Reference Sheet

To work out the bracket

Rule 1: $\left\langle \bigcirc \right\rangle = 1$ │ *The unknot has bracket 1* │

Rule 2: $\left\langle D\bigcirc \right\rangle = -\left(x^2 + \frac{1}{x^2}\right)\left\langle D \right\rangle$ │ D *represents any diagram* │

│ *The bracket of a diagram with an extra loop is* $-\left(x^2 + \frac{1}{x^2}\right)$ *times the bracket without the loop* │

Rule 3: $\left\langle \diagup\!\!\!\diagdown \right\rangle = x \left\langle \,)(\, \right\rangle + \frac{1}{x}\left\langle \asymp \right\rangle$

│ *Be sure to get it the right way round. Turning* **left** *as you go along the top string gives* x │

To work out the writhe

$\nearrow\!\!\!\nwarrow = +1$ $\nwarrow\!\!\!\nearrow = -1$

│ *Walk along the top in the direction of the arrow. If someone walking along the bottom is going from* **right to left** *count* $+1$*. Otherwise count* -1*. The* **writhe** *is the total over all the crossings.* │

To work out the square bracket

$$\left[D \right] = \left(-\frac{1}{x}\right)^{3\times \text{writhe}} \left\langle D \right\rangle$$

│ *The square bracket is the bracket times* $\left(-\frac{1}{x}\right)^{3\times writhe}$*. Remember that* $\left(-\frac{1}{x}\right)^{n}$ *is the same as* $\frac{1}{x^n}$*, with a minus sign if n is odd. Also,* $\left(-\frac{1}{x}\right)^{-n}$ *is the same as* x^n*, with a minus sign if n is odd.* │