TRADITIONAL JAPANESE GEOMETRY

A selection of problems

Most of these problems are taken from the following books:

H.Fukagawa and D. Pedoe, *Japanese Temple Geometry Problems*, The Charles Babbage Research Centre, Winnepeg 19&9.
ISBN 0-919611-21-4.

H.Fukagawa and J. F. Rigby, *Trrditiona1 Japanese Mathematics Problems of the 18th and 19th Centuries*, SOT Publishing, Singapore.
To appear.

The Sangaku in Gunma (in Japanese), Gunma Wasan Study Association 1987.

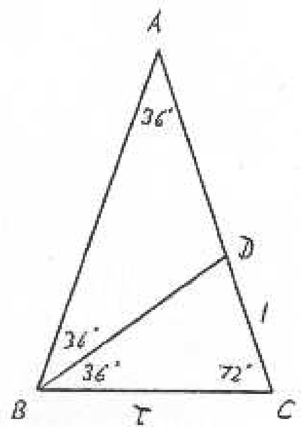
During the Edo period (l603-1867), when Japan was almost completely cut off from the western world, a distinctive style of mathematics, called *wasan*, was developed. Results and theorems were originally displayed in the form of problems, sometimes with answers but with no solutions, inscribed on wooden boards and accompanied by beautiful coloured figures. These boards, known as *sangaku*, were hung under the eaves in shrines and temples. Later, books appeared, either handwritten or printed from hand-carved wooden blocks, containing collections of sangaku problems with solutions.

John Rigby,
10 February 2002

A. THE GOLDEN RATIO

1. In the figure, where the angles have the values shown, the isosceles triangles ABC and BCD are similar, and DAB is also isosceles. If the length CD is 1, and the length BC is denoted by the Greek letter τ.

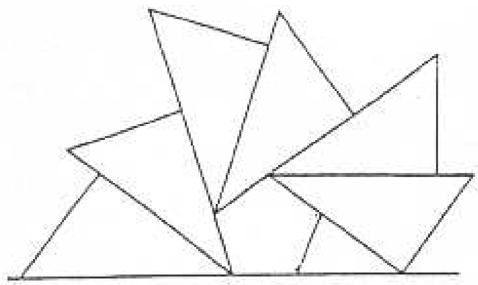
show that $\tau^2 = \tau + 1$, and hence find the value of τ



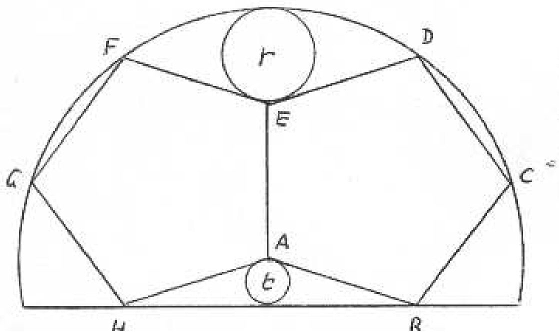
This number τ is called the *Golden Number,* or *Golden Ratio.* What is the connection between τ and the sides and diagonals of a regular pentagon?

Find the value of cos 36°

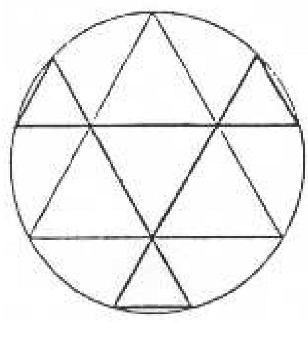
2. The next figure shows a regular pentagon with sides of length k, with six congruent right-angled triangles fanning out along the sides of the pentagon. Find the length of the hypotenuse of the triangles.



