Topology-changing transitions with gravity

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Talk outline

- moving around moduli space
- topological transitions and their consequences
- including gravity in cycle-collapses
- results
- conclusions

Moduli space motion

Higher dimensional perspective:

$$ds^2 = \mathrm{d}s^2_{us} + a^2(x)\mathrm{d}\theta^2 + b^2(x)\mathrm{d}\phi^2 + \ldots$$

Moduli space motion

Higher dimensional perspective:

$$ds^2 = \mathrm{d}s^2_{us} + a^2(x)\mathrm{d}\theta^2 + b^2(x)\mathrm{d}\phi^2 + ...$$

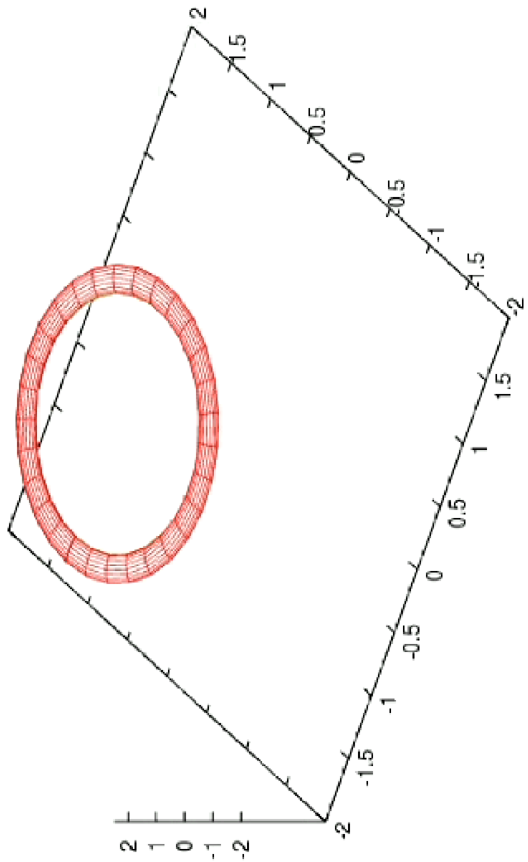
Our perspective:

$$\mathcal{L} = R - \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}\partial_\mu b \partial^\mu b - ...$$

Moduli space

Motion with constant topology:

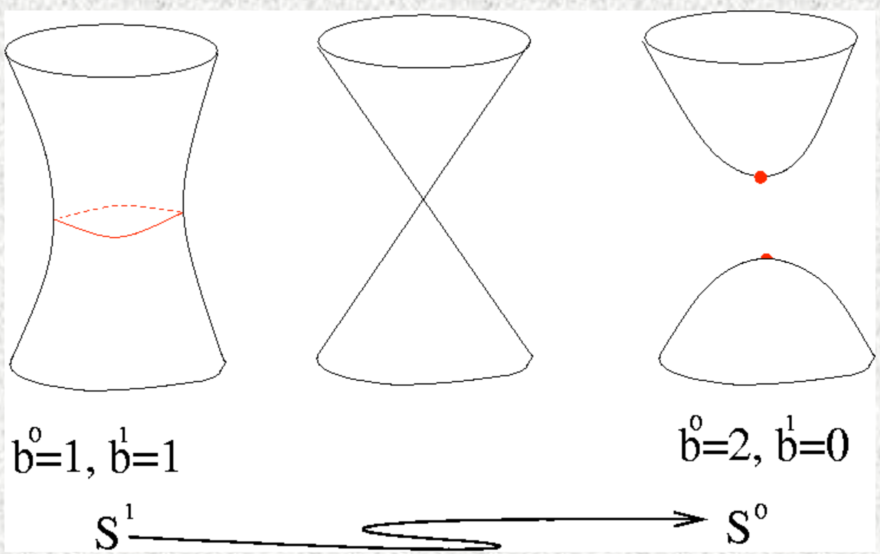
- Betti numbers unchanged
- intersection numbers unchanged

