Cosmological Signatures of Brane Inflation

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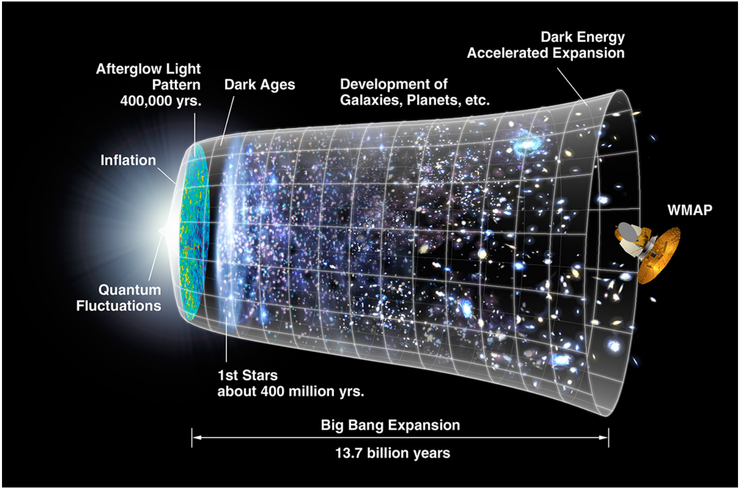
Milestones in the Evolution of the Universe



http://map.gsfc.nasa.gov/m_mm.html

Information about the Inflationary period

- The amplitude of the large-scale temperature fluctuations: $\delta_H = \frac{2}{5}\sqrt{\mathcal{P}_{\mathcal{R}}} = 1.9 \times 10^{-5}$, is determined by the energy density during inflation. The data suggests $V \sim 10^{15}$ GeV.
- The spectral index and the running of the spectral index:

$$
\begin{aligned}
n - 1 &= \frac{d \ln \delta_H^2(k)}{d \ln k} = 2\eta - 6\epsilon \\
\frac{dn}{d \ln k} &= -16\epsilon\eta + 24\epsilon^2 + 2\xi^2
\end{aligned}
$$

- Present value from WMAP (only) data: $n_{\text{scalar}} = 0.951^{+0.015}_{-0.019}$, $dn_{\text{scalar}}/d \log k = -0.055^{+0.029}_{-0.035}$.

Braneworld

- **Closed Strings**
- NS-NS: Graviton $G_{\mu\nu}$, Dilaton ϕ, Antisymmetric Tensor $B_{\mu\nu}$ and RR higher-spin fields $C_{\mu_0 \cdots \mu_p}$ couple to the brane world-volume.
- **Open Strings**
- NS and R: Gauge Bosons, Fermions and Tachyons
- The ends of the open strings cannot leave the branes.
- The distance between the branes becomes the mass of the particles:

$$M^2 = \frac{Y_\mu \cdot Y^\mu}{4\pi^2 \alpha'^2} + \frac{L_0}{\alpha'}$$

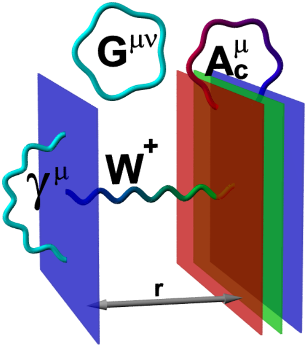
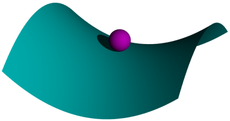


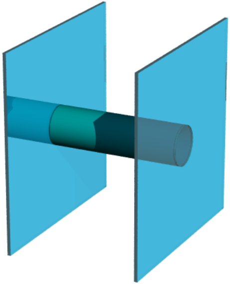
Figure: The Braneworld model can describe all the particles and interactions of the Standard Model. Gravity is present as well but being represented by a closed string mode is not confined to propagate only inside the branes.

