Maths Club - $25^{\text {th }}$ January 2003

## SEEING IS BELIEVING

## Aim

When you are working alongside someone else and both of you are looking for a solution to the same problem, the other person might suddenly say "It's obvious". Alas, it mightn't be obvious to you, possibly because you are looking at the problem in a different way. You just can't "see" it at all.

The aim of the session was to highlight the need to be able to "see" a mathematical problem in at least two ways. One way is necessary to derive a solution and a second way is then required in order to check this solution. When a solution is thus checked then it can be "believed" to be correct.

Preliminary tasks
Early arrivals were told that during the session there would be a short observation quiz on features of the building on the route from the entrance to the session-room. The arrivals were given a practice round to complete and thereafter the opportunity to go to look for themselves to judge how well they did.

Each arrival also measured using a Perspex rule the distance mid-pupil to mid-pupil of the eye-balls of a selected volunteer. Each measurement was recorded confidentially for later reference.

## Seeing things

Puzzle pictures were used to introduce the session. Each could be interpreted or "seen" in two different ways. They were used to demonstrate that if you and someone else are looking at the same thing you might both "see" it in different ways. Things are not always as they might seem at first sight.

## Observing things

The observation quiz involved five questions. These were of the type "Given that you are in Room 2.11, what is the number of the room directly next door?" and "How many Liver Birds appear on the University crest?" The comment was made that you tend to "see" only things that you think may be of interest or of value to you. Otherwise, observation can be poor. Maintenance of powers of observation is important for the study of mathematics.

## Counting things

A sentence was projected very briefly on a screen. Participants were required individually to record their count of occurrences of a particular letter. The results were collected and, inevitably, a large proportion were wrong!

Implications for giving evidence of a witnessed event were discussed.

## Measuring things

The eye-ball to eye-ball measurements collected at the start were presented and their inherent "variation" was noted.

In jest, it was commented that the participants were a group of young people who were not as good as they might be at observing, counting or measuring!

## Recognising things

Numeric and alphabetic sequences were projected. Based on recognised patterns, suggestions for the next member of each sequence were invited.

## Context

The idea of a learning process model was introduced. In this context the stages were noted where observable signs and signals could be missed, dismissed, ignored, rejected or, in time, simply forgotten. Being aware of how we think and how we learn is a start of elevating problem solving beyond the confines of the application of any particular taught techniques.

## And finally

The following exercise in "seeing" a mathematical problem in at least two ways was undertaken by the participants.
"A container whose external shape is a cube is built of smaller cubes, all the same size with dimensions $1 \times 1 \times 1$. The walls and the base of the container are all one single construction cube thick. The container has a completely open top.

The number of cubes required to build a container with external dimensions $n \times n \times n$ for various values of n are given.

| $n:$ | 3 | 4 | 5 |
| ---: | ---: | ---: | ---: |
| No. of construction cubes : | 25 | 52 | 89 |

Work out how many construction cubes are required to build a container with external dimensions $\begin{array}{lll}\text { (i) } 10 \times 10 \times 10 & \text { (ii) } n \times n \times n \text { " }\end{array}$

The results are (i) 424
(ii) $5 n^{2}-8 n+4$.

Two different ways of "seeing" the problem were highlighted. Firstly starting as a solid cube and then removing parts, secondly as structure made up of walls and a base. Both ways naturally led to the same results which could then be "believed" to be correct.

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