Monad Bundles in Heterotic String Compactification

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Overview

- Introduction: Heterotic Calabi-Yau compactifications
- Complete intersection Calabi-Yau manifolds
- Monad bundles
- Positive monads, stability and spectrum
- Semi-positive monads
- Conclusion and outlook

Heterotic Calabi-Yau compactifications

Bosonic fields in d=10

Background for N=1 in d=4

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metric g	$\longrightarrow$	$g = g(M_4) + g(X)$, where $g(X)$ Ricci- flat metric on CY X
NS 3-form H	$\longrightarrow$	$H = 0$ for now (possibly flux added later)
dilaton ϕ	$\longrightarrow$	$\phi = \mathrm{const}$
$E_8 \times E_8$ gauge fields A	$\longrightarrow$	A_{int} connection on holomorphic vector bundle V on X
5-brane embedding $X^I(\sigma)$	$\longrightarrow$	5-brane stretches across M_4 , wraps holomorphic curve $C \subset X$

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Data defining a heterotic vacuum:

- CY manifold X (Ricci-flat $g(X)$ exists from Yau's theorem)
- holom. bundle V on X (A_{int} exist from Donaldson-Uhlenbeck-Yau)
- 5-brane class $W = [C] \in H^2(X)$

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What is stability?

Slope of a bundle (coherent sheaf) $\mathcal{F}$: $\mu(\mathcal{F}) = \frac{1}{\text{rk}(\mathcal{F})} \int_X c_1(\mathcal{F}) \wedge J \wedge J$

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A bundle V is stable if $\mu(\mathcal{F}) < \mu(V)$ for all coherent sub-sheafs $\mathcal{F} \subset V$

Stability of bundles is usually hard to prove!

Model-building basics

Choose "observable"bundle V with structure group $G = \mathrm{SU}(n) \subset E_8$, where $n = 3, 4, 5$ such that $c_2(TX) - c_2(V) \in \text{Mori cone of } X$

Then anomaly constraint can be satisfied by a suitable 5-brane curve.

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$\mathrm{SU}(3) \times \mathrm{E}_6$	$\mathbf{248} \to (\mathbf{1}, \mathbf{78}) \oplus (\mathbf{3}, \mathbf{27}) \oplus (\overline{\mathbf{3}}, \overline{\mathbf{27}}) \oplus (\mathbf{8}, \mathbf{1})$
$\mathrm{SU}(4) \times \mathrm{SO}(10)$	$\mathbf{248} \to (\mathbf{1}, \mathbf{45}) \oplus (\mathbf{4}, \mathbf{16}) \oplus (\overline{\mathbf{4}}, \overline{\mathbf{16}}) \oplus (\mathbf{6}, \mathbf{10}) \oplus (\mathbf{15}, \mathbf{1})$
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Finally: Discrete symmetry, Wilson line to break to $G_{\mathrm{SM}} \times \mathrm{U}(1)^{n-3}$

Alternatively, use $\mathrm{U}(n)$ bundles. (Blumenhagen et al. '06)

Which CYs and which bundles?

- Complete intersections in $\mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_m}$

  (Hubsch, Green, Lutken, Candelas... '87)

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In particular: $h^0(X, \mathcal{O}_X(\mathbf{k})) > 0$ if all $k^r \geq 0$

$h^0(X, \mathcal{O}_X(\mathbf{k}))$ only non-zero cohomology if all $k^r > 0$

Monads

Definition: A monad bundle V on X defined by short exact sequence

$$0 \to V \to B \xrightarrow{f} C \to 0$$ (hence $V = \mathrm{Ker}(f)$)

where $B = \bigoplus_{i=1}^{r_B} \mathcal{O}_X(\mathbf{b}_i)\,,\quad C = \bigoplus_{a=1}^{r_C} \mathcal{O}_X(\mathbf{c}_a)$ and $\mathbf{c}_a > \mathbf{b}_i$.

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Properties: $n = \mathrm{rank}(V) = r_B - r_C \overset{!}{\in} \{3, 4, 5\}$

$$c_1^r(V) = \sum_i b_i^r - \sum_a c_a^r \overset{!}{=} 0$$

$$c_{2r}(V) = \frac{1}{2} d_{rst} \left(\sum_i b_i^s b_i^t - \sum_a c_a^s c_a^t\right) \overset{!}{\leq} c_{2r}(TX)$$

$$c_3(V) = \frac{1}{3} d_{rst} \left(\sum_i b_i^r b_i^s b_i^t - \sum_a c_a^r c_a^s c_a^t\right)$$

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where $B = \bigoplus_{i=1}^{r_B} \mathcal{O}_X(\mathbf{b}_i)\,, \quad C = \bigoplus_{a=1}^{r_C} \mathcal{O}_X(\mathbf{c}_a)$ and $\mathbf{c}_a > \mathbf{b}_i$.

Then V is a vector bundle on X!

The map f can be seen as a matrix of polynomials with degree $\mathbf{c}_a - \mathbf{b}_i$

Properties: $n = \mathrm{rank}(V) = r_B - r_C \overset{!}{\in} \{3,4,5\}$

$$c_1^r(V) = \sum_i b_i^r - \sum_a c_a^r \overset{!}{=} 0$$

$$c_{2r}(V) = \frac{1}{2} d_{rst} \left(\sum_i b_i^s b_i^t - \sum_a c_a^s c_a^t\right) \overset{!}{\leq} c_{2r}(TX)$$

$$c_3(V) = \frac{1}{3} d_{rst} \left(\sum_i b_i^r b_i^s b_i^t - \sum_a c_a^r c_a^s c_a^t\right)$$

Long exact sequence:
$$\begin{aligned} 0 &\to H^0(X,V) \to H^0(X,B) \to H^0(X,C) \\ &\to H^1(X,V) \to H^1(X,B) \to H^1(X,C) \\ &\to H^2(X,V) \to H^2(X,B) \to H^2(X,C) \\ &\to H^3(X,V) \to H^3(X,B) \to H^3(X,C) \to 0 \end{aligned}$$

Positive Monads, stability and spectrum

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For example, 126 positive monads on $\left[\begin{array}{c|cc} \mathbb{P}^1 & 0 & 2 \\ \mathbb{P}^4 & 4 & 1 \end{array} \right]$. First 10 of those:

$$\left\{ \left\{ \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}, \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix} \right\}, \left\{ \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix} \right\}, \right.$$
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Stability

Since $\mu(V) = 0$ we need $\mu(\mathcal{F}) < 0$ for all $\mathcal{F} \subset V$. We have

$$\mu(\mathcal{F}) \sim \int_X c_1(\mathcal{F}) \wedge J \wedge J = d_{rst} c_1^r(\mathcal{F}) t^s t^t = c_1(\mathcal{F}) \cdot \mathbf{s}$$

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Suppose $h^{11}(X) = 1$. Then $\mathcal{C}(X) = \{s \geq 0\}$. Now assume $\mathcal{F} \subset V$ is a destabilizing bundle, that is $\mu(\mathcal{F}) \geq 0$. Then $c_1(\mathcal{F}) \geq 0$. Define line bundle $L = \Lambda^{\mathrm{rk}(\mathcal{F})} \mathcal{F}$. It follows that $c_1(L) \geq 0$ so that $L = \mathcal{O}_X(k)$, $k \geq 0$ and $h^0(X, L) > 0$. Since $L \subset \Lambda^{\mathrm{rk}(\mathcal{F})} V$ this implies $h^0(X, \Lambda^{\mathrm{rk}(\mathcal{F})} V) > 0$.