Candidates for the Inflaton

- **Closed string modes**
- Radion/dilaton: gravitational strength couplings
- **Open string modes**
- Tachyons: their potential is too steep
- Brane separation as inflaton: brane interaction has a relatively flat potential
- In general, the brane interaction has gravitational strength → very weak → very flat potential
- But the inflaton has standard model strength coupling to standard model particles → can reheat efficiently

G. Dvali and H. Tye, hep-ph/9812483





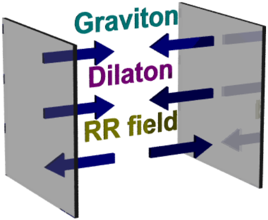
Brane Inflation

- Expansion of the universe driven by the energy of the *inflaton* field.
- Inflaton field identified with inter-brane separation.
- Parallel branes experience no force.

$$V(y) = \left(1_{NS-NS} \pm 1_{RR}\right) V_0(y)$$

- The interaction between branes gives the inflaton its potential.
- A brane and an anti-brane experience a Coulonb-type attractive force.

$$V(y) = C - \frac{V_0}{y^n}$$



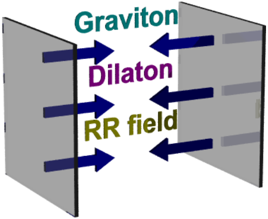
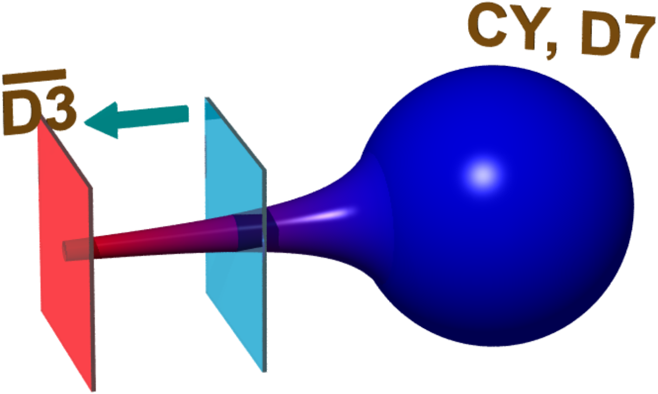


Figure: There in no interaction between parallel branes. The interaction between a brane and an anti-brane is attractive.

Stabilization of the compactification

- Stabilizing the compactification (GKP, hep-th/0105097) produces a warped geometry.
- $\overline{D3}$ branes sink to the bottom of the warped region.



KKLT

Stabilizing the compactification produces AdS vacua, hep-th/0301240.

- Use euclidean $D3$ branes or the gaugino condensation on wrapped $D7$ branes to generate the superpotential:

$$W = w_0 + Ae^{ia\rho}$$

- The volume modulus has a non-trivial Kähler potential:

$$K = -3 \ln \left[i \left(\rho - \overline{\rho} \right) \right]$$

- The F-term potential has the form:

$$V_F = e^K \left[K^{a\overline{b}} D_a W \overline{D_a W} - 3 \left| W \right|^2 \right]$$

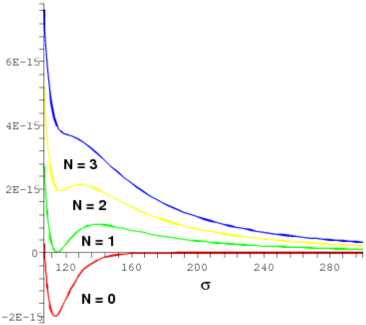
V_F has a negative niminum. One must add anti-branes to obtain Minkowski space.

Uplifting

- An anti-brane breaks SUSY and contributes a positive term to the potential.
- Its contribution is affected by the warp factor.

$$V_{\overline{D3}} = \frac{T_3 h_A^4}{\sigma^2}$$

- An anti-brane prefers to move to the bottom of the warped throat.



$$V = \frac{aAe^{-a\sigma}}{2\sigma^2}\left(\frac{1}{3}\sigma aAe^{-a\sigma} + W_0 + Ae^{-a\sigma}\right) + N\frac{T_3 h_A^4}{\sigma^2}$$

Adding mobile branes

If we now add brane-anti-brane pairs to this geometry:

- The anti-branes will sink to the bottom of the throat.
- They will uplift the potential to a *dS* minimum realizing Inflation.
- The mobile branes will be attracted towards the anti-branes, and will move towards the bottom of the throat.
- Inflation ends when the branes and the anti-branes collide and annihilate.

However: The Kahler potential for the mobile branes depends on the volume modulus:

$$K = -3 \ln \left[i \left(\rho - \overline{\rho} \right) + k \left(\phi, \overline{\phi} \right) \right]$$

This introdces a new η-problem.

Moduli Stabilization and the η Problem

KKLMMT, hep-th/0308055, argue that the potential for the inflaton takes the form:

- $$V = \frac{V_0\left(\sigma_c\right)}{\left(\sigma - \frac{1}{3}\left|\varphi\right|^2\right)^2} \simeq V_0\left(\sigma_c\right)\left(1 + \frac{2}{3}\left|\varphi\right|^2\right)$$
- This results in a value $\eta = 2/3$, so the slow-roll condition is *not* satisfied.
- A possible solution was suggested by Baumann et.al. arXiv:0705.3837:
- It is possible to find a value of the field where the η vanishes.

