$g-2$
Prospects for
Light by Light from the Lattice

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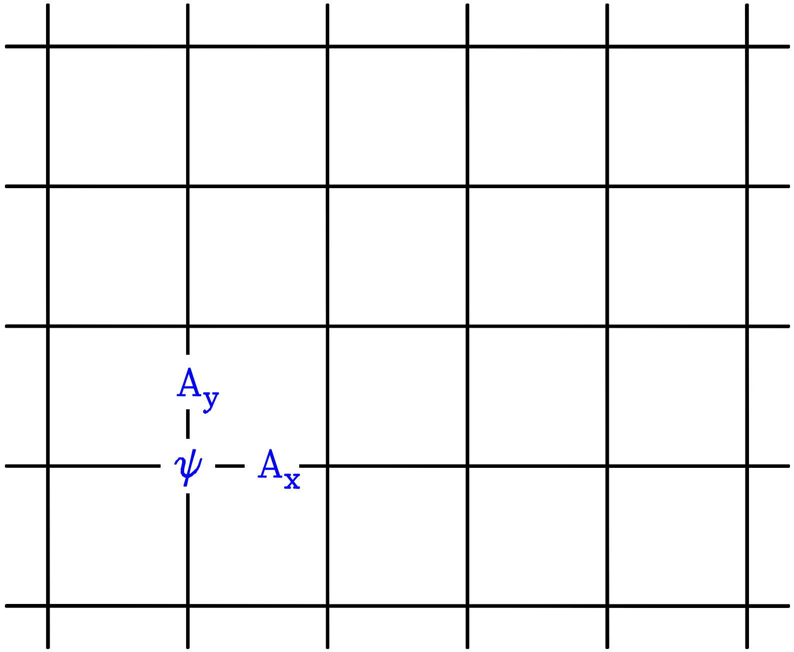
Introduction

- Non-technical introduction to the lattice
- Discussion of vacuum polarisation contribution to $g-2$
- Light by Light calculation (just starting, don't yet know how hard it will be).

Lattice Gauge Theory

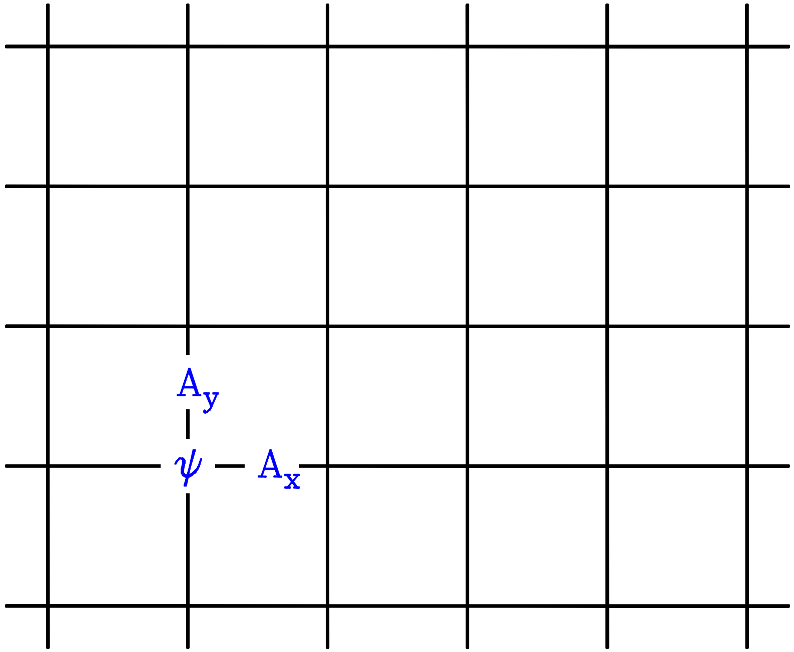


Computational field theory.

We can't put a continuum in a computer — divide up space and time into discrete steps (like solving a differential equation, or doing an integral).

Discrete 4-d Euclidean lattice, lattice spacing a.

Lattice Gauge Theory



Quark fields ψ on the sites.
Gauge fields A_μ on the links.