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Conclusion: A vector bundle V on a cyclic Cicy X ($h^{11}(X) = 1$) satisfying $h^0(X, \Lambda^q V) = 0$ for $q = 1, \ldots, \mathrm{rk}(V) - 1$ is stable.
(Hoppe's criterion)

Stability

Since $\mu(V)=0$ we need $\mu(\mathcal{F})<0$ for all $\mathcal{F}\subset V$. We have

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There are 5 cyclic Cicys with a total of 37 positive monad bundles and using this criterion we have shown they are all stable.

Not bad, but we want to get control over a large numbers of examples!

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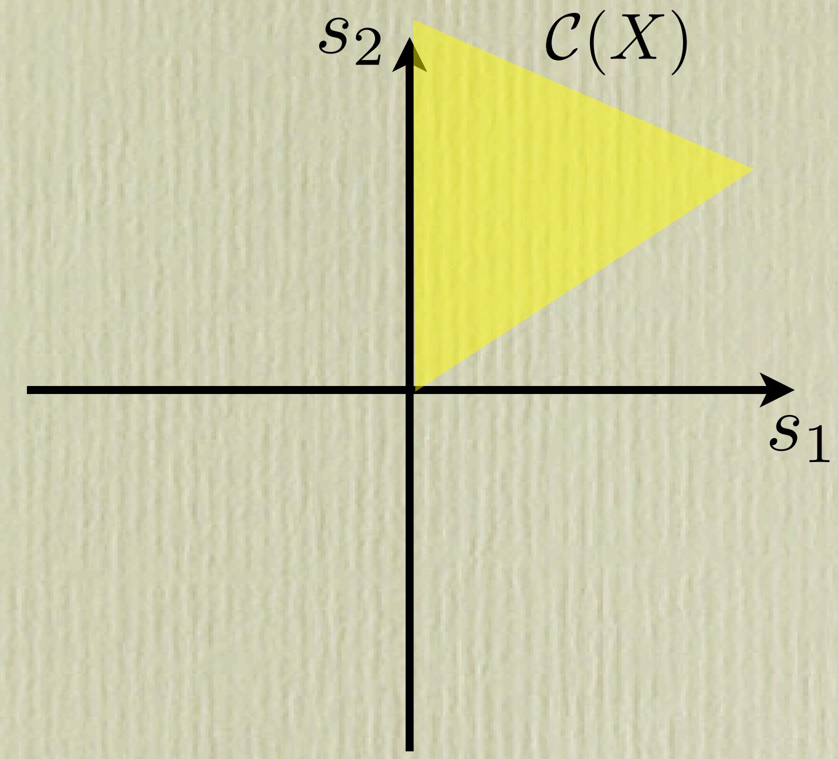
Next, consider Cicys with $h^{1,1} = 2$ (32 cases).



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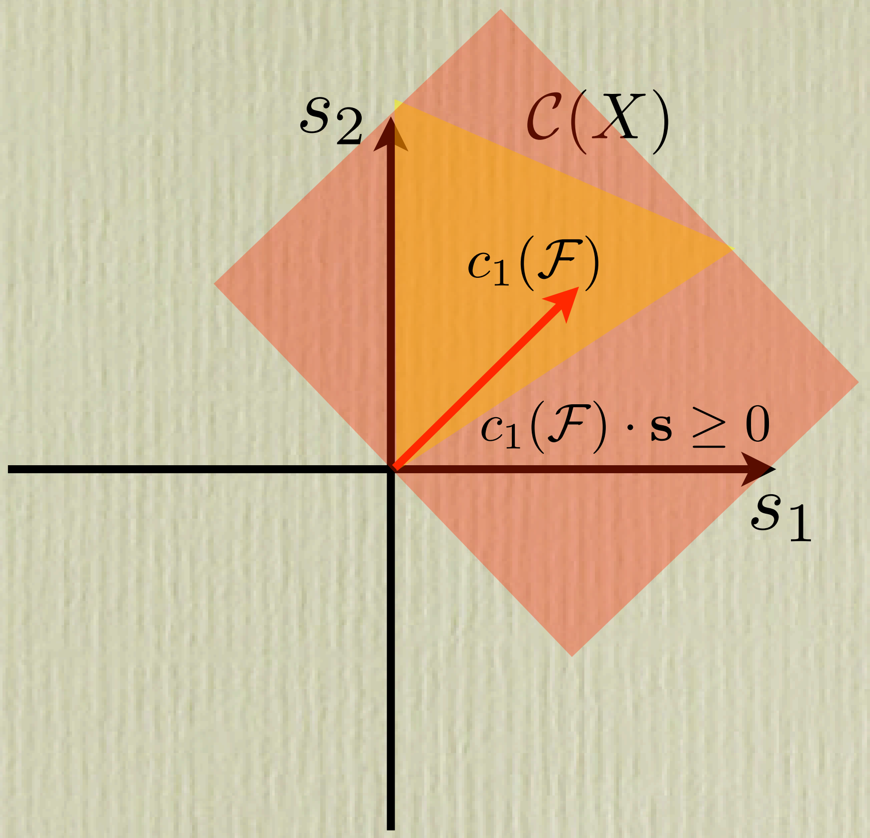