The curse of the inflexion point

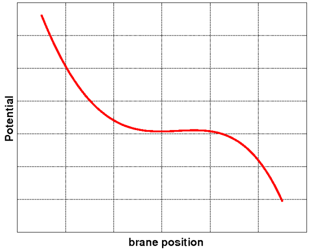
Take into account the embedding of the $D7$ in the CY.

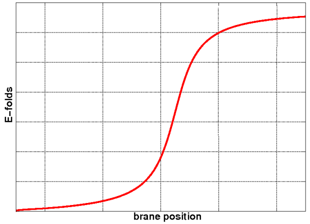
- The superpotential take the form:

$$W = w + A_0 \left(\frac{f(\phi)}{f(0)} \right)^{1/n} e^{-a\rho}$$

- The corresponding F-term potential features an inflexion point.
- Most of the e-folds come from the region around the inflexion point.
- Brane-anti-brane interaction also gives an inflexion point:

$$V_{\text{Coulomb}} = -\frac{k}{\left(\sigma - \frac{|\varphi|^2}{3}\right)^2 (\phi - \phi_0)^4}$$





Inflationary Dynamics

The effective action has the form:

$$S = \int d^3x dt \sqrt{-g} \left(\frac{1}{2} G_{ij} \partial_\mu \phi^i \partial^\mu \phi^j - V + R \right)$$

Assuming an FRW $4D$ metric the equations of motion are:

$$\ddot{\phi}^k + 3H\dot{\phi}^k + G^{ka} \frac{\partial V}{\partial \phi^a} + \Gamma^k_{ij} \dot{\phi}^i \dot{\phi}^j = 0$$

with the Hubble constant given by:

$$H^2 = \frac{1}{3} \left(G_{ij} \dot{\phi}^i \dot{\phi}^j + V \right)$$

Γ^k_{ij} are the connection coefficients obtained from the target space metric G_{ij}.

Density Perturbations

The power spectrum of a multi-field inflationary model is given by:

$$P\left(k\right) = \frac{V}{75\pi^2 M_P^2} G^{ij} \frac{\partial N}{\partial \phi^i} \frac{\partial N}{\partial \phi^j}\bigg|_{N=60}$$

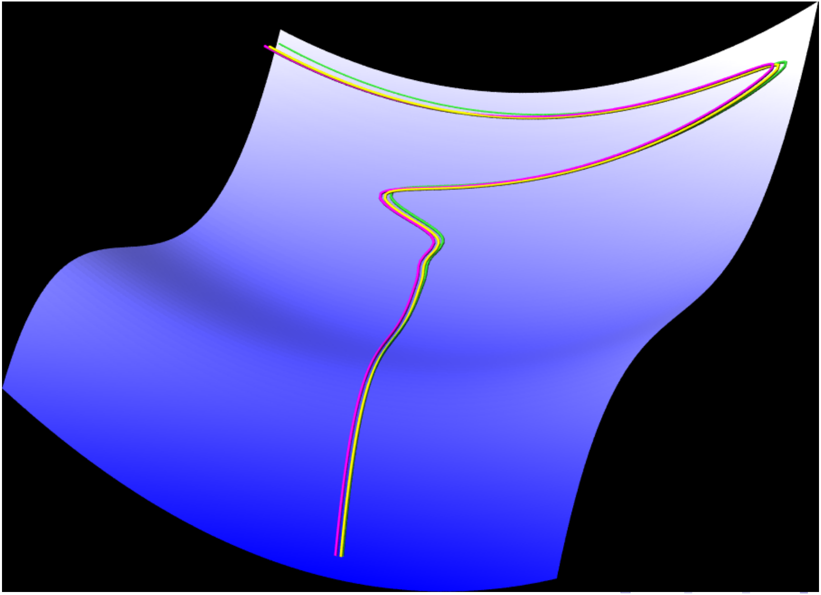
(M. Sasaki and E. D. Stewart, astro-ph/9507001).

- The COBE normalization implies $\sqrt{P\left(k_0\right)} \simeq 2 \times 10^{-5}$ at the scale $k_0 \sim 10^3$Mpc.
- In order to obtain the right amplitude for the density fluctuations, we have to rescale the potential, which amounts to rescaling the string mass.
- In our model we find:

$$M_S \simeq 4 \times 10^{15}\text{GeV}$$

which is about M_{GUT}.