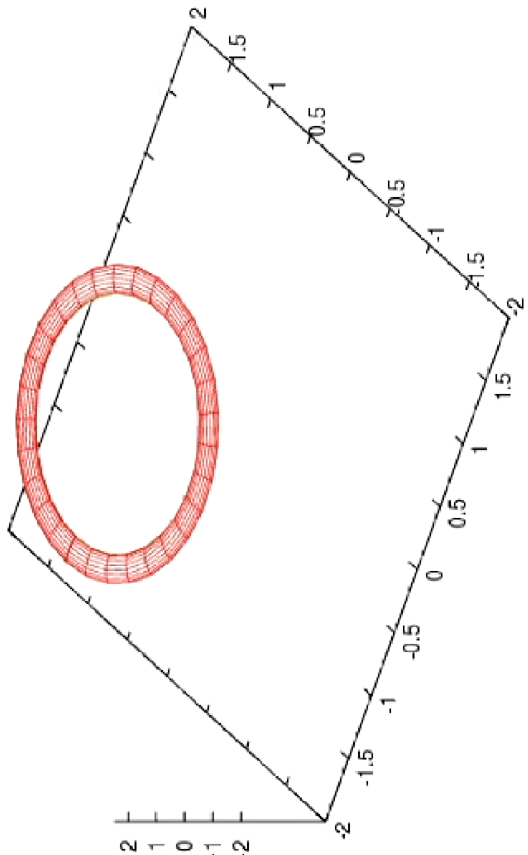


Moduli space

Motion with changing topology:

- Betti numbers changed
- e.g. conifold transition

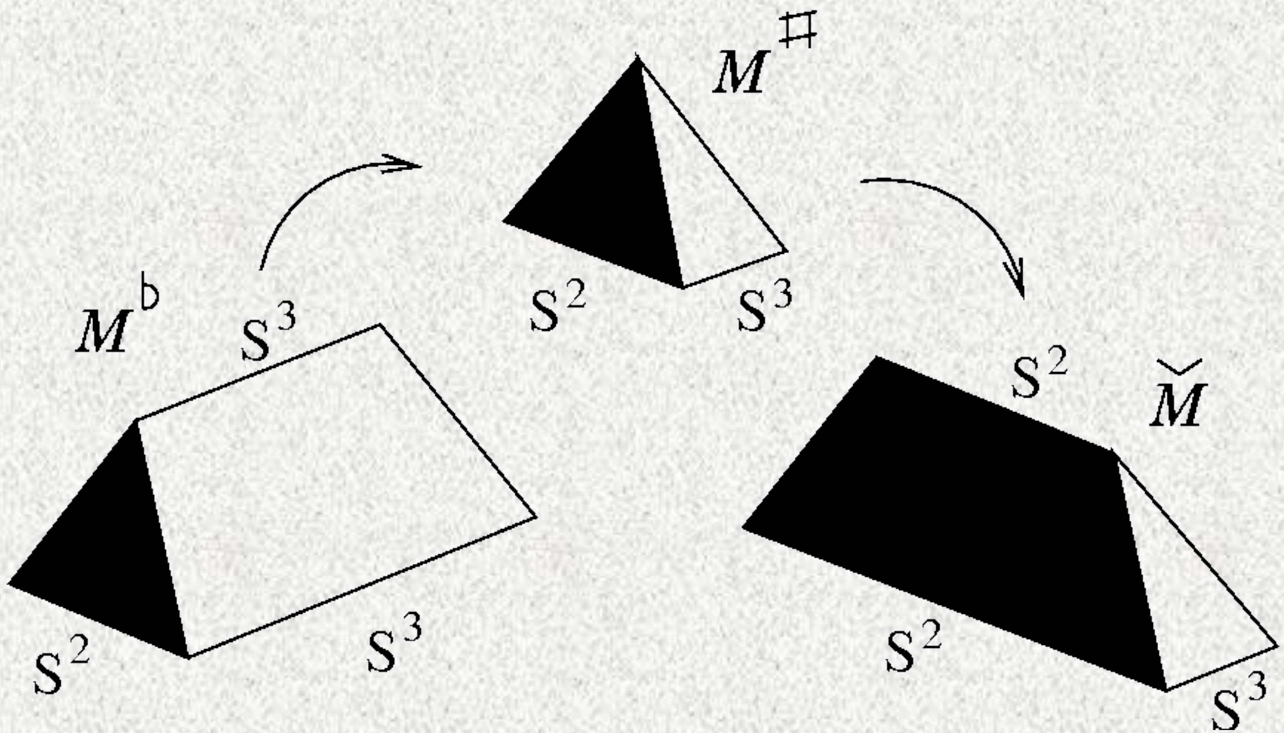




P. Candelas, P. Green, T. Hubsch,

Conifold transition

$$ds^2 = dr^2 + r^2 \left(d\Omega_{(2)} + d\Omega_{(3)}\right)$$



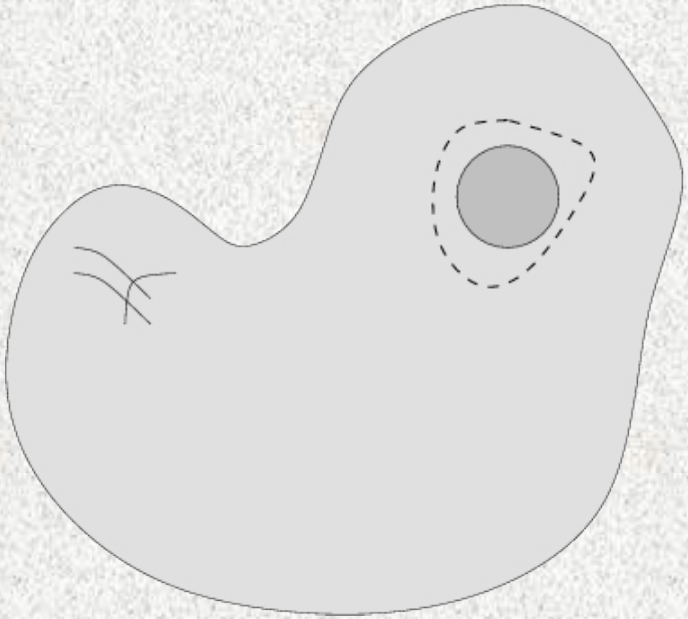
deformed

$$ds^2 = dr^2 + r^2 d\Omega_{(2)} + \epsilon d\Omega_{(3)}$$

resolved

$$ds^2 = dr^2 + \epsilon d\Omega_{(2)} + r^2 d\Omega_{(3)}$$

Regulating the transition



wrapped D-branes provide extra massless states which alter the effective theory

Strominger, Greene, Morrison, Vafa

Effective action

Use knowledge of effective theory to include extra degrees of freedom for the wrapped branes

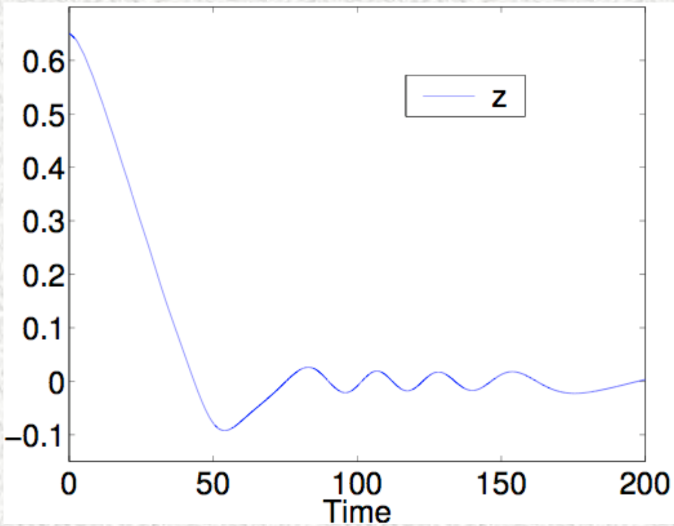
For the conifold in IIB we find 4D N=2 sugra, with one hypermultiplet for each degenerating cycle.

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \mathcal{R} \star 1 - g_{i\bar{j}} dz^i \wedge \star dz^{\bar{j}} - h_{uv} Dq^u \wedge Dq^v \\
& + \frac{1}{2} Im(\mathcal{N})_{IJ} F^I \wedge \star F^J - \frac{1}{2} Re(\mathcal{N})_{IJ} F^I \wedge F^J - \mathcal{V}(q,z)
\end{aligned}
$$

