Perennial Primes

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Contents:

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To look for all prime numbers up to 100 (Sieving)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

All prime numbers up to 100

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97					

All prime numbers up to 100

2 3 5 7 11 13 17 19 23 29

31 37 41 43 47 53 59 61 67 71

73 79 83 89 97

The largest known prime number is ($2^{13,466,917}-1$). This has 4,053,946 digits and was discovered in 2001. Also this is the largest Mersenne prime known and this gives rise to the largest even perfect number known.

All twin prime numbers up to 100

(3,5)
(5,7)
(11,13)
(17,19)
(29,31)
(41,43)
(59,61)
(71,73)

The largest known twin primes are (33,218,925* $2^{169,690}$ ∓ 1). This has 51,090 digits and was discovered in 2002.

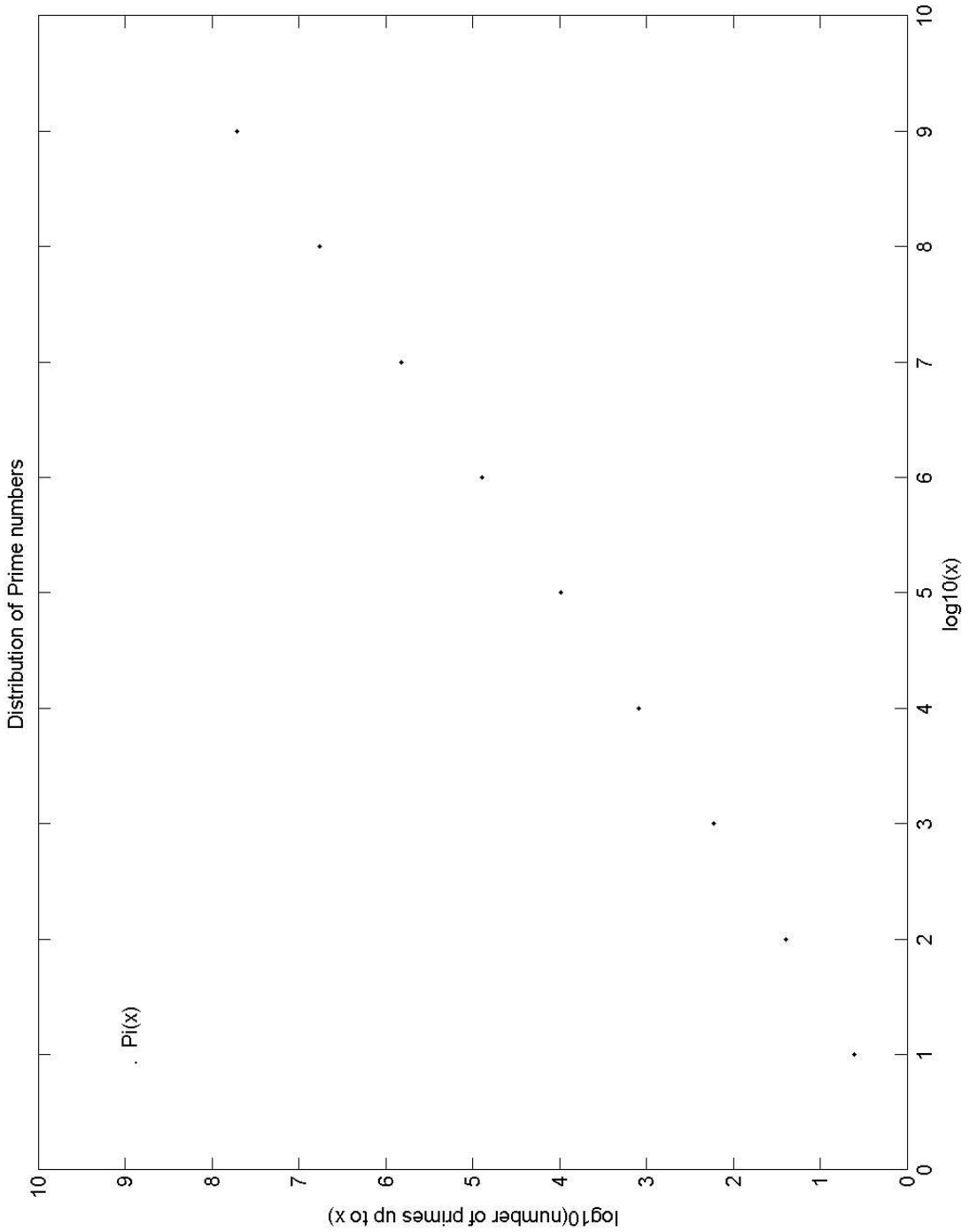
Distribution of Prime numbers and Approximations to this distribution

