Amplitudes meet the Swampland

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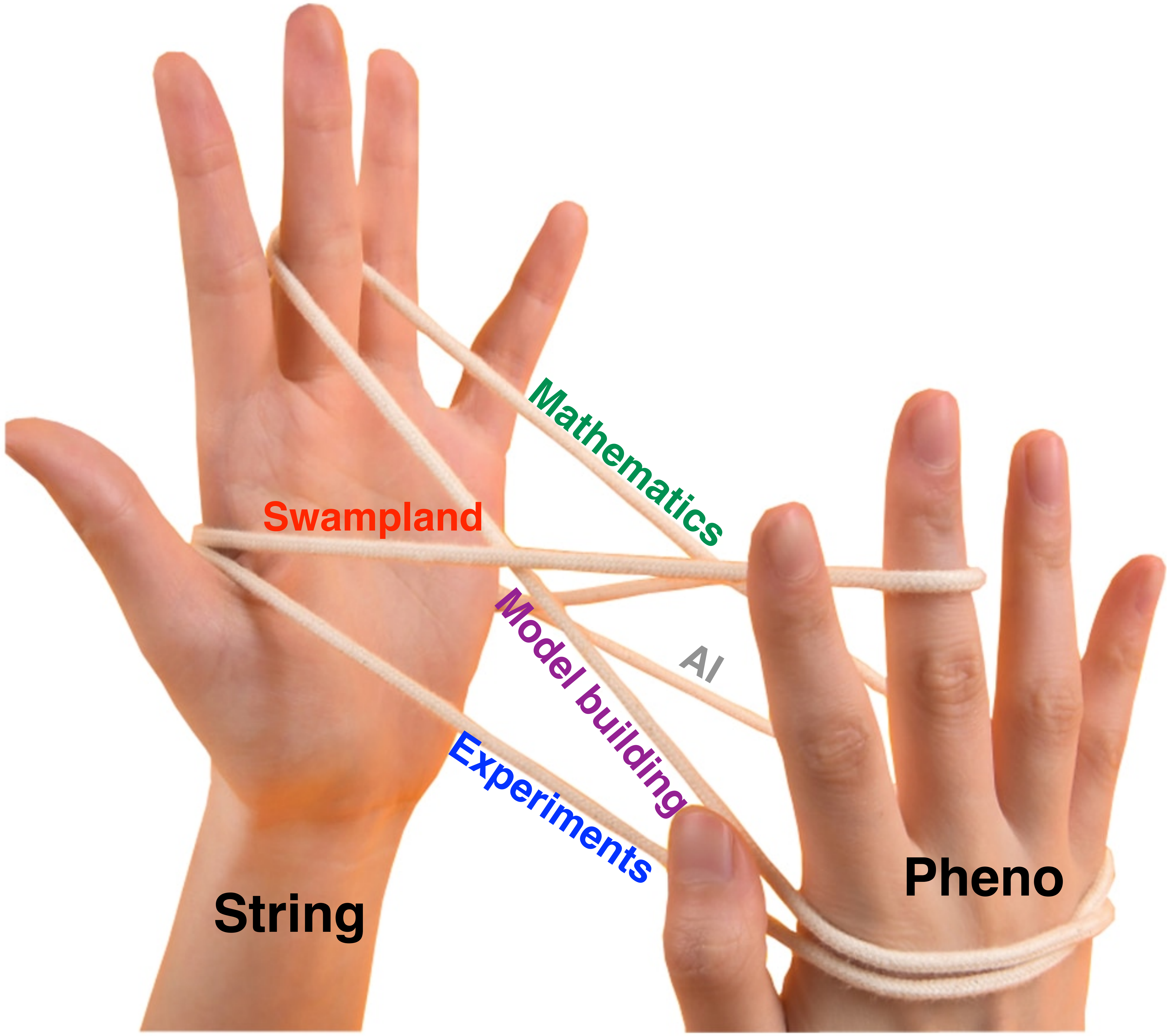
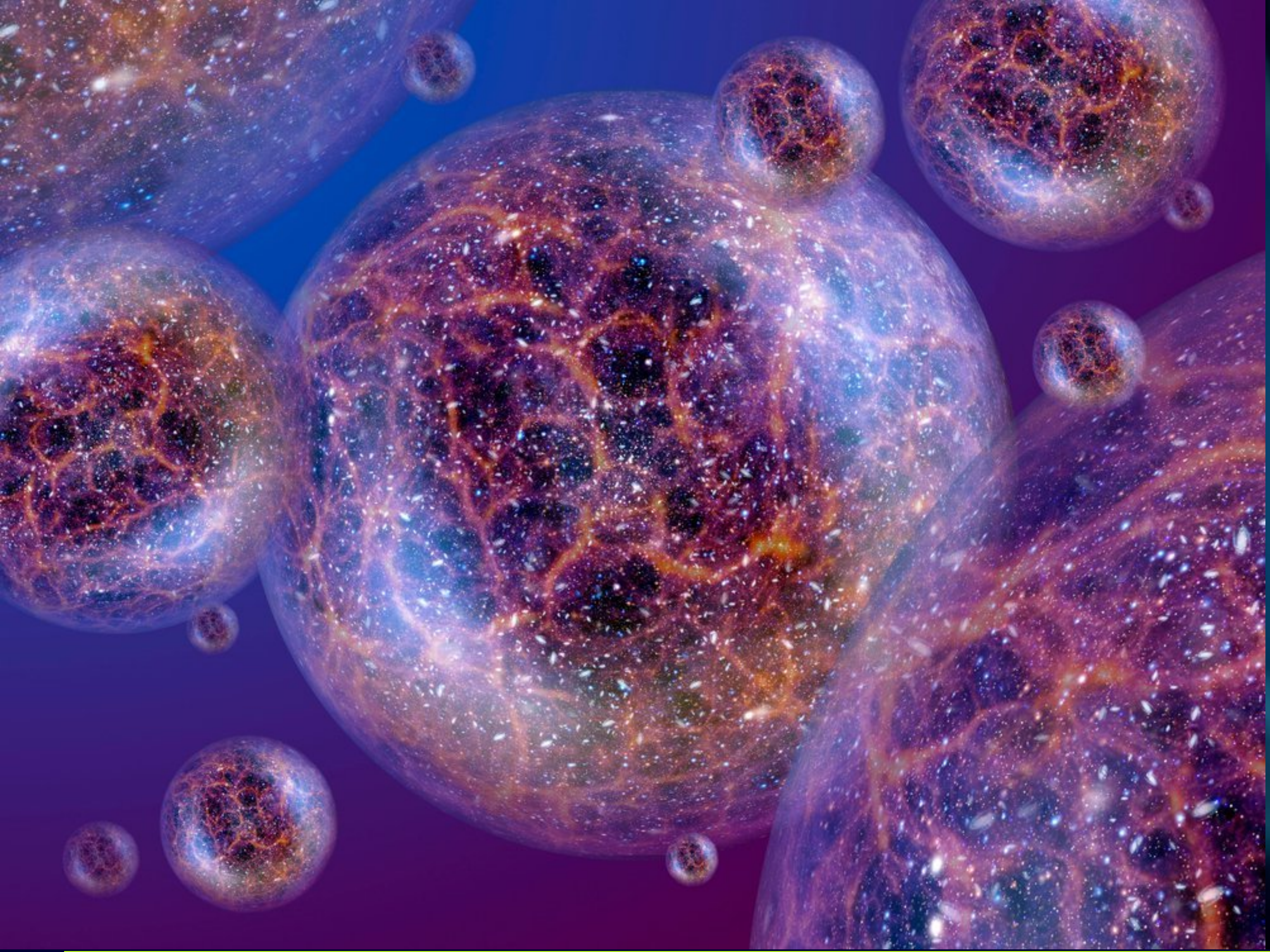
String Pheno turns legal (21)!





