Hadronic light-by-light scattering contribution to the muon $g-2$ from lattice QCD : Methodology

Masashi Hayakawa (Nagoya University)

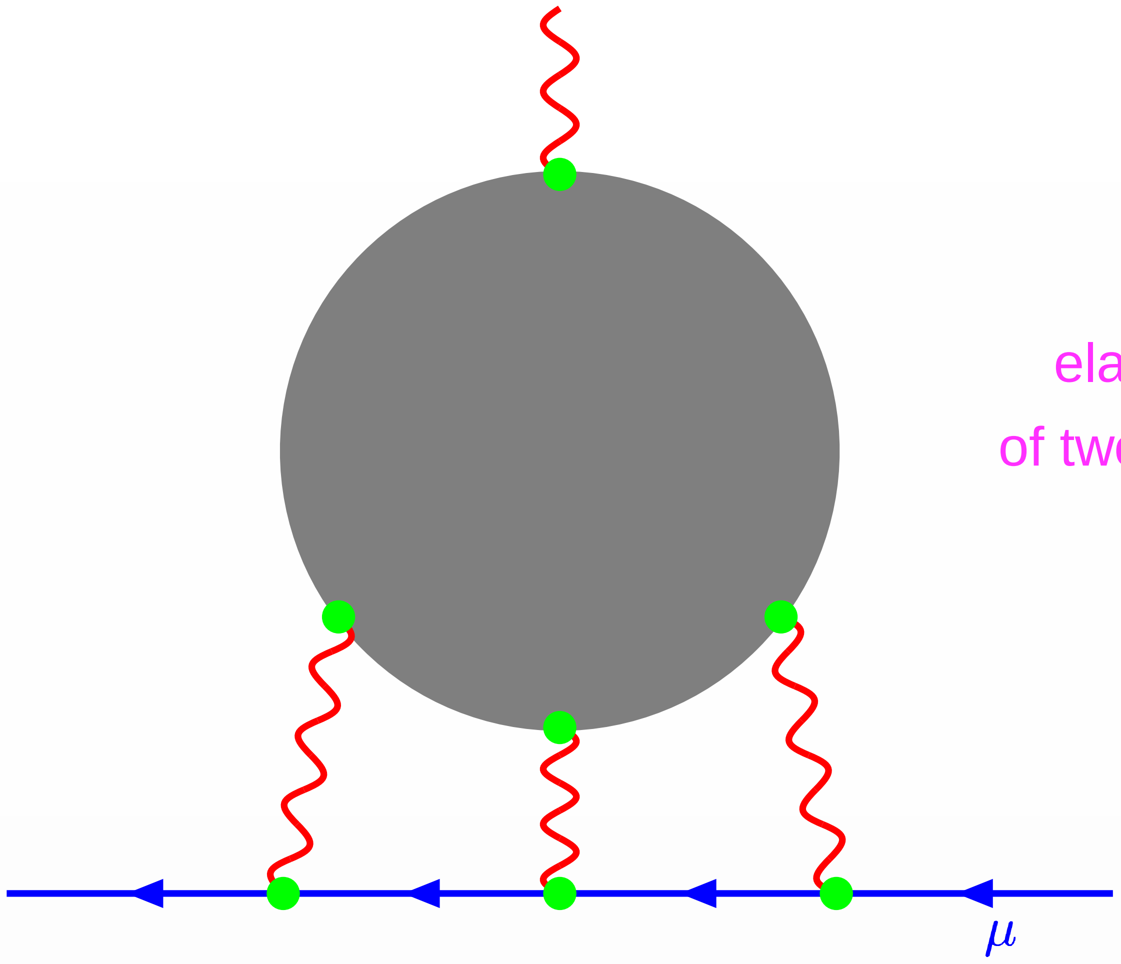
in collaboration with

T. Blum (Connecticut), S. Chowdhury (Connecticut), T. Doi (Kentucky),
T. Izubuchi (Kanazawa), T. Yamazaki (BNL) and N. Yamada (KEK)

@Glasgow University (25 Octorber, 2007)

Topic

Our target here is the hadronic light-by-light scattering contribution to the muon $g-2$;



elastic scattering amplitude
of two off-shell photons by QCD

Topic

Here I discuss a practical method to calculate the hadronic light-by-light scattering contribution by means of lattice QCD simulation.

The plan of my talk is as follows;

1. A basic material in lattice QCD simulation (pion mass measurement for instance)
2. Difficulty in a naïve method to calculate hadronic light-by-light scattering contribution
3. Nonperturbative QED method (hep-lat/0509016)
4. Hybrid method ($\Leftarrow$ nonperturbative QED method $+$ a basic idea in perturbative QED method)
5. Remark (on diagrams with multi-quark loops)

What is done in lattice simulation

(1) A basic material of lattice simulation

Lattice QCD simulation aims to study the properties of hadrons quantitatively through the virtual worlds realized in computers.

Here,

- Most of hadron properties are governed by nonperturbative QCD dynamics. The lattice QCD attempts to calculate them.
- The process of “study” is very similar to that of experiments;
  - ‡ strategy design ,
  - ‡ measurement ,
  - ‡ analysis .
- The limitation for experiment comes mostly from the capability of computational resources in simulations;

  statistical uncertainty

  systematic uncertainty finite volume, lattice artifacts, large quark masses, etc.

What is done in lattice simulation

Here I talk about a strategy in our hand that may be able to measure the hadronic light-by-light scattering contribution by means of lattice QCD simulation. We start with pion mass measurement to illustrate what is done there.
Note :

- I will use the language in the continuum theory apart from Wilson line, called link variable, as a dynamical variable of $SU(3)_C$ gauge field

$$U_\mu(x) \quad \simeq \quad P\exp\left(-\int_0^1 dt\, G_\mu(x+ta\widehat{\mu})\right) .$$

Pion mass measurement

To study the mass of (charged) pion, we compute

$$C(t) \;=\; \sum_{\vec{x}} \left(e^{i\vec{0}\cdot\vec{x}} = 1 \right) \left\langle \left(\overline{d}\gamma_5 u \right)(t,\, \vec{x})\; \left(\overline{u}\gamma_5 d \right)(0) \right\rangle_{\mathrm{QCD}} \,,$$

where, in Euclidean space,

$$\langle O(\overline{q},\, q\,;\, U) \rangle_{\mathrm{QCD}} \;\equiv\; \frac{1}{Z_0} \int DU \int Dq D\overline{q}\, \exp\left(-S_{\mathrm{QCD}}[\overline{q},\, q;\, U] \right)\, O(\overline{q},\, q;\, U)\,,$$

$$Z_0 \;\equiv\; \int DU \int Dq D\overline{q}\, \exp\left(-S_{\mathrm{QCD}}[\overline{q},\, q;\, U] \right)\,.$$

Since pion is the lightest particle in the pseudoscalar channel, we expect the following asymptotic behavior for $t \to \infty$

$$C(t) \to C_0\, \exp\left(-m_\pi t \right) \left(+C_1\, e^{-M_1 t} + \cdots \right) \quad \left(m_\pi \ll M_1,\, \cdots \right).$$

Pion mass measurement

To avoid the numerical integration over q, $\overline{q}$, we perform it by hand as follows;

$$\begin{aligned}
&\left\langle \left(\overline{d}\gamma_5 u\right)(x)\ \left(\overline{u}\gamma_5 d\right)(0)\right\rangle_{\mathrm{QCD}} \\
&= \lim_{\eta,\,\overline{\eta}\to 0} \left\langle \left(\overline{d}\gamma_5 u\right)(x)\ \left(\overline{u}\gamma_5 d\right)(0) \exp\left[\sum_{q=u,\,d}\int d^4y\ \left(\overline{\eta}_q q + \overline{q}\eta_q\right)\right]\right\rangle_{\mathrm{QCD}} \\
&= \lim_{\eta_q,\,\overline{\eta}_q\to 0} \left(\frac{\partial^R}{\partial\eta_d(x)}\gamma_5 \frac{\partial^L}{\partial\overline{\eta}_u(x)}\right)\left(\frac{\partial^R}{\partial\eta_u(0)}\gamma_5 \frac{\partial^L}{\partial\overline{\eta}_d(0)}\right)(0) \\
&\qquad \times \left\langle \exp\left[\sum_{q=u,\,d}\int d^4y\ \left(\overline{\eta}_q q + \overline{q}\eta_q\right)\right]\right\rangle_{\mathrm{QCD}} .
\end{aligned}$$

From

$$S_{\mathrm{QCD}}[\overline{q},\, q,\, U] = S_{\mathrm{YM}}[U] + \sum_{q=u,\,d,\,s}\int d^4y\, \overline{q}\left(\mathcal{D}[U] + m_q\right) q\,,$$

Pion mass measurement

the Gaussian integral over q, $\overline{q}$ can be carried out giving

$$\left\langle \left(\overline{d}\gamma_5 u\right)(x)\ \left(\overline{u}\gamma_5 d\right)(0)\right\rangle_{\mathrm{QCD}}$$
$$= \lim_{\eta_q,\,\overline{\eta}_q\to 0}\left(\frac{\partial^R}{\partial\eta_d(x)}\gamma_5\frac{\partial^L}{\partial\overline{\eta}_u(x)}\right)\left(\frac{\partial^R}{\partial\eta_u(0)}\gamma_5\frac{\partial^L}{\partial\overline{\eta}_d(0)}\right)(0)$$
$$\times\left\langle \exp\left[\sum_{q=u,\,d}\int d^4y\int d^4z\,\overline{\eta}_q(y)\left(\mathcal{D}[U]+m_q\right)^{-1}(y,\,z)\eta_q(z)\right]\right\rangle_{\mathrm{QCD}},$$

where $\langle,\ \rangle_{\mathrm{QCD}}$ should be regarded as

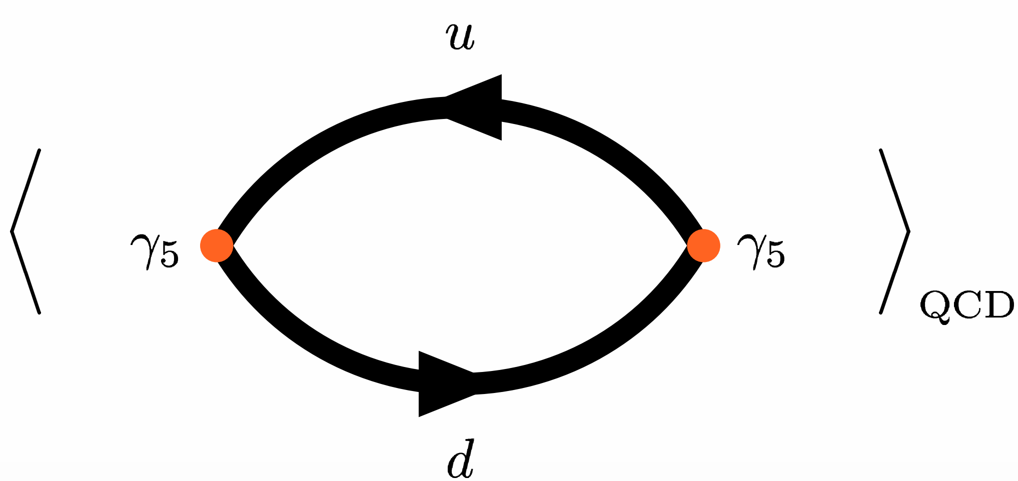
$$\langle O(q,\,\overline{q},\,U)\rangle_{\mathrm{QCD}} \equiv \frac{1}{Z_0}\int DU\prod_{q=u,\,d,\,s}\mathrm{Det}\left(\mathcal{D}[U]+m_q\right)\exp\left(-S_{\mathrm{YM}}[U]\right)\widehat{O}[U]\,.$$

