On a string of 33 pearls, the middle pearl is the largest and most expensive of all. Starting from one end, each pearl is worth £100 more than the one before, up to the middle. From the other end, each pearl is £150 more than the one before, up to the middle. The string of pearls is worth £65,000. What is the value of the middle pearl?

Solution:

There are exactly 16 pearls to the either side of the pearl in the center. Lets assume that the value of the pearl in the center is £x. Let’s add the pairs of pearls, following the formula given by the problem:

$$\begin{array}{cccc}
 & x & & \\
/ & & \backslash & \\
(x-100) & & (x-150) & \leftarrow 2x - 250 \\
| & & | & \\
(x-200) & & (x-300) & \leftarrow 2x - 500 \\
| & & | & \\
(x-300) & & (x-450) & \leftarrow 2x - 750 \\
| & & | & \\
(x-400) & & (x-600) & \leftarrow 2x - 1000 \\
: & & : & \\
: & & : & \\
(x-100n) & & (x-150n) & \leftarrow 2x - 250n
\end{array}$$

Adding them all up, we get

$$\text{Total value} = x + n(2x) - (250+500+750+ \ldots. + 250n)$$
$$= (2n+1)x - 250(n(n+1))/2$$

In this case we have 16 pearls on each side, so n = 16, and the total value is £65000.

$$65000 = (2*16 +1)x - 250(16*17/2)$$
$$= 33x - 34000$$

which gives the value of £3000 for the middle pearl.

A circle is completely divided into n sectors in such a way that the angles of the sectors are in arithmetic progression. If the smallest of these angles is 8 degrees, and the largest is 52 degrees, calculate n.

Solution:

Let n = the number of intermediate terms.

Let d = the common difference between terms.

We have,

$$8 + (8 + d) + (8 + 2d) + ... + (8 + nd) + 52 = 360$$

We also have that,

$$(8 + (n+1)d) = 52$$

because that would be the next term in the arithmetic progression.

For the first equation, subtract the first 8 and the 52 to get that

$$(8 + d) + ... + (8 + nd) = 300$$

Use the fact that the sum from 1 to n is 1/2 * n * (n+1), and use the fact that there are n 8's to get the equation

$$8n + 1/2 * n * (n+1) * d = 300$$

From the second equation you can subtract the 8 to get that

$$(n+1)d = 44$$

This equation can be substituted verbatim into the other equation to get

$$8n + 1/2 * n * 44 = 300$$

This gives

$$8n + 22n = 300, 30n = 300, \ n = 10$$

and substituting back, that d = 4. The n = 10 represents the intermediate steps, so there are a total of 12 arcs ... 8, 12, 16, 20, ... , 52.

Suppose a cockroach starts at one end of a 1000 meter elastic tightrope and runs towards the other end at a speed of one meter per second. At the end of every second, the tightrope stretches uniformly and instantaneously, increasing its length by 1000 meters each time throughout the whole string. Does the cockroach reach the other end and how long does it take?
Solution:
It sure seems like the roach would never get anywhere, with the rope stretching so fast and the roach running so slowly.

After 1 second, the roach travels 1 meter, or 1/1000 of the length. Then the rope stretches. But since it stretches uniformly, the roach has still made it 1/1000 of the way along the new 2000 foot rope.

In the next second, he travels 1 meter, which is 1/2000 of the length. Then the rope stretches. But since it stretches uniformly, the roach is still 1/1000 + 1/2000 of the total distance along the rope.

You want to pick some mangoes from a tree that is surrounded by seven walls with seven guards, one at each gated wall. To get to the tree you tell each guard that you will give him half of all the mangoes you have but that guard must give you back one mango. The question is, what is the minimum number of mangoes you must pick to satisy these conditions and have atleast one mango left when you exit the seventh gate?
Solution:

Let’s say you leave the tree with two mangoes. You get to the innermost guard. You give him half the mangoes - which is to say, one mango - and he gives one back to you. So now you have two mangoes again.

You can repeat this as often as you need to, whether the number of fences is 7 or 700. So you need to leave the tree with two mangoes.

Two trains leave two towns that are fifty miles apart. They travel towards each other at rate of 30 mph and 20 mph respectively. A bumblebee flying at the rate of 50 mph starts out just as the faster train departs the train station, and flies to the slower train. The bee then turns around and goes back to meet the faster train. Then it turns around again, and it keeps flying back and forth between the trains until the trains meet. How far does the tired bumblebee fly?
Solution:
The quick solution is to note that the trains will meet in 1 hour and the bee is flying at the rate of 50 mph. Hence, at the end of the hour the bee will have flown a distance of 50 miles.

An ant of negligible size walks out a distance of 1 from the origin. Down the x-axis. It then turns left and goes up ½ from its current point. If the ant continues turning left, going the half the distance it previously went, and repeating the pattern, where does the ant eventually end up?
Solution:
The sum of all the x-direction motions is a geometric series, that is, of the form

$$a + a*r + a*r^2 + a*r^3 + ... + a*r^{(n-1)} + ...$$
with a = 1 and r = ½.

The sum of an infinite geometric series is a/(1-r).
Thus, the solution is 2.

We are given the following information:
A happy bug splits into a sad bug and a blank bug. A sad bug splits into 2 happy bugs. A blank bug splits into a sad bug and a happy bug. The generation starts off with 1 happy bug which splits and forms the 2nd generation.The bugs die when they split. The bugs live the same amount of life.How many of each kind are there in the first five generations?
Solution:
The first step is to make a table, using the following notation:
n = generation, H = happy bug, S = sad bug, and B = blank bug.

n	1	2	3	4	5
H	1	0	3	2	9
B	0	1	0	3	2
S	0	1	1	3	5
Total	1	2	2^2	2^3	2^4

reference: http://mathforum.org/library/drmath/sets/high_seq_series.html