Toric resolution of Heterotic orbifolds

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+ work in progress

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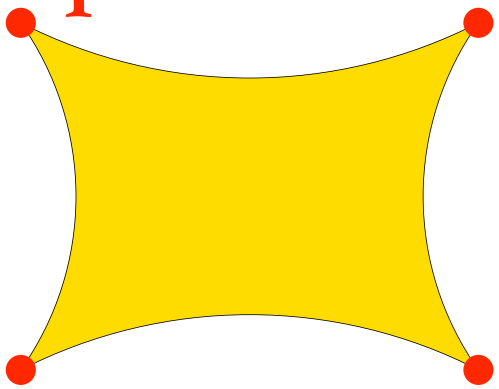
Introduction I: Motivations

Two main different paths to heterotic string phenomenology

Orbifold compactifications



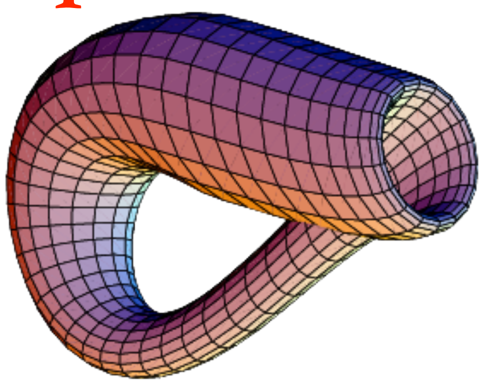
Exact quantization of string theory: CFT approach.

Buchmüller, Hamaguchi, Lebedev, Ratz; Liverpool group; Bonn group; ...

My task:

Reconciliate them!

Smooth manifold compactifications



Kaluza - Klein reduction of 10d SUGRA

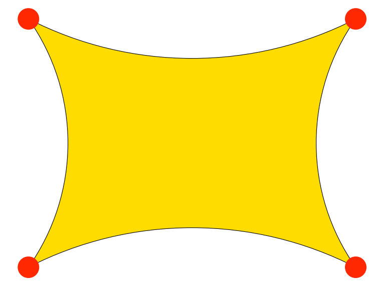
Ovrut, et al.; Oxford group, ...

Reproduce the orbifold models as compactifications of 10d SUGRA/SYM in the presence of gauge fluxes

Introduction II: the Spirit

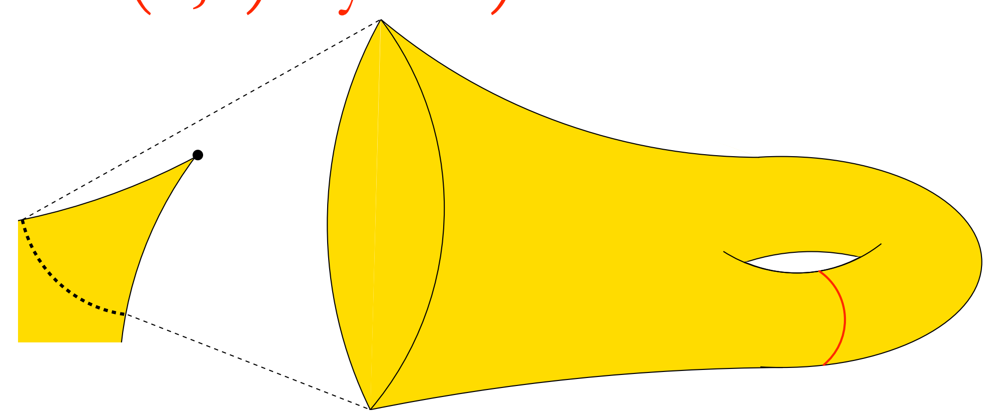
I - Resolve the orbifold geometry

Ia - Given the orbifold



Ib - Cut apart each singularity and resolve it:

characterize the local geometric structure “hidden” in the singularity (localized (1,1)-cycles)



Ic - Glue together the resolved singularities:

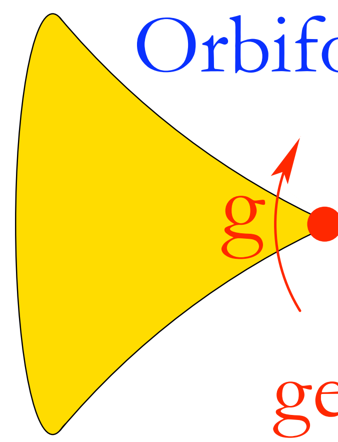
characterize the topology of the whole CY space (non-localized cycles)

Get a smooth compact CY space
(having the original orbifold as singular limit)

