(Non-)BPS bound states and multi-instantons

based on:

R. Blumenhagen, M. Cvetič, R. Richter, T.W., arXiv: 0708.0403

M. Cvetič, R. Richter, T.W., arXiv: 0803.2513

Timo Weigand

Department of Physics and Astronomy, University of Pennsylvania

Motivation

Non-perturbative corrections to effective action of 4D string compactifications play a prominent role
despite exponential suppression:
crucial if corresponding interactions forbidden perturbatively

- relevant for very definition of vacuum
  $\leftrightarrow$ moduli stabilisation
- determine phenomenological properties of vacuum:
  perturbatively forbidden important matter couplings
  $\rightsquigarrow$ Dynamical SUSY breaking
  $\rightsquigarrow$ natural generation of observed hierarchies,
  e.g. Majorana masses, certain Yukawas, μ-terms

This talk:
D-brane instantons in Type II orientifolds:
Which D-brane instantons correct the superpotential?

BPS instantons and superpotentials

According to general lore : instanton must wrap BPS cycle:
- volume minimizing in homology class
- preserves $\frac{1}{2}$ SUSY $\rightarrow$ minimal # of Goldstone fermions

BPS brane of (co)homological charge $\Gamma \leftrightarrow$ zentral charge $Z_\Gamma(m)$

$$Z \simeq \left\{ \begin{array}{cc} \int_\Pi \Omega & \mathrm{A-type\ branes} \\ \int_X e^J \mathrm{ch}(i\mathcal{F})\sqrt{\mathrm{td}(X)} & \mathrm{B-type\ branes} \end{array} \right\}$$

SUSY condition for Type II orientifolds:
$\varphi = Arg(Z) = 0 \leftrightarrow$ hypersurface $\mathcal{M}_{SUSY}$ in moduli space

BPS object can decay across hypersurface $\mathcal{M}_0$ where $|Z_\Gamma| = |Z_{\Gamma_1}| + |Z_{\Gamma_2}|$ for $\Gamma = \Gamma_1 + \Gamma_2$

BPS instantons and superpotentials

Distinguish 2 types of decay:

• line of threshold stability $\leftrightarrow$ $\exists$ BPS object on both sides
$\mathcal{M}_+, \mathcal{M}_-$
$\langle \Gamma_1, \Gamma_2 \rangle = n^+ - n^-, n^+ \neq 0 \neq n^-$ (non-minimal intersection)

• line of marginal stability $\leftrightarrow$ $\exists$ BPS object only on one side
Either $n^+ = 0$ or $n^- = 0$ (strictly chiral intersection)
spectrum of BPS cycles discontinuous

$\Rightarrow$Multi-instanton effects come in naturally

[Garcia-Etxebarria,Uranga 0711.1430]

Focus in this talk:
Can instantons decaying across line of marginal stability contribute to the superpotential?

BPS instantons and superpotentials

In $\mathcal{N}=1$ orientifolds on $X/(\Omega\sigma)$ distinguish:
instantons along invariant vs non-invariant cycles on X

1.) U(1) instantons in region $\mathcal{M}_0$ in moduli space:
E_p along cycle $\Xi \neq \Xi'$ on SUSY locus $\mathcal{M}_0$:

universal zero modes:
4 bosonic modes $x_E^i \leftrightarrow$ Poincaré inv. in 4D
2 + 2 Goldstinos θ_α, $\overline{\tau}_{\dot{\alpha}} \leftrightarrow$ broken SUSY

$\mathcal{N}=1$	$\mathcal{N}=1'$
θ_α	τ_α
$\overline{\theta}_{\dot{\alpha}}$	$\overline{\tau}_{\dot{\alpha}}$

2.) If $\Xi = \Xi'$: universal modes subject to orientifold projection
$O(1)$ instantons: x_E^i, θ_α survive, $\overline{\tau}_{\dot{\alpha}}$ projected out
$\Rightarrow$ superpotential contributions possible

BPS instantons and superpotentials

Can U(1) along $\Xi \neq \Xi'$ contribute as well?

Turns out: [BCRW, 0708.0403]
Yes, if $\exists$ modes in $E - E'$ sector that lift extra $\overline{\tau}^{\dot{\alpha}}$
$\leftrightarrow$ modes allow bound state out of Ξ and Ξ' of $O(1)$ type

works without problems if Ξ and Ξ' are at vector-like threshold - non-pert. superpotential provided bound state is rigid

[BCRW, 0708.0403], [G-E,U. 0711.1430]

for line of marginal stability:
puzzle since BPS state can disappear!
Compatible with holomorphic superpotential?