Pion mass measurement

By taking the derivative, we get

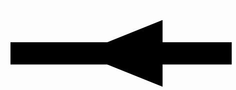
$$\left\langle \left(\overline{d}\gamma_5 u\right)(x)\ \left(\overline{u}\gamma_5 d\right)(0)\right\rangle_{\rm QCD}$$
$$= -\left\langle {\rm tr}\left(\left(\mathcal{D}[U]+m_u\right)^{-1}(x,\,0)\,\gamma_5 \left(\mathcal{D}[U]+m_d\right)^{-1}(0,\,x)\,\gamma_5 \right) \right\rangle_{\rm QCD},$$

where ${\rm tr}$ is over colors and spinors. This would be expressed by a diagram



,

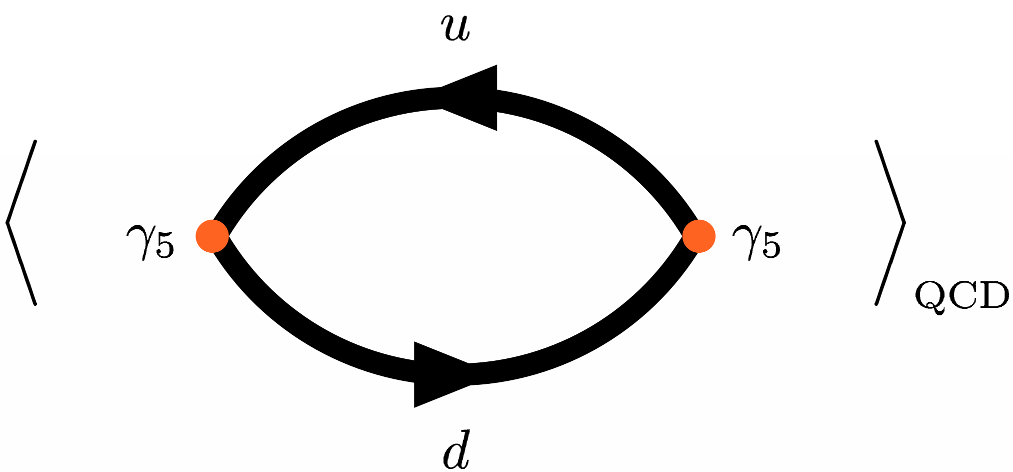
where



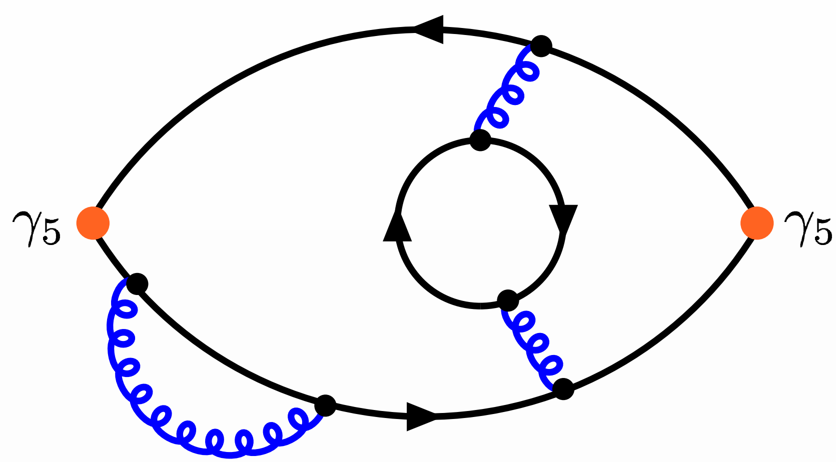
$$\Leftrightarrow \quad \left(\mathcal{D}[U]+m\right)^{-1}.$$

Pion mass measurement

If the average $\langle , \rangle$ is performed for the above diagram, we get the contribution from all relevant diagrams in QCD



$$= \sum_{\text{all connected diagrams}}$$



On average

The number of gluon configurations is infinite. The ensemble average over $U_\mu(x)$ is approximated by a sample average over a set $\mathfrak{U} \equiv \left\{U^{(i)}\right\}_i$;

$$\frac{1}{Z_0}\int DU \prod_{q=u,\,d,\,s} \mathrm{Det}\left(\mathcal{D}[U]+m_q\right)\,\exp\left(-S_{\mathrm{YM}}[U]\right)\,\widehat{O}[U] \quad\Rightarrow\quad \sum_{U^{(i)}\in\mathfrak{U}} \widehat{O}[U^{(i)}]\,.$$

Here $\mathfrak{U}$ is generated according to the probability density

$$\mathcal{P}[U] \equiv \frac{1}{Z_0} \prod_{q=u,\,d,\,s} \mathrm{Det}\left(\mathcal{D}[U]+m_q\right)\,\exp\left(-S_{\mathrm{YM}}[U]\right)\,.$$

On average

1. The generation of $\mathfrak{U}$ must resort to a big project of each lattice group.
2. Each user can study observables of his/her interest using commonly shared $\mathfrak{U}$.
3. I assume that we have $\mathfrak{U}$ which satisfies the conditions
   - sufficient number of configurations ($\Rightarrow$ good statistics) ,
   - sufficient number of $\mathfrak{U}(a, V, m_q)$ depending on parameters
     - ‡ a : lattice spacing (smaller one is better)
     - ‡ V : space-time volume (larger one is better)
     - ‡ m_q : current quark masses (smaller one is better)

   \+ fermions respecting chiral properties as possible ($\Rightarrow$ less systematic errors) .

On $(\mathcal{D}[U]+m)^{-1}$

The most time-consuming part in every measurement is to get "$(\mathcal{D}[U]+m)^{-1}$" for each $U \in \mathfrak{U}$.

- Note that we do not try to obtain all of the matrix elements $\left[(\mathcal{D}[U]+m)^{-1}\right]_{IJ}$ $(I = \left(x_{(1)}, c_{(1)}, \alpha_{(1)}\right), J = \left(x_{(2)}, c_{(2)}, \alpha_{(2)}\right))$. Rather, we calculate the vector

$$v[s]_I \equiv \sum_J \left[(\mathcal{D}[U]+m)^{-1}\right]_{IJ} s_J \,,$$

for a given "source vector" s by solving a linear difference equation

$$(\mathcal{D}[U]+m)\, v = s \,.$$

A machinery (subroutine) to solve this equation is called a "solver" here.

On $(\mathcal{D}[U]+m)^{-1}$

- In the present example, a point-like source can be used;

$$s^{(0,\,c_0,\,\alpha_0)}(x,\,c,\,\alpha) \quad\equiv\quad \delta_{x,\,0}\,\delta_{c,\,c_0}\,\delta_{\alpha,\,\alpha_0}\;,$$

By working the solver for this source vector for each $c_0 = 1,\,2,\,3,$ $\alpha_0 = 1,\,2,\,3,\,4,$ a part of matrix elements of $\left(\mathcal{D}[U]^{-1}+m\right)$ can be obtained from $v[s^{(0,\,c_0,\,\alpha_0)}]$ using the linearity of the equation;

$$(\mathcal{D}[U]+m)^{-1}\,(x,\,c,\,\alpha;\,0,\,c_0,\,\alpha_0) = v[s^{(0,\,c_0,\,\alpha_0)}](x,\,c,\,\alpha)\,.$$

Here only several column vectors are obtained. So, naïvely speaking, to get all of the matrix elements $\left[(\mathcal{D}[U]+m)^{-1}\right]_{IJ}$, the solver must be executed a huge number ($10^6 \sim 10^7$) times !

On $(\mathcal{D}[U]+m)^{-1}$

- For $(\mathcal{D}[U]+m_d)^{-1}(0,\, c_0,\, \alpha_0;\, x,\, c,\, \alpha)$, we use the γ_5-hermiticity for $\mathcal{D}[U]$ that is satisfied by most of lattice Dirac operator (Wilson fermion, Domain wall fermion, overlap fermion);

$$\mathcal{D}[U]^\dagger = \gamma_5 \mathcal{D}[U] \gamma_5 \,,$$

which leads

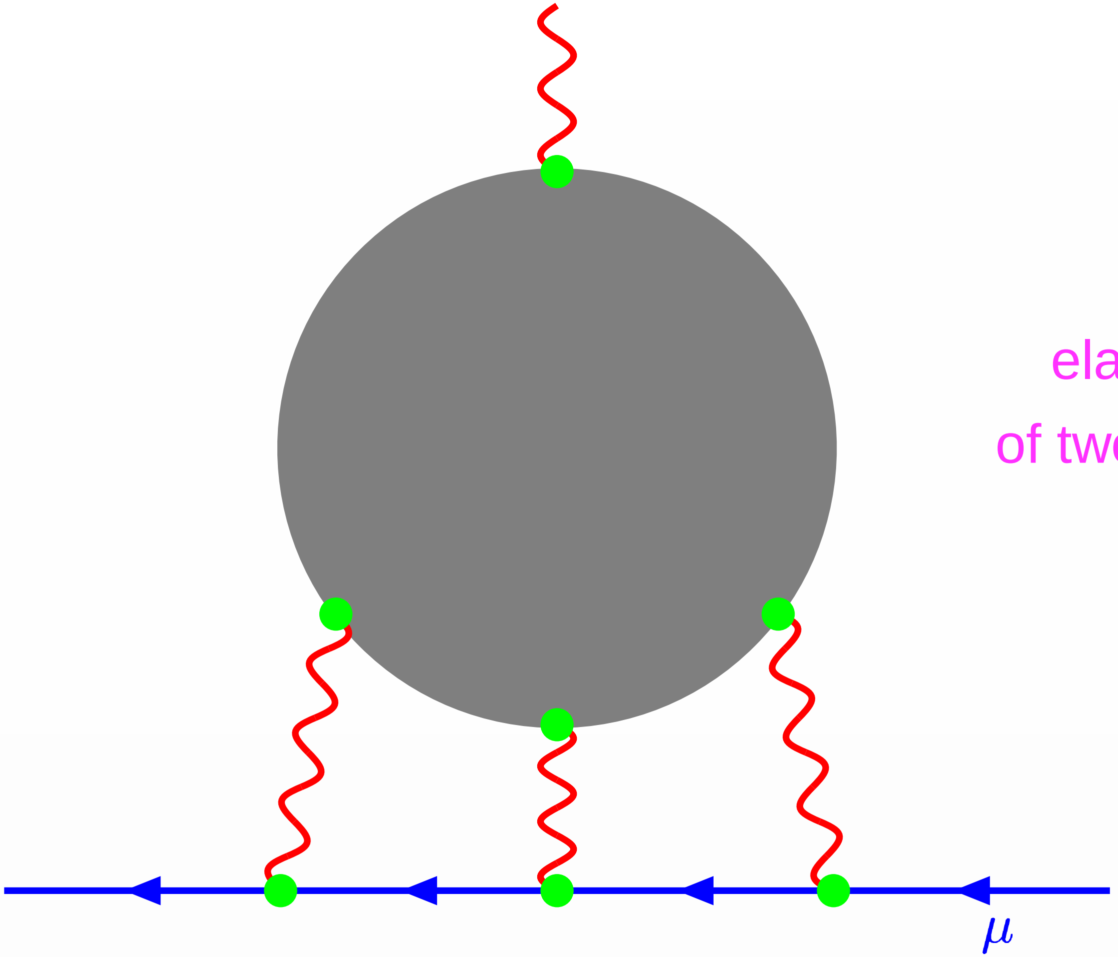
$$\begin{aligned}
&(\mathcal{D}[U]+m_d)^{-1}(0,\, c_0,\, \alpha_0,\, x,\, c,\, \alpha) \\
&\quad = \sum_{\beta,\,\delta=1}^{4} \left(\gamma_5^T\right)_{\alpha\delta} \Big((\mathcal{D}[U]+m_d)^{-1}(x,\, c,\, \delta;\, 0,\, c_0,\, \beta) \Big)^{\star} \left(\gamma_5^T\right)_{\beta\alpha_0} \,.
\end{aligned}$$