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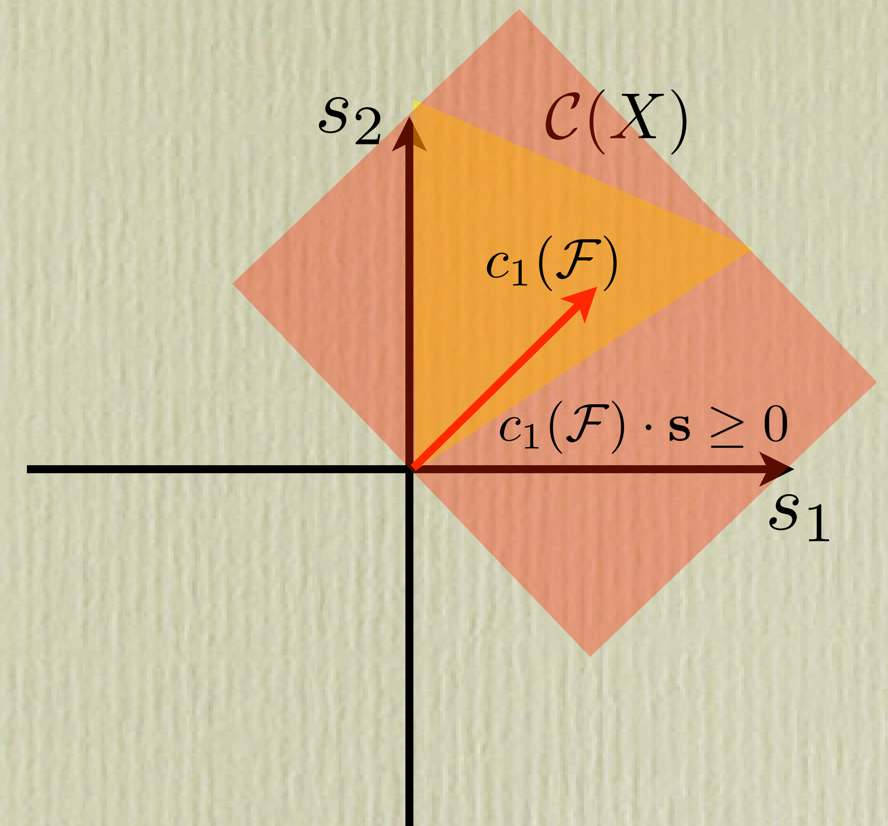

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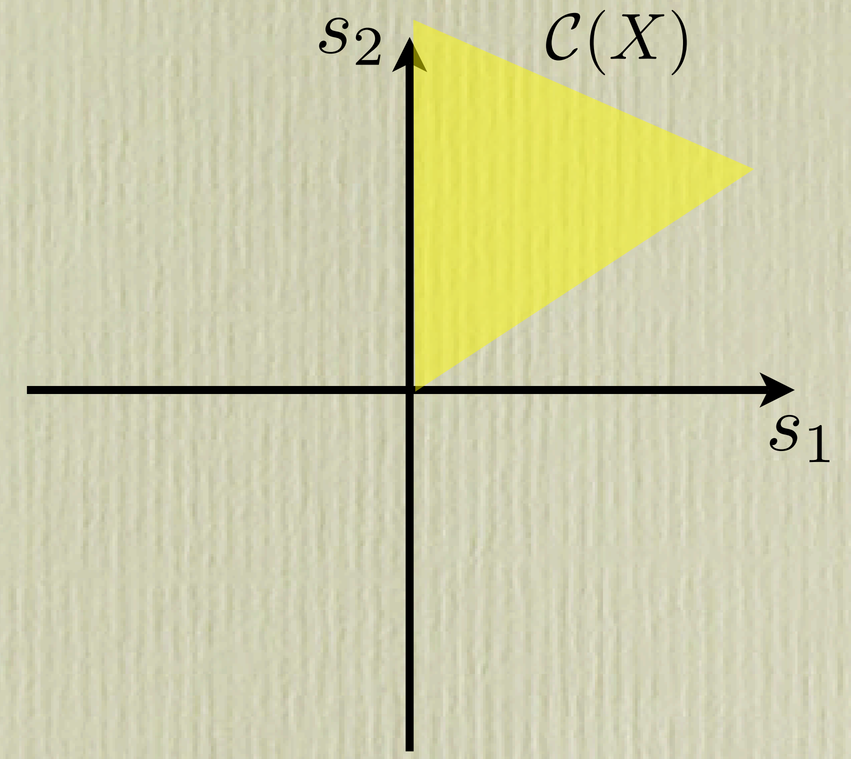

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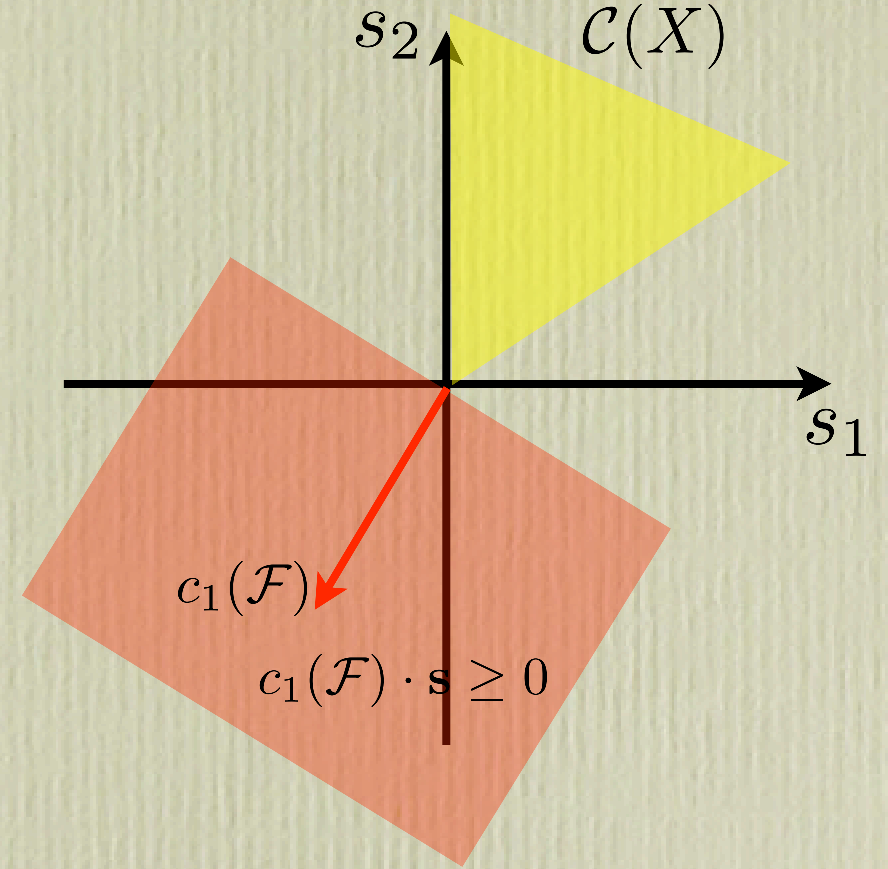