II - Compactify 10d SUGRA/SYM on the smooth CY

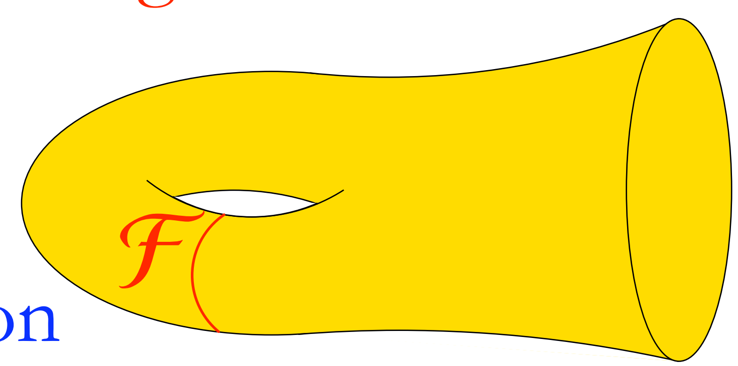
- A crucial detail:

Orbifold models:



Orbifold action g embedded in the gauge degrees of freedom.

The freedom in doing this generates a vast set of models!



SUGRA models:

U(1) gauge flux wrapped on the new localizes cycles, to be embedded in SO(32) or E_8 x E_8.

The freedom in the embedding generates a vast set of models

Reproduce each string orbifold model as a compactification of 10d SUGRA + SYM on a smooth CY embedding the "right" gauge flux

Introduction III - Outline

1) Getting the smooth CY space

- Basic facts in complex/toric geometry.
- Resolution of orbifold singularities using toric geometry
- Gluing of the resolved singularities
- A specific example: resolution of $\mathbf{T^4/Z_3}$

2) 10d SUGRA on the smooth CY space - only local study

- Consistency conditions (flux quantization, SYM e.o.m, ...)
- A specific example: $\mathbf{C^3/Z_4}$ models

3) Results and working plan

I - Resolution of Toroidal orbifolds using toric geometry

Lust, Reffert, Scheidegger, Stieberger '07

Basic facts in complex/toric geometry

Divisors

- Given a complex n-dim space (parameters z^i) a divisor X is locally an analytic hypersurface (e.g. $z^1 = 0$).
- To each divisor X we can associate a complex line bundle.

Linear equivalence

- Given two divisors X and Y we say that they are equivalent $X \sim Y$ if the associated line bundles differ by a trivial one.
- The set of divisors corresponds, modulo linear equivalence, to the (1,1)-forms on the space.

Intersection of divisors

- An intersection of divisors defines curves in the space.
- Intersecting n divisors we get points, the intersecting number $X_1 X_2 \ldots X_n = p$ means that the hypersurface X_1 intersects the curve $X_2 \ldots X_n$ in p points (or that X_2 intersects ...).
- Equivalently, we can read $X_1 X_2 \ldots X_n = p$ as the integral of the (1,1)-form X_1 on $X_2 \ldots X_n$ (or the integral of X_2 on ...).

Resolution of local singularities

- Each singularity (we treat) has form $\mathbf{C}^n/\mathbf{Z}_m$, with parameters z^i.
- Before resolution, the space has n divisors D_i, the surfaces $z^i = 0$.
- The singularity is resolved adding new exceptional divisors, E's to the set of D's.
- These exceptional divisors are not unrelated to the old ones: there are n linear relations $D_i \sim a_{ij} E_j$.
- In the resolution some basic intersection numbers are fixed by the procedure, the others can be re-obtained using the linear equivalence.

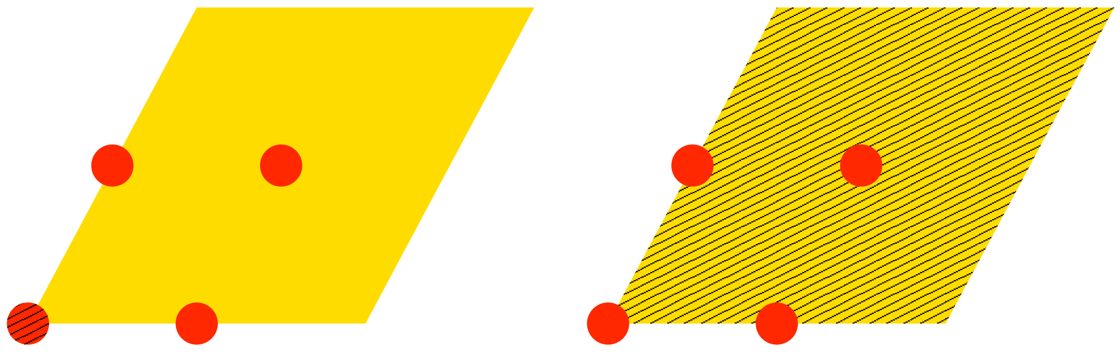
Gluing together the singularities into T^{2n}/Z_m

- Each resolved singularity is equipped with
  - a set of divisors $\{D_i, E_j\}$;
  - a set of linear equivalences $D_i \sim a_{ij} E_j$;
  - the local intersection numbers.
- Gluing:
  - "put together" the divisors in a single set
  - "put together" the linear equivalences in a single set
  - compute the intersections among the various divisors.

