Math Club November 29 2003

Problems from Moscow University Collection

1. A box contains sweets of three different colours. What is the minimal number of sweets to be taken (not looking at) from the box to be sure that at least three of the sweets are of the same colour ?

2. A farmer bought a new piece of land of a triangular shape very cheaply. He thought this was a good deal, since all sides of the triangle were longer than 5 hundred yards, but his wife believed the price of each square foot had been too high. Can she be right? Can you estimate the area the farmer bought.

3. Cut a square into rectangles so that any two neighbouring rectangles do not have a common side.

4. Two boys and two girls were picking up apples in a garden. Then they compared the numbers of apples collected. Annie collected most of all. The result of Betty was not the smallest. Is it true that girls together collected more than boys ?

Next day Katy and Jim joined the company. Again the result of Annie was the best, and the smallest result was neither Betty's nor Katy's. Same question.

5. Digits 123456789 are written in a row. Is it possible to insert signs + and − between all neighbouring digits so that the overall result of the sequence of these arithmetic operations is zero ? (Extra question: Is it possible to get zero inserting signs in *some* places?)

6. Two circles and a point are given on the plane. How can one construct a segment with the endpoints on the circles and the midpoint coinciding with the point given? Is this always possible ?
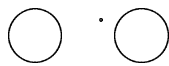
7. Is it possible to put whole numbers from 1 to 64 into squares of a chess board so that the differences between the numbers in any two neighbouring

squares (sharing a common side) do not exceed 4 ?

8. A telegraph message in the 19th century was a sequence of two signs "dot" and "dash." A small town in the Wild West was connected to the Capital by a long chain of low quality connections. Often two similar consecutive signs disappeared. The sequence "dot-dash-dot" inside a message was often mixed up with "dash-dot-dash" and vice versa. Sometimes many of these unpleasant transformations occurred during a sending. Sheriff Jenkins wants to find as many messages as possible which will, on arrival, definitely be different.

9. Four friends are chatting (Some of them always lie and others always tell the truth). Adam said to Billy " You are a liar," Jim said to Adam "You youself are a liar," Cris said to Jim " They both are liars ", then after thinking for a while he added, "By the way, you are also a liar". Which of these four are indeed liars?

10. Leaving the department professor Cleverly recalls that one of two exit doors is always locked, and the other is always open, but he forgot which is which. "Locked" was written on the first door. The other door was decorated by the following strange notice (apparently written by the professor of logic Stupidy) "Only one statement written on these doors is true." Where is the exit ?

11. A hundred toys are on sale at a market. Their prices are marked in pennies. Is it always possible to buy some toys having paid exactly a whole number of pounds?

12. Three circles are drawn so that there is a common region inside all of them. Three pairs of pairwise intersection points of the circles are connected by chords. Show that all the three chords meet at a point.

13. A huge round pizza is cut by 10 straight lines. What is the maximal number of pieces obtained ? Same question for a huge apple cut by 10 planes. (Start with few cuts, try to derive general method to count the pieces.)