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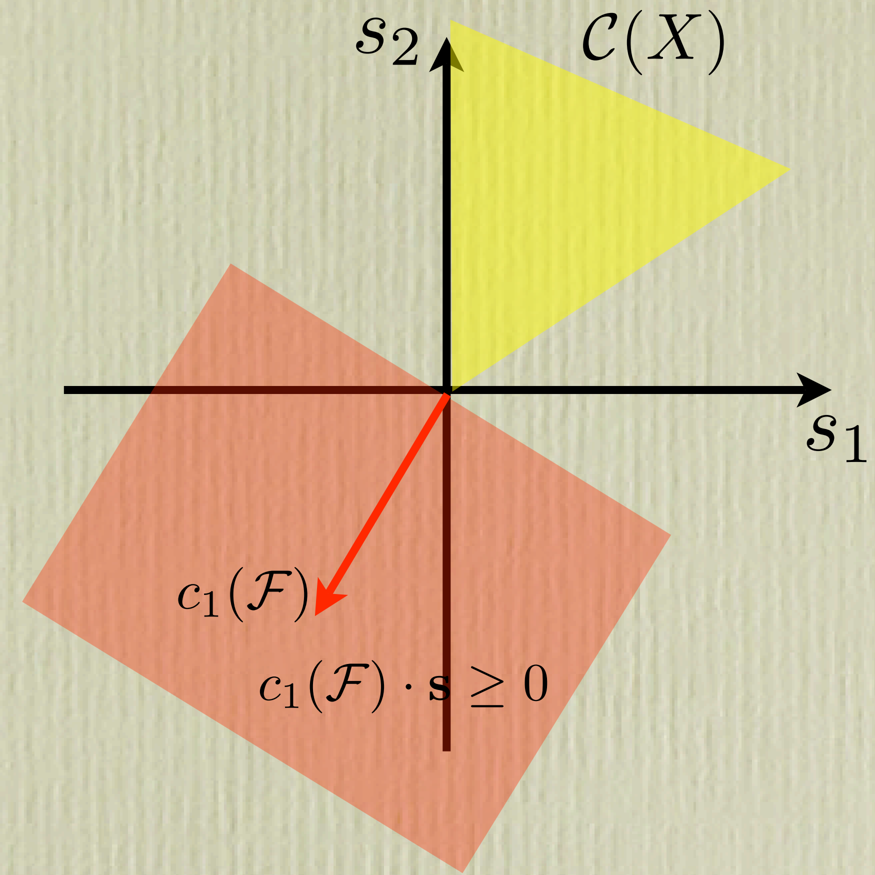

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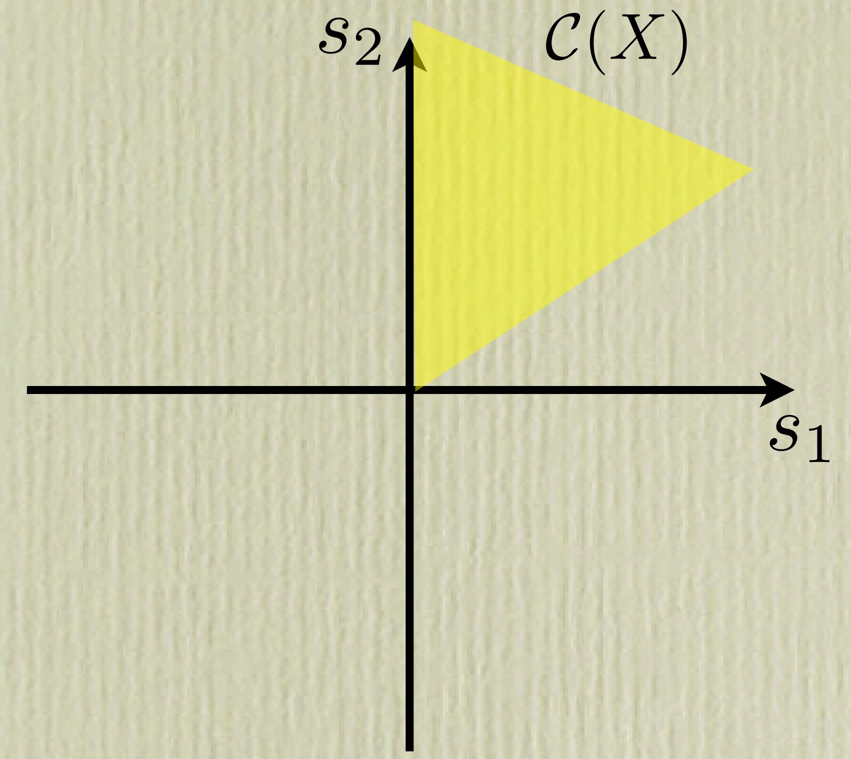

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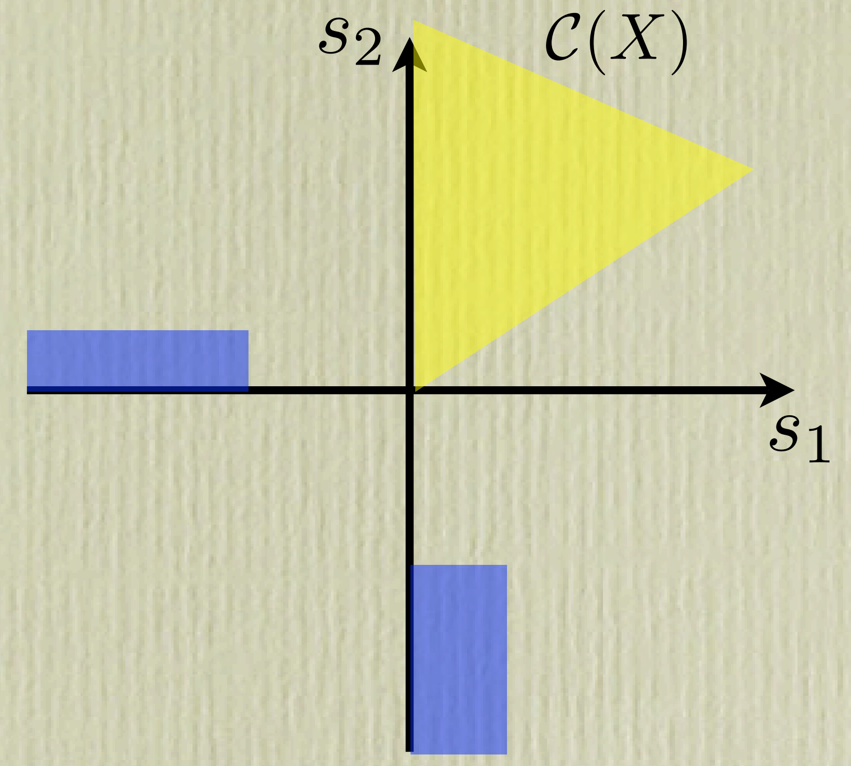