U(1) instantons in $IIA/\Omega\overline{\sigma}$

For concreteness: D2-brane instantons in Type IIA
consider pair of $E2 - E2'$ instantons at SUSY angle
Suppose intersection on top of orientifold:
$[\Xi' \cap \Xi]^+ = n^+ = [\Pi_{O6} \cap \Xi]^+, \qquad [\Xi' \cap \Xi]^- = n^- = [\Pi_{O6} \cap \Xi]^-$

recombination modes in $E - E'$ sector

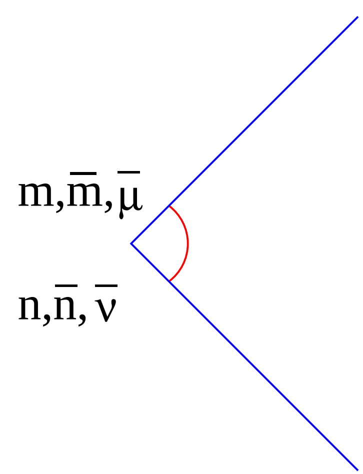
zero mode	Q_E	Multiplicity
$m, \overline{m}$	$2\ ,-2$	$\frac{1}{2}[\Xi' \cap \Xi + \Pi_{O6} \cap \Xi]^+$
$\overline{\mu}^{\dot\alpha}$	-2	$\frac{1}{2}[\Xi' \cap \Xi + \Pi_{O6} \cap \Xi]^+$
μ^{α}	2	$\frac{1}{2}[\Xi' \cap \Xi - \Pi_{O6} \cap \Xi]^+$
$n, \overline{n}$	$-2\ ,2$	$\frac{1}{2}[\Xi' \cap \Xi + \Pi_{O6} \cap \Xi]^-$
$\overline{\nu}^{\dot\alpha}$	2	$\frac{1}{2}[\Xi' \cap \Xi + \Pi_{O6} \cap \Xi]^-$
ν^{α}	-2	$\frac{1}{2}[\Xi' \cap \Xi - \Pi_{O6} \cap \Xi]^-$

Instantons and threshold stability

Minimal vector-like case: $n^+ = n^- = 1$
$E - E'$ modes: $m, \overline{m}$, $\overline{\mu}^{\dot{\alpha}}$, $n, \overline{n}$, $\overline{\nu}^{\dot{\alpha}}$

(re)combination governed by usual D-term in instanton effective action:
$S_D = \frac{1}{2g_E^2}(2m\overline{m} - 2n\overline{n} - \xi)^2$

in $\mathcal{M}_0$: $\xi = 0$, instanton (singular) union $E \cup E'$: U(1) locus
in $\mathcal{M}_+$: $\xi > 0$, condensation of $m \to$ bound state $E'\#E$
in $\mathcal{M}_-$: $\xi < 0$, condensation of $n \to$ bound state $E\#E'$



Instantons and threshold stability

Consider system on U(1) locus $\mathcal{M}_0$

fermionic instanton moduli action: [BCRW, 0708.0403]

$S_{fermionic} = m\,\overline{\mu}^{\dot\alpha}\,\overline{\tau}_{\dot\alpha} - n\,\overline{\nu}^{\dot\alpha}\,\overline{\tau}_{\dot\alpha}$
Integrate out $\overline{\tau}^{\dot\alpha}$ and combination $(\overline{\mu}^{\dot\alpha} - \overline{\nu}^{\dot\alpha})$

In absence of further interactions (e.g. toroidal orbifolds)
$\overline{\chi}^{\dot\alpha} = \overline{\mu}^{\dot\alpha} + \overline{\nu}^{\dot\alpha}$ unlifted $\Rightarrow$ no superpotential, but higher fermionic F-terms [BCRW, 0708.0403]

As pointed out in [G-E, U. 0711.1430]:
If exist quartic F-term couplings $(MN)^2$
$\Rightarrow$ $\overline{\chi}^{\dot\alpha}$ lifted and superpotential contributions possible
Presence of these terms equivalent to rigidity of O(1) bound state in $\mathcal{M}_+$ or $\mathcal{M}_-$
to be checked in conrete examples