The String Genome Project and ....
215 Billion Intersecting Brane Models

G. LOGES AND GS, 2112.08391 [HEP-TH], 2206.03506 [HEP-TH]

Sign Matters

- The goal of the swampland program is to delineate the landscape from the swamp. Swampland constraints often take the form of an inequality, e.g., the WGC:

$$\exists \text{ state with } \frac{q}{m} \geq \lim_{M\to\infty} \frac{Q}{M}\bigg|_{\text{ext}}$$

[Arkani-Hamed, Motl, Nicolis, Vafa ’06]

- Analyticity constraints on the S-matrix similarly give rise to inequalities (**positivity bounds**) that constrains the EFT coefficients.
- Natural to put 2+2 together. Indeed, some remarks were already made in [Arkani-Hamed, Motl, Nicolis, Vafa ’06];[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, ’06].
- Leading irrelevant operators shift the extremality bound of RN black hole:

$$-F_{\mu\nu}^2 + a(F^2)^2 + b(F\tilde{F})^2 + \cdots \xrightarrow[\text{(causality)}]{\text{analyticity}} M^2_{\text{extremal electric}} = Q^2 - \frac{2a}{5} < Q^2$$

Gravity Matters

- In the presence of dynamical gravity, the analyticity and boundedness properties of the S-matrix are less understood.

- An added assumption of **Regge boundedness**:

$$\lim_{s\to\infty,\ t<0:\text{fixed}} M(s,t)/s^2 = 0$$

- Graviton exchange in the t-channel: $M(s,t) \sim -\frac{1}{M_P^2}\frac{s^2}{t}$

- One can argue that the amplitude cannot grow faster than s^2 using the chaos bound [Maldacena, Shenker, Stanford, ’15] (or heuristically, using the `signal model’ [Camanho, Edelstein, Maldacena, Zhiboedov, ’14]).

- A more careful treatment can be found in [Chandorkar, Chowdhury, Kundu, Minwalla, ’21]. Establishing this behavior is relevant for the Classical Regge Growth Conjecture.

Gravity Matters

- More recently, gravitational Regge boundedness for $D \geq 5$ was shown in [Hairing, Zhiboedov, '22] to hold under some assumptions, including **subexponentiality**:

$$|M(s,t)| < e^{C|s|^{\beta}}, \quad \beta < 1 \quad \text{for fixed } t < 0$$

  everywhere in the region of analyticity in the upper half-plane $\arg(s) \in (0,\infty)$.

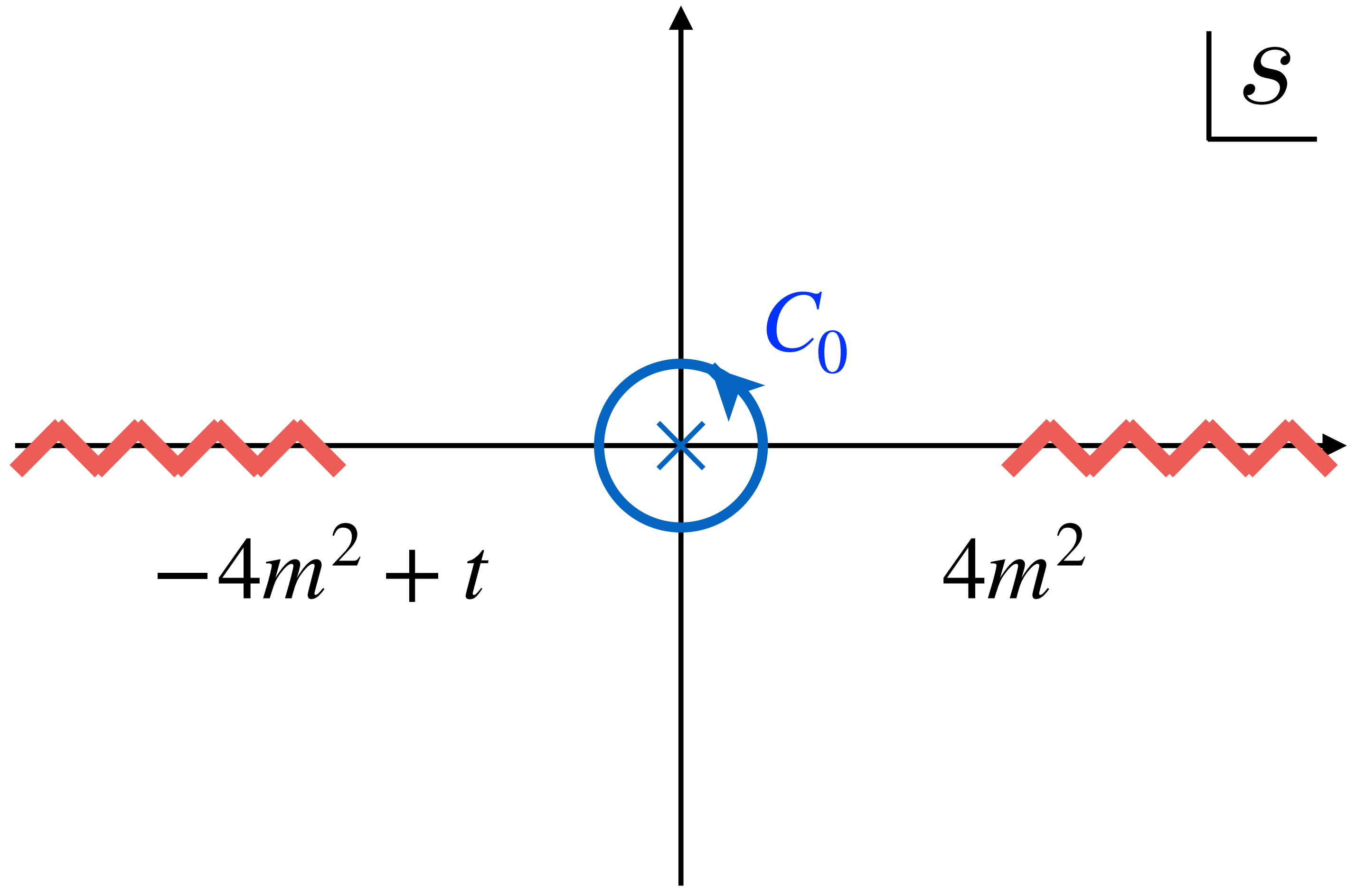
- Interestingly, arguments using large IR logs to show swampland constraints is 4d specific [Arkani-Hamed, Huang, Liu, Remmen, '21].