Difficulty in naïve approach

(2) Difficulty in naïve approach

Here I explain a naive method and why it is not practical for the lattice simulation

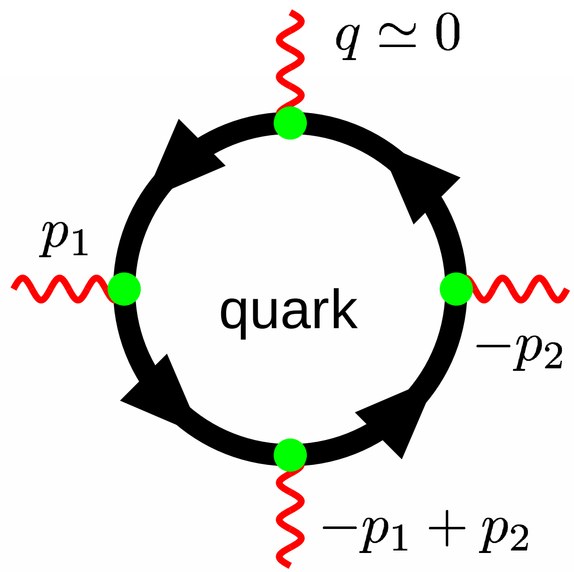
Let's recall our target



elastic scattering amplitude
of two off-shell photons by QCD

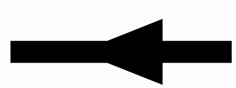
Difficulty in naïve approach

The issue would be how to evaluate the blob, i.e., the expectation value of four hadronic electromagnetic currents ($j_\mu^{\rm had} = \frac{2}{3}\overline{u}\gamma_\mu u - \frac{1}{3}\overline{d}\gamma_\mu d - \frac{1}{3}\overline{s}\gamma_\mu s + \cdots$) in QCD
From the lesson in Section (1), we are inclined to consider the quantity

$\Big\langle$ 

$\Big\rangle_{\rm QCD}$,

where

 $\Leftrightarrow \quad (\mathcal{D}[U] + m)^{-1}$,

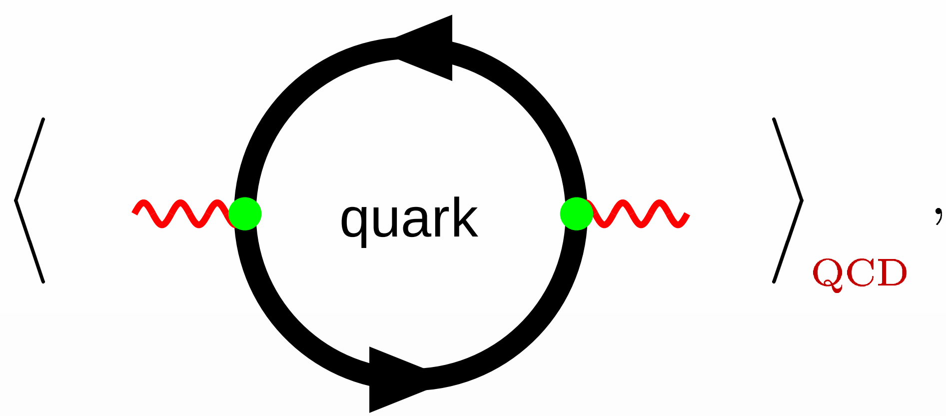
as before.

Difficulty in naïve approach

Recent works

- T. Blum, Phys. Rev. Lett. **91**, 052001 (2003) [arXiv:hep-lat/0212018].
- M. Gockeler, R. Horsley, W. Kurzinger, D. Pleiter, P. E. L. Rakow and G. Schierholz, Nucl. Phys. Proc. Suppl. **129**, 293 (2004).
- E. Shintani, *et.al.* [JLQCD Collaboration], arXiv:0710.0691 [hep-lat].

shows that hadronic vacuum polarization function (two-point function of hadronic electromagnetic currents)



can be calculated by means of lattice simulation.

This fact will turn out to be important for our methodology.

Difficulty in naïve approach

But,

1. Calculation of four-point function itself is a hard task for lattice simulation.
2. Since the light-by-light contribution involves two independent loop momenta, we have to repeat the calculation of four-point function an enormous number N times, where

$$\begin{aligned} N &\sim (\text{total number of independent momenta})^2 \\ &\sim (\text{total number of sites in a lattice})^2 \\ &\sim \left(32 \times 16^3\right)^2 \sim 1.7 \times 10^{10} \left(\left(64 \times 32^3\right)^2 \sim 4.4 \times 10^{12}\right) . \end{aligned}$$

3. More practical ways is to get full matrix elements $\left[(\mathcal{D}[U] + m)^{-1}\right]_{IJ}$ and to form a loop with four vertices inserted. As we have seen before, this requires us to work the solver $n \equiv 3 \times 4 \times \sqrt{N} \simeq (10^6 \sim 10^7)$ times for each $U \in \mathfrak{U}$. (All-to-all propagator method may reduce cost significantly.)

Nonperturbative QED method

(3) Nonperturbative method

Here I explain the nonperturbative method to calculate the hadronic light-by-light scattering contribution to the muon $g - 2$.

We put the photon field on the lattice in the following manner;

1. We employ non-compact lattice $U(1)$ gauge theory, where the dynamical variable for photons is $U(1)$ gauge potential $A_\mu(x)$.

2. We do quenched QED calculation, where no production of fermions occur through virtual photons. Note that it is not an approximation in our context. Rather, “quenched” is required for our method to work.

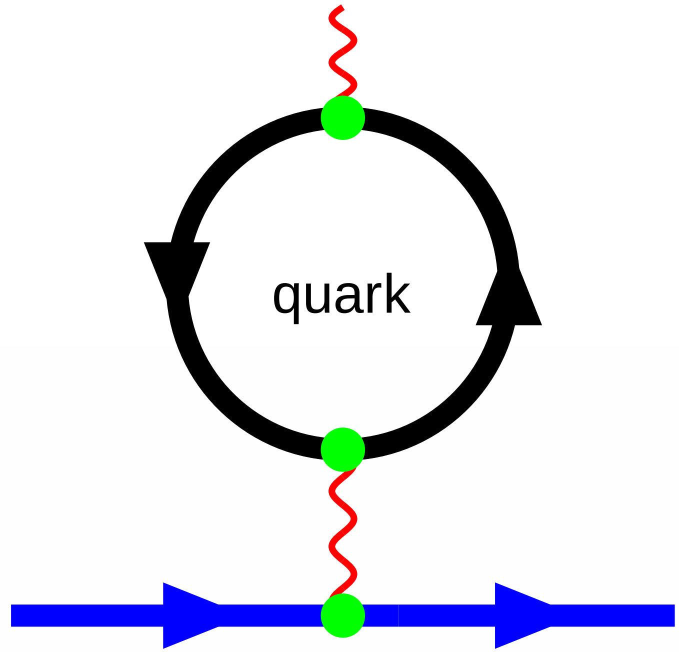
Nonperturbative QED method

The advantages of use of non-compact formulation are as follows;

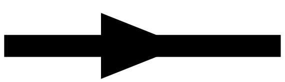
1. No self-coupling among photons.
   - Less diagrams that stem from lattice artifact.
   - We can obtain the free photon propagator in an analytic form.
2. Quenched non-compact QED formulation allows us to obtain a set of photon configurations with no auto-correlation even for weak coupling $\alpha \sim \frac{1}{137}$ very swiftly.

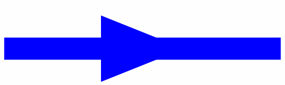
Nonperturbative QED method

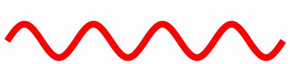
$$\Big\langle\Big\langle$$



$$\Big\rangle_{\mathrm{QCD}}\Big\rangle_{\text{quenched QED}} ,$$

 $\Leftrightarrow$ $(\mathcal{D}[U_{\mathrm{QCD}}, Q_q A] + m_q)^{-1} \left(Q_u = \frac{2}{3}, Q_d = Q_s = -\frac{1}{3}\right) ,$

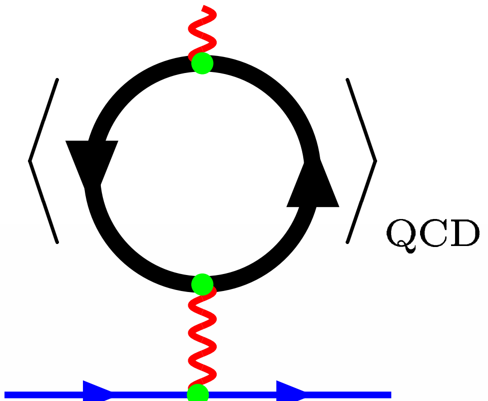
 $\Leftrightarrow$ $(\mathcal{D}[Q_\mu A]^{-1} + m_\mu) \quad (Q_\mu = -1) ,$

 $\Leftrightarrow$ free photon line .

Let's look at this quantity closely in perturbation theory w.r.t. QED.