Instantons and marginal stability

Now: chiral intersection $n^+ = 1, n^- = 0$ [BCRW, 0708.0403]

$E - E'$ modes: $m, \overline{m}, \overline{\mu}^{\dot\alpha}$ $\qquad S_D = \frac{1}{2g_E^2}(2m\overline{m} - \xi)^2$

in $\mathcal{M}_0$: $\xi = 0$, instanton (singular) union $E \cup E'$: U(1) locus

in $\mathcal{M}_+$: $\xi > 0$, condensation of $m \rightarrow$ bound state $E'\#E$

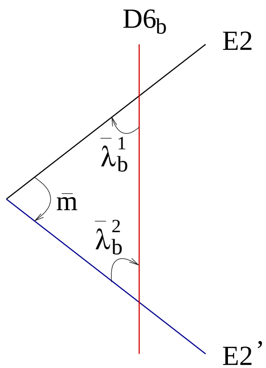
in $\mathcal{M}_-$: $\xi < 0$, no BPS state of charge $[E] + [E']$ exists!

Turns out: $E \cup E'$ and $E'\#E$ do not contribute F-terms:

Consider $E \cup E'$ on $\mathcal{M}_0$:

by tadpole cancellation $\exists$ charged fermionic zero modes λ^i in instanton - D-brane sector of $U(1)_E$ charge Q_E under $U(1)_E$

$\sum_i Q_E(\lambda^i) = -\sum_a N_a\, \Xi \circ (\Pi_a + \Pi_{a'}) = -4\ \Xi \circ \Pi_{O6} = 4$



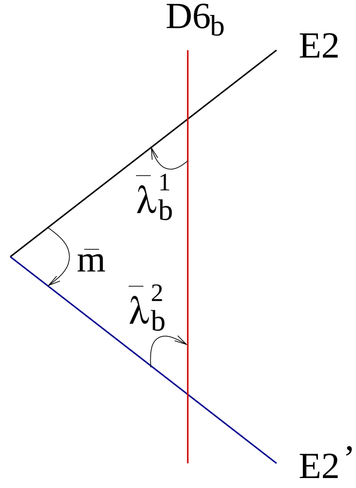
Instantons and marginal stability

No perturbative couplings in instanton effective action can lift these chiral excess modes λ^i! [BCRW, 0708.0403]

usual open string couplings $\lambda_a \, \phi_{ab} \, \lambda_b$ invariant under $U(1)_E$

$\leadsto$ 4 excess modes λ^i with $Q_E = 4$ cannot pair up this way

only gauge invariant combination: $\overline{m}^{-1} \, (\lambda)_b^{-1/2} \prod \phi^1_{b_i c_i} \, \lambda_c^{-1/2}$



These couplings are zero due to chiral ring structure

(cf. [Greene,Distler '88])

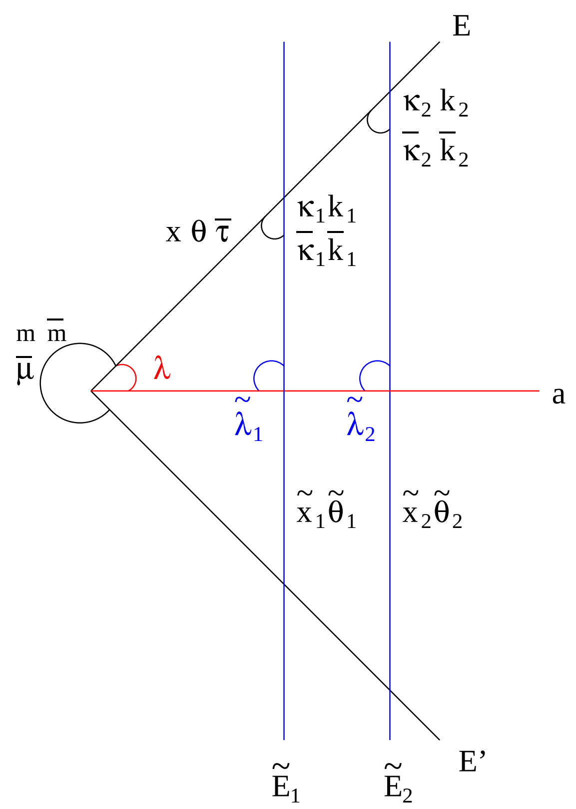
Multi-instantons and marginal stability

Non-perturbative lifting of λ^i via multi-instanton possible!