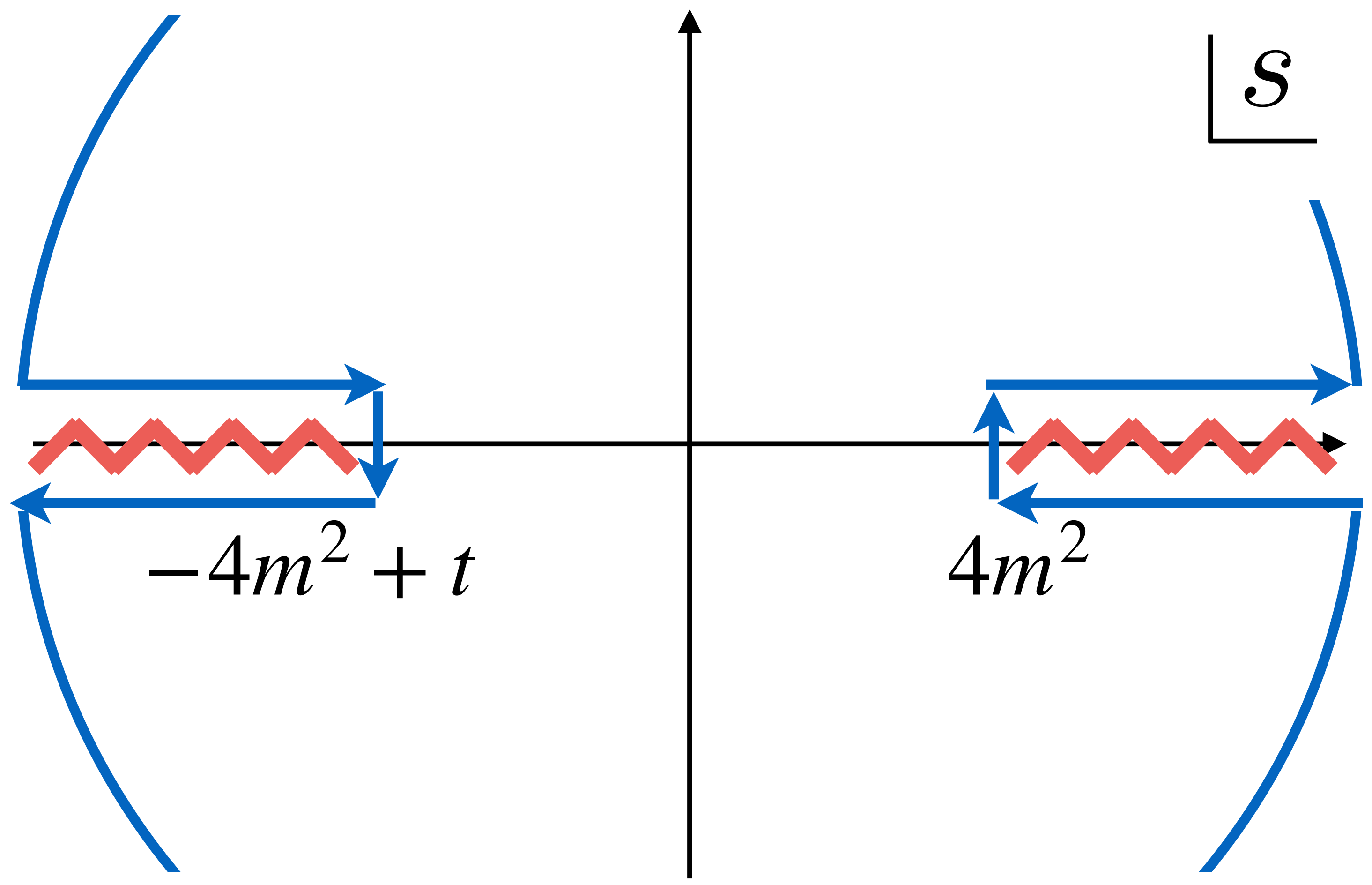
- Even though the Regge limit does not probe strong gravity (black hole exchange), understanding whether this is true for all UV completion may teach us lessons about the UV.

- **Swampland constraint?** Evidence: 1) perturbative string amplitude in flat space, 2) CFT argument [Caron-Huot, '17] for AdS scattering (leading $1/N$ gives $M \sim s^2$, to all orders $M \sim s$).

- The gravitational positivity bounds may be **only approximately positive** [Hamada, Noumi, GS, '18];[Alberte, de Rham, Jaity, Tolley, '20]; [Tokuda, Aoki, Hirano, '20];[Caron-Huot, Mazac, Rastelli, Simmons-Duffin, '21].

Gravitational Positivity Bounds





- How the graviton t-channel pole gets canceled depends on UV completion.

$$\mathcal{M}(s,t) = -\frac{4su}{M_{\rm Pl}^2 t} - \frac{4tu}{M_{\rm Pl}^2 s} - \frac{4ts}{M_{\rm Pl}^2 u} + \sum_{n=0}^{\infty} \frac{c_n(t)}{n!}\left(s+\frac{t}{2}\right)^n \qquad c_2(t) - \frac{8}{M_{\rm Pl}^2 t} = \frac{4}{\pi}\int_{4m^2}^{\infty} ds \frac{{\rm Im}\mathcal{M}(s,t)}{(s+t/2)^3}$$

- The leftover t^0 piece can be positive or negative, modifying the "positivity bounds".

[Hamada, Noumi, GS, '18]

Weak Gravity Conjecture

WGC from Unitarity and Causality

- With these caveats, one can "prove" the WGC from unitarity and causality [Hamada, Noumi, GS '18], [Cheung, Liu, Remmen, '18];[Bellanzini, Lewandowski, Serra ,'19], [Arkani-Hamed, Huang, Liu, Remmen, '21]

$$\mathscr{L} = \frac{1}{2}R - \frac{1}{4}F^2 + \frac{\alpha_1}{4}(F^2)^2 + \frac{\alpha_2}{4}(F\tilde{F})^2 + \frac{\alpha_3}{2}FFW$$

$$\frac{\sqrt{2(Q^2+P^2)}}{M} \leq 1 + \frac{32\pi^2}{5(Q^2+P^2)^3}\left[2\alpha_1(Q^2-P^2)^2 + 2\alpha_2(2QP)^2 - \alpha_3(Q^4-P^4)\right]$$

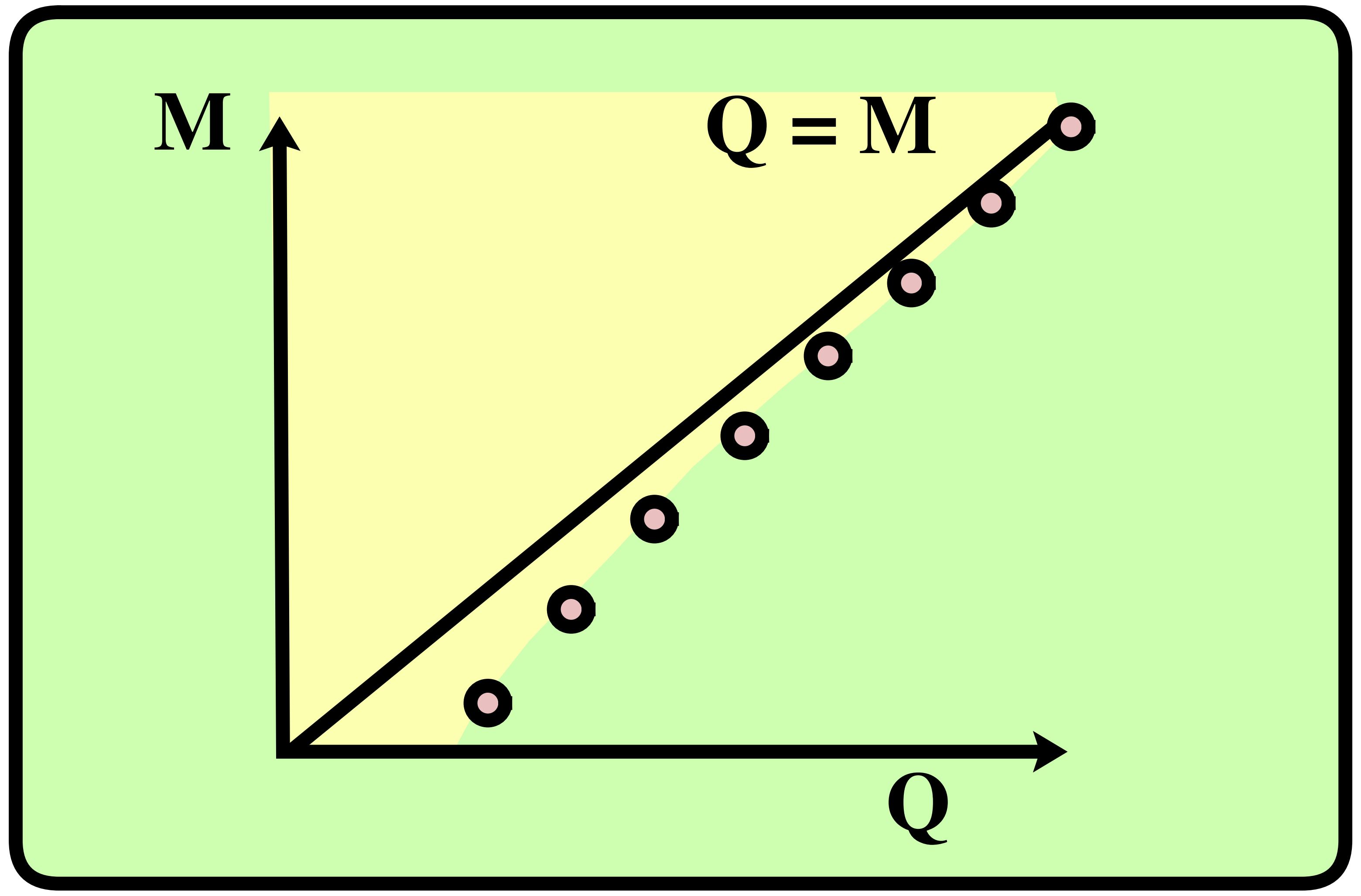
- Such arguments have been extended to more complicated charged black holes [Loges, Noumi, GS, '19];[Loges, Noumi, GS, '20].