3. * Two congruent regular pentagons with a side in common are inscribed in a large circle as shown A circle of radius r touches the large circle and the sides DE and EF of the pentagons, and the incircle of the triangle ABH has radius t. Show that r = 2t. *(There is* no need for *complicated calculations: try to inscribe the circle of radius r in a triangle.)*

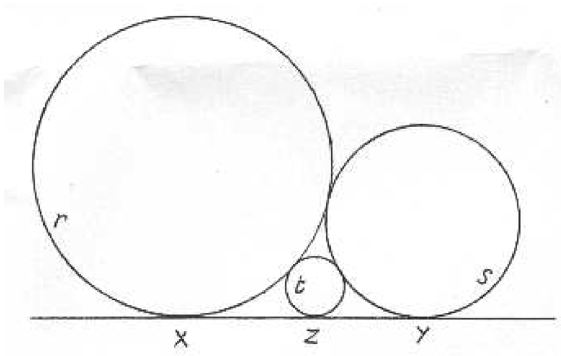


4. Find the ratios of the lengths of the sides of the equilateral triangles in this figure. The result is surprising: the golden ratio is usually associated not with equilateral triangles but with regular pentagons. (This is not a Wasan problem; it is a result due to George Odom.)



B.
TOUCHING CIRCLES AND TANGENT LINES

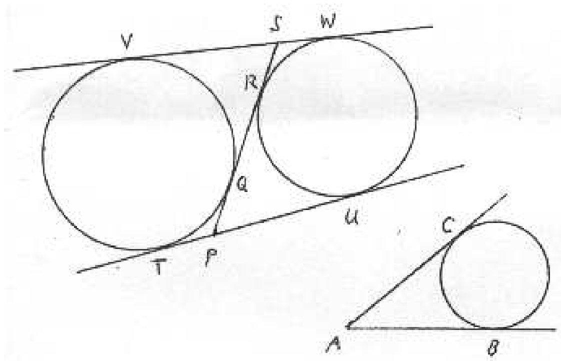
1. Two touching circles have radii r and s, and a common tangent touches them at X and Y, Use Pythagoras' theorem to find the length *XY* in terms of r and s.



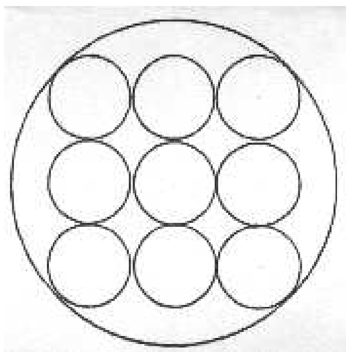
A third circle of radius t touches the other circles and XY as shown. Show that

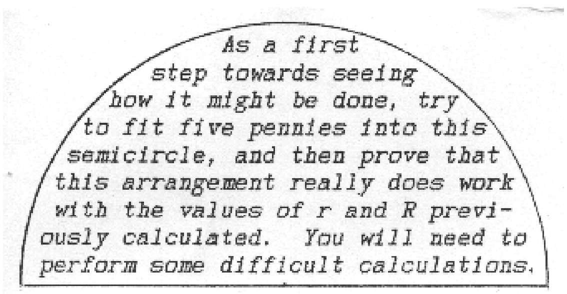
$$\frac{1}{\sqrt{t}} = \frac{1}{\sqrt{r}} = \frac{1}{\sqrt{s}}$$

2. In the next figure, show that the lengths TU and PS are equal.
(Remember that the two tangents to a circle from an external point are equal: AB = AC.)

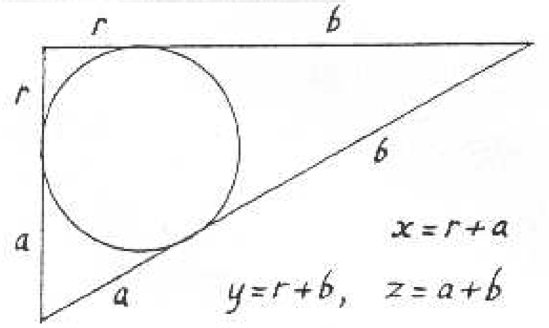


3. Nine circles of radius r are packed as shown in a circle of radius R. Express r in terms of R.
* Show that ten circles of this same radius r can be packed inside the same circle of radius R by an appropriate arrangement.



