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$Z_2 \times Z_2$ orientifolds of non factorisable tori

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Summary

1. Factorisable versus non factorisable $Z_2 \times Z_2$ orbifolds.

2. $Z_2 \times Z_2$ orientifolds with branes at angles.

3. The non factorisable case.

4. The case with discrete torsion - fractional branes.

5. Conclusions.

$Z_2 \times Z_2$ orbifolds

Factorisable tori:

$$T^6 = T^2 \times T^2 \times T^2$$

$$\theta_1 = (+\ ,\ \ -\ \ ,\ \ -)$$

$$\theta_2 = (-\ ,\ \ -\ \ ,\ \ +)$$

$$e_1 = (x, x, 0, 0, 0, 0)$$

$$e_2 = (x, x, 0, 0, 0, 0)$$

$$e_3 = (0, 0, x, x, 0, 0)$$

$$e_4 = (0, 0, x, x, 0, 0)$$

$$e_5 = (0, 0, 0, 0, x, x)$$

$$e_6 = (0, 0, 0, 0, x, x)$$

Non-factorisable tori: $T^6 \neq T^2 \times T^2 \times T^2$

$$\begin{pmatrix} x^1 \\ \vdots \\ x^6 \end{pmatrix} \rightarrow \theta_{1,2} \begin{pmatrix} x^1 \\ \vdots \\ x^6 \end{pmatrix}, \theta_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \theta_2 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$Z_2 \times Z_2$ orientifolds

$T^6/Z_2 \times Z_2 \times \Omega R, \quad R : x_{1,3,5} \longrightarrow x_{1,3,5}, \;\; x_{2,4,6} \longrightarrow -x_{2,4,6}$

$\Downarrow$

Four types of O6-planes : invariant under ΩR, $\Omega R\theta_1$, $\Omega R\theta_2$ and $\Omega R\theta_1\theta_2$.

$$\Omega R : \frac{\mathrm{vol}\,(\Lambda_{\mathcal{R},\perp})}{\mathrm{vol}\,(\Lambda_{-\mathcal{R},inv})} \cdot 32^2 + \frac{\mathrm{vol}\,(\Lambda_{\mathcal{R},inv})}{\mathrm{vol}\,(\Lambda_{-\mathcal{R},\perp})} \cdot N^2 - 2 \cdot N \cdot 32 \cdot \frac{\mathrm{vol}\,(\Lambda_{-\mathcal{R},inv})}{\mathrm{vol}\,(\Lambda_{\mathcal{R},inv})} = 0$$

$\Downarrow$

Introduce D6-branes for tadpole cancellation. D6-branes non parallel to the O6-planes give rise to chiral spectra.

The factorisable case. One-cycles:

$$[a_1] = (1,0,0,0,0,0) \qquad [b_1] = (0,1,0,0,0,0)$$
$$[a_2] = (0,0,1,0,0,0) \qquad [b_2] = (0,0,0,1,0,0)$$
$$[a_3] = (0,0,0,0,1,0) \qquad [b_3] = (0,0,0,0,0,1)$$

Orbifold invariant branes :

$$\begin{aligned} D6_a &= \left(m_a^1, n_a^1, 0,0,0,0\right) \times \left(0,0,m_a^2,n_a^2,0,0\right) \times \left(0,0,0,0,m_a^3,n_a^3\right) \\ &= \left(m_a^1\,[a_1] + n_a^1\,[b_1]\right) \times \left(m_a^2\,[a_2] + n_a^2\,[b_2]\right) \times \left(m_a^3\,[a_3] + n_a^3\,[b_3]\right) \end{aligned}$$

R-images, $D6_{a'} \quad : \quad n_a \rightarrow -\, n_a$

Consistency conditions

RR tadpole cancellation conditions

$$\sum_a N_a m_a^1 m_a^2 m_a^3 - 16 = 0$$
$$\sum_a N_a m_a^1 n_a^2 n_a^3 + 16 = 0$$
$$\sum_a N_a n_a^1 m_a^2 n_a^3 + 16 = 0$$
$$\sum_a N_a n_a^1 n_a^2 m_a^3 + 16 = 0$$

$\mathcal{N} = 1$ supersymmetry

$$\mathcal{N} = 2 \xrightarrow{10D \rightarrow 4D} \mathcal{N} = 8 \xrightarrow{Z_2 \times Z_2} \mathcal{N} = 2 \xrightarrow{\Omega} \mathcal{N} = 1$$