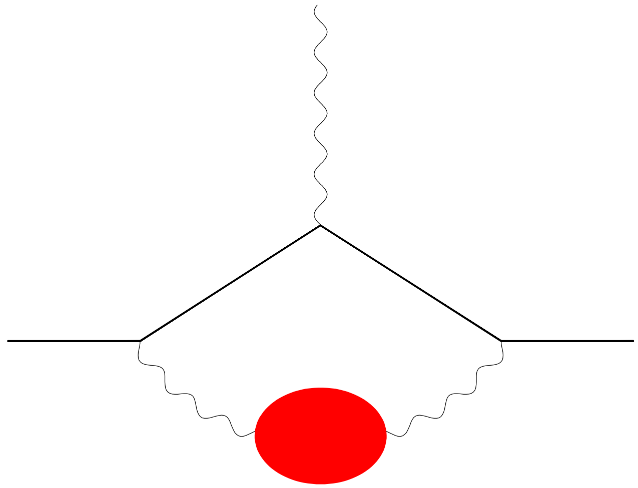
A regularisation of field theory, preserving exact gauge invariance.

Lattice Gauge Theory

We generate examples of vacuum gluon-field A_μ configurations. (Nowadays with the effects of virtual quark loops taken into account).

We can then explicitly propagate quarks through these vacuum configurations, and “measure” any quantity built up from these quark fields.

Vacuum Polarisation



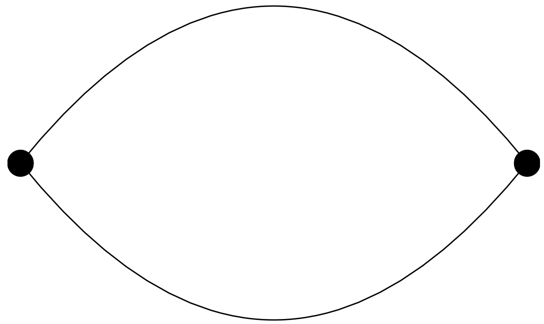
Example: Calculating the vacuum polarisation in lattice gauge theory.

$$\langle J_\mu(0) J_\nu(x) \rangle$$

Vacuum Polarisation

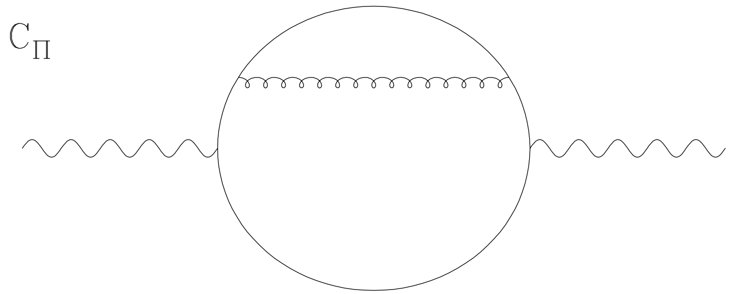
$$\langle J_\mu(0) J_\nu(x) \rangle$$

Introduce a quark at the origin, allow it to propagate through a vacuum configuration. Get the propagator from 0 to all x. Use symmetry to find anti-quark propagator too, no extra work.



Stitch together the propagators (multiply and trace), get $\langle J_\mu(0) J_\nu(x) \rangle$.
Can Fourier transform to find $\langle J_\mu J_\nu \rangle$ in momentum space.
Each propagator calculated gives vacuum polarisation at all q.
One measurement, all q.

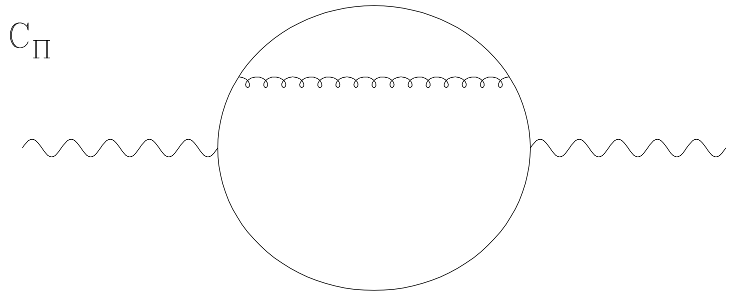
Vacuum Polarisation

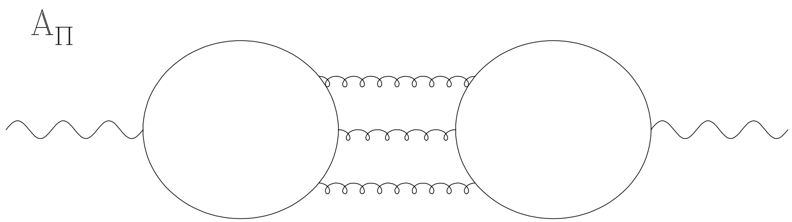


Although I only drew the quarks we explicitly add in by hand, because we propagate through vacuum configurations, these automatically become dressed by all possible gluon and gluon + quark bubble diagrams.