Caveats

1) There are extra (1,1)-forms/divisors inherited from $\mathbf{T}^{2n}$: R_i.
2) Some of the divisors (all R's and D's, some of the E's) extend in planes in the orbifold: they are "shared" by different singularities.

T^4/Z_2 example: the divisors D_1 corresponding to the surface $z_1=0$ extends along z_2 and is shared among the 4 singularities (0, x).



Example I: K3 as a T^4/Z_3 orbifold

- $\mathbf{T}^4 = \mathbf{T}^2 \times \mathbf{T}^2$, complex coordinates z_1, z_2.
- $\mathbf{Z}_3$ has 3 x 3 equivalent fixed points (singularities).

Local information:

- Each singularity has form $\mathbf{C}^2/\mathbf{Z}_3$, with 2 divisors (pre-resolution):

  D_1 corresponding to $z^1=0$ (fills the second **C**-plane)

  D_2 corresponding to $z^2=0$ (fills the first **C**-plane)
- Resolution: add two exceptional divisors E_1 and E_2.

  Linear equivalences: $0 \sim 3\,D_1 + E_1 + 2\,E_2$

  $0 \sim 3\,D_2 + E_2 + 2\,E_1$

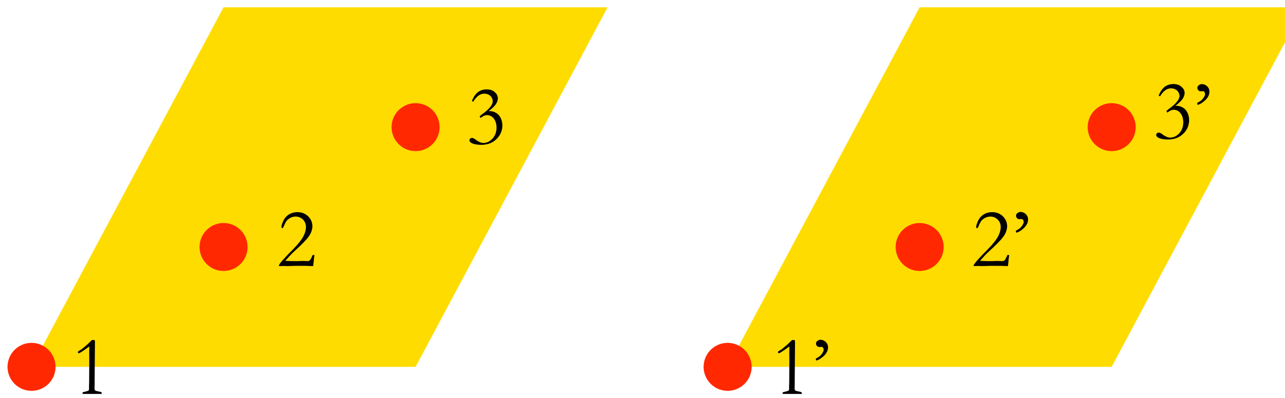
  Intersections: $D_1E_2 = E_2E_1 = E_1D_2 = 1$

  $D_1E_1 = D_2E_2 = 0 \quad E_1E_1 = E_2E_2 = -2$

Gluing:

1) "Assign fixed point indices"

- The E_i's are "localized" in the singularities, named 11', 12', 32', ...



for each E_i we assign two extra indices: $E_i^{jk'}$.

- D_1 extends in the second torus and is localized in the first:

we assign an extra index: D_1^i, similarly for D_2: $D_2^{j'}$.

- The D's are shared among various fixed points!

