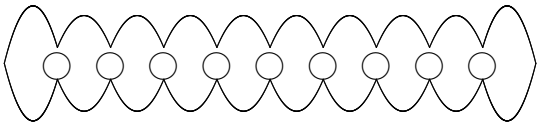
Math Club November 24 2001

Solutions

1. *Draw this figure tracing any arc only once not taking the pen away from the paper*



There are many possibilities, e.g. see the picture.

2. *Two boys are runing down an escalator. Bill is faster than Victor. Who will count more steps ?*

Bill will count more. When Victor reaches the end of the escalator some of the steps Bill touched will disappear under the floor.

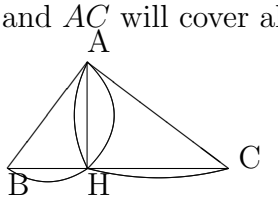
3. *The height of any tree in a garden is greater than 30 but less than 150 feet. The distance between any two trees does not exceed the difference of their heights. Decide if it is possible to walk at most 240 feet having touched each tree and returning to the starting point.*

Order all trees $T_1, T_2, \dots T_n$ by their heights $h_1 \leq h_2 \leq \dots h_n$. Let us walk from T_1 to T_2 then to T_3 and so on. Distance $|T_1T_2|$ does not exceed $h_2 - h_1$, and $|T_2, T_3| \leq h_3 - h_2$. After summation we get $|T_1T_2| + |T_2T_3| \leq h_2 - h_1 + h_3 - h_2 = h_3 - h_1$. The intermediate value h_2 cancells. Following this pattern, the overall length of the path $T_1 \to T_2 \to \dots \to T_n$ does not exceed $h_n - h_1 \leq 120$ feet. Coming back we cover altogether at most 240 feet.

4. *A spot on a table has the form of a quadrilateral. A hostess wants to cover the spot with four round napkins whose diameters are equal to the lengths of the sides of this quadrilateral. Will there remain any uncovered*

part of the spot if she put the centers of the napkins at the middlepoints of the corresponding sides?

Nothing remains. Take a triangle ABC. The half-discs with diametersAB and AC will cover all the triangle: the circles will intersect at the base H of



the perpendicular AH to the line BC.

Draw a diagonal in the quadrilateral. We get two triangles. Each of them will be covered by two napkins.

5. *Lady Elfly said when boarding a bus : -" The product of ages of my three sons is* 36 *while the sum of their ages is the number of this bus service."*

Her friend miss Marple replied : –" This information is not sufficient to find the ages. "

Then Lady Elfly added: –" Recently my older boy caught a cold."

–"Now I know how old they are –" exclaimed Miss Marple. And what about you, can you determine the bus service number ?

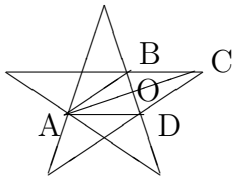
All possible factorizations of 36 in triples are :

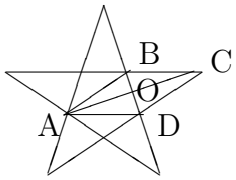
$$1 \times 1 \times 36, \qquad 1 \times 2 \times 18, \qquad 1 \times 3 \times 12, \qquad 1 \times 4 \times 9$$

$$1 \times 6 \times 6, \qquad 2 \times 2 \times 9, \qquad 2 \times 3 \times 6, \qquad 3 \times 3 \times 4.$$

In all cases except $2 + 2 + 9 = 1 + 6 + 6 = 13$ the sums are distinct. So the only possibility when miss Marple (she knew the sum) fails to find the answer without additional information is one of those two cases. Hence the older son is 9 and there are twins of 2. The bus service is 13.

6. *Eltsin and Putin divided the Kremlin star made of chocolate. It had the regular pentagon shape. Putin took the part dashed. Who took more?*





The figure $ABCD$ is a rhombus since $AB||CD, \quad BC||AD, \quad AB = AD$. Hence the areas of triangle AOD and BOC are the same. Hence the area of triangle ACD equals the area of triangle BCD. Dash the upper triangle of the star instead of ACD we will get exactly the half of the star. The other part will be symmetric about AC. So both took exactly 1/2.

7. *The product of successive numbers* $1 \times 2 \times 3 \times \ldots \times (n-1) \times n$ *is denoted by* $n!$ *(this is read " n factorial"). It is known that*

$$35! = 10333147966386144929 * 66651337523200000000$$

Here $*$ *stands for some digit. Find this digit.*

The answer is $* = 6$. This is easy to calculate using the well known property: a number is divisible by 9 if and only if the sum of its digits is divisible by 9. Also you can use the divisibility by 11.

8. *Two clever pirates John Silver and Billy Bones are playing the following game: There is a strip of leather divided into twenty squares. There is a golden coin at the leftmost square. They move the coin in turns to any other square (to the right or to the left). They agree that each time the length of the shift must be different from all the previous shifts. The person who has no further possibility to move the coin fails. Who will win if Silver is to start?*

At first Silver will move the coin to the rightmost corner. After any shift of Billy (say by a squares to the left) Silver will shift the coin to the leftmost square (by $19-a$ squares). Split all numbers from 1 to 18 into pairs $a, 19-a$.

If Billy uses one number from a pair – Silver replies by its counterpart. So Silver always has a possibility to move. He will win.

9. *Coins of 1, 15 and 50 Manats circulate in a certain country. Visiting this country Mike bought a book for several coins. The change contained one coin more. Decide what could be the least possible price of this book.*

The answer is 6 Manats. For example Mike paid by $1+50$ and got change $15+15+15$. The difficult part of this problem is to show that 6 is the minimal solution. We give a hint : All these coin values $1, 15, 50$ give remainder 1 when divided by 7. Prove that the remainder of the price of the book is 6 when divided by 7.