Vacuum Polarisation

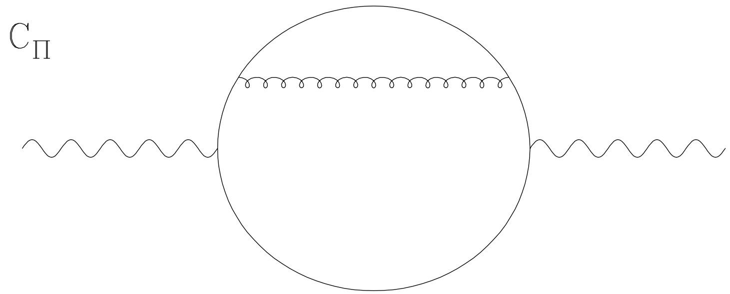
Really should be two terms present:

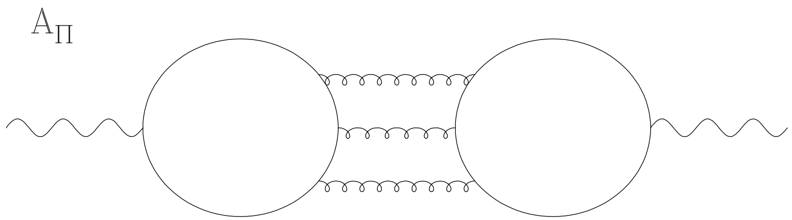




$$
\begin{aligned}
-12\pi^2\Pi(Q^2) &= \sum_f e_f^2\, C_\Pi(Q^2, m_f) \\
&+ \sum_{f,f'} e_f e_{f'}\, A_\Pi(Q^2, m_f, m_{f'})
\end{aligned}
$$

Vacuum Polarisation





$$e_u + e_d + e_s = \frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$$

A_Π term vanishes in flavour $SU(3)$ limit.
Quark-line connected piece is easiest.

Light by Light

For light by light, we need QCD contribution to

$$\langle J_{\nu_1}(x_1) J_{\nu_2}(x_2) J_{\nu_3}(x_3) J_\mu(x_q) \rangle$$

or its Fourier transform.
Rest of calculation by QED perturbation theory. (See other lattice talk for alternative approach.)
More complicated than $\langle JJ \rangle$, which just depends on a single momentum.

Light by Light

For a given q the light by light amplitude depends on two internal photon momenta (eg k_1, k_2). Final momentum fixed by momentum conservation.

Our hope is to measure $\langle JJJJ \rangle$ for enough values of the photon momentum to be able to calculate the light by light contribution to $g - 2$.

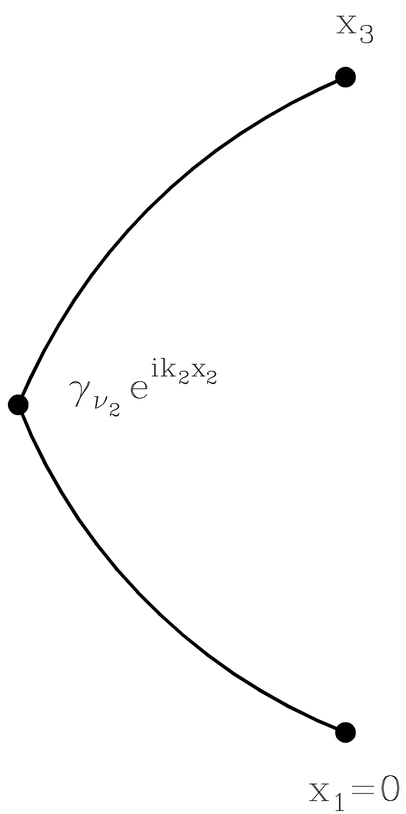
Light by Light

At first sight this sounds daunting - looks like we will have to map out an 8-dimensional function (two internal photon momenta).