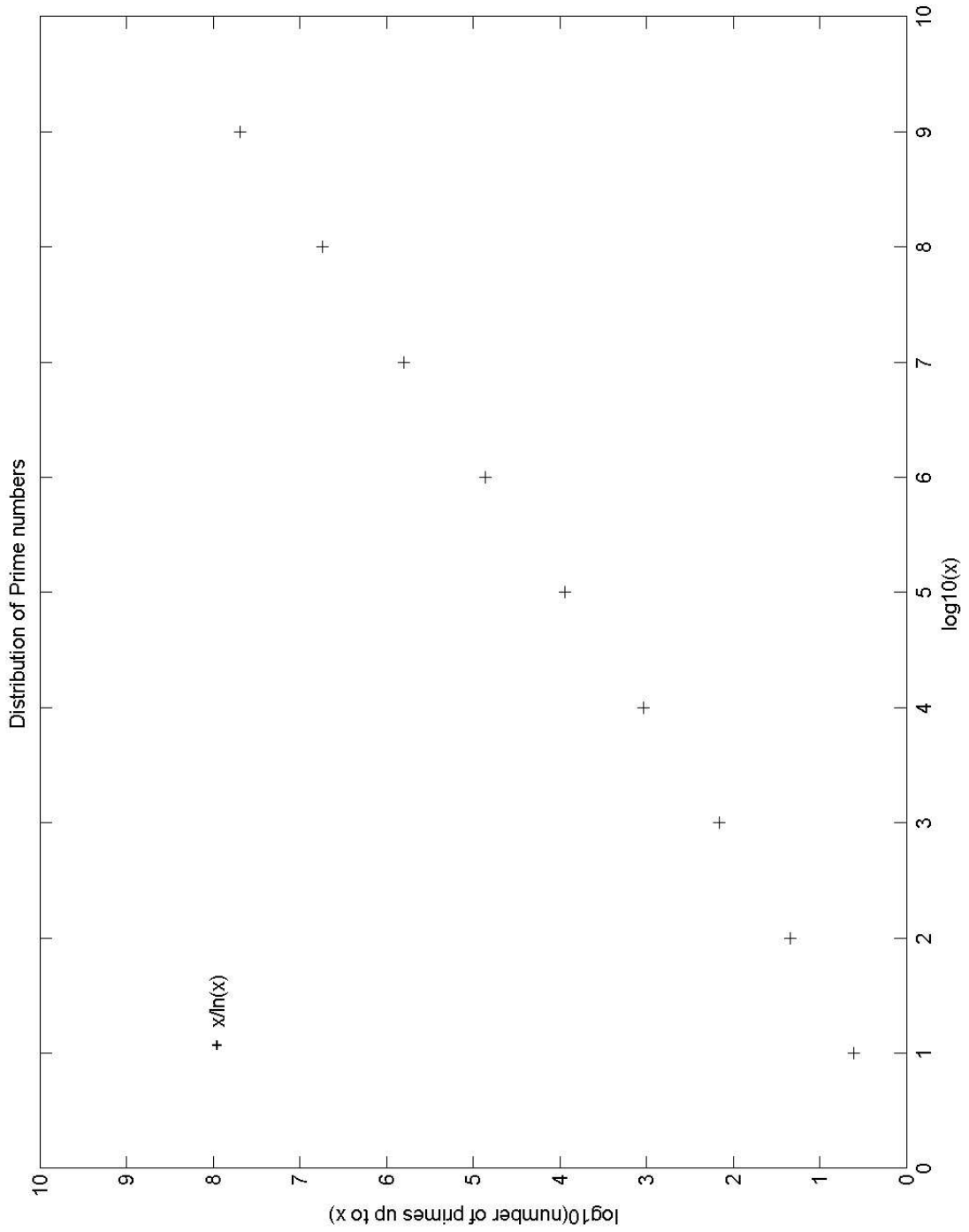
Pi(x) = Actual number of primes up to x