[CRW, 0803.2513]

Consider in addition 2 $O(1)$ instantons $\widetilde{E}_1, \widetilde{E}_2$ along $\widetilde{\Xi}_1, \widetilde{\Xi}_2$

$[\widetilde{\Xi}_1 \cap \Pi_a]^+ = 2 = [\widetilde{\Xi}_2 \cap \Pi_a]^+, \qquad [\Xi \cap \widetilde{\Xi}_1]^+ = 1 = [\Xi \cap \widetilde{\Xi}_2]^+.$



Example on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ in [arXiv:0803.2513]

Multi-instantons and marginal stability

Extra modes:

zero mode	sector	repr.	multiplicity
$k_i,\ \kappa_i^{\alpha}$	$\widetilde{E}_i - E$	$(1_{\widetilde{E}_i}, -1_E)$	$[\Xi \cap \widetilde{\Xi}_i]^+ = 1$
$\overline{k}_i,\ \overline{\kappa}_i^{\dot\alpha}$	$\widetilde{E}_i - E$	$(1_{\widetilde{E}_i}, 1_E)$	$[\Xi \cap \widetilde{\Xi}_i]^+ = 1$
$\widetilde{\lambda}_1^i$	$\widetilde{E}_1 - D6_a$	$(1_{\widetilde{E}_1}, 1_a)$	$[\widetilde{\Xi}_1 \cap \Pi_a]^+ = 2$
$\widetilde{\lambda}_2^i$	$\widetilde{E}_2 - D6_a$	$(1_{\widetilde{E}_2}, 1_a)$	$[\widetilde{\Xi}_2 \cap \Pi_a]^+ = 2$

Consider system $E \cup E' \cup \widetilde{E}_1 \cup \widetilde{E}_2$ on $\mathcal{M}_0$:
All fermionic modes can be lifted:

$$S_1 \simeq \kappa_1^{\alpha} \widetilde{\theta}_{1\alpha}\, \widetilde{\lambda}_1^i\, \lambda^j \qquad + \qquad (1 \leftrightarrow 2)$$

$$S_2 \simeq \overline{\mu}^{\dot\alpha}\, \overline{\kappa}_{1\dot\alpha}\, \overline{k}_1 + \overline{m}\, \overline{\kappa}_1^{\dot\alpha}\, \overline{\kappa}_{1\dot\alpha} + m\, \kappa_1^{\alpha}\, \kappa_{1\alpha} \qquad + \qquad (1 \leftrightarrow 2),$$

$$S_3 \simeq m\, \overline{\mu}^{\dot\alpha}\, \overline{\tau}_{\dot\alpha} + \overline{\kappa}_1^{\dot\alpha}\, \overline{\tau}_{\dot\alpha}\, k_1 + \overline{\kappa}_2^{\dot\alpha}\, \overline{\tau}_{\dot\alpha}\, k_2$$

Multi-instantons and marginal stability

$\Rightarrow$ non-zero path integral over bosonic modes:

$$\int d^2k_1\ d^2k_2\ d^2m\ \ (|k_1|^2\,|k_2|^2+|m|^4)\ \ exp(-S_D-S_F)$$

$$S_D=\frac{1}{2g_E^2}\,(2m\overline{m}-k_1\overline{k}_1-k_2\overline{k}_2-\xi)^2,$$

$$S_F=l^2\left((k_1\,\overline{k}_1)^2+|m\,k_1|^2+(k_2\,\overline{k}_2)^2+|m\,k_2|^2\right)$$

$\Rightarrow W\simeq e^{-\left(U(\Xi)+U(\widetilde{\Xi}_1)+U(\widetilde{\Xi}_2)\right)}$, $\qquad U(\Pi)=\frac{2\pi}{\ell_s^3}(\int_\Pi \frac{1}{g_s}\Omega+iC_3)$

How does this match results away from $\mathcal{M}_0$?