Brandle, Greene, Jarv, Lukas, Mohaupt, Morrison, Palti, Saffin, Saueressig, Vafa

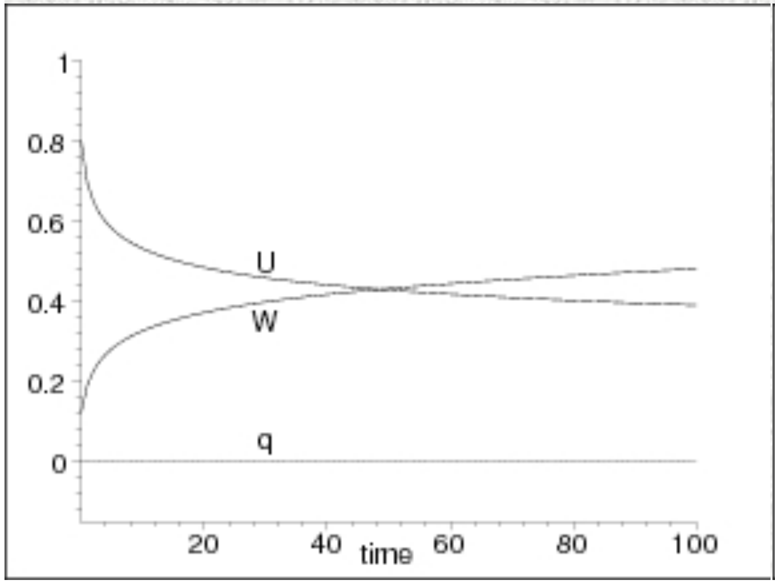
Dynamical consequences

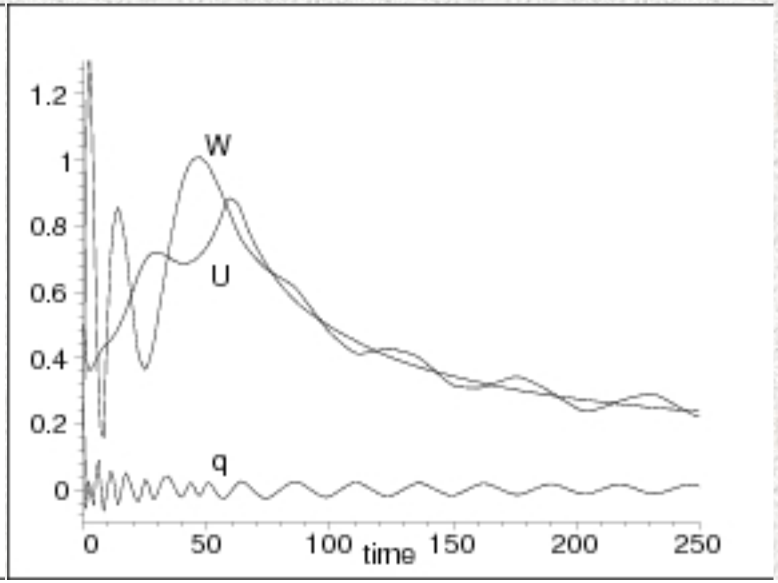
Trapping at the transition point



IIB conifold: Lukas, Palti, Saffin

M-theory flop: Lukas, Brandle





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• the cycle stops collapsing, then expands

• the cycle continues to collapse, classical geometry requires the formation of a curvature singularity

• does this create a naked singularity?
• is a horizon formed?
• what is the final state?

Cycle collapse - a simpler model

$$
\begin{aligned}
ds^2 &= \alpha^{-1}(\rho) d\rho^2 + \frac{1}{4}\rho^2 \left[(\sigma_1^2 + \sigma_2^2) + \alpha(\rho)\sigma_3^2\right] \\
\alpha(\rho) &= 1 - \left(\frac{L}{\rho}\right)^4
\end{aligned}
$$

Eguchi, Hanson

two-cycle:
$$\rho = L + \frac{R^2}{L}$$
$$\mathrm{d}s^2 \simeq \mathrm{d}R^2 + R^2\sigma_3^2 + \frac{L^2}{4}\left(\sigma_1^2 + \sigma_2^2\right)$$

Cycle collapse - a simpler model

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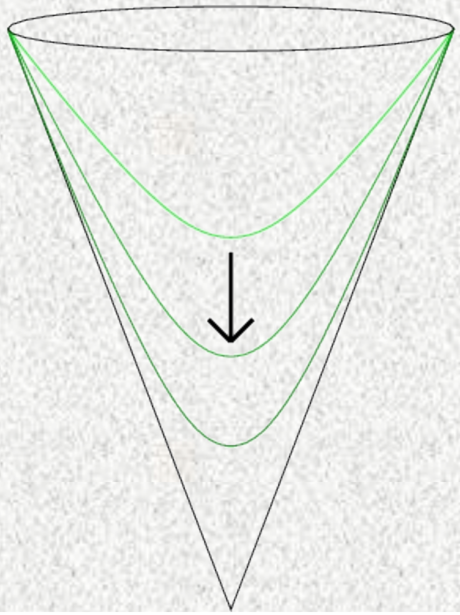
Moduli space approximation:

$$L \longrightarrow L(t)$$



$$S = \int \mathrm{d}t\mathrm{d}^4x\sqrt{-g}\mathcal{R}$$

$$S_{eff} = \frac{\pi^2}{8}\int \mathrm{d}t\left(\frac{d}{dt}L^2\right)^2$$

Cycle collapse - a simpler model

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t,r)\mathrm{d}r^2 + b^2(t,r)(\sigma_1^2 + \sigma_2^2) + c^2(t,r)\sigma_3^2$$

Cycle collapse - a simpler model

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apparent horizon:

$$\frac{d}{dt}(area)\Big|_{null\ ray} = 0$$

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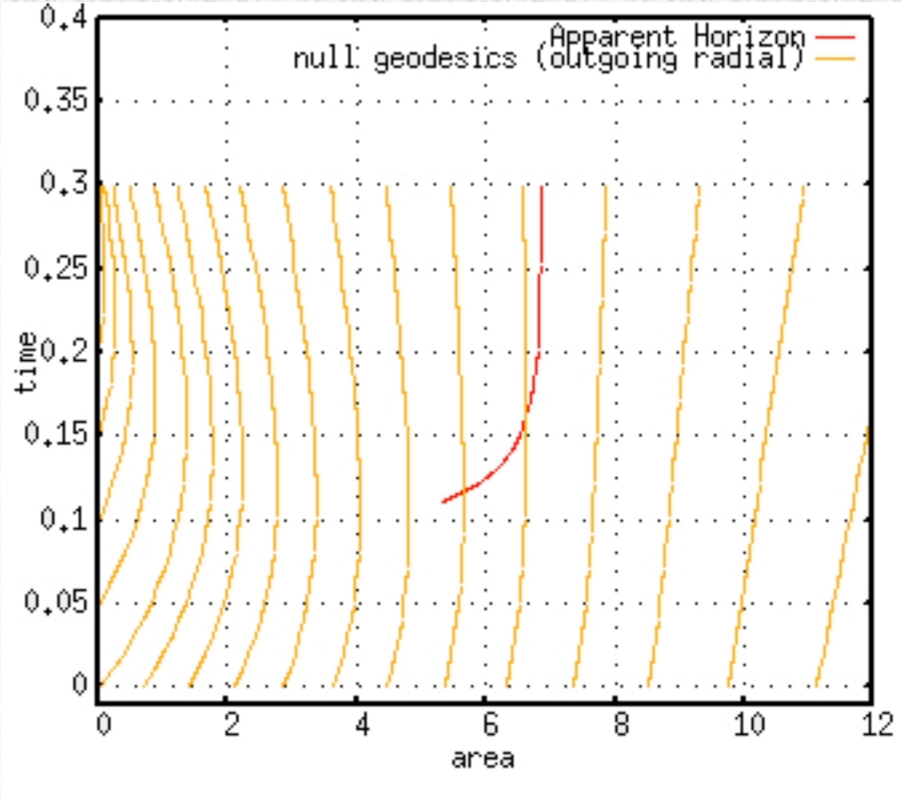

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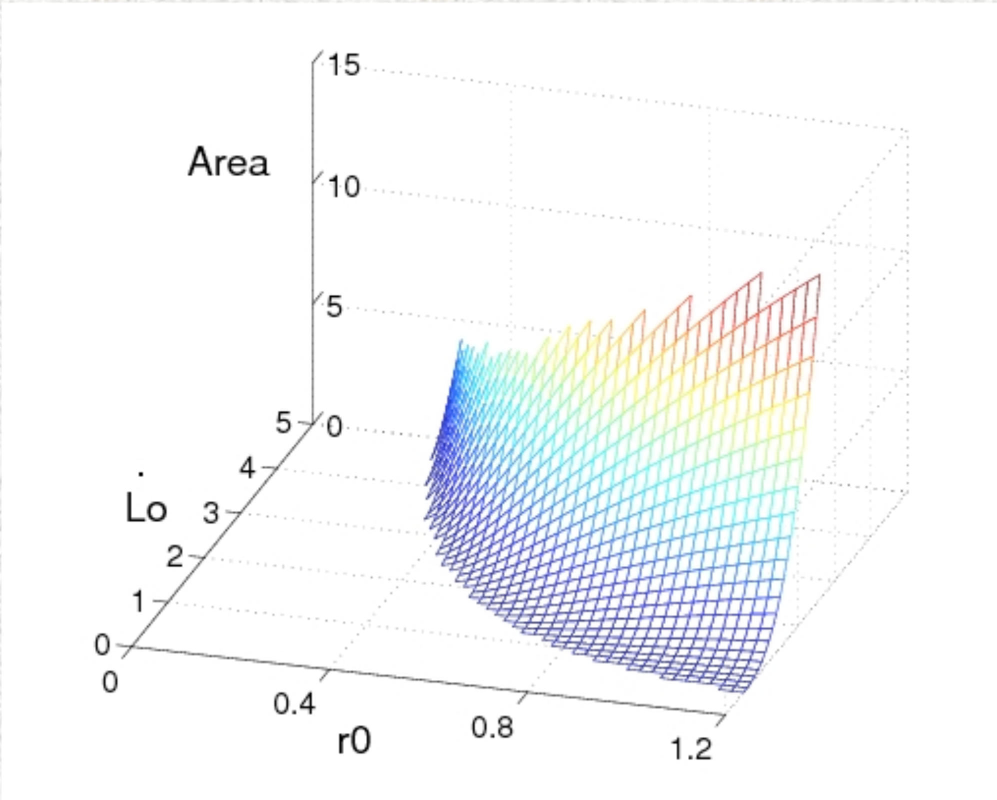