- In some cases, positivity bounds alone do not imply the WGC; additional symmetries of the EFT (well motivated from UV completions like $SL(2,\mathbb{R})$ and $O(d,d,\mathbb{R})$) are needed to enforce the WGC. [Loges, Noumi, GS, '20].

Strong forms of the WGC

- Consistency with dimensional reduction and duality suggests stronger versions of the WGC known as the sub-lattice WGC [Montero, GS, Soler '16], [Heidenreich, Reece, Rudelius '16] and tower WGC [Andriolo, Junghans, Noumi, GS '18].

- The strongest evidence comes from string theory, suggesting a monotonic behavior.



Can we upgrade the scattering positivity bound arguments to show this monotonicity?

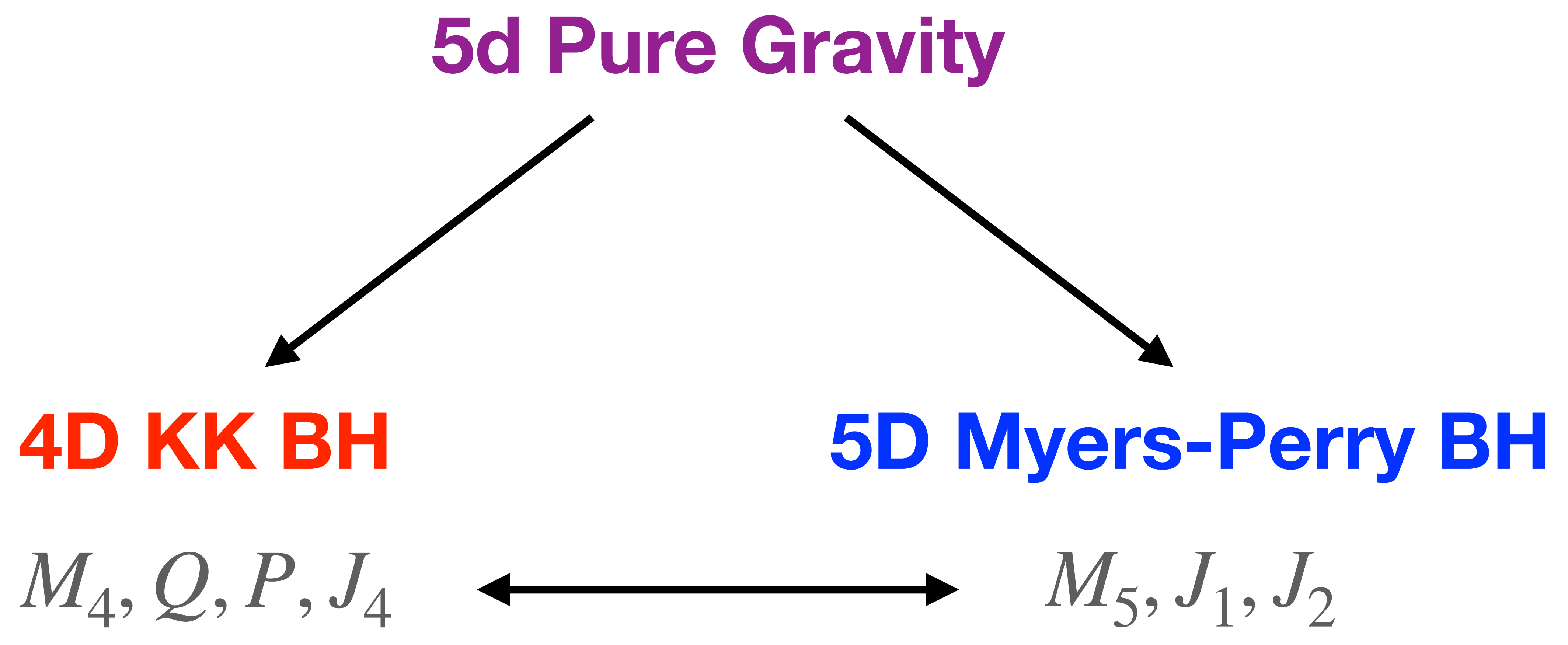
- The correspondence principle [Horowitz, Polchinski '96] does not guarantee that the extremal curve stays on one side. True for BH with near-horizon BTZ geometry [Aalsma, Cole, GS '19].

Spinning WGC?

[See also Aalsma's talk]

Rotating BHs

- Could there be similar constraints for rotating BHs? [Aalsma, GS, '22]. **Maybe not:**
  - Rotating BHs can lose energy via **superradiance**.
  - For pure gravity in $D \geq 6$, BHs w/ a given mass can have arbitrarily large J [Myers, Perry, 86].
- **But ….**
  - Spinning WGC for BTZ BH follows from c-theorem of the dual CFT [Aalsma, Cole, Loges, GS, '20]
  - In string theory, spin can sometimes be mapped to charge, e.g.,



Rotating BHs

- However, in string theory, spin can sometimes be mapped to charge, e.g.,

5d Pure Gravity

4D KK BH M_4, Q, P, J_4 $\longleftrightarrow$ **5D Myers-Perry BH** M_5, J_1, J_2

- The leading correction to Einstein gravity is the Gauss-Bonnet term:

$$L_{5D} = \frac{1}{2}R + \lambda\left(R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2\right)$$

Can consider KK BH with $J_4 = 0$ to fix the sign of λ using the WGC

[Aalsma, GS, '22]

Mapping Spins to Charges

[Aalsma, GS, ’22]

- Consider a **rotating dyonic** KK BH with (M_4, Q, P, J_4)

  $\rightarrow$ BH sitting at the tip of Taub-NUT $(r_{BH} \ll R_5)$

- Taking a **decompactification** limit, this becomes a 5D MP BH with (M_5, J_1, J_2).
- Map between parameters:

$$M_5 \sim M_4 - M_{\text{monopole}}$$

$$QP \sim \frac{1}{2}\left(J_1 + J_2\right)$$

$$J_4 \sim \frac{1}{2}\left(J_1 - J_2\right)$$

Can map 5d rotations to pure charges when $J_1 = J_2$!