2) Include the inherited divisors:

- The R's and D's are linked, on the singular space: $R_i \sim 3D_i$.
- This link is the same for each of the D's : $R_1 \sim 3D_1^i, \; R_2 \sim 3D_2^{j'}$
- After resolution this linear equivalence is modified as

$$R_2 \sim 3D_2^{j'} + \sum_{i=1}^{3} \left(E_2^{ij'} + 2E_1^{ij'}\right), \; R_1 \sim 3D_1^{i} + \sum_{j'=1'}^{3'} \left(E_1^{ij'} + 2E_2^{ij'}\right)$$

3) Compute the global set of intersections:

- Use of the local information
- Input on the intersection of the R's

$$\mathrm{E}_1^{\mathrm{ij}'}\mathrm{E}_2^{\mathrm{pq}'} = \delta^{\mathrm{ip}}\delta^{\mathrm{j}'\mathrm{q}'}, \quad \mathrm{E}_1^{\mathrm{ij}'}\mathrm{E}_1^{\mathrm{pq}'} = \mathrm{E}_2^{\mathrm{ij}'}\mathrm{E}_2^{\mathrm{pq}'} = -2\delta^{\mathrm{ip}}\delta^{\mathrm{j}'\mathrm{q}'},$$

$$\mathrm{R}_1\mathrm{R}_2 = 3, \quad \mathrm{R}_1\mathrm{R}_1 = \mathrm{R}_2\mathrm{R}_2 = 0, \quad \mathrm{R}_\mathrm{i}\mathrm{E}_\mathrm{j}^{\mathrm{pq}'} = 0.$$

Outcome:

- Number of (1,1)-forms:

9 x 2 exceptional divisors
+ 2 x 3 "normal divisors"
– 2 x 3 equivalences
+ 2 inherited divisors

= **20**

- Characteristic classes (splitting principle)

$$c(\mathcal{R}) = (1+\mathrm{R}_1)(1+\mathrm{R}_2)\prod_{i=1}^{3}(1+\mathrm{D}_2^i)\prod_{j'=1'}^{3'}(1+\mathrm{D}_1^{j'})\prod_{i=1}^{3}\prod_{j=1'}^{3'}(1+\mathrm{E}_1^{ij'})(1+\mathrm{E}_2^{ij'})$$

from linear equivalence and intersections:

$$c_1(\mathcal{R}) = 0, \quad c_2(\mathcal{R}) = 24.$$

II - 10d SUGRA/SYM on the resolved CY: U(1) gauge flux on the singularities

Consistency conditions

1) Flux quantization: $\int_\gamma F \in \mathbf{Z}$

2) Equations of motion/SUSY:

- F must be a (1,1)-form, fulfilling the DUY condition

3) The Bianchi Identity for H must be fulfilled

$$\int_{C_2} (\mathcal{R} \wedge \mathcal{R} - F \wedge F) = 0$$

In the language of divisors:

- F can be written as $F = E_i V^{iI} H^I$
  - E_i the localized (1,1)-forms (flux invisible in blow-down)
  - H^I elements in the Cartan algebra of SO(32) or E_8 x E_8
- Quantization: V_i^I must be integers (half-integers)
- E.o.m.: conditions on the Kaehler moduli
- Bianchi Identity: use the splitting principle and the intersections

model dependent conditions

Ex: C^3/Z_4

1) Informations on the (local) geometry:

Set of divisors: D_1, D_2, D_3, E_1, E_2

Linear equivalences: $4\,D_1 + E_1 + 2\,E_2 \sim 0,$

$4\,D_2 + E_1 + 2\,E_2 \sim 0,\ 2\,D_3 + E_1 \sim 0.$

Intersections: $E_1^2\,E_2 = 0,\ E_2^2\,E_1 = 0,\ E_1^3 = 8,\ E_2^3 = 2.$

Chern classes: $c(\mathcal{R}) = \prod_{i=1}^{3}(1 + D_i)\prod_{j=1}^{2}(1 + E_i)$

3) Flux: $F = E_1\,V^{1I}\,H^I + E_2\,V^{2I}\,H^I$

4) Consistency conditions

- Quantization: the V's must be integer (half-integer)
- Bianchi Identity on the compact 4-cycle E_1:

$$\int_{E_1}(\mathcal{R}\wedge\mathcal{R} - F\wedge F) = E_1\left(\prod_{i=1}^{3}(1 + D_i)\prod_{j=1}^{2}(1 + E_i)\right)_2 - E_1 F^2 = 0$$

using the intersection numbers: $V^1V^1 + V^1V^2 = 4$

- Bianchi Id. on the non-compact 4-cycle E_2: $V^1V^2 = -2$
- Bianchi Id. on the sub-variety $\mathbf{C^2/Z_2}$: $V^2V^2 = 6$