Multibrane trajectories



Signatures of Multiple Branes

We now consider the most general perturbations around the background:

- For the metric:

$$g_{\mu\nu} + \delta g_{\mu\nu} =$$
$$\begin{pmatrix} -\left(1+2A\left(t,\mathbf{x}\right)\right) & a\left(t\right)B_{,i}\left(t,\mathbf{x}\right) \\ a\left(t\right)B_{,i}\left(t,\mathbf{x}\right) & a^{2}\left(t\right)\left[\left(1-2\psi\left(t,\mathbf{x}\right)\right)\delta_{ij}+2E_{,ij}\left(t,\mathbf{x}\right)\right] \end{pmatrix}$$

- and the fields ϕ^{I} as:

$$\phi^{I} = \phi^{I}\left(t\right) + \delta\phi^{I}\left(t,\mathbf{x}\right)$$

Perturbations of the Scalar field

The equation for the scalr field perturbations:

$$\delta\ddot{\phi}^K + 3\frac{\dot{a}}{a}\delta\dot{\phi}^K + \frac{k^2}{a^2}\delta\phi^K + \left[G^{KI}\frac{\partial^2 V}{\partial\phi^I\partial\phi^J} + \frac{\partial G^{KI}}{\partial\phi^J}\frac{\partial V}{\partial\phi^I}\right]\delta\phi^J$$
$$= -2AG^{KI}\frac{\partial V}{\partial\phi^I} + \left[\dot{A} + 3\dot{\psi} + k^2\dot{E} - \frac{k^2}{a}B\right]\dot{\phi}^K$$
$$-2\Gamma^K_{IJ}\dot{\phi}^I\delta\dot{\phi}^J - \frac{\partial\Gamma^K_{IJ}}{\partial\phi^L}\dot{\phi}^I\dot{\phi}^J\delta\phi^L = 0$$

and similarly for the metric fluctuations.
The equation simplifies if we use the Mukhanov-Sasaki variables.

Mukhanov-Sasaki Variables

$$Q^I \equiv \delta\phi^I + \frac{\dot{\phi}^I}{H}\psi$$

For these varibles the equation of motion becomes:

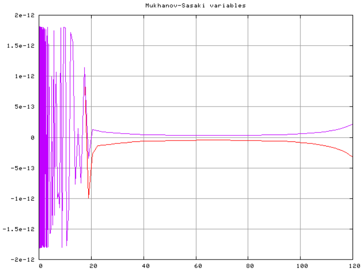
$$\ddot{Q}^K + 3H\dot{Q}^K + \frac{k^2}{a^2}Q^K + 2\Gamma^K_{IJ}\dot{\phi}^I\dot{Q}^J +$$
$$\left[G^{KI}\frac{\partial^2 V}{\partial\phi^I\partial\phi^J} + \frac{\partial G^{KI}}{\partial\phi^J}\frac{\partial V}{\partial\phi^I} + \frac{\partial\Gamma^K_{IJ}}{\partial\phi^L}\dot{\phi}^I\dot{\phi}^L\right.$$
$$-\left(\frac{8\pi G}{a^3}\right)G_{IJ}\frac{\partial}{\partial t}\left(\frac{a^3\dot{\phi}^K\dot{\phi}^I}{H}\right) -$$
$$\left.\left(\frac{8\pi G}{H}\right)\Gamma_{ILJ}\dot{\phi}^K\dot{\phi}^I\dot{\phi}^L - \left(\frac{8\pi G}{H}\right)\Gamma^K_{LM}\dot{\phi}^L\dot{\phi}^M G_{IJ}\dot{\phi}^I\right] Q^J = 0$$

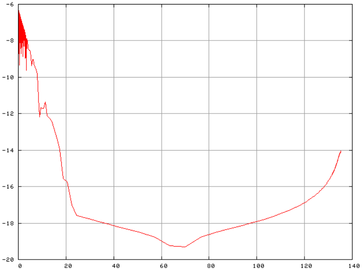
Initial Conditions

We take the initial conditions to be given by the Buch-Davies vacuum

$$Q_k^I(\eta) = \frac{H}{\sqrt{2k^3}} e^{-i\eta k}$$

In practical terms we evolve the each mode from "just inside" the horizon until the end of inflation. There is some non-trivial evolution outside the horizon:





Adiabatic and Entropy Directions

Velocities of the background fields define a vector in field space:

$$\overrightarrow{\phi} = \left(\dot{\phi}^1 \ldots \dot{\phi}^N\right)$$

One can now decompose the M-S vector $\overrightarrow{Q}$ in two components:

adiabatic $\parallel$ $\overrightarrow{\phi}$

$$Q_\sigma = \frac{G_{IJ}\dot{\phi}^I Q^J}{\sqrt{G_{KL}\dot{\phi}^K\dot{\phi}^L}} + \frac{\sqrt{G_{kl}\dot{\phi}^k\dot{\phi}^l}}{H}\psi$$

entropy $\perp$ $\overrightarrow{\phi}$

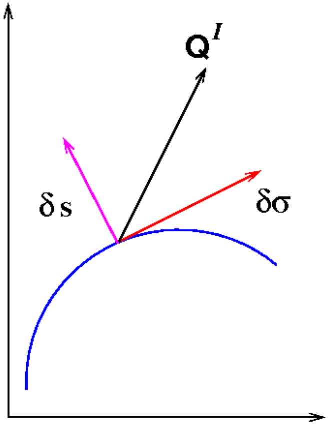
$$\delta s = \frac{G_{ij}\left[Q^i - \left(\frac{G_{kl}\dot{\phi}^k\delta\phi^l}{G_{mn}\dot{\phi}^m\dot{\phi}^n}\right)\dot{\phi}^i\right]Q^j}{\sqrt{\left|Q^p - \left(\frac{G_{ab}\dot{\phi}^a Q^b}{G_{cd}\dot{\phi}^c\dot{\phi}^d}\right)\dot{\phi}^p\right|}}$$

We can now define the curvature and entropy perturbations

Adiabatic and Entropy Spectra

The adiabatic and entropy perturbations are defined as:

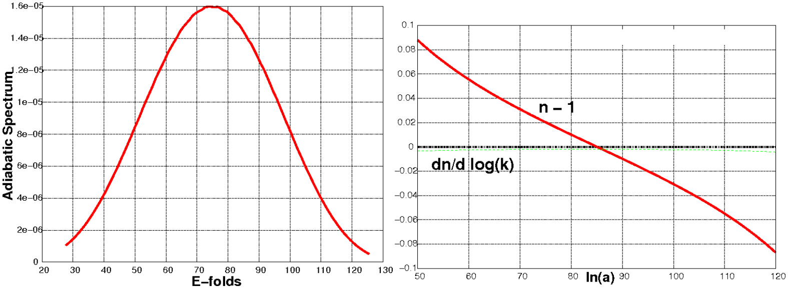
$$\begin{aligned} R &= \frac{H}{\sqrt{G_{ij}\dot{\phi}^i\dot{\phi}^j}} Q_\sigma \\ S &= \frac{H}{\sqrt{G_{ij}\dot{\phi}^i\dot{\phi}^j}} \delta s \end{aligned}$$



The corresponding spectra for the two variables are:

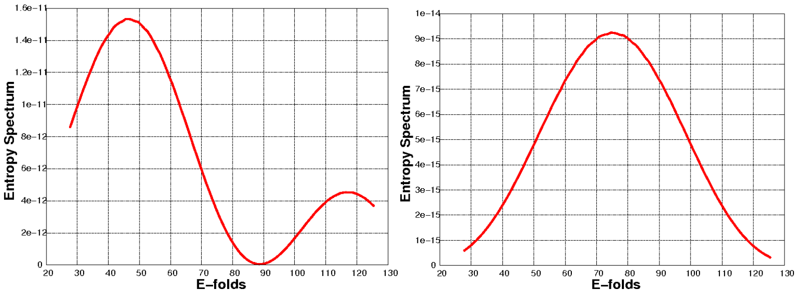
$$\begin{aligned} P_{\mathcal{R}}(\mathbf{k}) &= \frac{k^3}{2\pi^2}\left|R(k)\right|^2 \\ P_{\mathcal{S}}(\mathbf{k}) &= \frac{k^3}{2\pi^2}\left|S(k)\right|^2 \end{aligned}$$

Results



Field trajectories, adiabatic spectrum.

More Results



Entropy spectra for different brane trajectories. Left: non-coincident branes. Right: coincident branes.

Conclusions

- The Brane World model accomodates both inflation and the "Standard Model"
- It also offers a natural mechanism for ending Inflation.
- The potential features an inflexion point: η vanishes there.
- Many models fit the present data.
- Entropy perturbations are too small to be observable.