Thermal and Quantum Superstring Cosmologies

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C. Kounnas

in supergravity : arXiv 0706.0728; 0705.3206 (Nucl. Phys. B)

and

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Introduction

-Cosmological models in string theory are difficult to construct.
-Most of the classical backgrounds are static : flat or AdS-like.
-This neglects quantum and finite temperature effects.

Classical flat supertring background
where susy is spontaneously broken

Quantum corrections

Thermal corrections

Backreaction on metric and moduli :
Cosmological evolution

Heterotic realization

-When susy is broken, the 1-loop effective action in 4D is, in string frame [Atick,Witten(88)]

$$S = \int d^4x\sqrt{-g}\left(e^{-2\phi}(\frac{1}{2}R + 2\partial_\mu\phi\partial^\mu\phi + \cdots) - \mathcal{V}_{\text{String}}\right)$$

dilaton   other moduli

$$\frac{Z}{V_4} = -\mathcal{V}_{\text{String}}$$

-At finite T, $\frac{Z}{V_4} = -\mathcal{F}_{\text{String}} = P_{\text{String}} = -\mathcal{V}_{\text{String}}$ ← thermal effective

-Write Z for compactification on S^1 x T^3 x S^1xT^5(or T^6/Z_2)

Euclidean time x^0 Space $x^{1,2,3}$ Internal x^4, $x^{5,6,7,8,9}$

-At finite temperature in field theory, KK bosons are periodic, while KK fermions are antiperiodic along S^1. In string theory, one couples the fermionic number to the momenta and windings i.e. generalized Scherk-Schwarz compactification on S^1

[Kounnas, Rostand (90)] $T \sim 1/2\pi R_0$

-Spontaneous breaking of $\mathcal{N} = 4$ (or $\mathcal{N} = 2$) susy implemented by Scherk-Schwarz compactification on S^1 : use an R-symmetry charge. $M \sim 1/2\pi R_4$

-To find the back reaction, work in Einstein frame

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R-g^{\mu\nu}\,K_{I\bar{J}}\,\partial_\mu\Phi^I\partial_\nu\bar{\Phi}^{\bar{J}}-\frac{1}{(\mathrm{Im}S)^2}\mathcal{V}_{\mathrm{String}}(T,\Phi^I,\bar{\Phi}^{\bar{I}})\right]$$

$$\mathcal{O}(e^{-c_0R_0^2-c_4R_4^2})$$

numerator interpreted as $1\,/\,T$ (Einstein frame)

where $$\mathcal{V}_{\mathrm{Ein}}=\frac{1}{(\mathrm{Im}S)^2}\frac{F\left(\frac{R_0}{R_4}\right)}{(2\pi R_4)^4}+\cdots=\frac{F\left(\frac{2\pi\sqrt{\mathrm{Im}S}\,R_0}{2\pi\sqrt{\mathrm{Im}S\,\mathrm{Im}T_1\,\mathrm{Im}U_1}}\right)}{\left(2\pi\sqrt{\mathrm{Im}S\,\mathrm{Im}T_1\,\mathrm{Im}U_1}\right)^4}$$

-Only one flat direction is lifted. No back reaction on other moduli : We can still chose them constant.

$$K=-\ln\left(\frac{S-\bar{S}}{2i}\right)\left(\frac{T_1-\bar{T}_1}{2i}\right)\left(\frac{U-\bar{U}_1}{2i}\right)\equiv-\ln\left(\frac{Y-\bar{Y}}{2i}\right)^3$$

-It is associated to the susy mass scale (or gravitino mass), since

$$M\equiv\frac{e^{K/2}}{2\pi}=\frac{1}{2\pi\sqrt{\mathrm{Im}S\,\mathrm{Im}T_1\,\mathrm{Im}U_1}}=\frac{1}{\sqrt{\mathrm{Im}S}\,2\pi R_4}=\frac{1}{\sqrt{2\pi(\mathrm{Im}Y)^3}}:=e^{\alpha\Phi}$$

$$K_{Y\bar{Y}}\,\partial_\mu Y\partial^\mu\bar{Y}-\mathcal{V}_{\mathrm{Ein}}=\frac{\alpha^2}{3}\partial_\mu\Phi\partial^\mu\Phi-M^4\,F\left(M/T\right)+\cdots$$

($\alpha^2 = 3/2$)

-The classical flat direction of the "no-scale" modulus Φ is the only one lifted.