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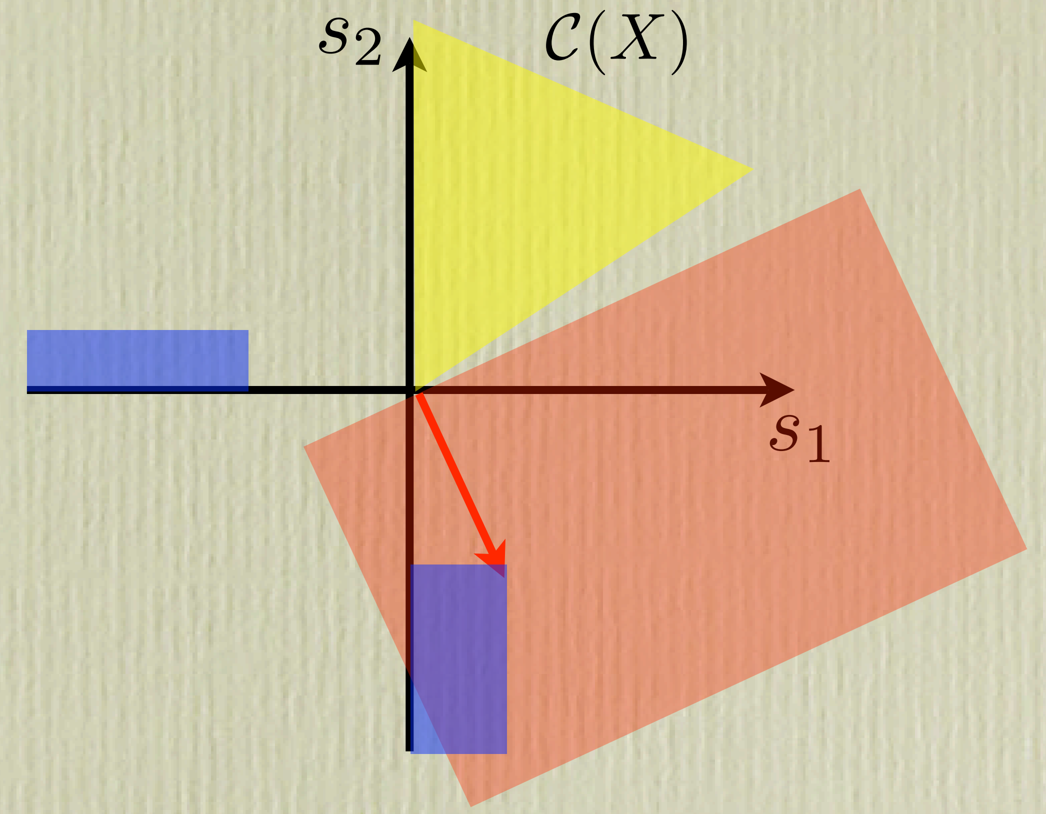

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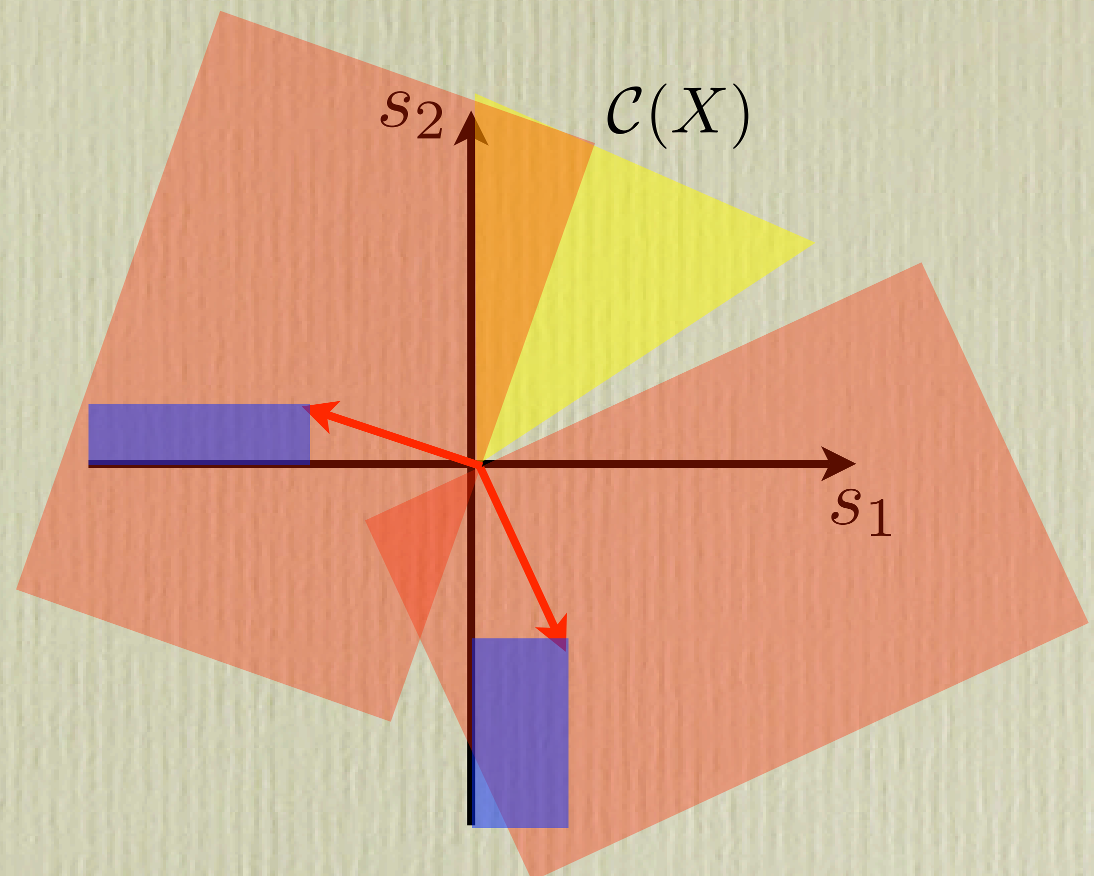