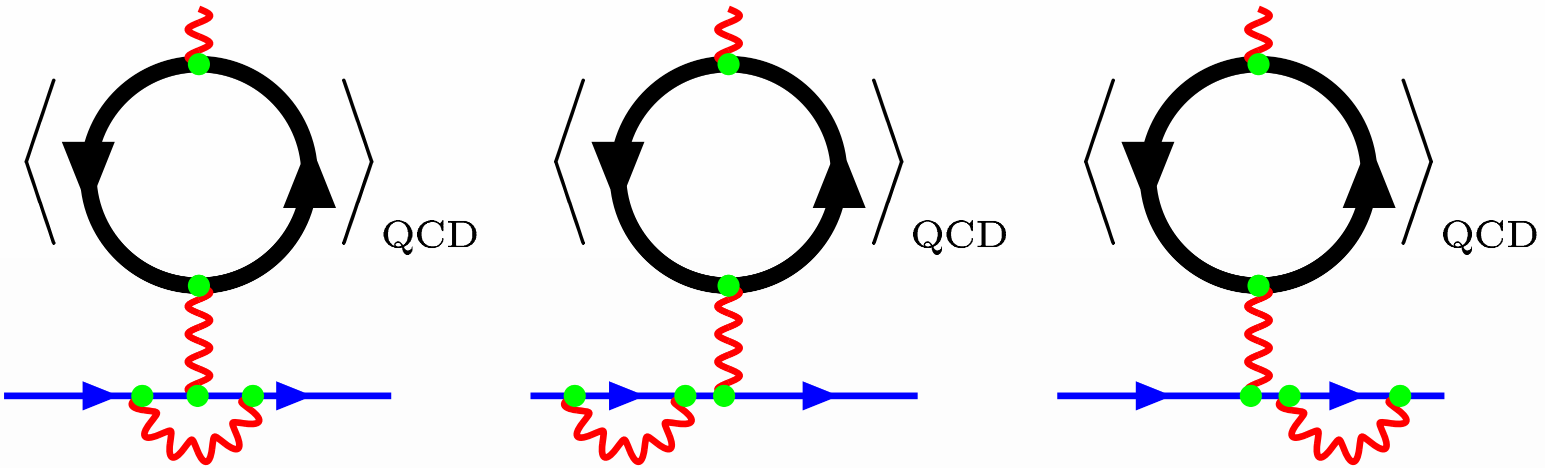
Nonperturbative method

At $\mathcal{O}(\alpha_{\rm em})$,



In the above, the bold black line denotes $(\mathcal{D}[U_{\rm QCD}]+m)^{-1}$.

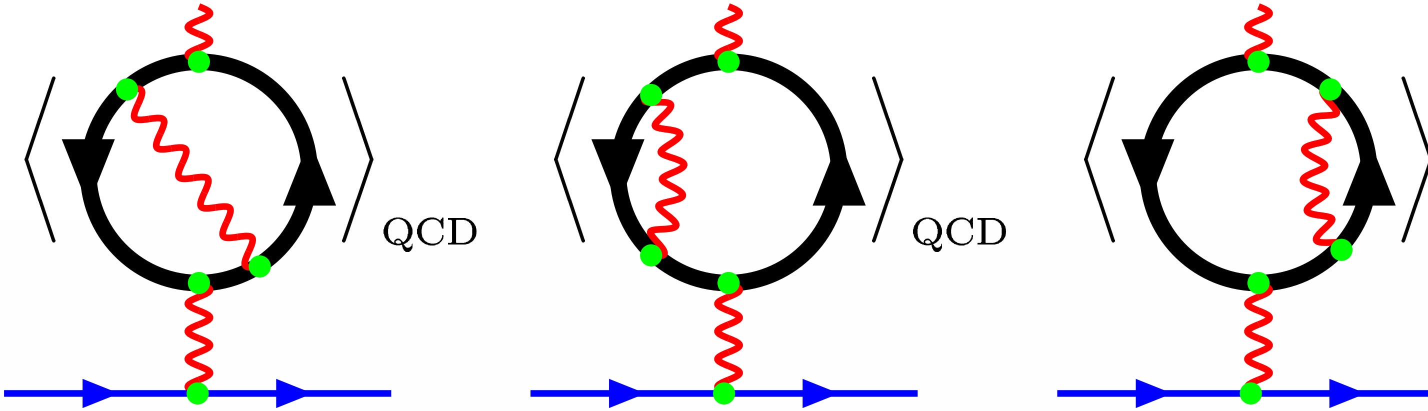
At $\mathcal{O}(\alpha_{\rm em}^{2})$,



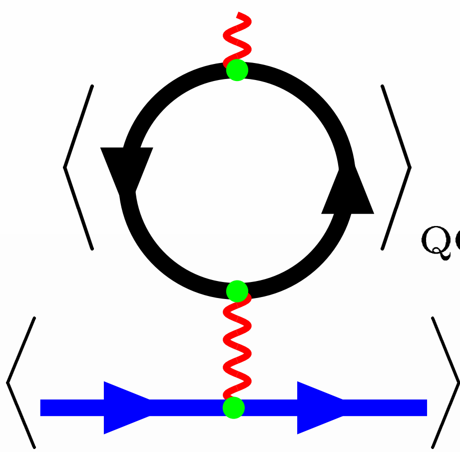
(to be continued to the next page)

Nonperturbative QED method

At $\mathcal{O}(\alpha_{\rm em}^2)$,

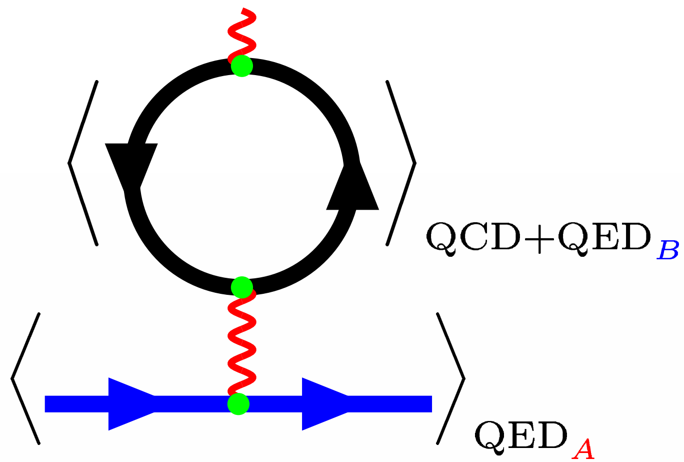


All of those diagrams at $\mathcal{O}(\alpha_{\rm em}^2)$ can be summarized as $\mathcal{O}(\alpha_{\rm em}^2)$-term in the following diagram

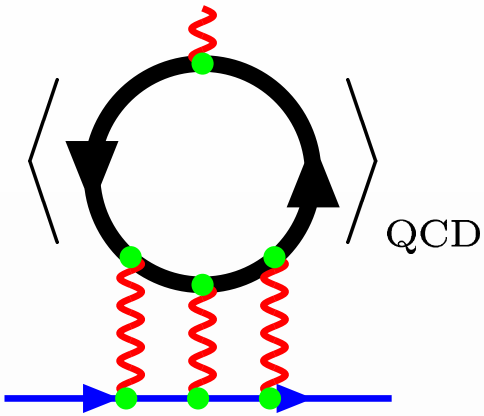


Nonperturbative QED method

The $\mathcal{O}(\alpha_{\mathrm{em}}^3)$ contribution comes from two types of diagrams.
The first type is $\mathcal{O}(\alpha_{\mathrm{em}}^3)$-term in the previous diagram

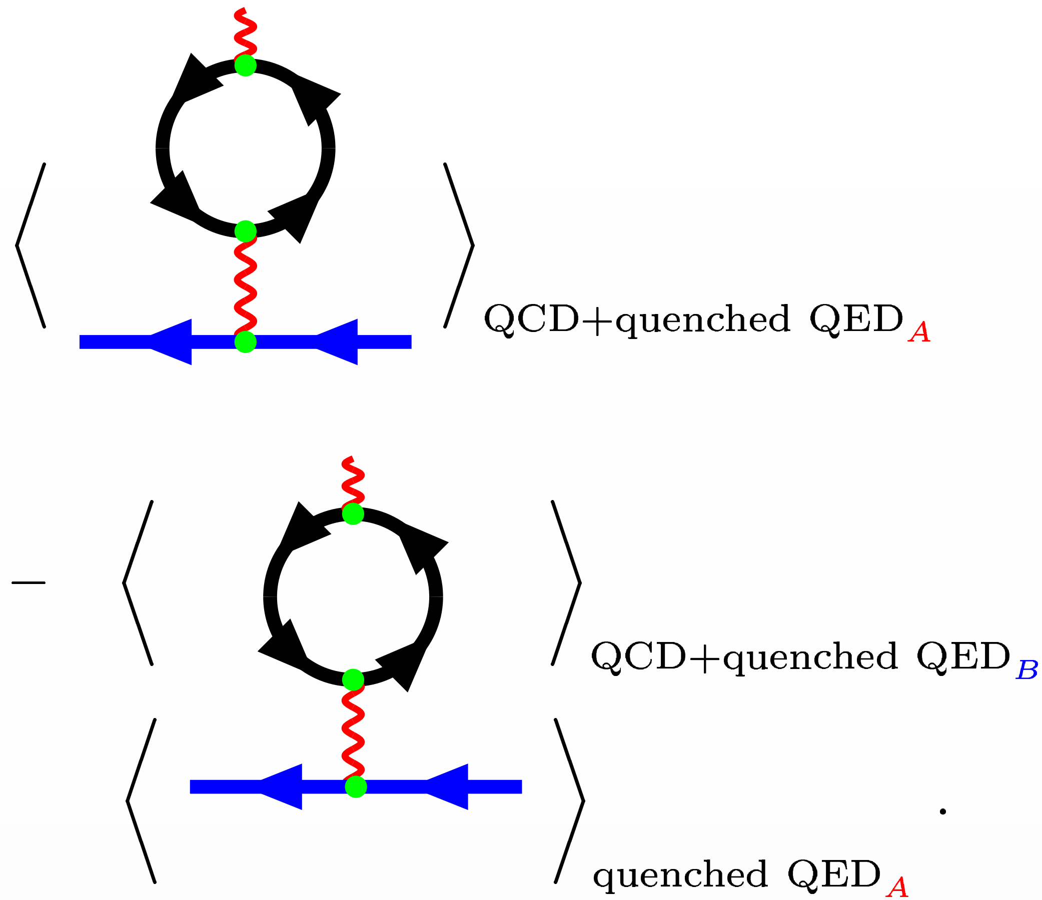


The other type is exactly what we want



Nonperturbative QED method

Thus, the hadronic light-by-light scattering diagram will emerge as the leading term of



