

Homework Sheet 8

MATH431 — Introduction to Modern Particle Theory

(Dr Thomas Teubner)

1. Suppose that we assign the proton and the neutron to the Isospin doublet in the fundamental triplet representation of $SU(3)$, and identify the electric charge with the combination

$$Q_{electric} = \alpha T_{3_{Isospin}} + \beta Y_{hypercharge},$$

where $T_{3_{Isospin}}$ and $Y_{hypercharge}$ are the diagonal generators of $SU(3)$ with the normalisation as used in the lecture.

- (a) Determine the coefficients α and β .
 - (b) Given the coefficients found in (a), work out the charge assignments for the sextet, octet and decuplet representations of $SU(3)$.
2. Consider the special unitary group $SU(5)$.
- (a) How many diagonal generators of the Lie algebra are there? Write down a representation of the diagonal generators in terms of 5×5 hermitian matrices.
 - (b) What is the dimension of the group?
 - (c) What is the fundamental representation of $SU(5)$? Write down its decomposition in terms of the maximal subgroup $SU(3) \times SU(2) \times U(1)$.
 - (d) Find the product and the decomposition under the maximal subgroup $SU(3) \times SU(2) \times U(1)$ of the fundamental times the anti-fundamental representations of $SU(5)$.
3. Weak decays of leptons:

- (a) Show that the (charge-lowering) weak current of the form

$$\bar{u}_e \gamma^\mu \frac{1}{2} (1 - \gamma^5) u_\nu$$

involves only left-handed electrons (or right-handed positrons).

- (b) Draw the (lowest order) Feynman diagram for the muon decay in the Standard Model,

$$\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e.$$

Name and explain the different elements of the Feynman diagram and give their corresponding algebraic expressions. (You can leave out the labels for the momenta and spins of the external particles.)

- (c) Given that the weak coupling g is of the order of the electromagnetic coupling e ($g \sin \theta_W = e$ with the sine of the Weinberg angle $\sin \theta_W \approx 0.5$), why are the weak decays suppressed compared to typical electromagnetic or strong decays?
- (d) In analogy to the μ decay, discuss the possible decays of the τ lepton.