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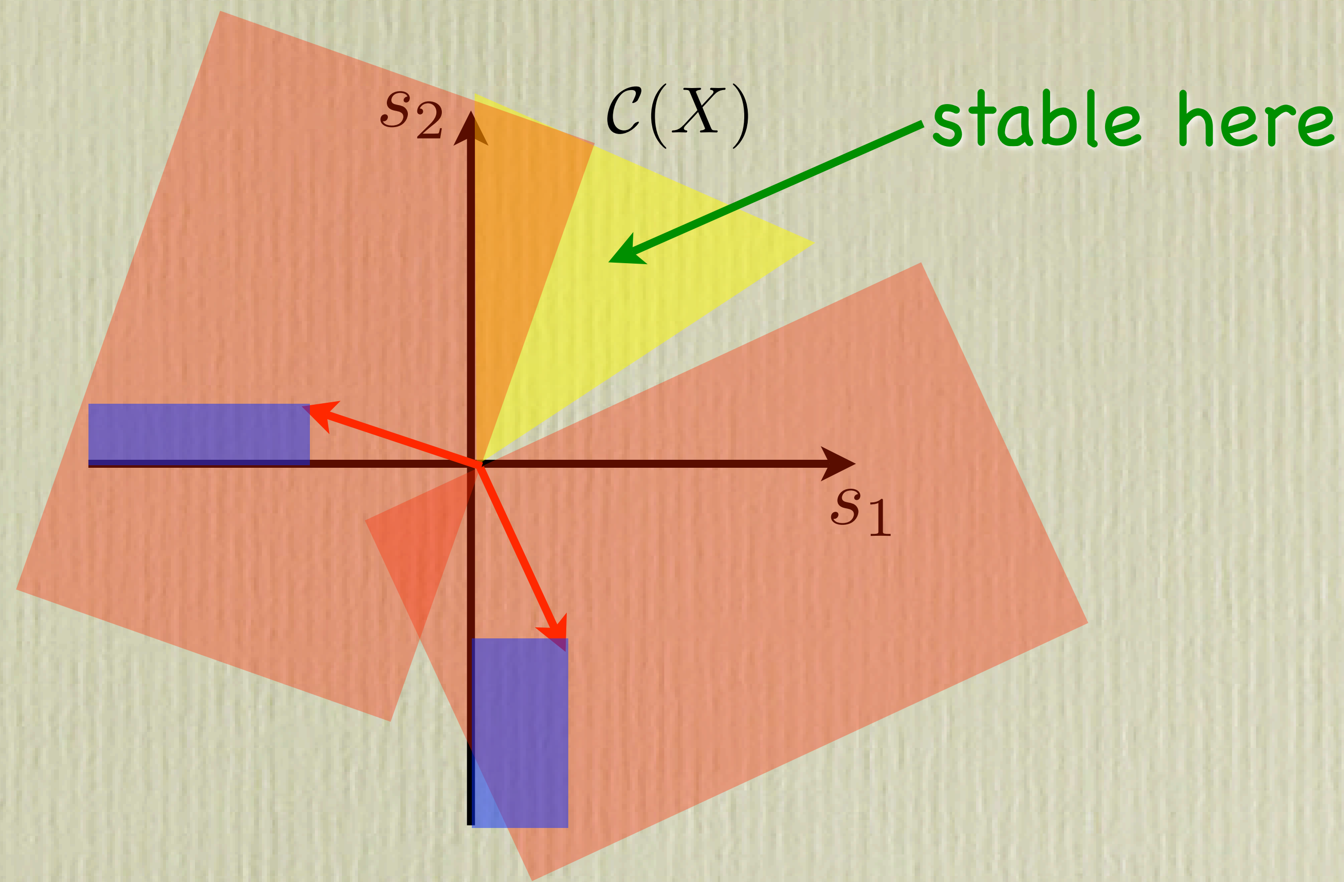

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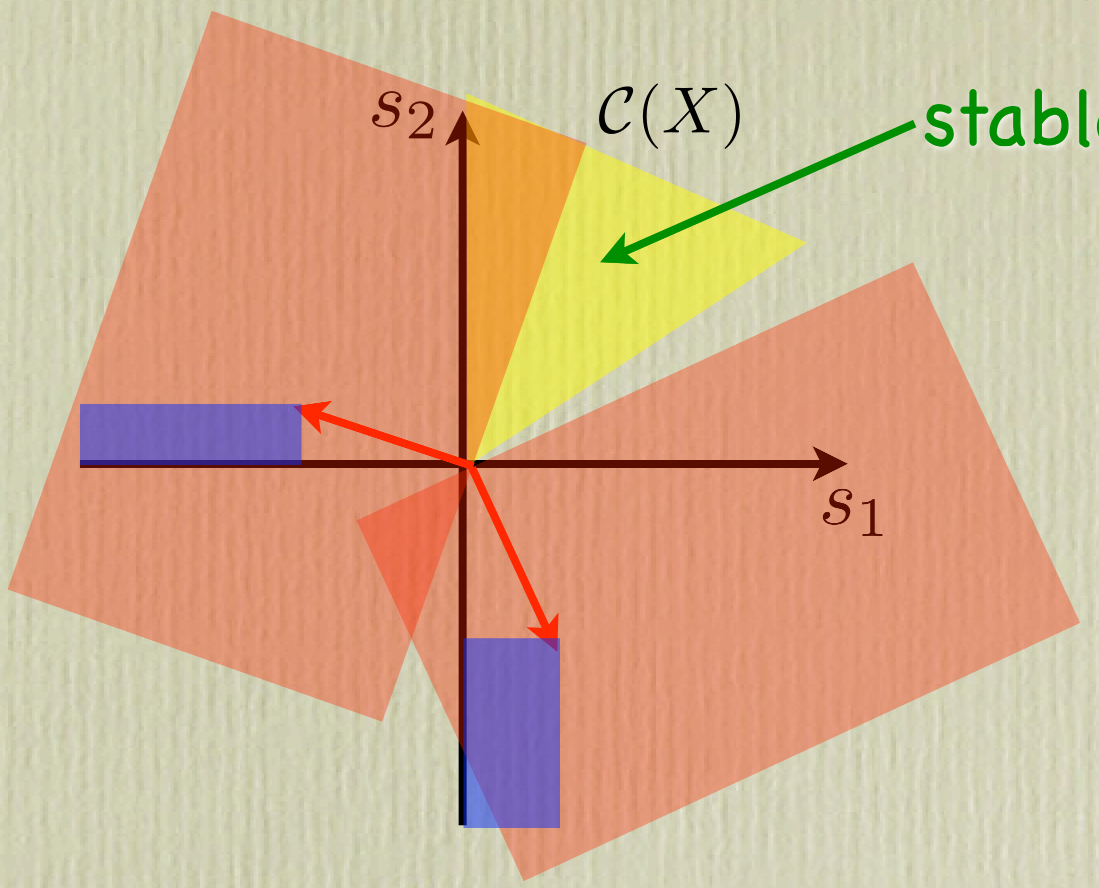