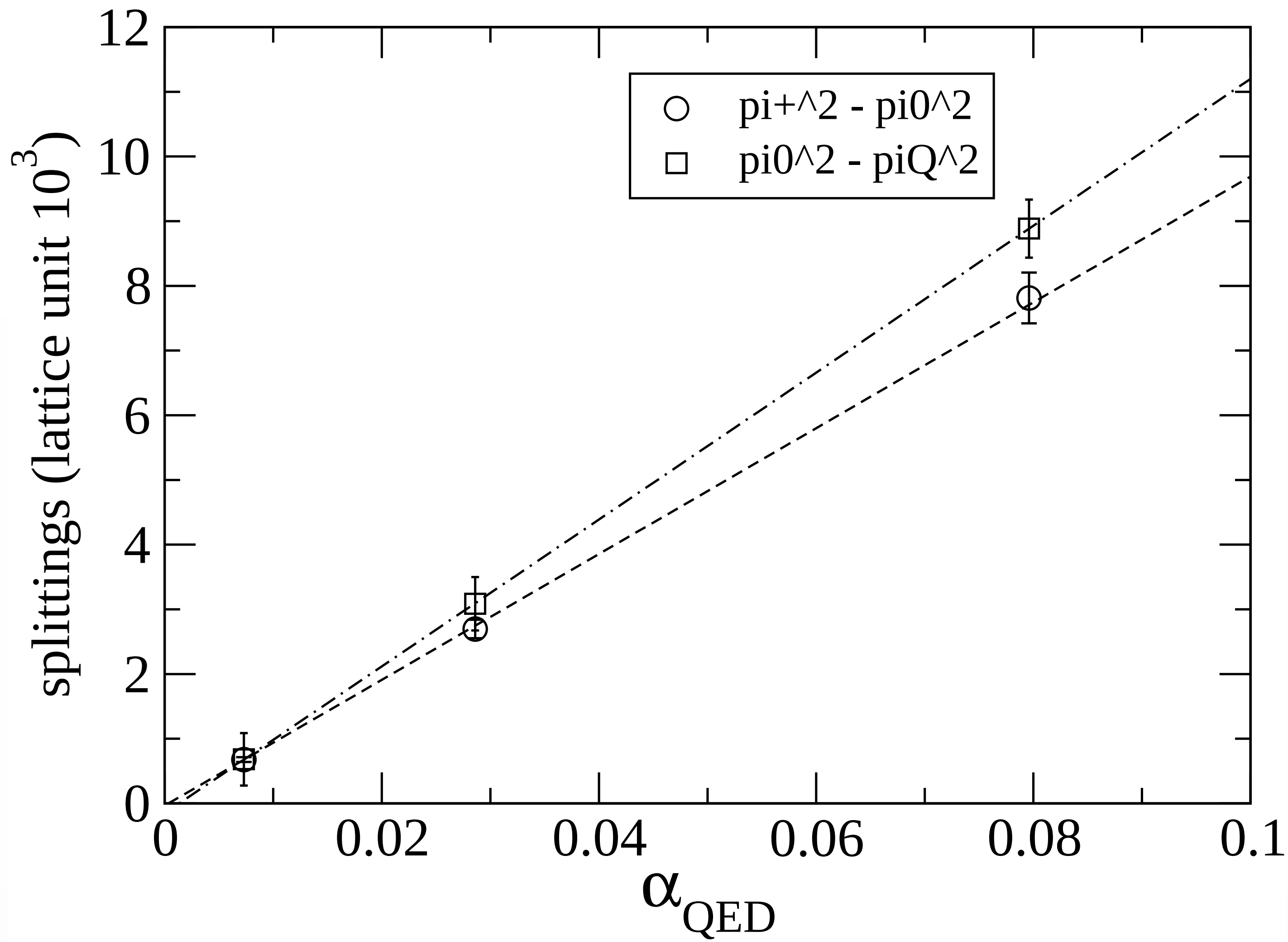
(Remark : The factor $\frac{1}{3} Z_\mu$ must be multiplied.)

Nonperturbative QED method

- The measured quantities are the same in the first and second terms. The difference is just the ways to average over photon configurations. The hadronic light-by-light scattering contribution should emerge as such a subtle difference in averaging procedure.
- The same QED configurations A should be used for the muonic part, expecting that the $\mathcal{O}(\alpha_{\rm em}^2)$-"component" in the magnetic projected quantity vanishes at the level of each set of QCD and QED configurations as precisely as possible.
- The calculation itself will be done non-perturbatively w.r.t. QED while our above consideration is entirely based on perturbation w.r.t. QED. Thus, the following question arises :
  - ‡ Can the result of the nonperturbative QED calculation be interpreted according to perturbative consideration ? Almost equivalently, will higher-order contributions be negligibly small ?

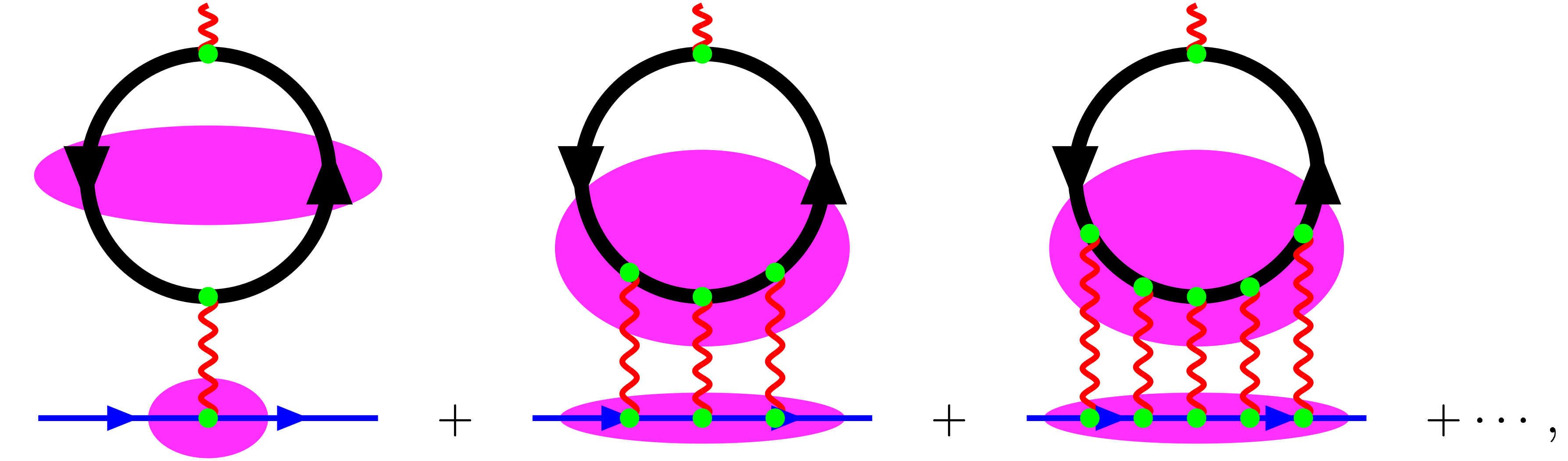
  Yes, at least in the lattice (QCD + QED) study of $(m_{\pi^+}^2 - m_{\pi^0}^2)$.

Nonperturbative QED method

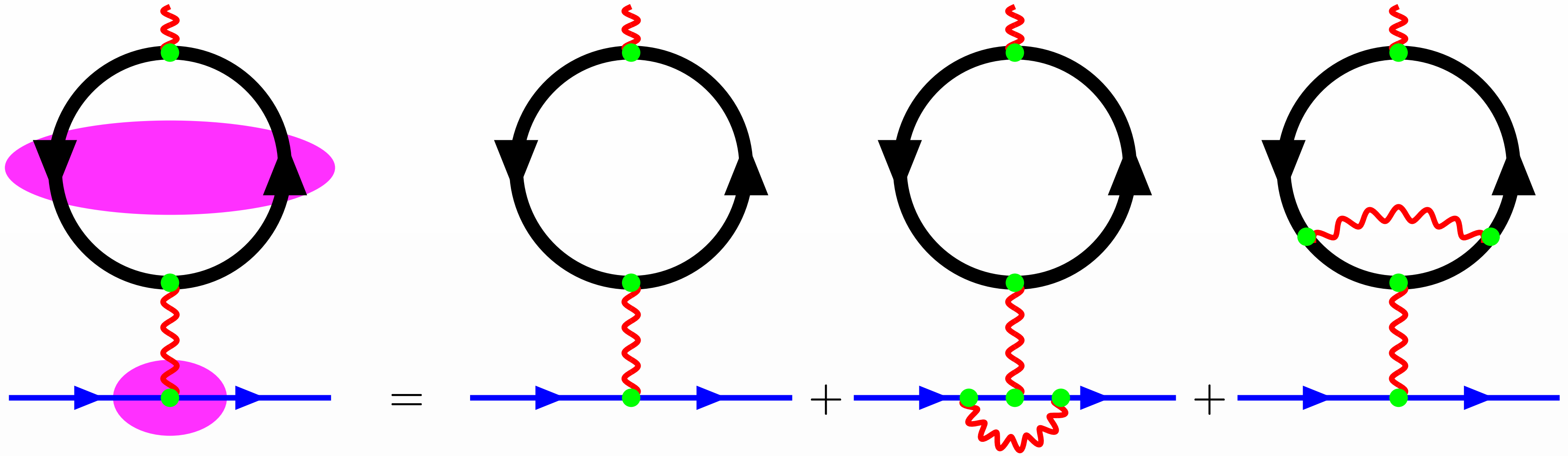


from Blum, Doi, M.H., Izubuchi and Yamada, arXiv:0708.0484 [hep-lat]

Nonperturbative QED method



where $\langle , \rangle_{QCD}$ is still present in each term, and



$+ \cdots .$

Hybrid method

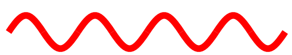
From the latest discussion with my collaborators, nonperturbative QED method revisited as the most practical method.
But, before it, I had been worried about the following point and invented the hybrid method. The point is that

- Nonperturbative QED method still seems to require us to work the solver $n = \sqrt{N} \approx (\text{total number of lattice sites})$ times, because
  1. Recall that the free photon attached by hand can carry all available four-momenta p. Its number is equal to the total number of lattice sites n.
  2. Thus, a quark loop depends on p so that the solver must be worked out for each p.
- But, the second point, 2, turned to be wrong.

In spite of this fact, let me explain the hybrid method.

Hybrid method

Hybrid method incorporates an idea used in our perturbative QED method, where we try to reproduce a free photon propagator in a stochastic manner



$$
\begin{aligned}
&= D_{\lambda\nu}(x, y) \\
&= \frac{1}{Z_A} \int DA\, A_\lambda(x)\, A_\nu(y) \exp\left(-S_{U(1)\ \mathrm{Yang\text{-}Mills}}\right) ,
\end{aligned}
$$

For a given photon configuration $\{A_\mu(x)\}$, we can assign $A_\lambda(x)$ ($A_\nu(y)$) to the vertex inserted in the quark loop (the muon line).