Made easier by the fact that a δ function contains all momenta. In the ordinary vacuum polarisation this meant that one measurement gave all q. Now this means that when we put in a value for k_2 and q, each measurement we make gives $\langle JJJJ \rangle$ for all possible k_1.

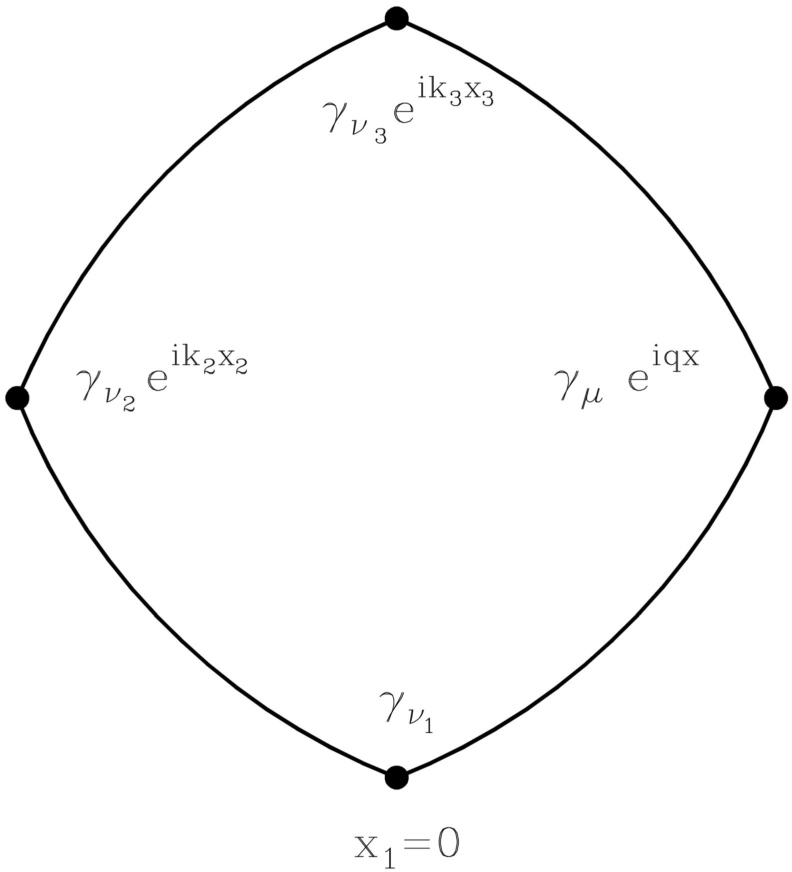
So we only need enough measurements to map out the dependence of $\langle JJJJ \rangle$ on k_2. Do we need to map out all 4 dimensions of k_2? No, if we exploit rotation invariance we only need enough k_2 values to map out k_2^2 and $k_2 \cdot q$. (Virtuality and angle with q)

Light by Light



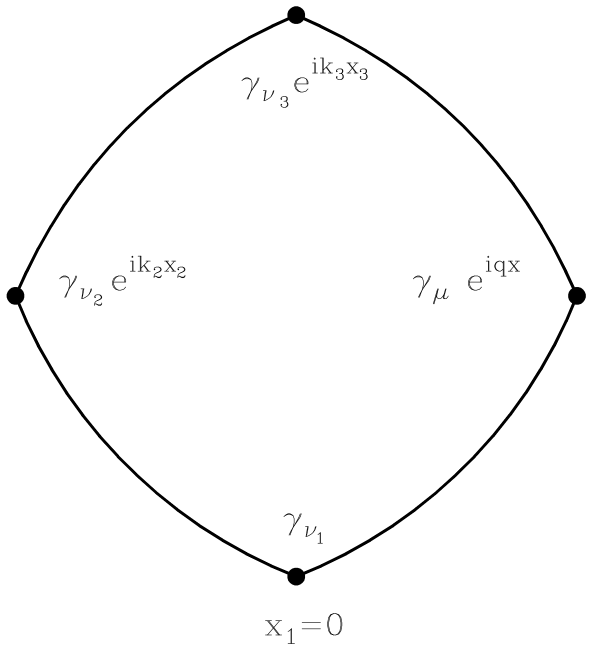
Basic object to program: a 3-point function, point source (contains all momenta), followed by a momentum transfer k_2.