- Calculate the leading higher derivative corrections to the extremality bound for the KK BH, either in 5D or its reduction to 4D, $R_{GB} \rightarrow 4-$derivative terms involving R, F, ϕ.

KK Black Hole

[Aalsma, GS, '22]

- **KK black hole:**

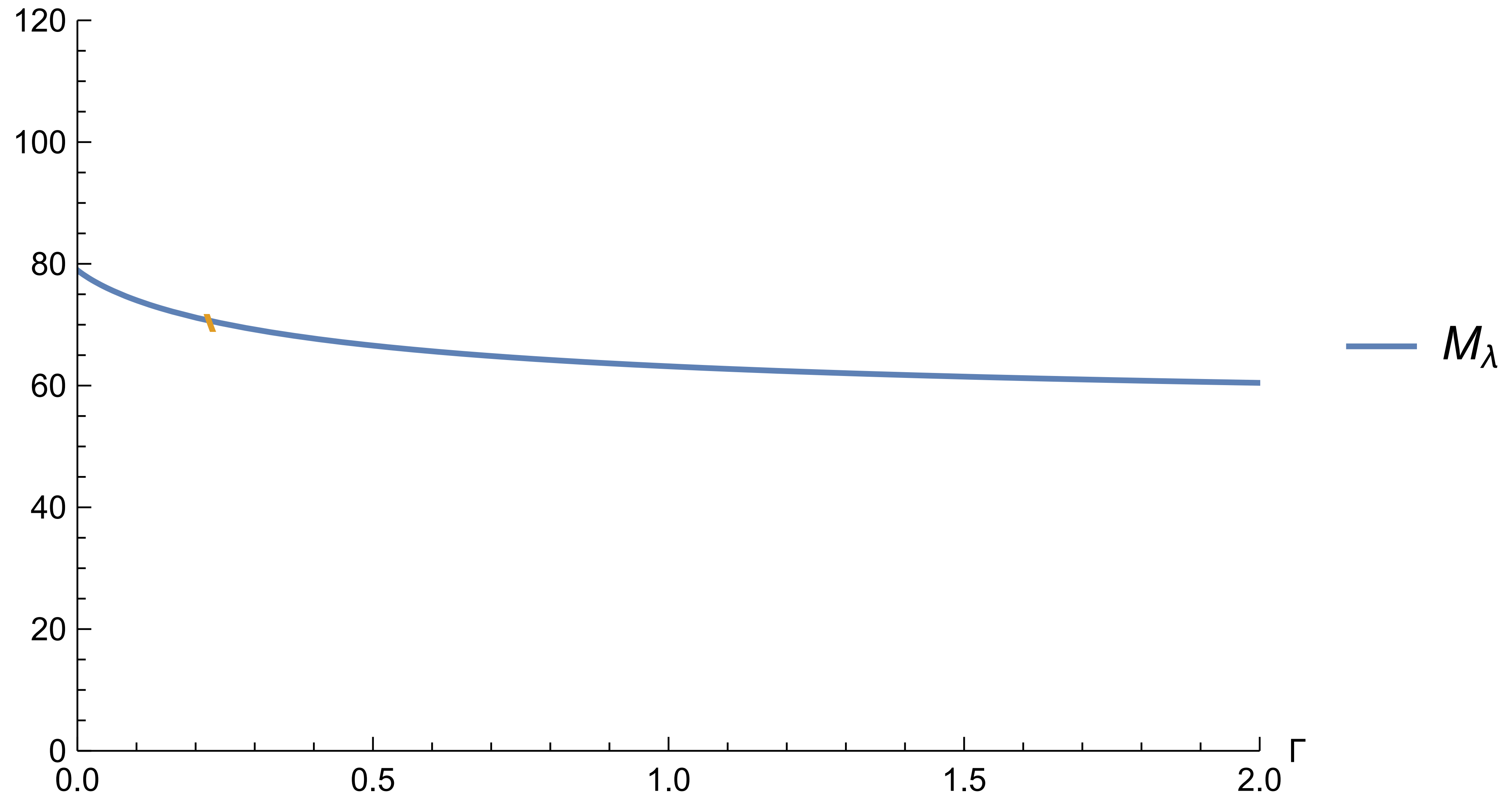
$$\delta M_4 \sim \int d^5x \sqrt{-g}\lambda R_{GB} = -\frac{8\pi^2\lambda R}{p}\frac{(1+\Gamma)}{(1-\Gamma)^2\sqrt{\Gamma^2-1}}\left(3\pi\Gamma^2 \text{sgn}(\Gamma-1) + (1-4\Gamma)\sqrt{\Gamma^2-1} + 6\Gamma^2 \arctan\left[\sqrt{\frac{\Gamma+1}{\Gamma-1}}\right]\right)$$

where $\Gamma = q/p$ $\quad Q = 4\pi\sqrt{\frac{q(q^2-4m^2)}{p+q}}\,, \quad P = 4\pi\sqrt{\frac{p(p^2-4m^2)}{p+q}}\,.$ Extremal BH: $m = 0$

- For $Q = P$, the expression simplifies:

$$\delta M_4^{\text{KK}}\Big|_{p=q} = -\frac{32\pi^2 R\lambda}{5qL}$$

- **δM_4 does not change sign for a fixed λ.**
- The WGC $\Rightarrow \lambda \geq 0$.



Myers-Perry Black Hole

[Aalsma, GS, '22]

- **MP black hole**:

$$\delta M_5 \sim \int d^5x\sqrt{-g}\,\lambda R_{GB} = -\frac{4\pi^2\lambda}{L}\left(\frac{J_1^2+J_2^2-6\,|J_1J_2|}{|J_1J_2|}\right)$$

indefinite sign!

- However, for $J_1 = \pm J_2$

$$\delta M_5 = +\frac{16\pi^2\lambda}{L} \geq 0$$

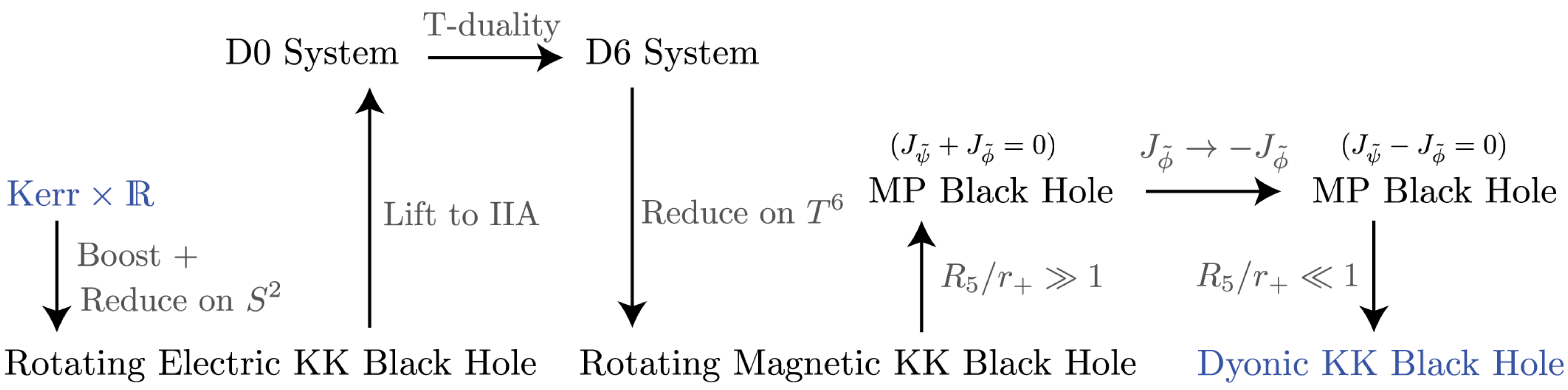
where we used the Charge WGC $\Rightarrow \lambda \geq 0$

- The extremality bound for rotating BH is **shifted negatively**.