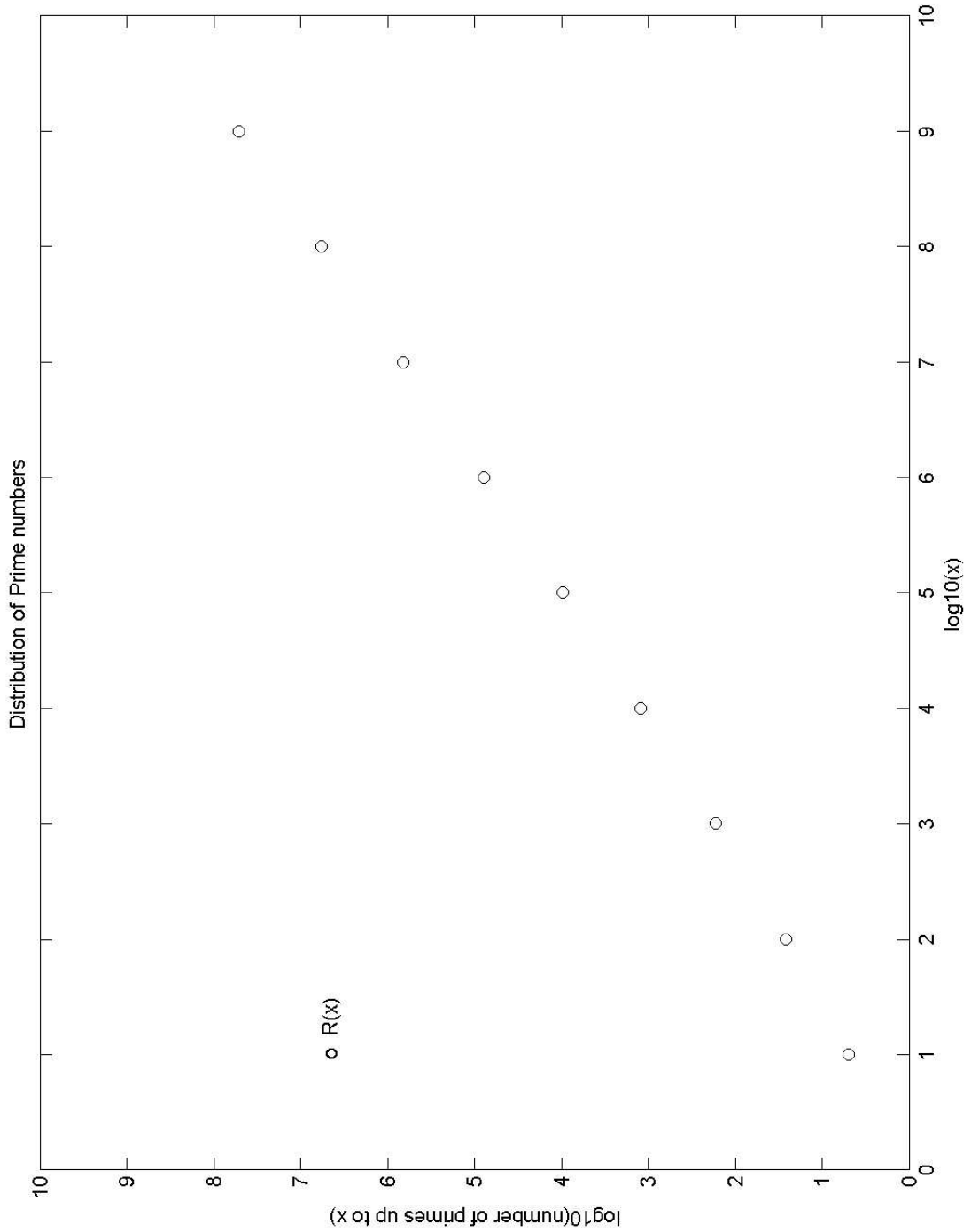
Gauss's approximation to Pi(x) ~ x/ln(x)