Light by Light



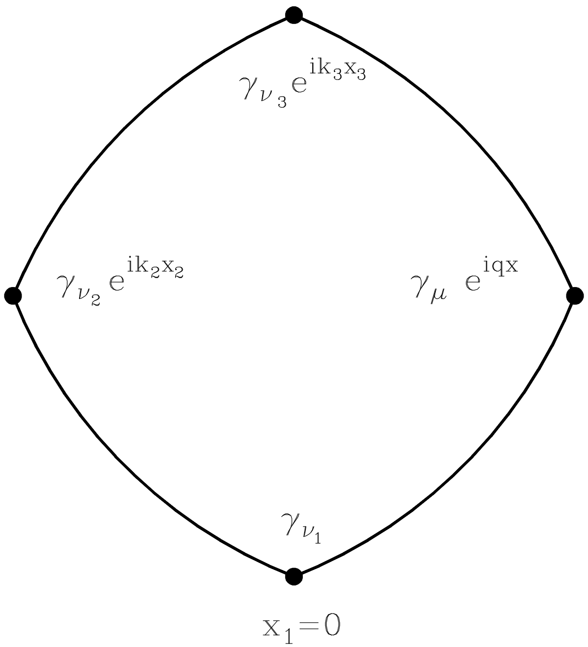
Measure two of these 3-point objects, one with momentum transfer k_2, and one with momentum transfer q, and join them together. Gives a $\langle JJJJ \rangle$ measurement (for all k_1).

Light by Light



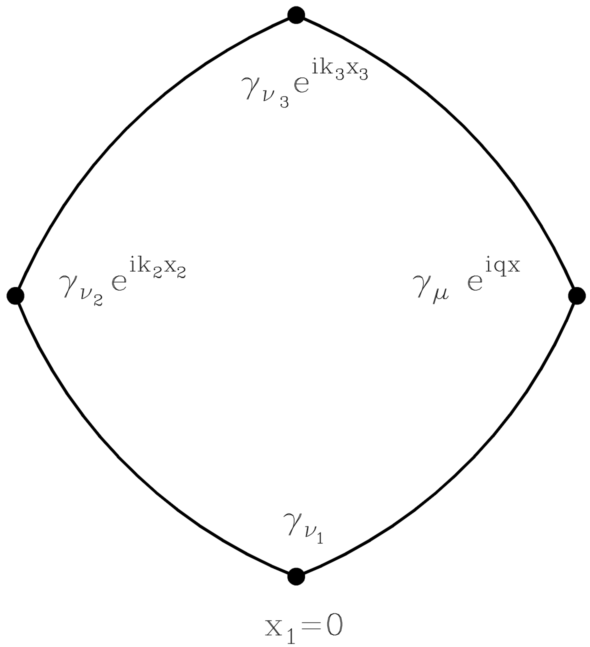
Remember, though I've only drawn in the quarks we add in the measuring process, the fact that we propagate them through an ensemble of vacuum configurations means that you should imagine this loop dressed with all possible gluon transfers, and quark bubbles, so that we get the full QCD hadron $\langle JJJJ \rangle$.

Light by Light



Not quite full: Current calculation only considers the diagrams with all 4 photons attached to the same quark line, $\sum_f e_f^4$
Flavour $SU(3)$ suppresses most other diagrams, because there are quark loops with a single photon attached. $\sum_f e_f$ argument.

Light by Light



However there are diagrams with 2 photons on one quark line, and the other two on another quark line, which we are missing, and which aren't suppressed by interference between flavours. Some hope from large N_c. Should think about ways to include neglected diagrams.

Extrapolations

Even after measuring $< JJJJ >$ will be faced with extrapolations before we have a physical number.

- Lattice spacing to zero.
- Sea quark mass to m_u, m_d, m_s.
- Momentum extrapolations to $k \sim m_\mu$. Lowest momentum allowed by boundary conditions about 450 MeV.

Conclusions

- We have been given some computer resources in Germany for this calculation Thanks Jülich!. Calculation just beginning. Don't know yet how difficult it will be, what problems we may encounter.
- Currently just looking at the single quark-line connected contribution. Is this sufficient to give a useful result?
- Should try to find ways to include the other diagrams.
- Limitations in computer time still mean that lattice results need to be extrapolated to physical points.