Hybrid method

$$\sum_{\nu,\,\lambda=0}^{4} \sum_{x} \sum_{y} D_{\lambda\nu}(x,\,y)$$
$$\times \sum_{\vec{z}} e^{-i\vec{q}\cdot\vec{z}} \mathrm{tr}\left(\gamma_\mu \left(\mathcal{D}[U,\,Q_q A_{(B)}] + m_q\right)^{-1} ((t_C,\,\vec{z}),\,x)\right.$$
$$\left.\times \gamma_\lambda \left(\mathcal{D}[U,\,Q_q A_{(B)}] + m_q\right)^{-1} (x,\,(t_C,\,\vec{z}))\right)$$
$$\times \sum_{\vec{w}} e^{-i\vec{p}_I\cdot\vec{w}} \left(\mathcal{D}[Q_\mu A_{(A)}] + m_\mu\right)^{-1} ((t_F,\,\vec{0}),\,y)\,\gamma_\nu$$
$$\times \left(\mathcal{D}[Q_\mu A_{(A)}] + m_\mu\right)^{-1} (y,\,(t_I,\,\vec{w}))\,.$$

Hybrid method

$$
\begin{aligned}
= \quad & \frac{1}{Z_A} \int DA_{(C)} \, \exp\left(-S_{U(1)\ \text{Yang-Mills}}[A_{(C)}]\right) \\
& \times \sum_{\vec{z}} e^{-i\vec{q}\cdot\vec{z}} \sum_{x} \mathrm{tr}\Bigg(\gamma_\mu \left(\mathcal{D}[U,\, Q_q A_{(B)}] + m_q\right)^{-1} ((t_C,\, \vec{z}),\, x) \\
& \qquad \times \sum_{\lambda=0}^{3} \gamma_\lambda A_{(C),\,\lambda}(x) \left(\mathcal{D}[U,\, Q_q A_{(B)}] + m_q\right)^{-1} (x,\, (t_C,\, \vec{z}))\Bigg) \\
& \times \sum_{\vec{w}} e^{-i\vec{p}_I\cdot\vec{w}} \sum_{y} \left(\mathcal{D}[Q_\mu A_{(A)}] + m_\mu\right)^{-1} ((t_F,\, \vec{0}),\, y) \sum_{\nu=0}^{3} \gamma_\nu A_{(C),\,\nu}(y) \\
& \qquad \times \left(\mathcal{D}[Q_\mu A_{(A)}] + m_\mu\right)^{-1} (y,\, (t_I,\, \vec{w}))\,.
\end{aligned}
$$

Hybrid method

- On quark loop side, A gives the A-dependent vertex

$$\Gamma[A](x) \equiv \sum_{\mu=0}^{3} A_\mu(x)\gamma_\mu \quad \text{(I work in continuum language)}\,.$$

- Similarly, on muon line, A gives the A-dependent vertex $\Gamma[A](y)$.
- We form the following quantities by the sequential source method

Hybrid method

$$\sum_{\vec{z}} e^{-i\vec{q}\cdot\vec{z}} \sum_{x} \mathrm{tr}\left(\gamma_\mu \left(\mathcal{D}[U,\, Q_q A_{(B)}] + m_q\right)^{-1} ((t_C,\, \vec{z}),\, x) \right.$$

$$\left. \times\Gamma[A_{(C)}](x) \left(\mathcal{D}[U,\, Q_q A_{(B)}] + m_q\right)^{-1} (x,\, (t_C,\, \vec{z}))\right) ,$$

$$\sum_{\vec{w}} e^{-i\vec{p}_I\cdot\vec{w}} \sum_{y} \left(\mathcal{D}[Q_\mu A_{(A)}] + m_\mu\right)^{-1} ((t_F,\, \vec{0}),\, y)\, \Gamma[A_{(C)}](y)$$

$$\times \left(\mathcal{D}[Q_\mu A_{(A)}] + m_\mu\right)^{-1} (y,\, (t_I,\, \vec{w}))\, .$$

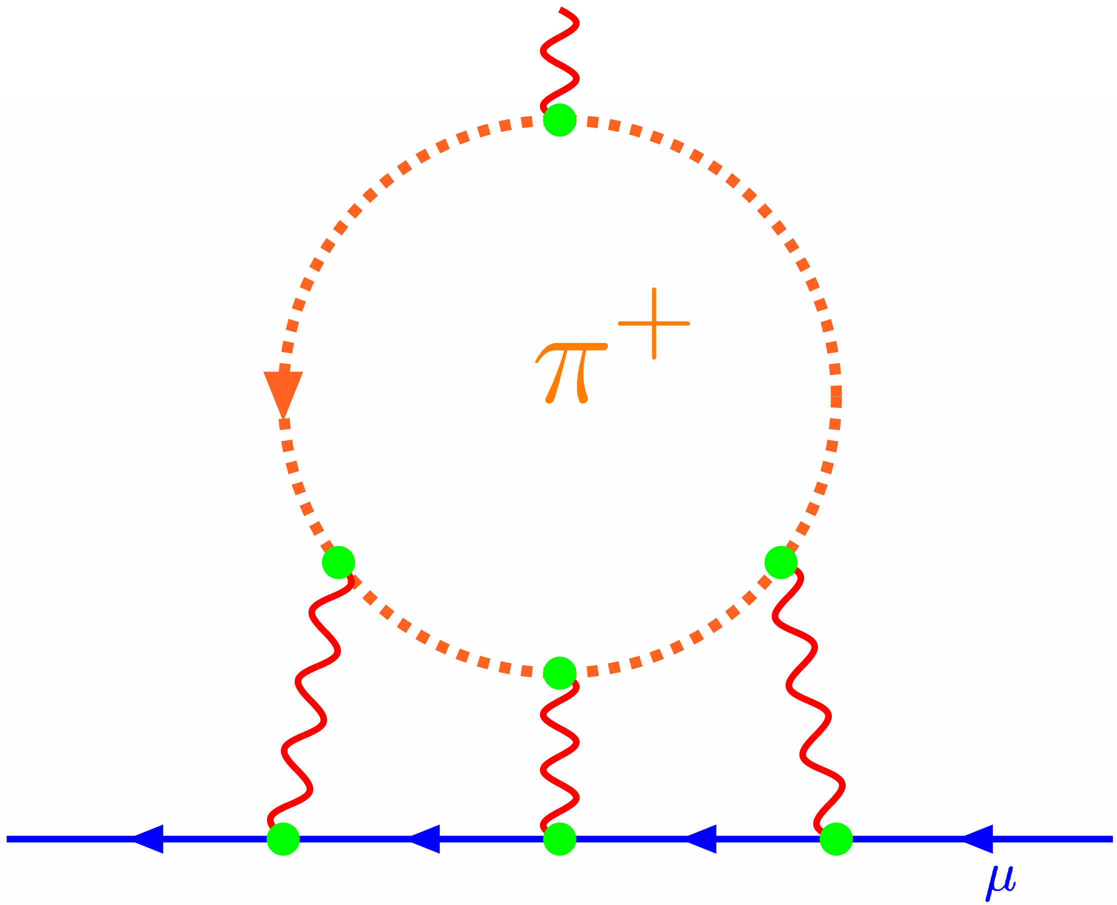
The expected reduction of computational cost is ($\Leftarrow$ "was" now)

$$\frac{\text{Number of } A_{(C)} \text{ to reproduce } D_{\lambda\nu}(x,\, y)}{n = 3 \times 4 \times \text{Total number of lattice sites}} = \frac{\mathcal{O}(10)}{10^6 \sim 10^7}\, .$$

Multi-quark loop contribution

There are actually other types of diagrams !

To elucidate this point, let's return to the low-energy hadron picture, and observe the charged pion loop contribution,



from the quark model picture, for simplicity.

Multi-quark loop contribution (cont')

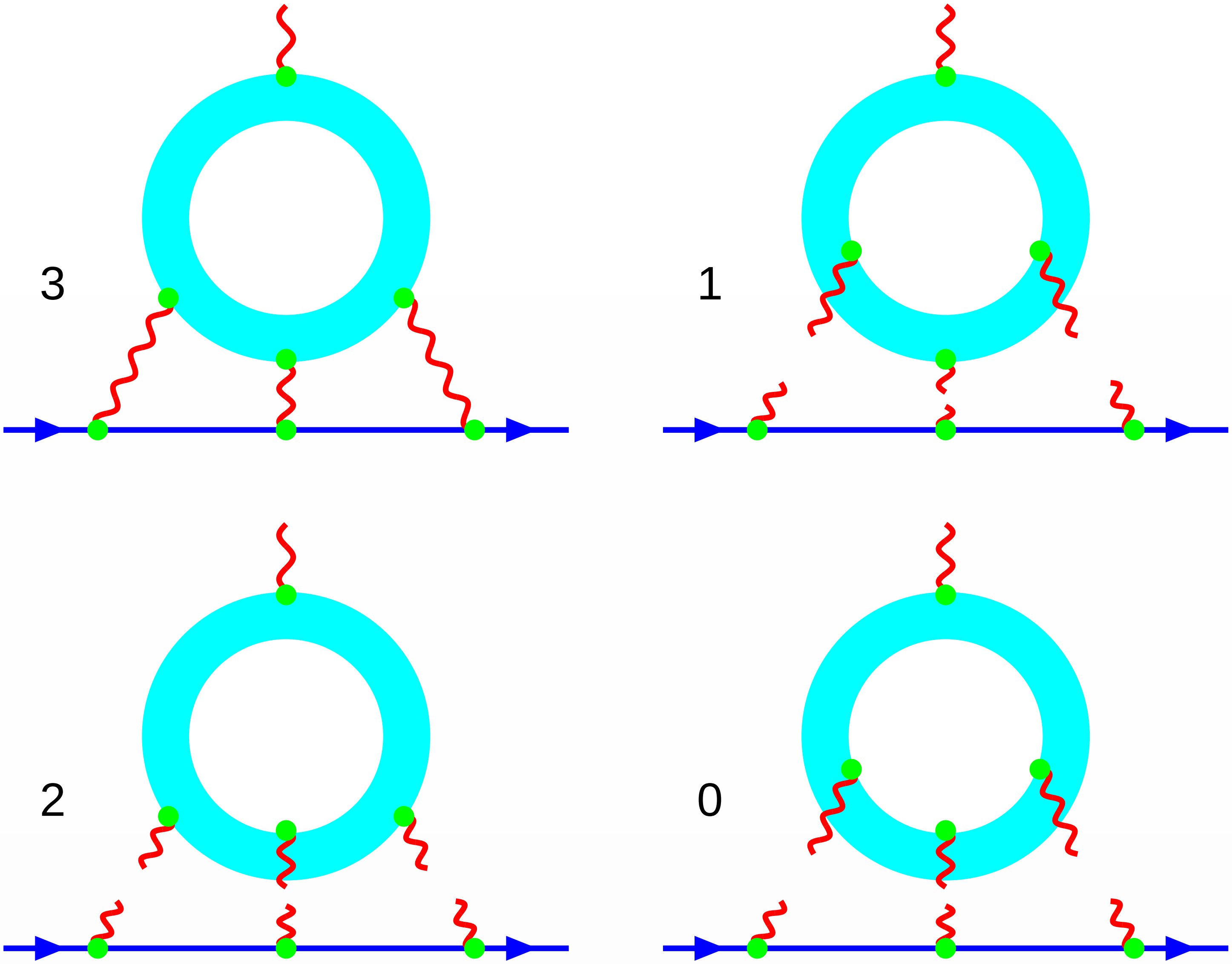
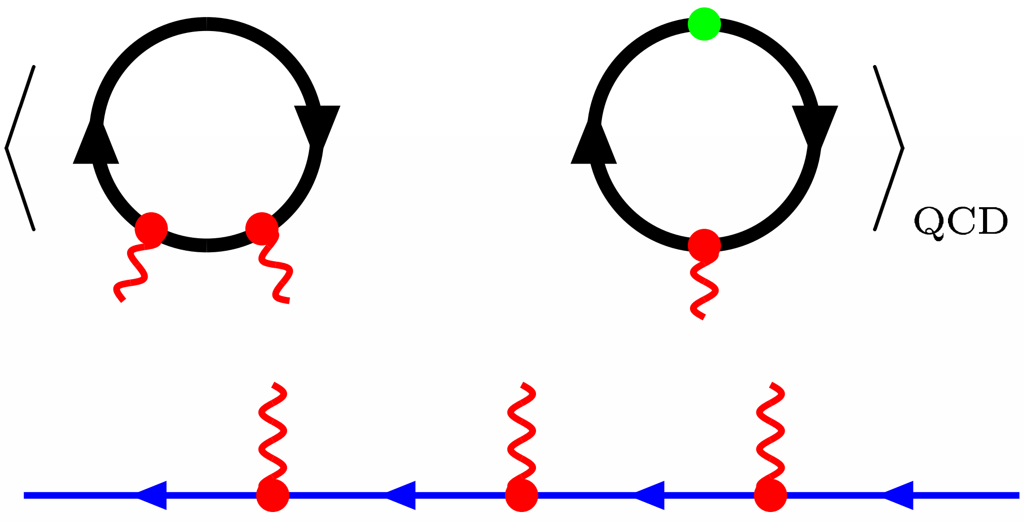