Kerr BH

[Aalsma, GS, '22]

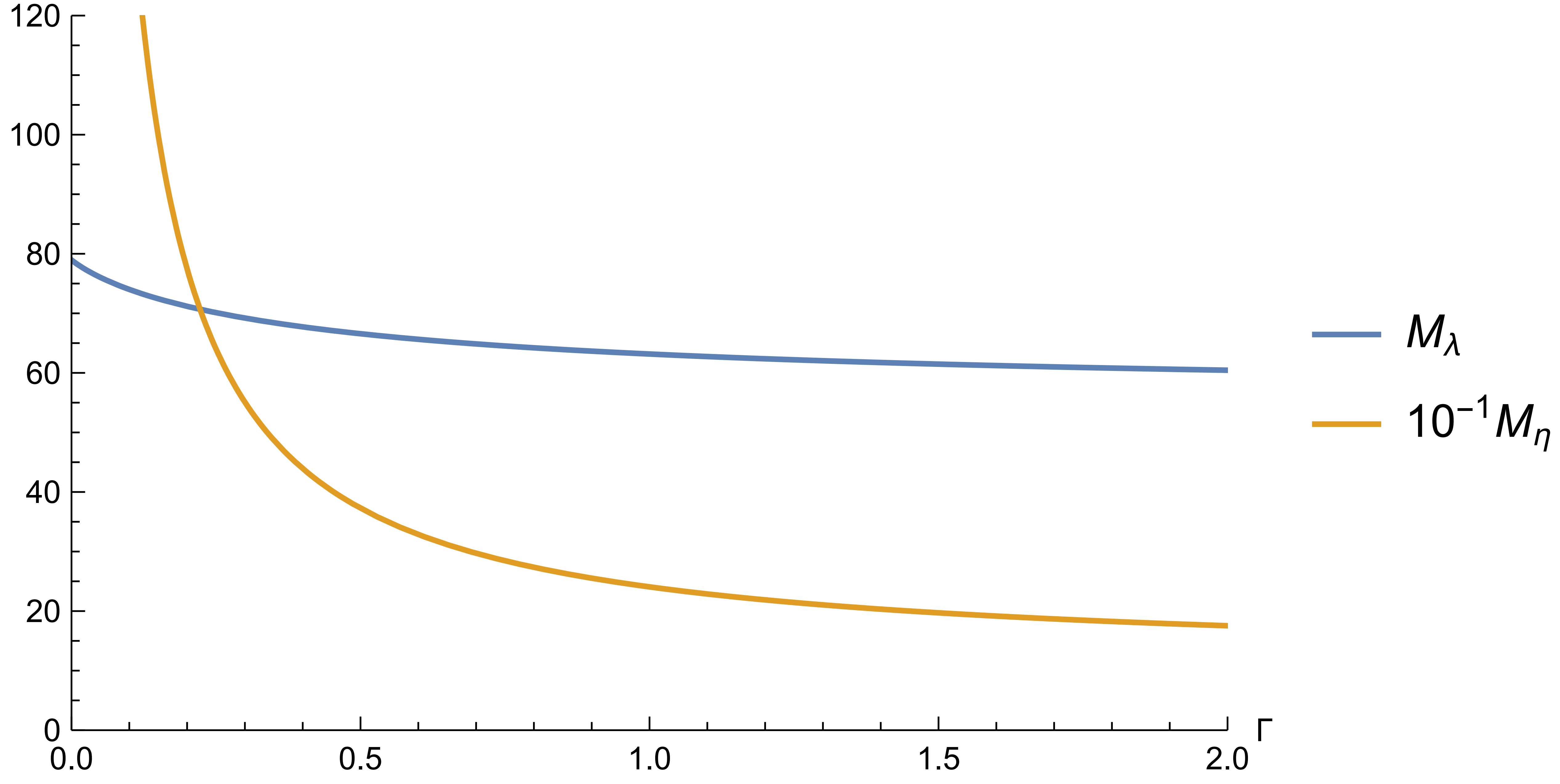
- A chain of dualities maps a Kerr BH to a non-rotating charged dyonic BH:



- Similar logic can fix corrections to Kerr BH. However, the Gauss-Bonnet term is topological in 4D, and so the leading correction is the 6-derivative operator:

$$\delta L = \frac{\lambda}{L} R_{abcd} R^{abcd} + \eta L \, R_{ab}{}^{cd} R_{cd}{}^{ef} R_{ef}{}^{ab}$$

Corrections to Extremality Bounds



	$\frac{\lambda}{L}\mathcal{R}^2$	$\eta L \mathcal{R}^3$
δM_4^{KK}	$-\frac{\lambda}{L}\mathcal{M}_\lambda$	$\eta L \mathcal{M}_\eta$
WGC:	$\lambda \geq 0$	$\eta \leq 0$
δM_5^{MP}	$\frac{\lambda}{L} 16\pi^2$	$\eta L \frac{192\pi^2}{7a^2}$
Sign:	$+$	-
$\delta M_4^{\mathrm{Kerr}}$	0	$\eta L \frac{8\pi}{7\hat{\alpha}^3}$
Sign:	n.a.	-

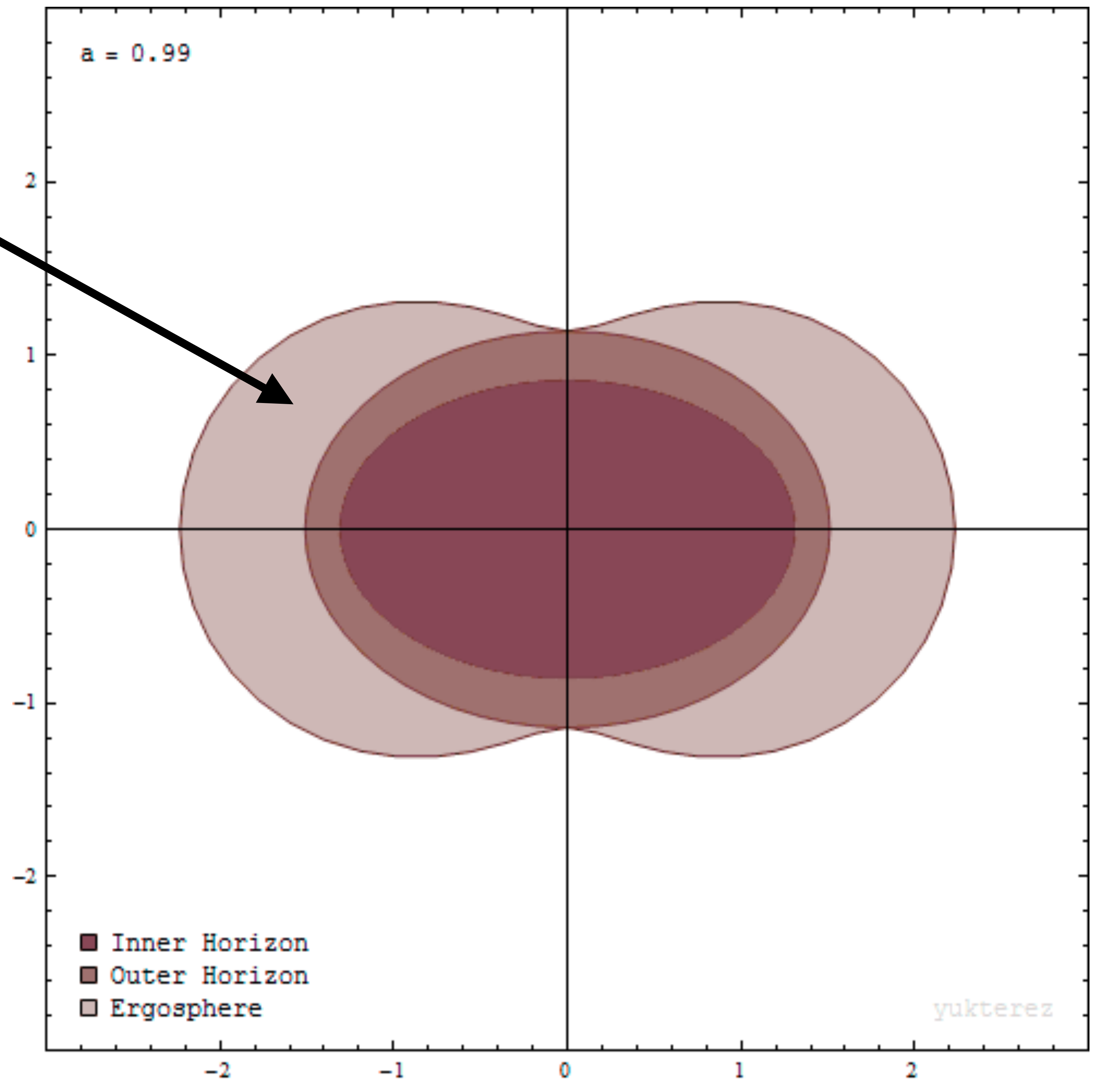
[Aalsma, GS, '22]