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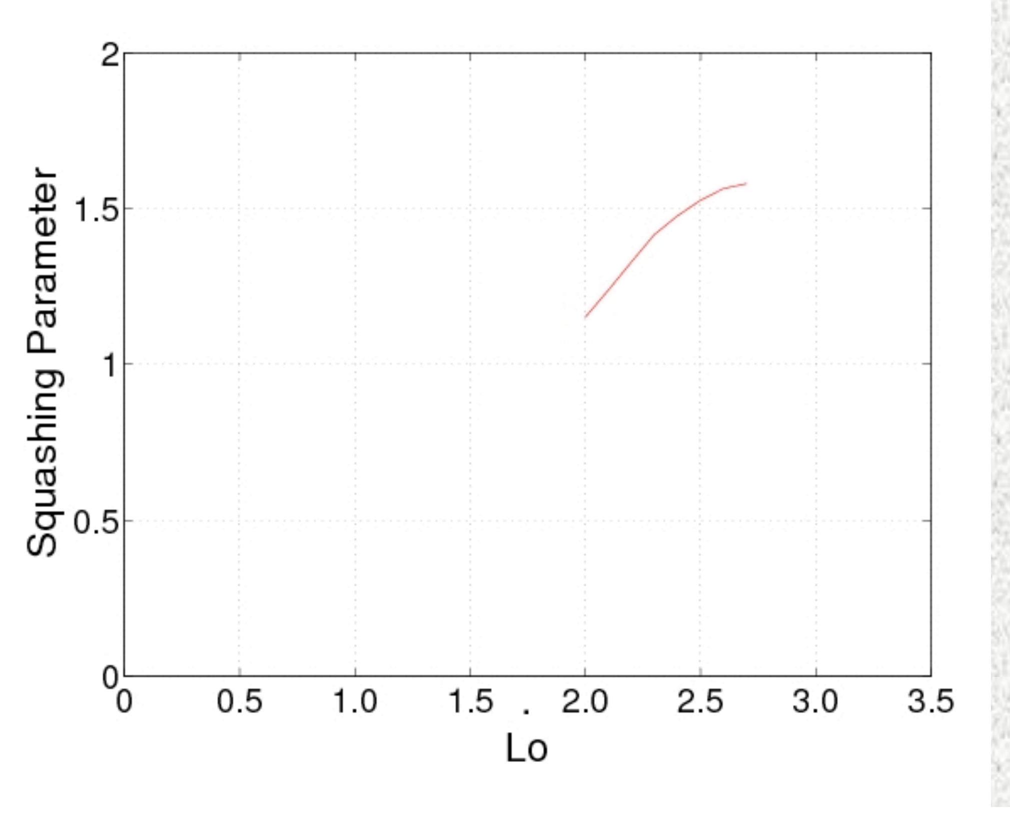


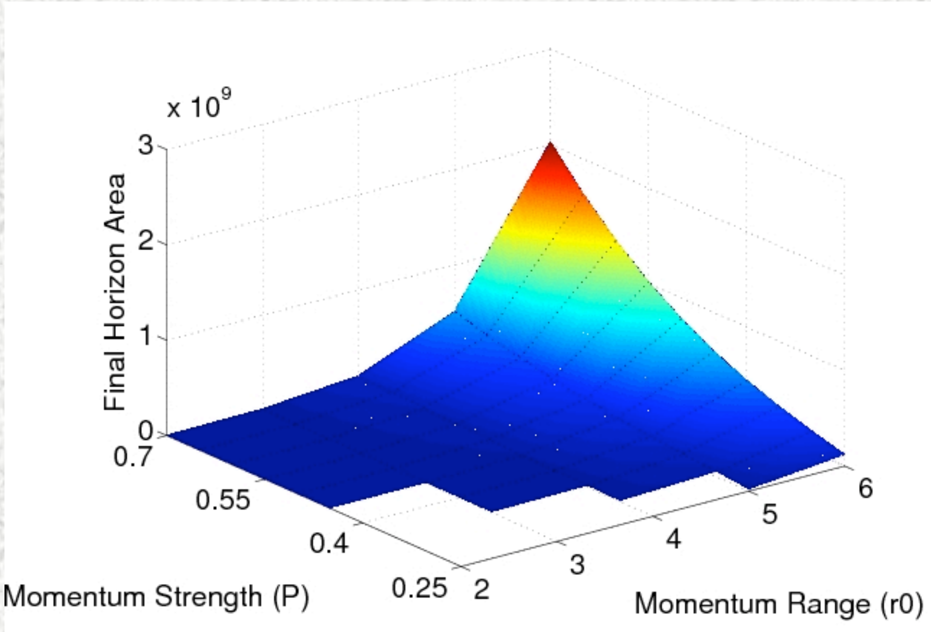
Final state?