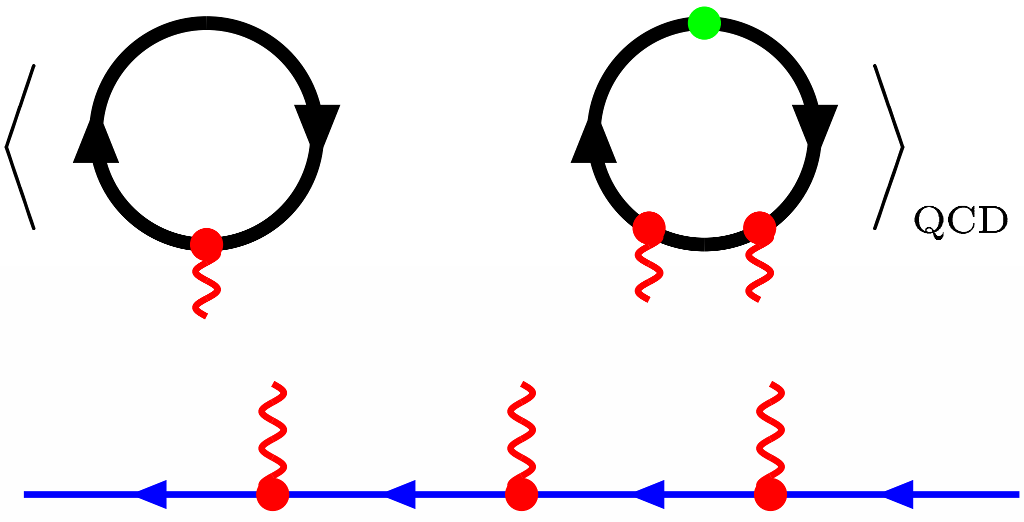
Diagram with 2 quark loops

There are three types of hadronic light-by-light scattering contribution with two quark loops with internal electromagnetic vertex (vertices) ($\Leftrightarrow$ red dot(s));



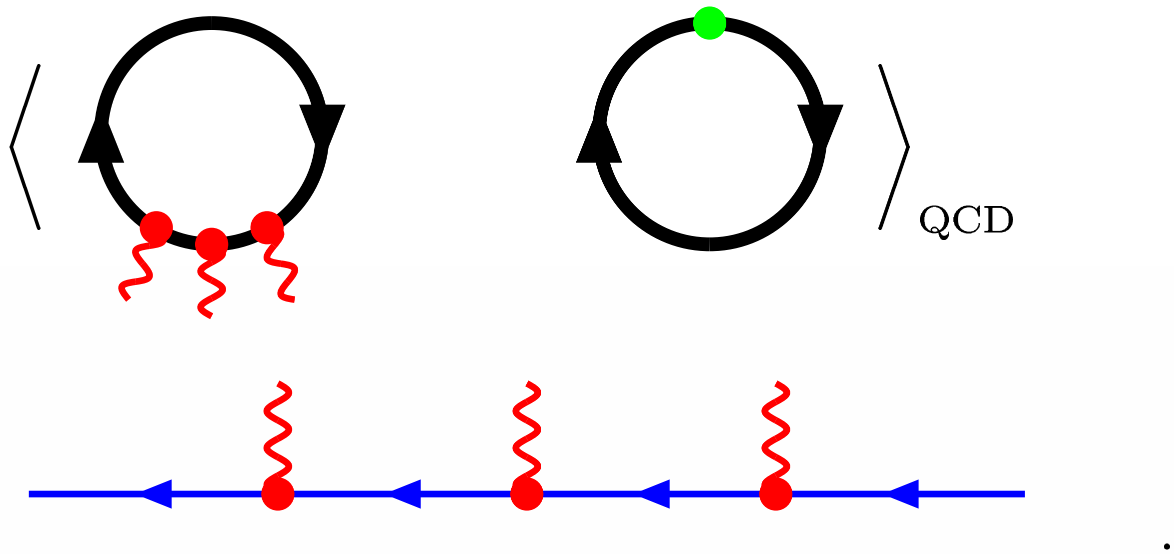
.

Diagrams with 2 quark loops (cont)



,

Diagrams with 2 quark loops (cont)



The last two diagrams including two quark loops vanish in the (flavor) $SU(3)_V$

limit, $m_u = m_d = m_s$, because $\mathrm{tr}(Q) = Q_u + Q_d + Q_s = 0$.

Diagrams with 2 quark loops (cont)

An economical way to compute those three diagrams is to compute

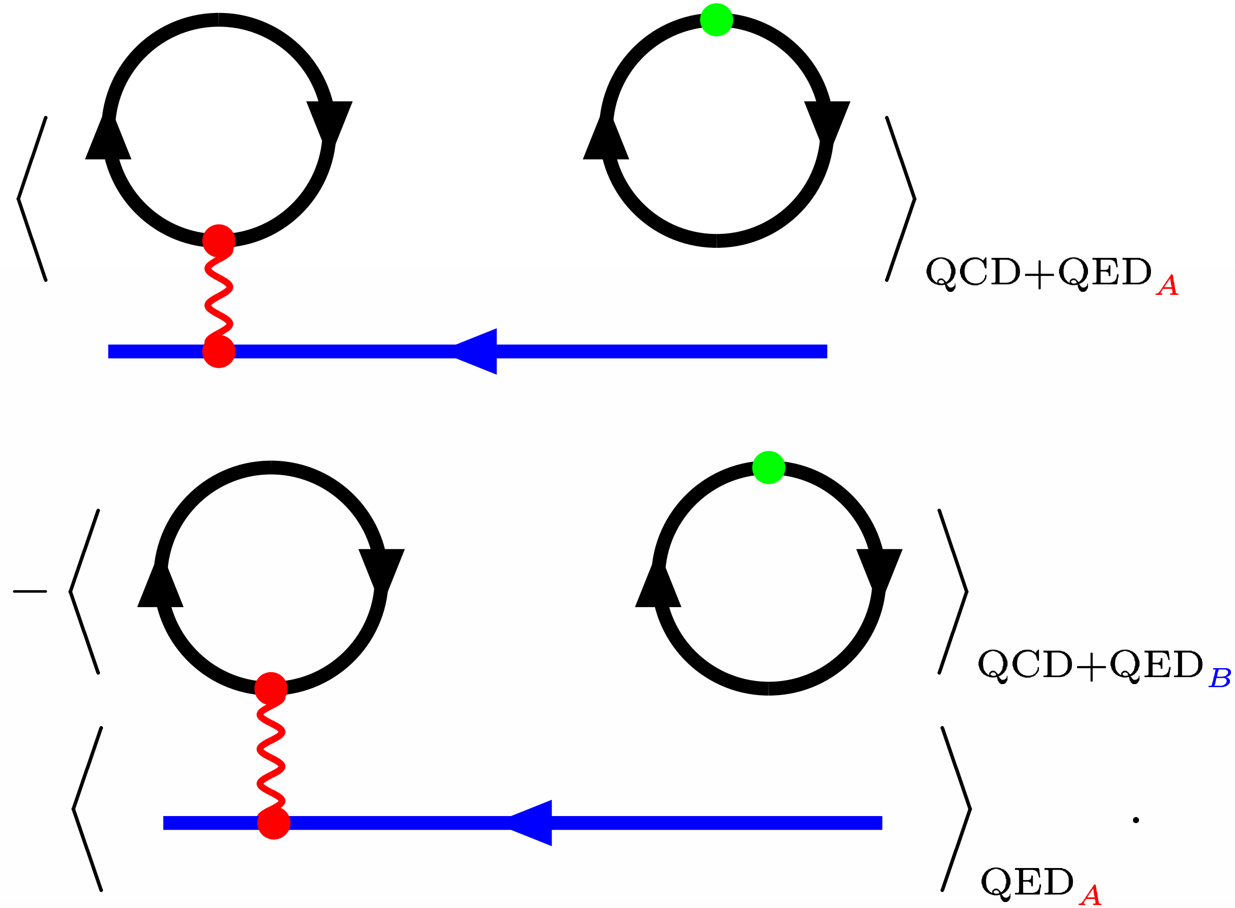
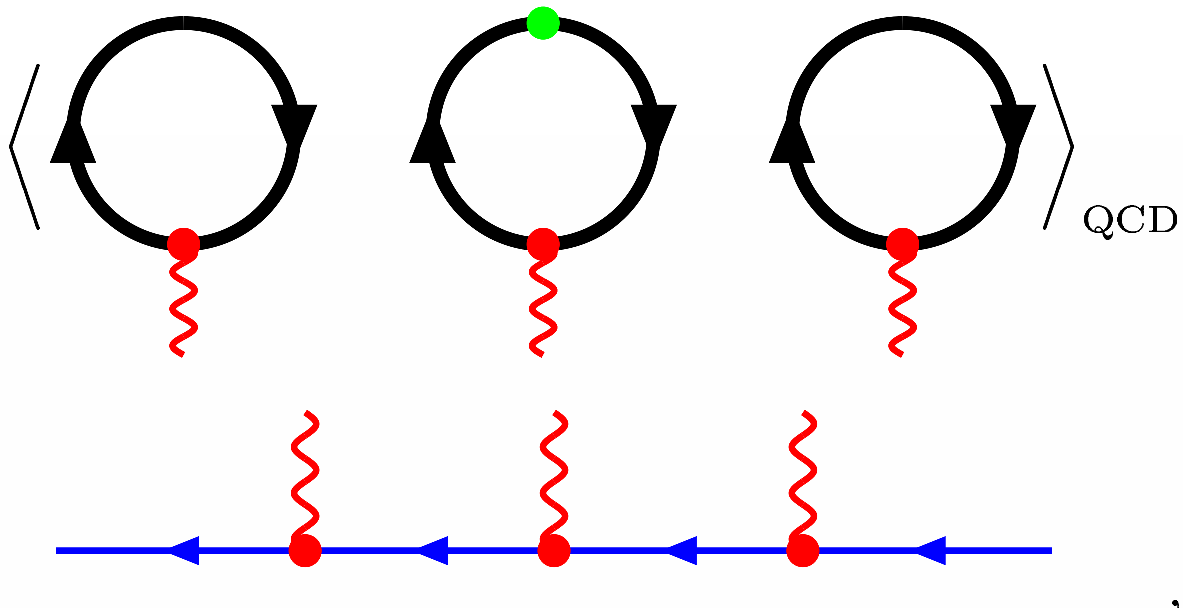
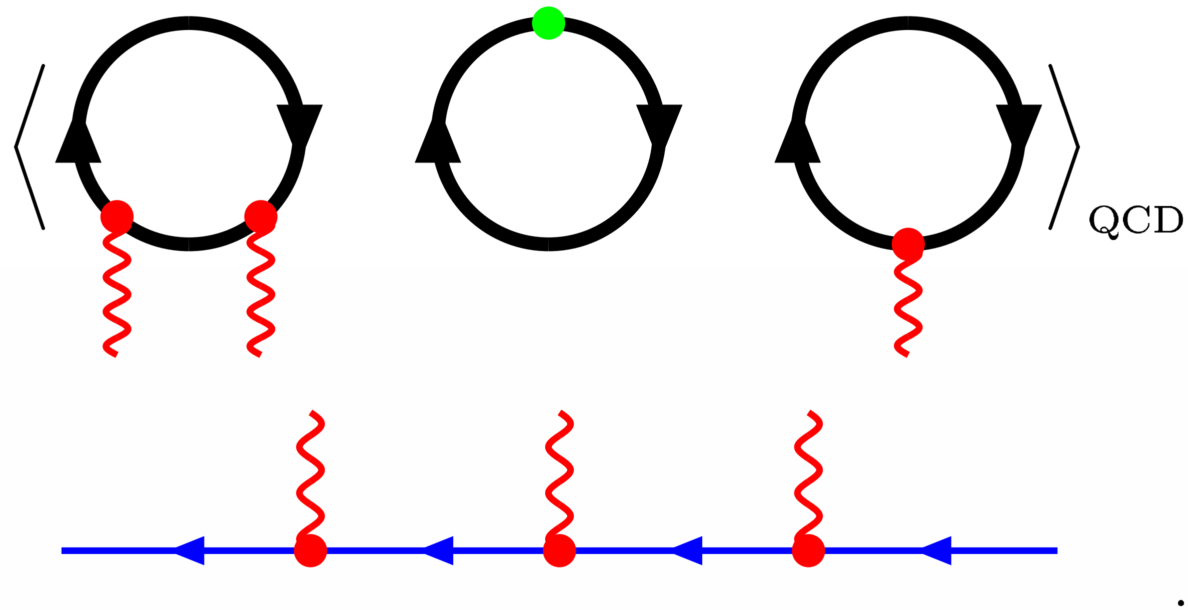


