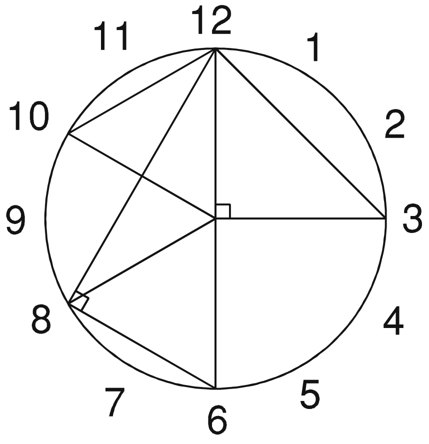
105 Chords Squared



Here is a circle of *radius* 1, with twelve points equally spaced around it, labelled from 1 to 12, as in a clock-face.

The *distance* between two points of the circle is the length of the *chord*, or straight line, joining them.

A number, multiplied by itself, is its *square*.

What is the *square* of the distance from 12 to 3? ....

What is the *square* of the distance from 12 to 6? ....

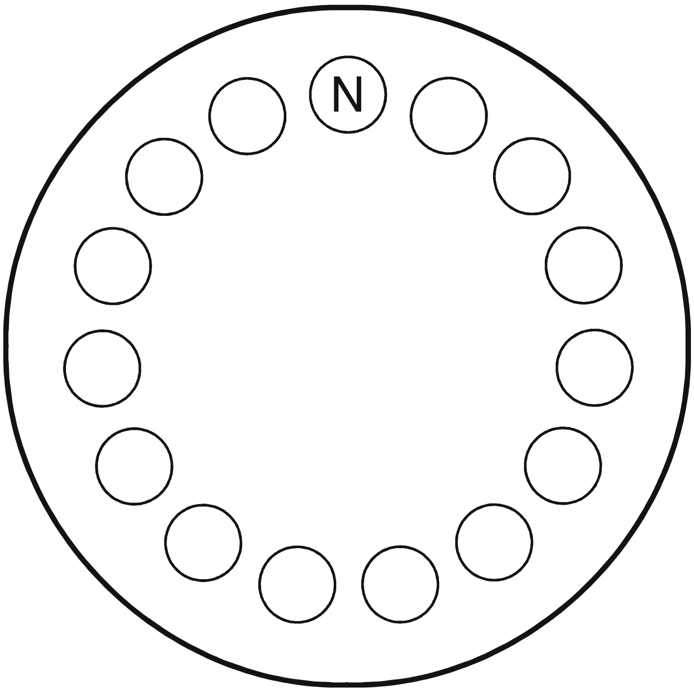
What is the *square* of the distance from 12 to 8? ....

What is the *square* of the distance from 12 to 10? ....

Be sure that you can justify *every one* of these.
Two are surely trivial. To answer the other two
you will need to use Pythagoras's Theorem.

*Resources: Four numbers [from templates]: **1**, **2**, **3**, **4***

122 Stepping Stones



Think of these fifteen spots as stepping stones set in a circle in a pond.

Arrange A, B, C & D on them so that measured the same way round the circle, *either clockwise or anticlockwise* (it doesn't matter which!):

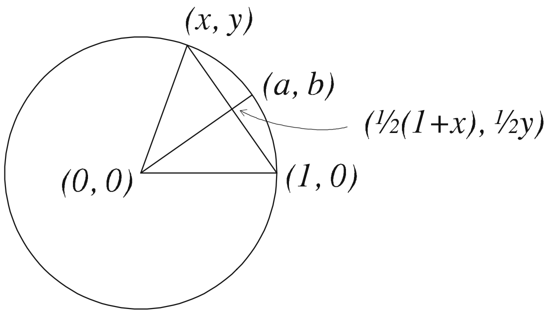
the distance from N to B is twice that from N to A,
the distance from N to C is twice that from N to B,
the distance from N to D is twice that from N to C,
and the distance from N to A is twice that from N to D,

A being further from N than any of the others.

*Resources: Four letters [from templates]: **A**, **B**, **C**, **D***

140 A Circular Argument

The points $(1, 0)$, (a, b) and (x, y) lie on the circle with centre $(0, 0)$ and radius 1, with the angles between adjacent radii equal, as shown in the diagram. Place the equations provided to prove that $x = a^2 - b^2$ and $y = 2ab$.



Since the points (a, b) and (x, y) lie on the circle with radius 1,

.... and ....

Also, the mid-point of the line joining $(1, 0)$ to (x, y) lies on the line from the origin to (a, b). This is the point $\left(\frac{1}{2}(1+x), \frac{1}{2}y\right)$, so that, evaluating the gradient of the radius in two ways,

....

But, since $x^2 + y^2 = 1$, $y^2 = 1 - x^2 = (1 - x)(1 + x)$, implying

that .... , so that .... Then, eliminating y,

$\frac{b^2}{a^2} = \frac{1-x}{1+x}$. Solving for x, using $a^2 + b^2 = 1$, we find that ....

Moreover, $1 + x = 2a^2$, implying that $y = (1+x)\frac{b}{a} = 2ab$.

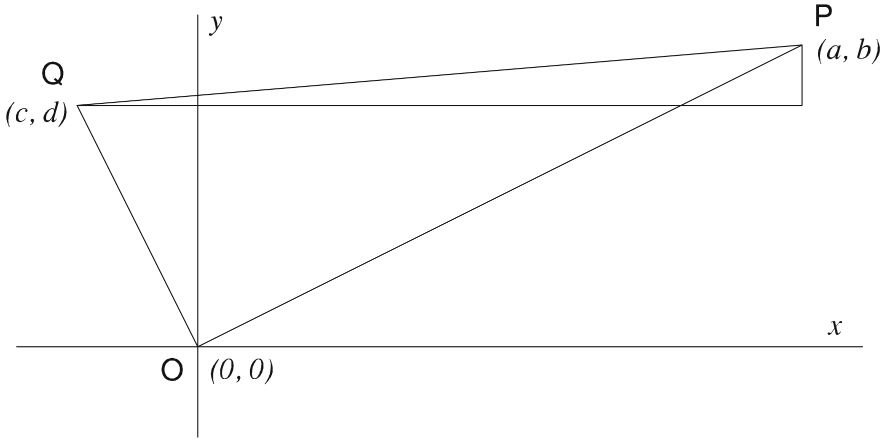
This completes the proof.

Resources: Six equations [from templates]

191 Orthogonality

A vector in the (x, y) coordinate plane is an ordered pair of numbers (a, b). This vector determines a point P of the plane, namely the point with x-coordinate a and y-coordinate b. The vector (a, b) is said to *represent* the point P. The vector $(0, 0)$ represents the origin O.

Two vectors, (a, b) representing a point P, and (c, d) representing a point Q, are said to be mutually *orthogonal* if the line segments OP and OQ form a *right angle* at the origin.



In that case, by Pythagoras' Theorem, the square of the distance from the point P to the point Q is

$|PQ|^2$ = ...

= ....

= ....

= $|OP|^2 + |OQ|^2 - 2(ac + bd)$.

But, again by Pythagoras' Theorem, $|PQ|^2 = |OP|^2 + |OQ|^2$. So the condition for the vectors (a, b) and (c, d) to be orthogonal is that:

....

Resources: Three formulas and an equation [from templates]