$\mathcal{M}_+$: $\xi>0\rightsquigarrow\langle m\rangle=\sqrt{\xi/2}\Rightarrow$ BPS bound state $Y=E'\#E$

BPS multi-instanton $Y\cup\widetilde{E}_1\cup\widetilde{E}_2$ contributes:

$\langle m\rangle$ renders modes $m,\overline{\mu},\overline{\tau},\overline{k},\overline{\kappa}$ massive

charged modes lifted via $\langle\widetilde{\theta}_1^\alpha\,\widetilde{\theta}_1^\beta\ \widetilde{\lambda}^i\lambda^j\widetilde{\lambda}^k\lambda^l\rangle+1\leftrightarrow 2$

Multi-instantons and marginal stability

What happens on $\mathcal{M}_- \leftrightarrow \xi < 0$?

$$S_D = \frac{1}{2g_E^2}\left(2m\overline{m} - k_1\overline{k}_1 - k_2\overline{k}_2 - \xi\right)^2,$$
$$S_F = l^2\left((k_1\,\overline{k}_1)^2 + |m\,k_1|^2 + (k_2\,\overline{k}_2)^2 + |m\,k_2|^2\right)$$

Classical vacuum $\widetilde{\Psi}$ for

$$|k_1| = |k_2| = \sqrt{-\frac{\xi}{2+a}}, \qquad m = 0, \qquad (a = 2g_E^2\, l^2 << 1)$$

D- and F-flatness broken!

There exists no true BPS configuration in usual sense

$\widetilde{\Psi}$ is non-calibrated cycle

Consider instead non-BPS state

$\Psi = \widetilde{E}_1\#(E \cup E')\#\widetilde{E}_2 \leftrightarrow |k_1| = |k_2| = \sqrt{-\frac{\xi}{2}}$

$\mathcal{O}(g_s^{-1})$: D-flat, k_i massive, m massless (modulus)

$\mathcal{O}(g_s^0)$: F-flatness broken, m massive ('obstructed')

Multi-instantons and marginal stability

$\rightsquigarrow$ Ψ not dissimilar to quasi-instanton in field theory:

- solution to field equations only at leading order in coupling
- VEV of scalars invalidate solution at higher order

For holomorphicity of superpotential this object has to contribute on $\mathcal{M}_-$!

Summary:

- $\mathcal{M}_+$: superpotential W corrected by BPS configuration $(E'\#E) \cup \widetilde{E}_1 \cup \widetilde{E}_2$
- $\mathcal{M}_0$: $(E'\#E)$ meets line of marginal stability, BPS multi-instanton $E \cup E' \cup \widetilde{E}_1 \cup \widetilde{E}_2$ contributes to W
- $\mathcal{M}_-$: $\exists$ no BPS state of charge $[E] + [E'] + [\widetilde{E}_1] + [\widetilde{E}_1]$ superpotential corrected by quasi-instanton $\Psi = \widetilde{E}_1\#(E \cup E')\#\widetilde{E}_2$

Type I/Heterotic picture

$O(1)$ instantons $\leftrightarrow$ $E1$ instantons along holomorphic curves
$U(1)$ instantons $\leftrightarrow$ $E5$ instantons with gauge bundle $L \oplus L^\vee$
recombination modes in E5-E5' sector: extension moduli
$Ext^1(L^\vee, L) = H^1(L^2)$ or $Ext^1(L, L^\vee) = H^2(L^2)$
$n^+ = h^1(L^2)$, $n^- = h^2(L^2)$

Multi-instantons vs. bound states as we vary Kähler moduli J:
$\mathcal{M}_0$: $L \oplus L^\vee$
$\mathcal{M}_+$: $0 \to L \to V \to L^\vee \to 0$
$\mathcal{M}_-$: $0 \to L^\vee \to U \to L \to 0$

All couplings can be analysed in a similar spirit:
• quartic couplings for $n^+ = 1 = n^-$ $\leftrightarrow$ $H^1(V \otimes V^\vee) = 0$
Depends on concrete bundles!
• chiral case $n^+ = 1, n^- = 0$ bound states of two
E1-instantons and one E5 w/ $L \oplus L^\vee$

Conclusions

More types of instantons contribute to superpotential

Type I/Heterotic: magnetised E5 or NS5 instantons are relevant for superpotential!
$\rightsquigarrow$ affects vanishing results for certain heterotic backgrounds?

BPS decay in moduli space $\Leftrightarrow$ multi-instantons
Discussed explicit decay of BPS multi-instanton into non-BPS configuration
Conjecture: even non-BPS instantons related to BPS instantons somewhere in moduli space contribute to superpotential
concrete examples were non-BPS in subtle way (destabilised by F-terms)
Can this effect be demonstrated in other examples?