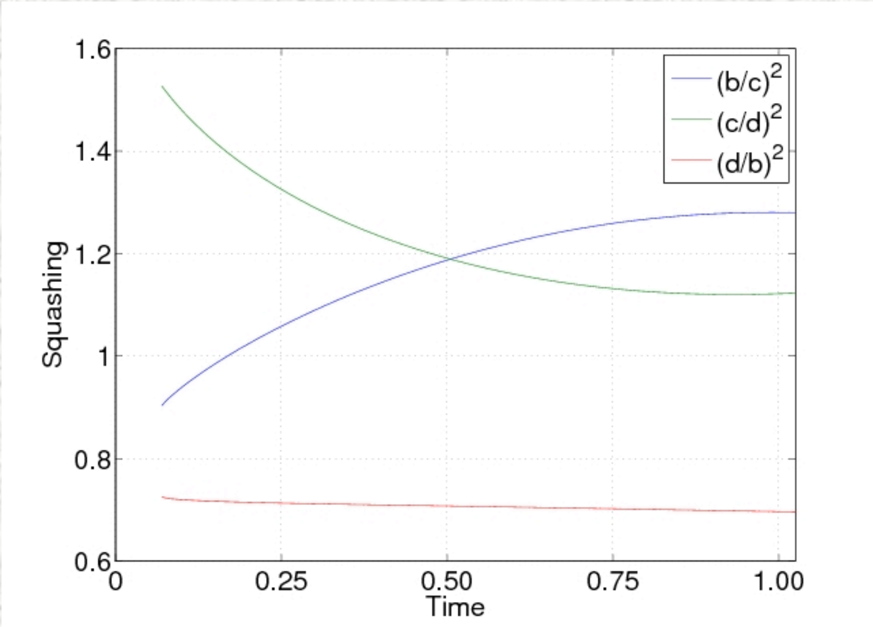
$$
\begin{aligned}
\mathrm{d}s^2 &= -f\mathrm{d}t^2 + \frac{k^2}{f}\mathrm{d}r^2 + \frac{r^2}{4}\left[k(\sigma_1^2 + \sigma_2^2) + \sigma_3^2\right] \\
f(r) &= \frac{r^2 - r_+^2}{r^2} \\
k(r) &= \frac{(r_\infty^2 - r_+^2)r_\infty^2}{(r_\infty^2 - r^2)^2}
\end{aligned}
$$

Gibbons, Maeda,
Ishihara, Matsuno

Collapsing conifold (preliminary results)

$$ds^2 = -dt^2 + a^2(t,r)dr^2 + b^2(t,r)(\sigma_1^2 + \sigma_2^2) + c^2(t,r)(\Sigma_1^2 + \Sigma_2^2) + d^2(t,r)(\sigma_3 + \Sigma_3)^2$$





conclusions

• branes can regulate the certain singularities

• can derive effective actions for topology changes using the moduli space approximation

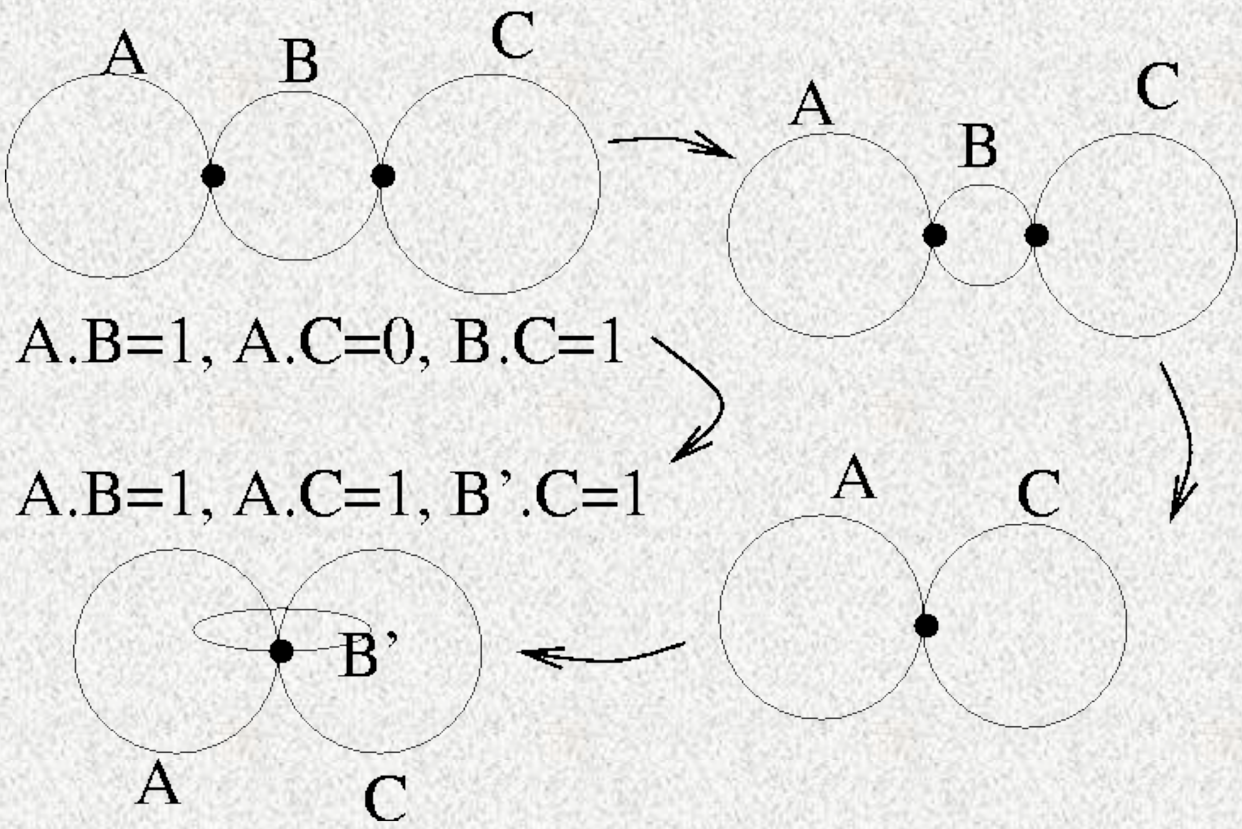
• the moduli space approximation breaks down as cycles collapse

