Mathematics Club

Sequences And Series

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Outline

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- Geometric Progression

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  - nth term of a Geometric Progression

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  - Sum of the first n terms

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  - Sum of the first n terms
- The Sigma $(\sum)$ Notation
- The Infinite Geometric Series
- Binomial theorem
- Some challenging examples

Geometric Progression

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For a GP:

- first term is denoted by a.
- common ratio is denoted by r.
- the nth term is given by ar^{n-1}.

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$$1 + 2 + 3 + \ldots + n \quad = \sum_{r=1}^{n} r$$

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The sum of the first n natural numbers,

$$\begin{aligned} 1+2+3+\ldots+n &= \sum_{r=1}^{n} r \\ &= \frac{1}{2}n(n+1) \end{aligned}$$

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$$\begin{aligned} 1^2+2^2+3^2+\ldots+n^2 &= \sum_{r=1}^{n} r^2 \\ &= \tfrac{1}{6}n(n+1)(2n+1) \end{aligned}$$

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$$\begin{aligned} 1^3 + 2^3 + 3^3 + \ldots + n^3 &= \sum_{r=1}^{n} r^3 \\ &= \frac{1}{4} n^2 (n+1)^2 \end{aligned}$$

Infinite Geometric Series

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Consider the series

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots + \left(\frac{1}{2}\right)^{n-1} + \ldots$$

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n	
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n	2
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S_n	1.5	

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As $n \to \infty$, $S_n \to 2$. This limit is the **sum to infinity** of the GP.

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Sum to infinity of a GP:

$$S_\infty = \frac{a}{1-r}, \qquad |r| < 1.$$

Binomial Theorem

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Pascal's triangle.

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		1		1		
	1		2		1	
1		3		3		1

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$$
\begin{array}{ccccccccc}
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$