The branes must preserve $\mathcal{N} = 1$ supersymmetry:

$$\arctan\left(\frac{m_1}{n_1}\right) + \arctan\left(\frac{m_2}{n_2}\right) + \arctan\left(\frac{m_3}{n_3}\right) = 0$$

Non-factorisable example: $SO(12)$

$$\begin{aligned} e_1 &= (1,-1,0,0,0,0)\,, \\ e_2 &= (0,1,-1,0,0,0)\,, \\ e_3 &= (0,0,1,-1,0,0)\,, \\ e_4 &= (0,0,0,1,-1,0)\,, \\ e_5 &= (0,0,0,0,1,-1)\,, \\ e_6 &= \quad (0,0,0,0,1,1)\,. \end{aligned}$$

$$\begin{aligned} D6_a &= \left(m_a^1\,[a_1] + n_a^1\,[b_1]\right) \times \left(m_a^2\,[a_2] + n_a^2\,[b_2]\right) \times \left(m_a^3\,[a_3] + n_a^3\,[b_3]\right) \\ &= \left(m_a^1 e_1 + (m_a^1 + n_a^1)\left(e_2 + e_3 + e_4 + \frac{1}{2}e_5 + \frac{1}{2}e_6\right)\right) \times \\ &\left(m_a^2 e_3 + (m_a^2 + n_a^2)\left(e_4 + \frac{1}{2}e_5 + \frac{1}{2}e_6\right)\right) \times \left(\frac{m_a^3}{2}(e_5+e_6) + \frac{n_a^3}{2}(e_6 - e_5)\right) \end{aligned}$$

$$\begin{aligned} O_{\Omega\mathcal{R}} &= (1,0,\text{-}1,0,0,0) \times (0,0,1,0,\text{-}1,0) \times (0,0,1,0,1,0) = 2\,[a_1] \times [a_2] \times [a_3]\,, \\ O_{\Omega\mathcal{R}\theta} &= (0,1,0,1,0,0) \times (0,0,0,\text{-}1,\text{-}1,0) \times (0,0,0,\text{-}1,1,0) = -2\,[b_1] \times [b_2] \times [a_3]\,, \\ O_{\Omega\mathcal{R}\omega} &= (1,0,0,\text{-}1,0,0) \times (0,0,0,1,0,1) \times (0,0,0,1,0,\text{-}1) = -2\,[a_1] \times [b_2] \times [b_3]\,, \\ O_{\Omega\mathcal{R}\theta\omega} &= (0,1,\text{-}1,0,0,0) \times (0,0,1,0,0,1) \times (0,0,1,0,0,\text{-}1) = -2\,[b_1] \times [a_2] \times [b_3]\,. \end{aligned}$$

Intersection number

Intersecting number of a stack $D6_a$ and a stack $D6_b$ =
Jacobian(**lattice where the the D-branes intersect once** $\longrightarrow$**compactification lattice)**

Depends on the lattice!

$$I_{ab} = \det \begin{pmatrix} m_a^1 & n_a^1 & 0 & 0 & 0 & 0 \\ m_b^1 & n_b^1 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_a^2 & n_a^2 & 0 & 0 \\ 0 & 0 & m_b^2 & n_b^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_a^3 & n_a^3 \\ 0 & 0 & 0 & 0 & m_b^3 & n_b^3 \end{pmatrix} = \prod_{i=1}^{3} (m_a^i n_b^i - n_a^i m_b^i)$$

Model building rules

Gauge group factors :

- N D6-branes not parallel to an O6-plane $\rightsquigarrow$ $U(N/2)$.
- N D6-branes parallel to an O6-plane $\rightsquigarrow$ $USp(N)$.

Chiral spectrum (comes from strings stretched between branes tilted with respect to the O-planes) :

- Strings stretching between the brane-stacks N_a and N_b $\rightsquigarrow$ I_{ab} multiplets in the $\left(\frac{N_a}{2}, \frac{\overline{N}_b}{2}\right)$ representation of $U(N_a/2) \times U(N_b/2)$.

- Strings stretching between the stack N_a and the $\mathcal{R}$-image $N_{b'}$ $\rightsquigarrow$ $I_{ab'}$ multiplets in the $\left(\frac{N_a}{2}, \frac{N_{b'}}{2}\right)$ representation of $U(N_a/2) \times U(N_{b'}/2)$.

- Strings stretching between the stack N_a and its $\mathcal{R}$ image, $N_{a'}$, $\rightsquigarrow$ $\frac{1}{2}(I_{aa'} + 4I_{aO6})$ multiplets in the anti-symmetric representation and $\frac{1}{2}(I_{aa'} - 4I_{aO6})$ in the symmetric representation of $U(N_a/2)$.