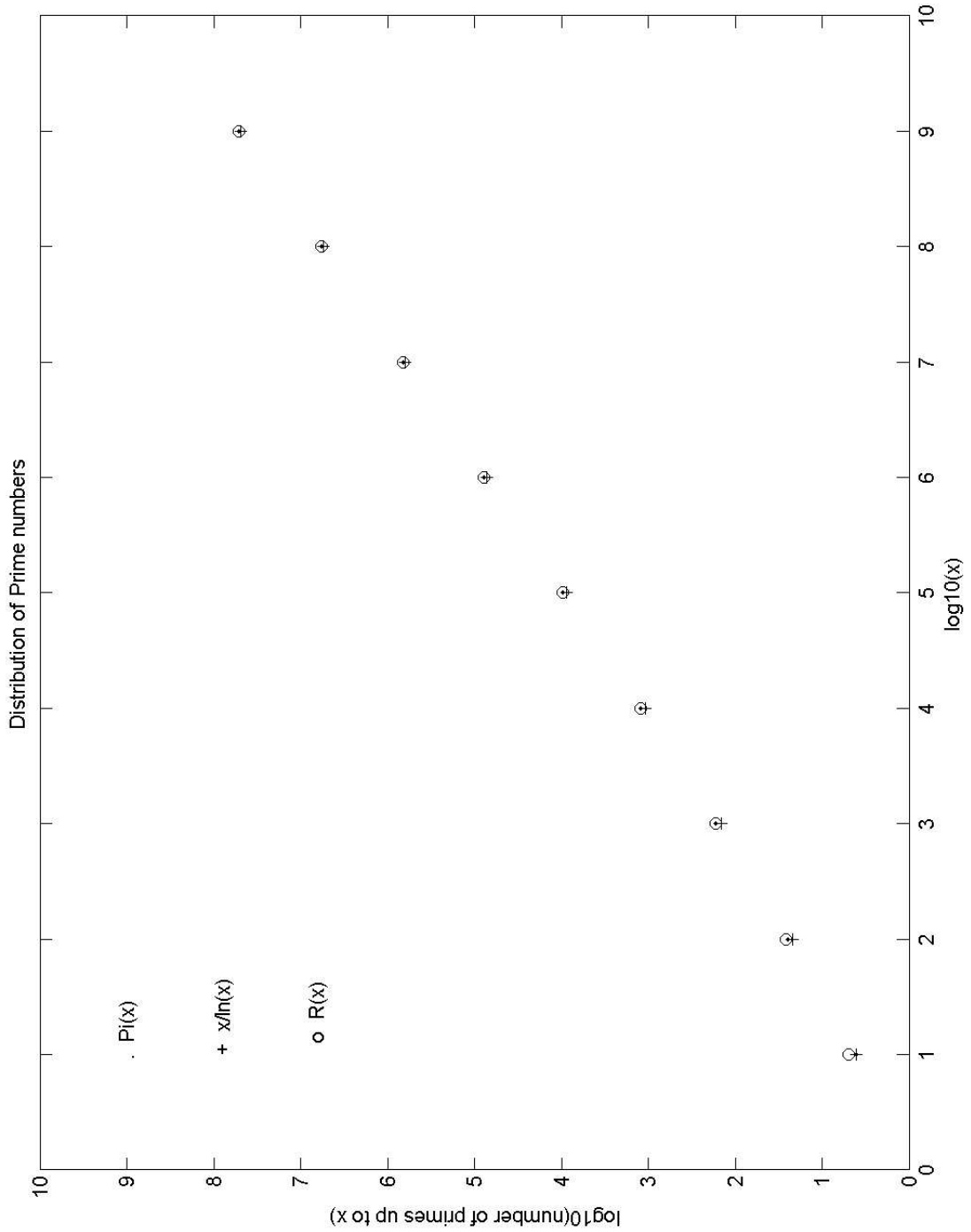
R(x) = Riemann function which is another approximation to Pi(x) involving the logarithmic integral

x	Pi(x)	round(x/ln(x))	round(R(x))
10	4	4	5
100	25	22	26
1,000	168	145	168
10,000	1,229	1,086	1,227
100,000	9,592	8,686	9,587
1,000,000	78,498	72,382	78,527
10,000,000	664,579	620,421	664,667
100,000,000	5,761,455	5,428,681	5,761,552
1,000,000,000	50,847,534	48,254,942	50,847,455


Distribution of Prime numbers
Pi(x)
log10(number of primes up to x)
log10(x)
0
1
2
3
4
5
6
7
8
9
10


Distribution of Prime numbers
+ x/ln(x)
log10(number of primes up to x)
log10(x)
0
1
2
3
4
5
6
7
8
9
10


Distribution of Prime numbers
o R(x)
log10(number of primes up to x)
log10(x)
0
1
2
3
4
5
6
7
8
9
10


Distribution of Prime numbers
log10(number of primes up to x)
log10(x)
. Pi(x)
+ x/ln(x)
o R(x)
0
1
2
3
4
5
6
7
8
9
10

A few of the unsolved problems:

Primes:

Mersenne Primes: *How many are there?*

Perfect Numbers: *How many are there?*
Does an odd perfect number exist?

Goldbach Conjecture: *Is this true?*

Twin Primes: *Are there infinitely many of them?*

Distribution of Primes: *Is there any deep connection with physics?*

The Riemann Hypothesis: *Is this true?*

Perennial Primes

You will find below a very small selection of some web addresses that relate to, and extend, what you have been doing with me today. One of the problems with web addresses is that these sometimes have a tendency to 'disappear' or 'be unavailable'. I can assure you that these have been available this week. You are very welcome to explore these but do obtain permissions of your parents. I hope that you derive some pleasure from these.

Primes:
http://www.utm.edu/research/primes/largest.html

Mersenne Primes:
http://www.utm.edu/research/primes/mersenne/

Perfect Numbers:
http://pachome1.pacific.net.sg/~novelway/MEW2/lesson1.html

Goldbach Conjecture:
http://mathworld.wolfram.com/GoldbachConjecture.html

http://www.ieeta.pt/~tos/goldbach.html

Distribution of Primes:
http://www.maths.ex.ac.uk/~mwatkins/zeta/tutorial.htm

http://klein.math.okstate.edu/~wrightd/4713/nt_essay/node17.html

The Riemann Hypothesis:
http://www.utm.edu/research/primes/notes/rh.html

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