-Solve the eqs. of motion for Φ and the gravity.

-E.g. : T^6 , R-symmetry charge = Fermionic number

$$Z = \frac{V_3}{(2\pi)^3} \int \frac{d\tau d\bar{\tau}}{\tau_2^{5/2}} \; \frac{1}{2} \sum_{a,b} \frac{(-)^{a+b+ab}\theta[^a_b]^4}{\eta(\tau)^{12}\bar{\eta}(\bar{\tau})^{24}} \; \Gamma_{(5,21)}(R_5,\dots,R_9) \; \times$$

$$\sum_{h_0,g_0} \Gamma_{(1,1)} \left[^{h_0}_{g_0}\right] (R_0) \; (-)^{g_0 a + h_0 b + g_0 h_0} \; \sum_{h_4,g_4} \Gamma_{(1,1)} \left[^{h_4}_{g_4}\right] (R_4) \; (-)^{g_4 a + h_4 b + g_4 h_4}$$

where $$\Gamma_{(1,1)} \left[^{h_0}_{g_0}\right] (R_0) = \frac{R_0}{\tau_2^{1/2}} \sum_{m_0,n_0} \; e^{-\frac{\pi R_0^2}{\tau_2}|(2m_0+g_0)+(2n_0+h_0)\tau|^2}$$

-The parity of $(2m_0+g_0)$ is coupled to a = 0 (bosons), 1 (fermions) : ~~Susy~~

-The parity of the windings is coupled to b : Reversed GSO.

Tachyons occur for $\frac{1}{R_H} < R_{0,4} < R_H = \frac{\sqrt{2}+1}{2}$: Hagedorn transition.

-Consider R_0,R_4 a bit > 1 for a string description to be valid (i.e. small T, M).

-In this limit, the contribution of any mode with oscillators is exponentially suppressed.

-It is thus the case for any odd winding mode of the directions 0 or 4 (i.e. h_0=1 or h_4=1), due to the reversed GSO.

-The lattice modes $\Gamma_{(5,21)}(R_5,\dots,R_9)$ are exponentially suppressed, if $R_5, \dots , R_9 = O(1)$.

The only significant contributions to Z come from the massless states and their towers of KK and even winding modes in the directions 0 and 4.

$$P \equiv \frac{Z}{V_4} = \frac{\Gamma\left(\frac{5}{2}\right)}{\pi^{\frac{5}{2}}} \left(n_T \, T^4 \, f_{\frac{5}{2}}(u) + n_V \, M^4 \, \frac{f_{\frac{5}{2}}(1/u)}{u} \right)$$

where $u = \frac{R_0}{R_4} \equiv \frac{M}{T}$

and $$f_k(u) = \sum_{m_0, m_4} \frac{u^{2k-1}}{|(2m_0+1)iu + 2m_4|^{2k}}$$

-The constant n_T is the number of massless boson-fermion pairs in the original susy model.

-Here, we have $n_T = n_V$ because we chose the two ways to break susy to be identical ⇨ symmetry $R_0 \leftrightarrow R_4$ ie $T \leftrightarrow M$.

Duality **finite temperature effects** ⟷ **quantum corrections**

-For a general R-symmetry : $-n_T \leq n_V \leq n_T$

Wilson lines deformations

-The originally supersymmetric models have E_8xE_8 gauge symmetry. What is happening when constant Wilson lines are switched on ?

-In directions I = 5, … , 9,

$$m_I \longrightarrow m_I + y_I^a Q_a$$

16 charge operator of Cartan generators

-This deforms the contribution of $\Gamma_{(5,21)}(R_5,\ldots,R_9)$,

$$1 \longrightarrow e^{-\pi\tau_2\left(\frac{y_I^a Q_a}{R_I}\right)} \simeq 1 - \pi\tau_2\left(\frac{y_I^a Q_a}{R_I}\right)$$

$$\Rightarrow \quad P = \frac{\Gamma\left(\frac{5}{2}\right)}{\pi^{\frac{5}{2}}}\left(n_T\, T^4 f_{\frac{5}{2}}(u) + n_V\, M^4 \frac{f_{\frac{5}{2}}(1/u)}{u}\right) - \frac{\Gamma\left(\frac{3}{2}\right)}{\pi^{\frac{3}{2}}}\left(M_T^2\, T^2 f_{\frac{3}{2}}(u) + M_V^{(2)}\, M^2 \frac{f_{\frac{3}{2}}(1/u)}{u}\right)$$

where $M_T^2 = \frac{1}{4\pi}\sum_{s=1}^{n_T}\left(\frac{y_I^a Q_a^s}{R_I}\right)^2$ and $-M_T^2 \leq M_V^{(2)} \leq M_T^2$

Cosmological solution

-We have non trivial eqs. of motion for gravity and $M = e^{\alpha\Phi}$,
i.e. Einstein gravity coupled to a scalar field with source $P(T, M/T)$.