Getting the Standard Model :

Total number of families $= I_{ab} + I_{ab'}$.

If the stack $D6_a$ generates the U(3) gauge group and

the stack $D6_b$ gives the U(2) gauge group,

in order to have three copies of the $(3, 2)$ representation of SU(3)×SU(2)

we need either

(i) $I_{ab} = 3$ and $I_{ab'} = 0$ or

(ii) $I_{ab} = 2$ and $I_{ab'} = 1$

The factorisable case

$$I_{ab} = \prod_{i=1}^{3}(m_a^i n_b^i - n_a^i m_b^i)$$
$$I_{ab'} = -\prod_{i=1}^{3}(m_a^i n_b^i + n_a^i m_b^i)$$

$$I_{ab} + I_{ab'} = -2\left[m_a^1 m_a^2 n_a^3 n_b^1 n_b^2 m_b^3 + m_a^1 n_a^2 m_a^3 n_b^1 m_b^2 n_b^3 + n_a^1 n_a^2 n_a^3 m_b^1 m_b^2 m_b^3 + n_a^1 m_a^2 m_a^3 m_b^1 n_b^2 n_b^3\right]$$

One tilted torus :

$$\begin{aligned} e_1 &= (1, -1, 0, 0, 0, 0) \\ e_2 &= (1,\ 1, 0, 0, 0, 0) \\ e_3 &= (0, 0, 1, 0, 0, 0) \\ e_4 &= (0, 0, 0, 1, 0, 0) \\ e_5 &= (0, 0, 0, 0, 1, 0) \\ e_6 &= (0, 0, 0, 0, 0, 1) \end{aligned}$$

$$\begin{aligned} I_{ab} &= \det \begin{pmatrix} \frac{m_a^1 - n_a^1}{2} & \frac{m_a^1 + n_a^1}{2} & 0 & 0 & 0 & 0 \\ \frac{m_b^1 - n_b^1}{2} & \frac{m_b^1 + n_b^1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_a^2 & n_a^2 & 0 & 0 \\ 0 & 0 & m_b^2 & n_b^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_a^3 & n_a^3 \\ 0 & 0 & 0 & 0 & m_b^3 & n_b^3 \end{pmatrix} \\ &= \frac{1}{2} \prod_{i=1}^{3} \left(m_a^i n_b^i - n_a^i m_b^i \right) \end{aligned}$$

$$\begin{aligned} D6_a &= \left(m_a^1 [a_1] + n_a^1 [b_1] \right) \times \left(m_a^2 [a_2] + n_a^2 [b_2] \right) \times \left(m_a^3 [a_3] + n_a^3 [b_3] \right) \\ &= \left(\frac{m_a^1}{2} (e_1 + e_2) + \frac{n_a^1}{2} (e_2 - e_1) \right) \times \left(m_a^2\, e_3 + n_a^2\, e_4 \right) \times \left(m_a^3\, e_5 + n_a^3\, e_6 \right) \end{aligned}$$

Closed cycles condition : $m_a^1 + n_a^1 = \text{even}$ *(if not then wrap twice)*

- $m_{a,b}^1,\ n_{a,b}^1 = \text{odd} \longrightarrow$ factor of 2 in I_{ab}
- $m_{a,b}^1,\ n_{a,b}^1 = \text{even} \longrightarrow$ factor of 4 in I_{ab}

Examples with three generations on factorisable lattices

Case	Stack	N_a	m_a^1	n_a^1	m_a^2	n_a^2	m_a^3	n_a^3
$I_{ab} = 2$	a	6	3	1	1	−1	1	0
$I_{ab'} = 1$	b	4	1	1	1	0	1	−1
$I_{ab} = 3$	a	6	1	1	0	−1	1	1
$I_{ab'} = 0$	b	4	1	−1	3	1	1	0

Non factorisable tori

$$\begin{aligned} e_1 &= (1,0,-1,0,0,0) \\ e_2 &= (0,1,0,0,0,0) \\ e_3 &= (1,0,1,0,0,0) \\ e_4 &= (0,0,0,1,0,0) \\ e_5 &= (0,0,0,0,1,0) \\ e_6 &= (0,0,0,0,0,1) \end{aligned}$$

$$I_{ab} = \frac{1}{2}\prod_{i=1}^{3}\left(m_a^i n_b^i - n_a^i m_b^i\right)$$

$$D6_a = \left(\frac{m_a^1}{2}\,(e_1+e_3) + n_a^1\,e_2\right)\times\left(\frac{m_a^2}{2}\,(e_3-e_1) + n_a^2\,e_4\right)\times\left(m_a^3\,e_5 + n_a^3\,e_6\right)$$

Closed cycles condition : $m_a^1, m_a^2 = \text{even}$

Each condition gives a factor of 2, hence we get a factor of 4 in I_{ab}.