Diagram with 3 quark loops

There are two types of hadronic light-by-light scattering diagrams with three quark loops



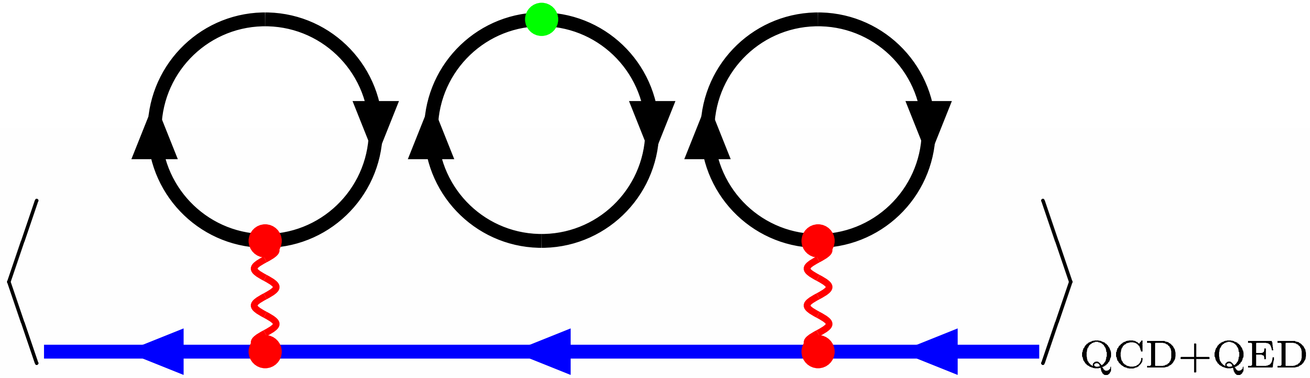
Diagrams with 3 quark loops (cont)



Each of both diagrams vanishes in the exact $SU(3)_V$ limit.

Diagrams with 3 quark loops (cont)

An economic way to calculate those two diagrams is to calculate



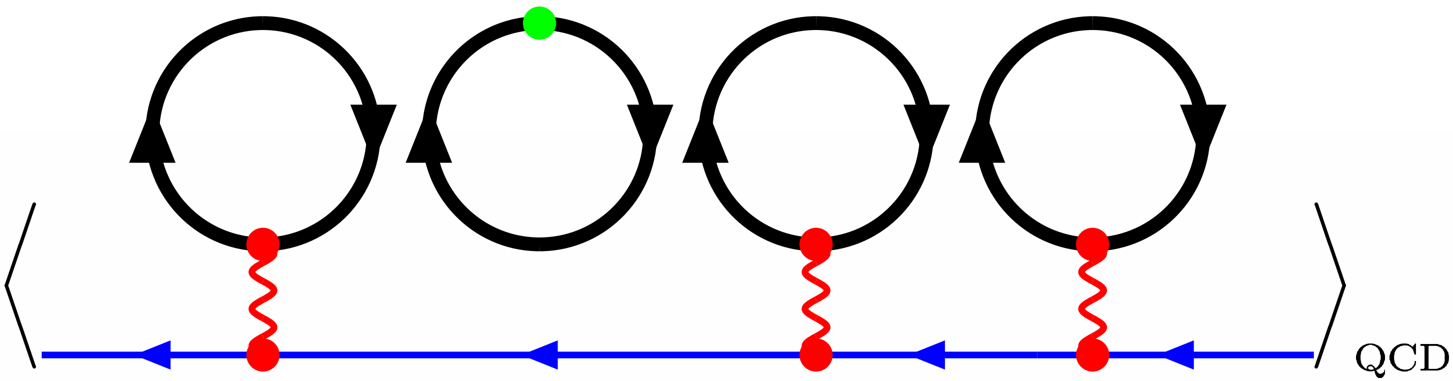
.

We note that no subtraction is necessary, because the correlation function o

three electromagnetic currents vanishes.

Diagram with 4 quark loops

There is a diagram with 4 quark loops which is the most difficult to calculate;



.

Calculation of quark loop

We use stochastic method to calculate the quark loop part.
The quark loop part can be summarized in the following form

$$\mathrm{Tr}\left(\mathcal{ASBS}\right) ,$$

where $\mathcal{S} \equiv (\mathcal{D}[U] + m)^{-1}$, and $\mathcal{A}$ and $\mathcal{B}$ denote the external vertex and the internal vertex respectively, including the factors depending on their positions.
Let η_I be the colored spinor scalar fields with the action

$$S_\eta = \sum_I \eta_I^\dagger \frac{1}{\kappa_I} \eta_I .$$

Then, the propagator for η_I becomes

$$\left\langle \eta_I \eta_J^\dagger \right\rangle_\eta \equiv \frac{1}{Z_\eta} \int D\eta D\eta^\dagger \, e^{-S_\eta} \, \eta_I \, \eta_J^\dagger = \kappa_I \delta_{IJ} .$$

We actually put $\kappa_I = 0$ for I outside the specified time slice, for which $\eta_I = 0$.

Calculation of quark loop

Using η_I, the trace can be written as

$$
\begin{aligned}
\mathrm{Tr}\left(\mathcal{A}\mathcal{S}\mathcal{B}\mathcal{S}\right) &= \sum_{I,\,J} \left(\mathcal{A}\mathcal{S}\mathcal{B}\mathcal{S}\right)_{JI} \delta_{IJ} \frac{1}{\kappa_J} \\
&= \sum_{I,\,J} \left\langle \left(\mathcal{A}\mathcal{S}\mathcal{B}\mathcal{S}\right)_{JI} \eta_I \eta_J^\dagger \frac{1}{\kappa_J} \right\rangle_\eta \\
&= \left\langle \eta^\dagger \frac{1}{\kappa} \mathcal{A}\mathcal{S}\mathcal{B}\mathcal{S}\eta \right\rangle_\eta .
\end{aligned}
$$

We can calculate this quantity in one of the following two ways;

- Sequential source method starting with the source vector $s_I = \eta_I$.

Calculation of quark loop

- We divide the above quantity in the three parts; $\mathcal{S}\eta$, $\eta^\dagger \frac{1}{\kappa}\mathcal{A}\mathcal{S}$ and $\mathcal{B}$. The first part will be obtained by working the solver with the source vector $s_I = \eta_I$. The second part will also be obtained with the source vector depending on η using the γ_5-hermiticity.

Does triviality of QED matter to prediction ?

- Our interest is limited to the prediction of the observable from the standard model, that are saturated by its low-energy dynamics.
- Here the "low-energy" E implies that $E \ll \Lambda_{\mathrm{QED}}$, where Λ_{QED} is the (perturbative) Landau-pole.
- As long as we perform renormalization and set the values of parameters, e, m_e, etc, correctly, without taking $\Lambda_{\mathrm{QED}} \to \infty$, we can get the prediction universally for the observables of our interest.
- The fact does not depend on the regularization scheme as well as the calculation method, in so far as they respect the gauge symmetry. The lattice regularization with finite cutoff for QED can thus also give qualitative prediction to the observables of our interest.