-Actually, thermodynamical system : T , P and energy density ρ,

$$\frac{\rho + P}{T} = \frac{\partial P}{\partial T}$$ where we rewrite $$P = T^4 p_4(u) + T^2 p_2(u)$$

$$\Rightarrow \quad \rho = T^4 r_4(u) + T^2 r_2(u)$$

-Homogeneous isotropic: $ds^2 = -N(t)^2 dt^2 + a(t)^2 \, d\Omega_k{}^2$, $\quad H = \left(\frac{\dot{a}}{a}\right)$

constant space curvature

Friedmann eq. : $$3H^2 = -\frac{3k}{a^2} + \frac{1}{2}\dot{\Phi}^2 + \rho$$

Eq. for $a(t)$: $$2\dot{H} + 3H^2 = -\frac{k}{a^2} - \frac{1}{2}\dot{\Phi}^2 - P$$

-Linear sum :

$$\dot{H} + 3H^2 = -\frac{2k}{a^2} + \frac{1}{2}(\rho - P) = -\frac{2k}{a^2} + \frac{1}{2}\left(T^4(r_4 - p_4) - T^2(r_2 - p_2) \right)$$

functions of $u = M / T$

-Eq. for Φ :

$$\ddot{\Phi} + 3H\dot{\Phi} = \frac{\partial P}{\partial \Phi} = -\alpha \left(T^4(r_4 - 3p_4) + T^2(r_2 - p_2) \right)$$

-Anzats : $M(t) = u\, T(t)$, with **constant** u.

Entropy conservation (consistency of thermodynamics) : $T(t) = \dfrac{1}{\gamma\, a(t)}$

-Proportionality of M and $1/a \Longrightarrow H = -\alpha\, \dot{\Phi}$ and the l.h.s. of the eqs. are proportional.

-Degree 4 fixes u : $r_4(u) = \dfrac{6\alpha^2 - 1}{2\alpha^2 - 1} p_4(u)$

Degree 2 fixes : $k\gamma^2 = -\dfrac{2\alpha^2 - 1}{4} \dfrac{r_2(u) - p_2(u)}{u^2}$

-The modulus and gravity eqs. are now equivalent.
The last thing to do is to solve the Friedmann gravity equation

$$3H^2 = -\frac{3k}{a^2} + \frac{1}{2}\dot{\Phi}^2 + \rho \quad \leftarrow \quad T^4 r_4 + T^2 r_2$$

that we rewrite as

$$3H^2 = -\frac{3\hat{k}}{a^2} + \frac{c_r}{a^4}$$

where

$$\begin{cases} \hat{k} = -\dfrac{1}{\gamma^2}\,\dfrac{2\alpha^2}{6\alpha^2-1}\,\dfrac{1}{u^2}\left(\dfrac{3(2\alpha^2-1)}{4}(r_2-p_2)+r_2\right) \\ c_r = \dfrac{1}{\gamma^4}\,\dfrac{6\alpha^2}{2\alpha^2-1}\,\dfrac{p_4}{u^4} \end{cases}$$

-In our $\mathcal{N}=4$ or $\mathcal{N}=2$ models, $\hat{k} > 0,\ = 0,\ \text{or}\ < 0$, while $c_r > 0$.

-At low energy, the string cosmology cannot be distinguished from a universe of constant space curvature, filled of thermalized radiation.

Conclusions

- We have insisted on the fact that finite temperature effects and quantum corrections are of "same nature".

- Start with any classical superstring background, which is flat and where $\mathcal{N} = 4$, $\mathcal{N} = 2$ or even $\mathcal{N} = 1$ susy is spontaneously broken, i.e. **no-scale structure** [Cremmer, Ferrara, Kounnas, Nanopoulos, (83)]. At one loop, the model is cosmological.

- $e^{\alpha\Phi} = M(t)$, $T(t)$, $1/a(t)$ are proportional, i.e. Φ is not stabilized. We have extremized the full action and not the potential alone.