Idea : $m_a^1, m_a^2 = \text{even}$ and $m_b^1, m_b^2 = \text{odd}$ and wrap twice cycle $D6_b$.

$\Downarrow$

factor of 2. One can get $I_{ab} = \text{odd}$, but $I_{ab'}$ will be odd as well.

Keep conditions in the form $m_a^1 + n_a^1 = \text{even}$

$$e_1 = (1, -1, 0, 0, 0, 0)$$
$$e_2 = (0, 1, -1, 0, 0, 0)$$
$$e_3 = (0, 1, \ 1, 0, 0, 0)$$
$$e_4 = (0, 0, 0, 1, 0, 0)$$
$$e_5 = (0, 0, 0, 0, 1, 0)$$
$$e_6 = (0, 0, 0, 0, 0, 1)$$

$$D6_a \ = \ \left(m_a^1 \left(e_1 + \frac{e_2 + e_3}{2} \right) + \frac{n_a^1}{2} (e_2 + e_3) \right) \times \left(\frac{m_a^2}{2} (e_3 - e_2) + n_a^2 e_4 \right) \times (m_a^3 e_5 + n_a^3 e_6)$$

Conditions : $m_a^1 + n_a^1 = \text{even}, \ m_a^2 = \text{even}$

..again too many factors of 2

Different orientifold action

$$\mathcal{R} : \ \ z^1 \to \mathrm{i}\bar{z}^1, \ \ z^i \to \bar{z}^i, \ \ i = 2, 3$$

$\mathcal{R}$ image branes : $n \leftrightarrow m$

Interesting conditions are $m_a^i = \text{even}$, since $I_{ab'} \sim \prod_{i=1}^3 (m_a^i m_b^i - n_a^i n_b^i)$, but they are incompatible with $\mathcal{R}$.

Non-invariant branes:

$$
\begin{aligned}
e_1 &= (1, 0, -1, 0, 0, 0) \\
e_2 &= (0, 1, 0, 0, 0, 0) \\
e_3 &= (1, 0, 1, 0, 0, 0) \\
e_4 &= (0, 0, 0, 1, 0, 0) \\
e_5 &= (0, 0, 0, 0, 1, 0) \\
e_6 &= (0, 0, 0, 0, 0, 1)
\end{aligned}
$$

$$
D6_a = (m_a^1, 0, n_a^1, 0, 0, 0) \times (0, m_a^2, 0, n_a^2, 0, 0) \times (0, 0, 0, 0, m_a^3, n_a^3)
$$

$I_{ab} + I_{ab'} = 3$ is possible, but supersymmetry is broken.

Outlook

$Z_2 \times Z_2$ orientifolds with discrete torsion of non-factorisable lattices :

- Admits fractional branes, which might yield an odd number of families.

$$\Pi_a^F = \frac{1}{4}\Pi_a^B + \frac{1}{4}\left(\sum_{i,j\in S_\Theta^a} \epsilon_{a,ij}^{\Theta}\,\Pi_{ij,\,a}^{\Theta}\right) + \frac{1}{4}\left(\sum_{j,k\in S_{\Theta'}^a} \epsilon_{a,jk}^{\Theta'}\,\Pi_{jk,\,a}^{\Theta'}\right) + \frac{1}{4}\left(\sum_{i,k\in S_{\Theta\Theta'}^a} \epsilon_{a,ik}^{\Theta\Theta'}\,\Pi_{ik,\,a}^{\Theta\Theta'}\right)$$

- Admits rigid cycles.

- Extra (twisted) tadpole conditions.

Conclusions

$Z_2 \times Z_2$ orientifolds of non-factorisable lattices with branes at angles :

- Admit chiral $N = 1$ models.
- Tadpoles conditions change according to the lattice.
- Gauge groups become smaller.
- Non-factorisable lattices $\rightsquigarrow$ constraints on the wrapping numbers of the D6-branes $\rightsquigarrow$ even intersection numbers $\rightsquigarrow$ even number of families.
- Odd intersection numbers possible with non-invariant branes $\rightsquigarrow$ supersymmetry is broken.

Other options :

- fractional branes
- shift orientifolds
- higher order orbifolds