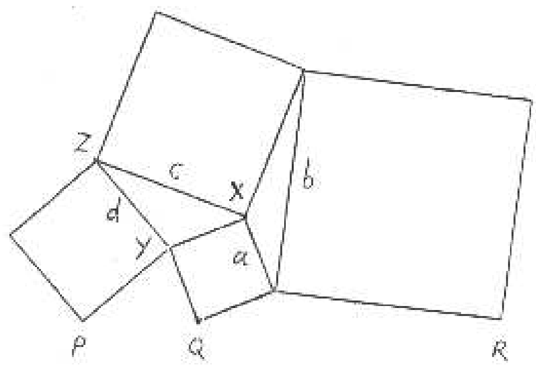


4. Show that if the sides of a right-angled triangle have integer lengths then the inradius of the triangle is also an integer. (Using *the notation in the figure, express the inradius r in terms of the sides x, y and z.)*

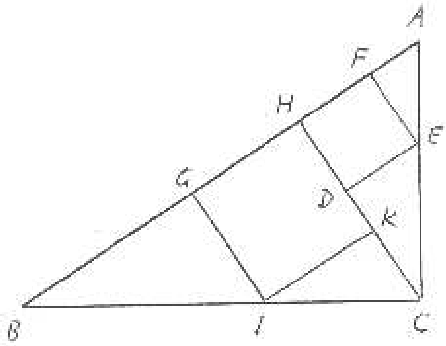


C.
TRIANGLES, SQUARES AND RECTANGLES

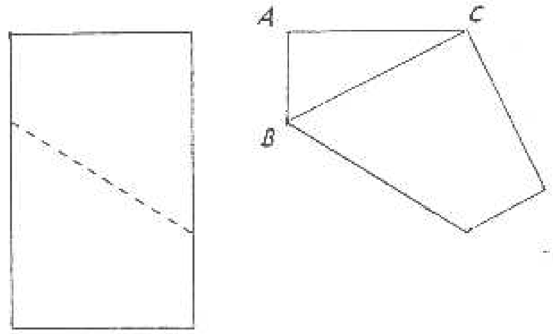
1. There are four squares in the figure with sides of lengths a, b, c, d as shown. Without using trigonometrical formulae, show that the two triangles in the figure have equal areas. Show also that $b^2 + d^2 = 2(a^2 + c^2)$ (you may use the cosine formula to prove this, but see *Math. Gazette* 83, July 1999, p.283).



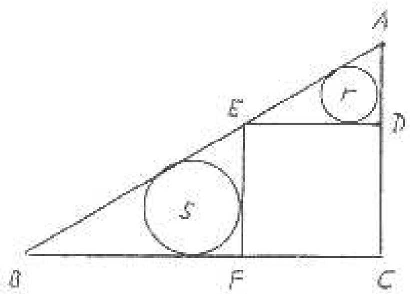
2. * Show also that, if 2a = c, the points P, Q and R lie on a line. (Hint. By rotating the triangle YQP clockwise about Y through 9O°, show that PQ is parallel to the bisector of the angle <ZXY.)



3. In the right-angled triangle ABC, H is the foot of the perpendicular from C to AB, and squares FHDE and GHIK are inscribed in ΔAHC and ΔHCB.
Show that CI = CE. *(Hint.* why are *the points C, E,* H, *I concyclic?)*



4. A rectangle of paper is folded so that two opposite corners coincide. If the longer side of the rectangle is of given fixed length, what shape of rectangle gives the greatest value of the area ABC ?



5. * A square CDEF is inscribed in the right-angled triangle ABC, and the incirdes of the two smaller right-angled triangles have radii r and s. Find the side-length t of the square in terns of r and s. *(You will need to use the formula for r in Problem B4.)*