• event horizons can form from cycles collapsing

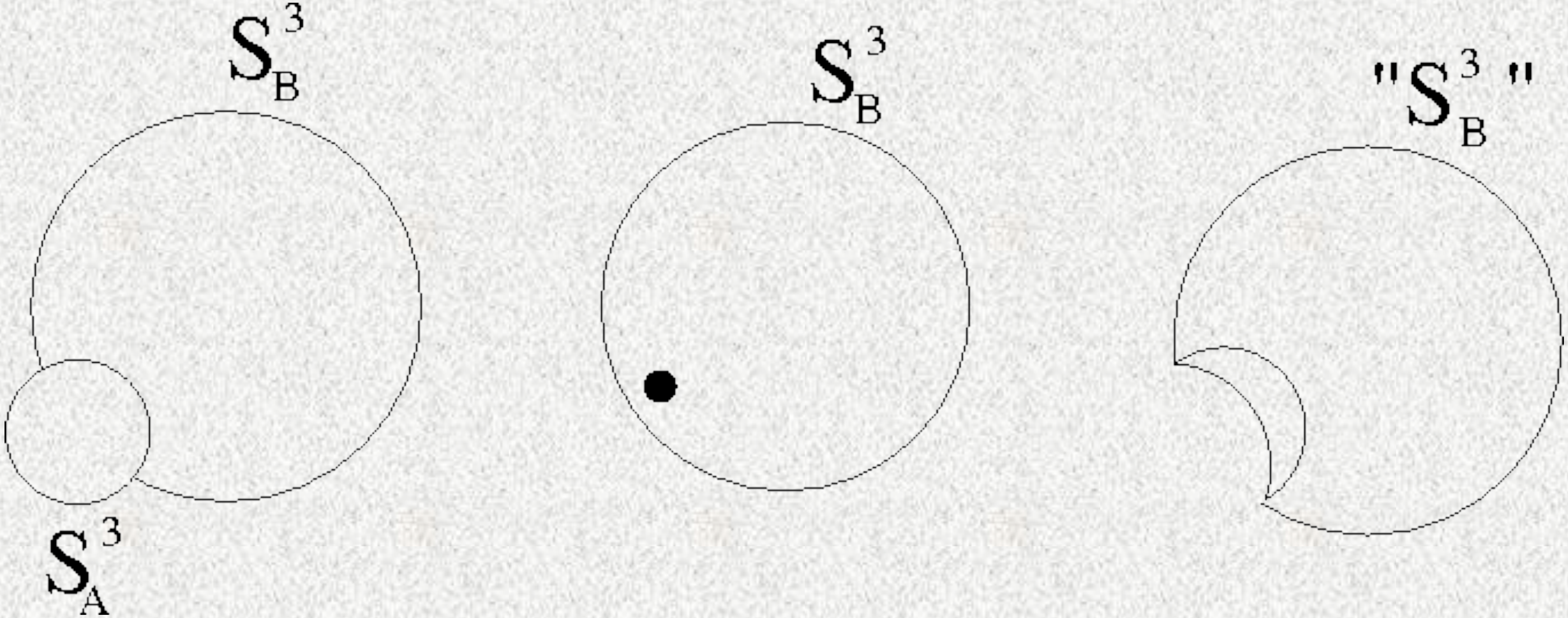
• cycle collapses give a novel form of dynamical compactification

Moduli space

- Betti numbers unchanged
- Intersection numbers changed
- e.g. flop transition



Conifold transition



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