Homework Sheet 1

MATH431 — Introduction to Modern Particle Theory

(Dr Thomas Teubner)

1. Write down the Lagrangian for a particle of mass m in a potential $V(r,\phi)$ when referred to planar polar coordinates (r,ϕ). Show that the equations of motion are

$$m\ddot{r} - mr\dot{\phi}^2 = -\frac{\partial V}{\partial r} \quad \text{and} \quad mr\ddot{\phi} + 2m\dot{r}\dot{\phi} = -\frac{1}{r}\frac{\partial V}{\partial \phi}\,.$$

Derive the conservation of angular momentum in the plane and obtain the usual formula v^2/r for centripetal acceleration.

2. (a) Show that if the Hamiltonian is independent of a generalised coordinate q_0, then the conjugate momentum p_0 is a constant ot the motion. Such coordinates are called *cyclic coordinates*. Give two examples of physical systems which have a cyclic coordinate.

(b) Show that in 3 dimensional spherical polar coordinates the Hamiltonian of a particle of mass m moving in a potential $V(\vec{x})$ is

$$H = \frac{1}{2m}\left(p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_\phi^2}{r^2\sin^2\theta}\right) + V(\vec{x})\,.$$

Now show that $p_\phi = \textit{constant}$ if $\partial V/\partial\phi \equiv 0$ and give a physical interpretation of this result.

3. The potential function of a two-dimensional harmonic oscillator is

$$V(x,y) = \frac{1}{2}\,k\,(x^2+y^2)\,.$$

(a) Write down the Lagrangian of this system.

(b) Write down the Euler-Lagrange equations of motion.

(c) Write down the Hamiltonian.

(d) Write down the Lagrangian and Hamiltonian in polar coordinates (r,ϕ) with $(x = r\cos\phi\,,\ y = r\sin\phi)$.

(e) How many constants of the motion are there?
What are they?