Superradiance

[Aalsma, GS, '22]

- Rotating BHs are unstable due to superradiance which occurs when there is an **ergosphere**:

$\omega < 0$



Extract energy ω and angular momentum j, BH can lose its mass if

$$dM = TdS + \Omega_i dJ^i \quad = TdS\frac{\omega}{\omega - j^i\Omega_i} \quad \leq 0$$

$$\Rightarrow \quad \omega \leq j^i \Omega_i$$

guaranteed if ∃ ergosphere!

- **How does the superradiant instability of rotating BHs manifest in the charged BH?**

Superradiance vs WGC

[Aalsma, GS, '22]

- Charged BHs have no ergosphere, but can lose energy in a similar sense if

$$\mathrm{d}M = T\mathrm{d}S + \Psi_q \mathrm{d}Q + \Psi_p \mathrm{d}P \quad \leq 0$$

- If the particle extracting energy ω and electric, magnetic charges (k_q, k_p) from the BH:

$$\frac{16\pi G_4 \omega}{k_q\sqrt{1+(P/Q)^{2/3}} + k_p\sqrt{1+(Q/P)^{2/3}}} \leq 1$$

charged superradiance

- This stronger charged superradiance condition **implies the WGC**: $\frac{16\pi G_4 \omega}{(k_q^{2/3}+k_p^{2/3})^{3/2}} \leq 1$

- The superradiance condition and the WGC coincide when $k_q/k_p = Q/P$.

- Phrased in term of superradiance, rotating and charged BHs are treated in unified manner.

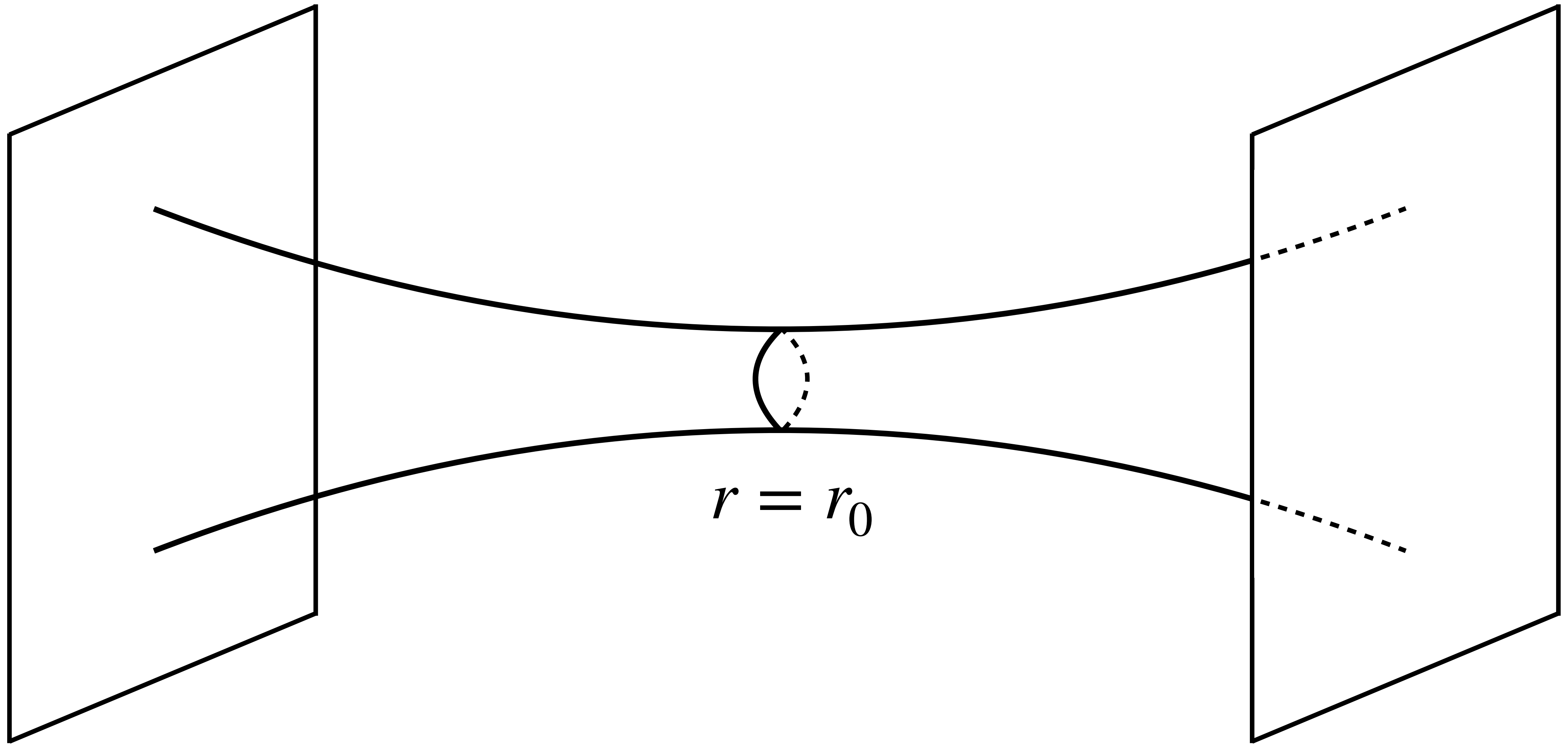
Axion WGC

Axionic WGC and Wormholes

- Without a clear notion of extremality for -1 form symmetries, wormholes have been used to set the WGC $f \cdot S_{inst} < \mathcal{O}(1) M_P$ [Andriolo, Huang, Noumi, Ooguri, GS '20]; [Andriolo, GS, Soler, Van Riet '22].