Matching the orbifold models

orbifold shift $4\,v$	blowup vector V_2	blowup vector V_1	Nr.
$(0^{13},1^2,2)$	$(0^{13},1^2,2)$	$(0^{13},1^2,\text{-}2)$	1a
	$(0^{13},1^2,2)$	$(0^{12},2,\text{-}1^2,0)$	1b
	$(0^{13},1^2,2)$	$(0^{11},2,1,0^2,\text{-}1)$	1c
$(0^{11},1^2,2^3)$	$(0^{13},1^2,2)$	$(0^{10},1^4,\text{-}1^2)$	2a
	$(0^{13},1^2,2)$	$(0^{11},1^2,\text{-}2,0^2)$	2b
$(0^{9},1^2,2^5)$	$(0^{13},1^2,2)$	$(0^{8},1^5,0^2,\text{-}1)$	3a
	$(0^{13},1^2,2)$	$(0^{9},1^4,\text{-}1^2,0)$	3b
$(0^{7},1^2,2^7)$	–	–	4
$(0^{10},1^6)$	$(0^{10},1^6)$	$(0^{10},1^2,\text{-}1^4)$	5a
	$(0^{10},1^6)$	$(0^{13},1,\text{-}1,\text{-}2)$	5b
$(0^{10},1^5,3)$	$(0^{10},1^6)$	$(0^{9},2,\text{-}1^2,0^4)$	6
$(0^{8},1^6,2^2)$	$(0^{10},1^6)$	$(0^{8},1^3,\text{-}1^3,0^2)$	7a
	$(0^{10},1^6)$	$(0^{8},1^2,\text{-}2,0^5)$	7b
$(0^{6},1^6,2^4)$	$(0^{10},1^6)$	$(0^{6},1^4,\text{-}1^2,0^4)$	8

orbifold shift $4\,v$	blowup vector V_2	blowup vector V_1	Nr.
$(0^{5},1^{10},2)$	$(0^{10},1^6)$	$\frac{1}{2}(\text{-}3,1^{10},\text{-}1^5)$	9
$(0^{3},1^{10},2^3)$	$(0^{10},1^6)$	$\frac{1}{2}(1^{12},\text{-}1^3,\text{-}3)$	10
$(1^{14},2^2)$	$(0^{13},\text{-}2,1^2)$	$\frac{1}{2}(1^{15},\text{-}3)$	11
$(1^{13},\text{-}1,2^2)$	$(0^{13},1^2,2)$	$\frac{1}{2}(1^{15},\text{-}3)$	12a
	$(0^{13},1^2,2)$	$\text{-}\frac{1}{2}(\text{-}3,1^{15})$	12b
$\frac{1}{2}(1^3,3^{12},\text{-}3)$	$\frac{1}{2}(\text{-}3,1^{15})$	$\text{-}(0^{13},1^2,2)$	13a
	$\frac{1}{2}(1^{15},\text{-}3)$	$(0^{13},1^2,2)$	13b
	$\frac{1}{2}(1^{15},\text{-}3)$	$\frac{1}{2}(1^3,\text{-}1^{11},3,1)$	13c
$\frac{1}{2}(1^7,3^8,\text{-}3)$	$\frac{1}{2}(1^{15},\text{-}3)$	$(\text{-}1^5,1,0^{10})$	14a
	$\frac{1}{2}(1^{15},\text{-}3)$	$\frac{1}{2}(1^6,\text{-}1^8,\text{-}3,1)$	14b
$\frac{1}{2}(1^{11},3^4,\text{-}3)$	$\frac{1}{2}(1^{15},\text{-}3)$	$(0^{10},1^3,\text{-}1^3)$	15
$\frac{1}{2}(1^{15},\text{-}3)$	$\frac{1}{2}(1^{15},\text{-}3)$	$(0^{13},\text{-}2,1^2)$	16a
	$\frac{1}{2}(1^{15},\text{-}3)$	$\frac{1}{2}(\text{-}1^{14},3,\text{-}1)$	16b

III - Results & Working plan

1) We show how to resolve all the $\mathbf{C^n/Z_m}$ and $\mathbf{C^n/Z_m \times Z_p}$ singularities, how to wrap U(1) flux on them and match heterotic orbifold models.
   - Recover and extend the results of Groot Nibbelink, MT, Walter ($\mathbf{C^n/Z_n}$ singularities)

2) The new approach (toric geometry) allows to glue the singularities and recover compact $\mathbf{T^n/Z_m}$ and $\mathbf{T^n/Z_m \times Z_p}$ orbifolds.

3) Study of compact heterotic models
   - done the $\mathbf{T^6/Z_3}$ model.
     Groot Nibbelink, Klevers, Ploger, MT, Vaudrevenge
   - <u>work in progress:</u> the K3 models
     - reobtain the results of Honecker, MT with explicit control on the line bundles
     - tool for a study of Heterotic/IIA duality
   - <u>next step:</u> the phenomenologically appealing $\mathbf{T^6/Z_{6\text{-}II}}$ model