- The Giddings-Strominger wormhole is a solution to the Euclidean eoms for axion gravity:

$$\mathrm{d}s^2 = \left(1 - \frac{r_0^4}{r^4}\right)^{-1} \mathrm{d}r^2 + r^2\, \mathrm{d}\Omega_3^2 \qquad H = \frac{n}{2\pi^2}\mathrm{vol}_3 \qquad 24\pi^4 r_0^4 = n^2$$



Evidence for Axionic WGC

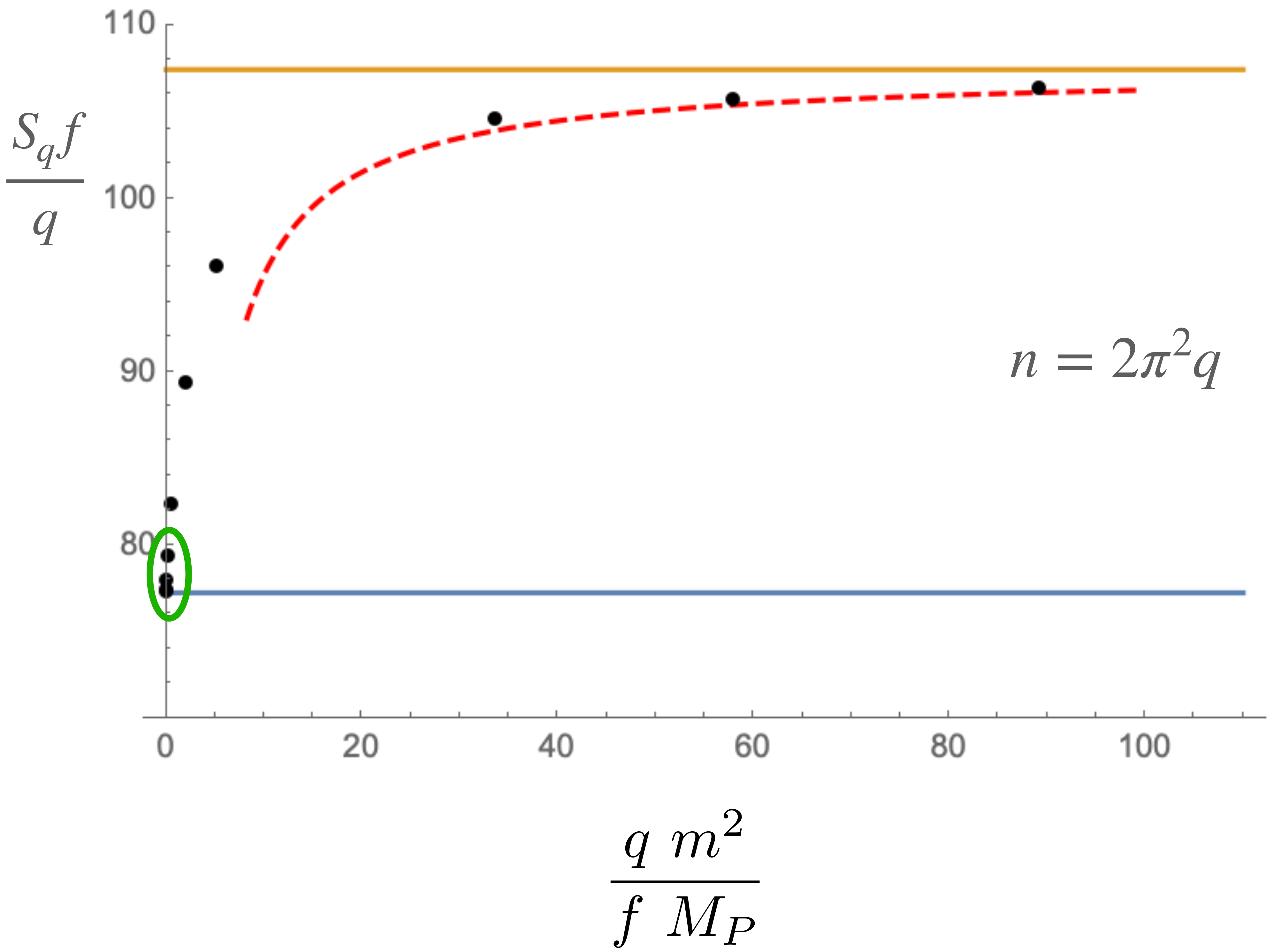
- The WGC is set by the action-to-charge ratio of a macroscopic semi-wormhole (considering axion-gravity and axion-dilaton-gravity) [Andriolo, Huang, Noumi, Ooguri, GS '20]; [Andriolo, GS, Soler, Van Riet '22].

$$\frac{S_{instanton}}{n} \lesssim \frac{S_{wormhole}}{n} = \frac{\sqrt{6}\pi}{4} \cdot \frac{M_P}{f}$$

axion gravity

- Action-to-charge ratio was shown to decrease with charge by considering leading irrelevant operators with signs fixed by unitarity/causality [Andriolo, Huang, Noumi, Ooguri, GS '20] and further by numerically solving wormhole solutions with general dilaton mass [Andriolo, GS, Soler, Van Riet '22]

[See Soler's talk]



Wormhole Stability

[Loges, GS, Sudhir, '22]

- Previous works (25+ years) on perturbative stability of axion wormholes have led to contradictory claims, casting doubts on their contributions to the Euclidean path integral.

	Frame	**Stable**	**Gauge-inv**	**j=0,1**	**B.C.**
Rubakov, Shvedov, '96	axion	No	No	physical	✗
Alonso, Urbano, '17	axion	Yes	Yes	physical	✗
Hertog, Truijen, Van Riet, '18	axion	No	Yes	pure gauge	✗
Loges, GS, Sudhir, '22	3-form	Yes	Yes	pure gauge	✓

Boundary Conditions and Gauge Invariance

- Under diffeomorphism, metric and axion/3-form perturbations are mixed. Physically meaningful conclusions can only be drawn on gauge-invariant perturbations.

- In analyzing scalar perturbations around the GS wormhole, the boundary conditions in the 3-form picture can be imposed more straightforwardly. Finite energy perturbations:

$$\int \delta H \wedge \star \delta H < \infty \,,$$

  which corresponds to:

$$\int \mathrm{d}\delta\theta \wedge \star \mathrm{d}\delta\theta < \infty \,,$$

- Metric perturbations vanish at the boundaries. Gauge invariant perturbations are Dirichlet in the H_3 picture [Loges, GS, Sudhir, '22], while in the θ picture, gauge invariant perturbations involve mixed b.c. [Hertog, Meanaut, Tielemans, Van Riet, to appear].

Wormhole Stability

[Loges, GS, Sudhir, '22]

- We determine the stability of GS wormhole by carrying out the following steps:
  - Parametrization of scalar perturbations and their boundary conditions.
  - Diffeomorphisms and physical degrees of freedom.
  - Quadratic action.
  - Integrate out non-dynamical and unphysical, gauge-dependent modes.
  - Analyze spectrum of remaining physical modes.

Steps akin to analyzing gauge invariant perturbations in inflationary cosmology.

But as we shall show, not only is the spectrum of perturbations

but **on-shell value of the quadratic action** is important for determining stability.

Conclusion: the Giddings-Strominger wormhole is perturbatively stable.

Summary

- The S-matrix bootstrap program and the Swampland program both aim to make precise the boundaries between consistent and inconsistent theories.
- The Swampland program provides some clear targets for positivity bounds.
- Sharpening the gravitational positivity bounds is important for proving swampland constraints.
- No spinning WGC because of superradiance, but dualities mapping rotation to charges $\Rightarrow$ charged superradiance $\Rightarrow$ WGC.
- WGC on charged BHs $\Rightarrow \lambda_{GB} \geq 0, \eta_{R^3} \leq 0 \Rightarrow$ correction of extremality bound of MP/Kerr BH.
- Axionic WGC which constrains axion inflation is a statement about wormhole fragmentation.
- Swampland constraints (if established) can be used in combination with duality to obtain new positivity bounds which are otherwise difficult to prove directly with amplitude techniques.