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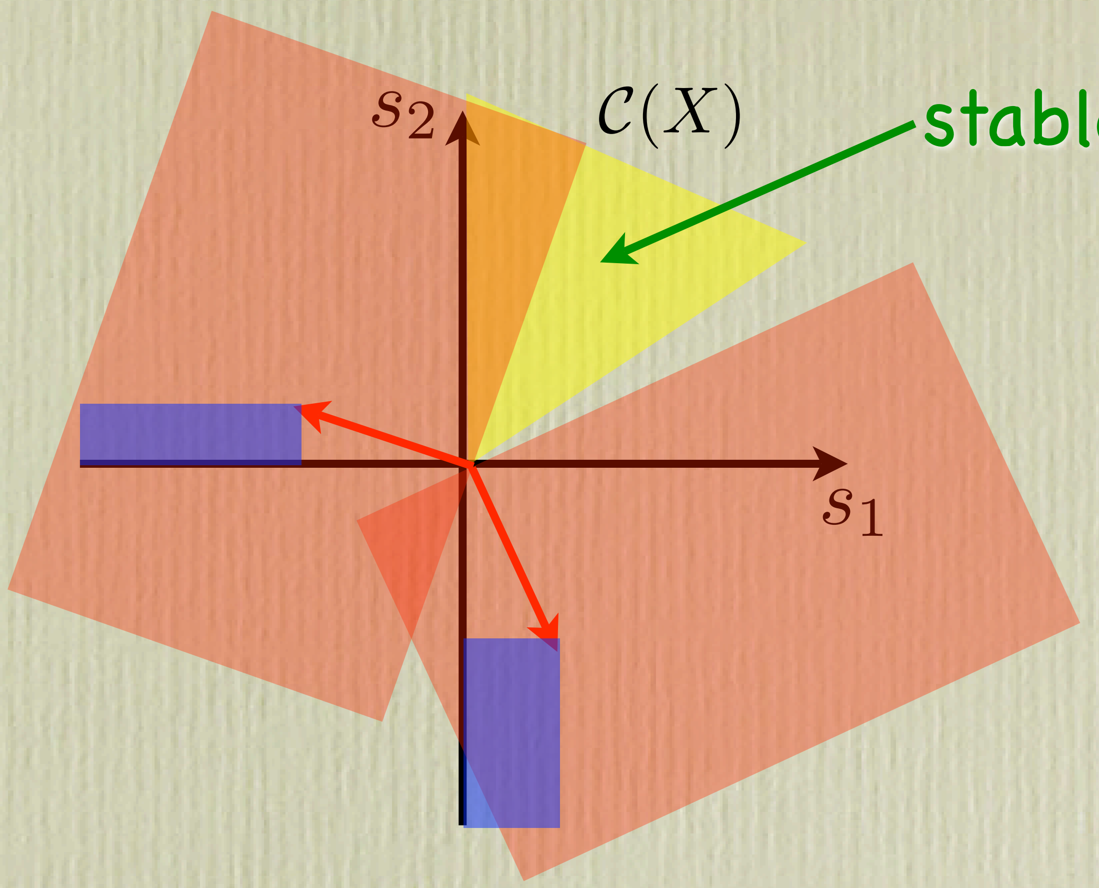

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We conjecture that all positive monad bundles on Cicys are stable.

Spectrum

Families, anti-families:

$$
\begin{aligned}
0 \;&\to\; H^0(X,V) \to H^0(X,B) \to H^0(X,C)\\
&\to\; H^1(X,V) \to H^1(X,B) \to H^1(X,C)\\
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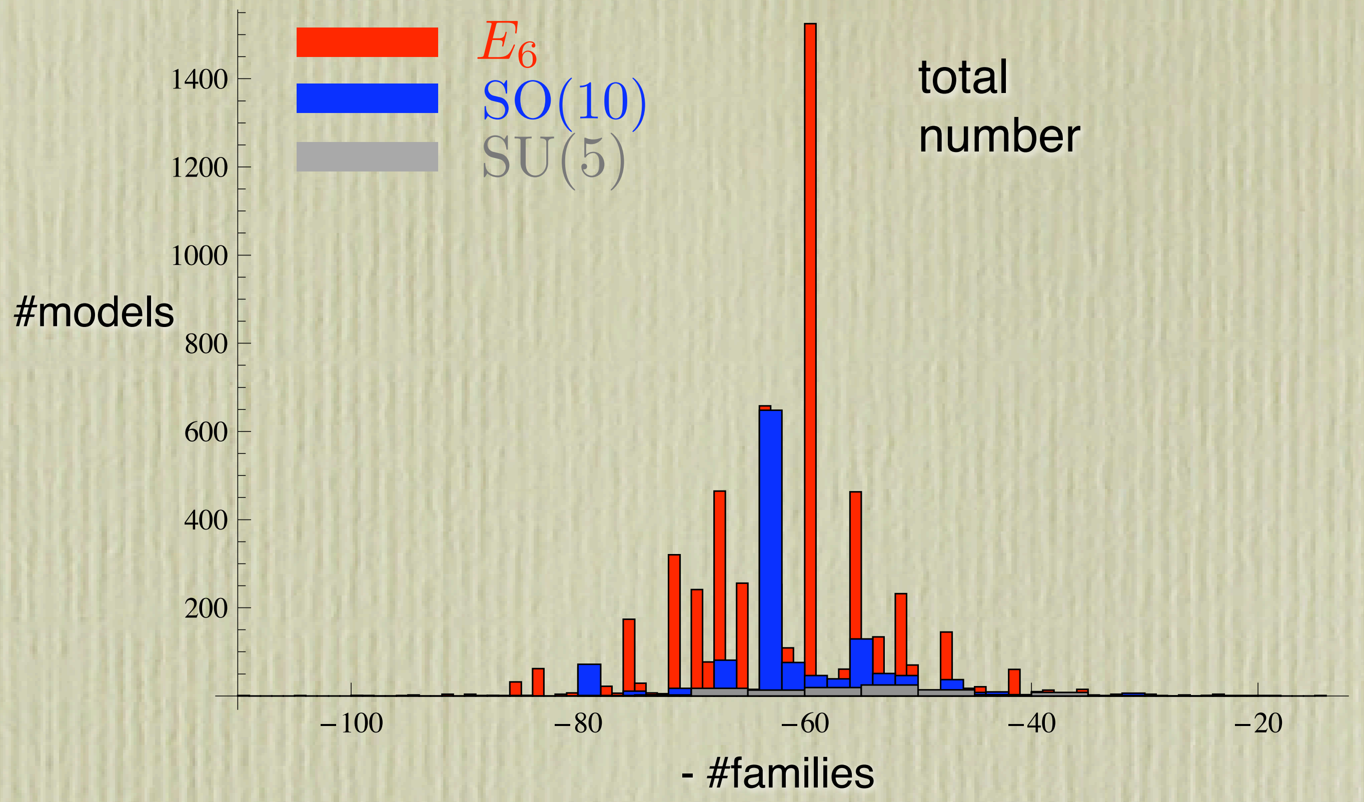
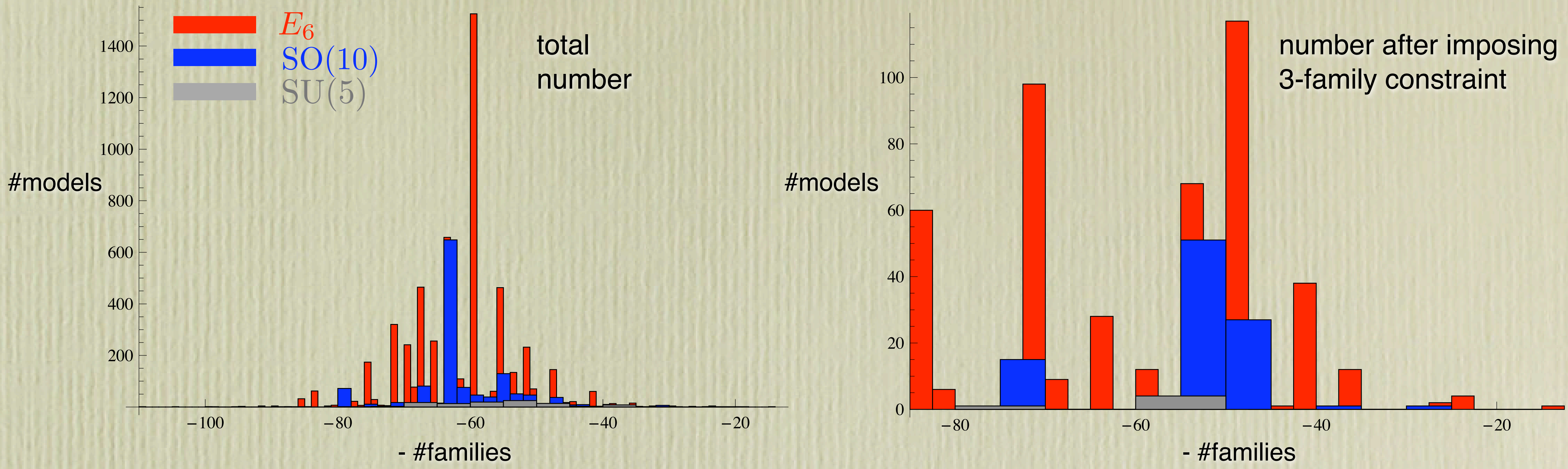
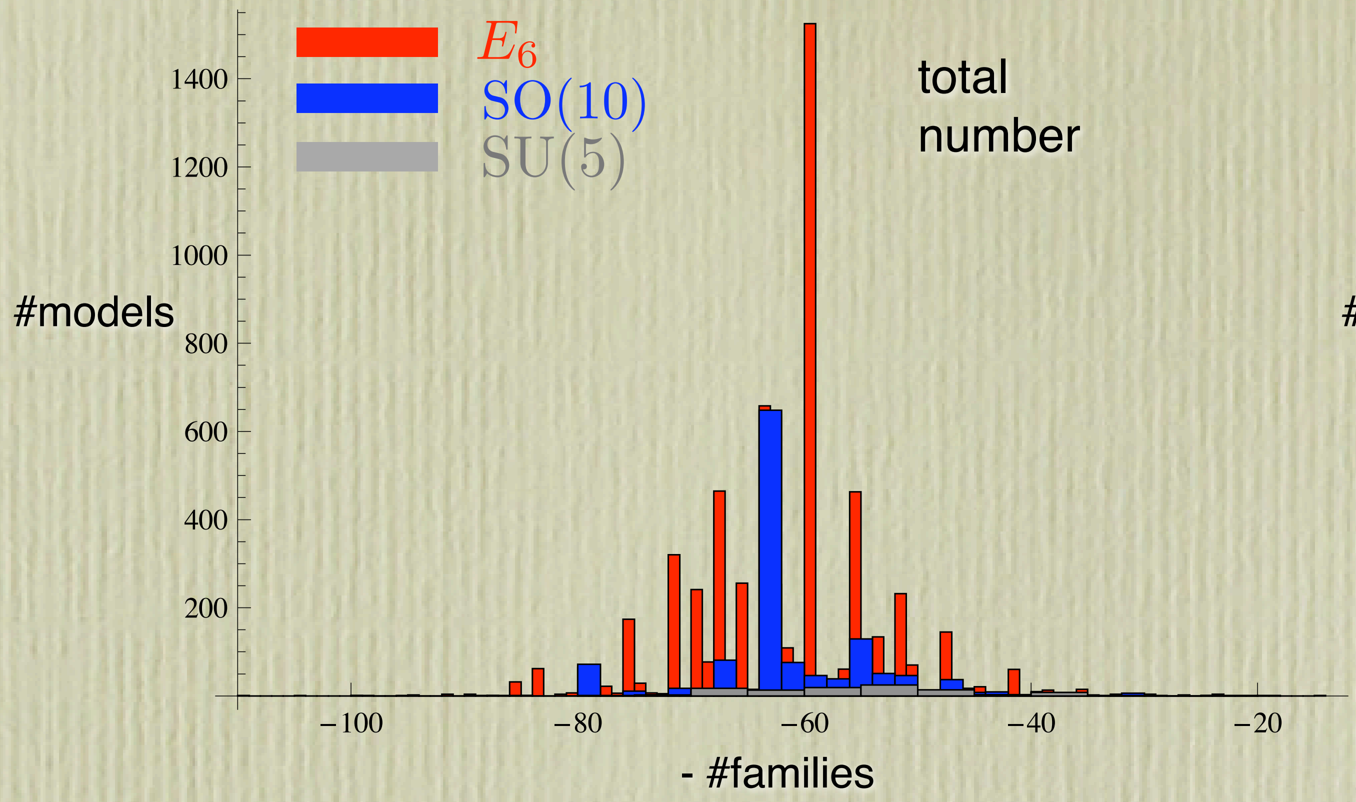
	E6	SO(10)	SU(5)	total
total	5680	1334	104	7118
#families | 3	3091	207	52	3350
Euler number | 3	458	96	5	559

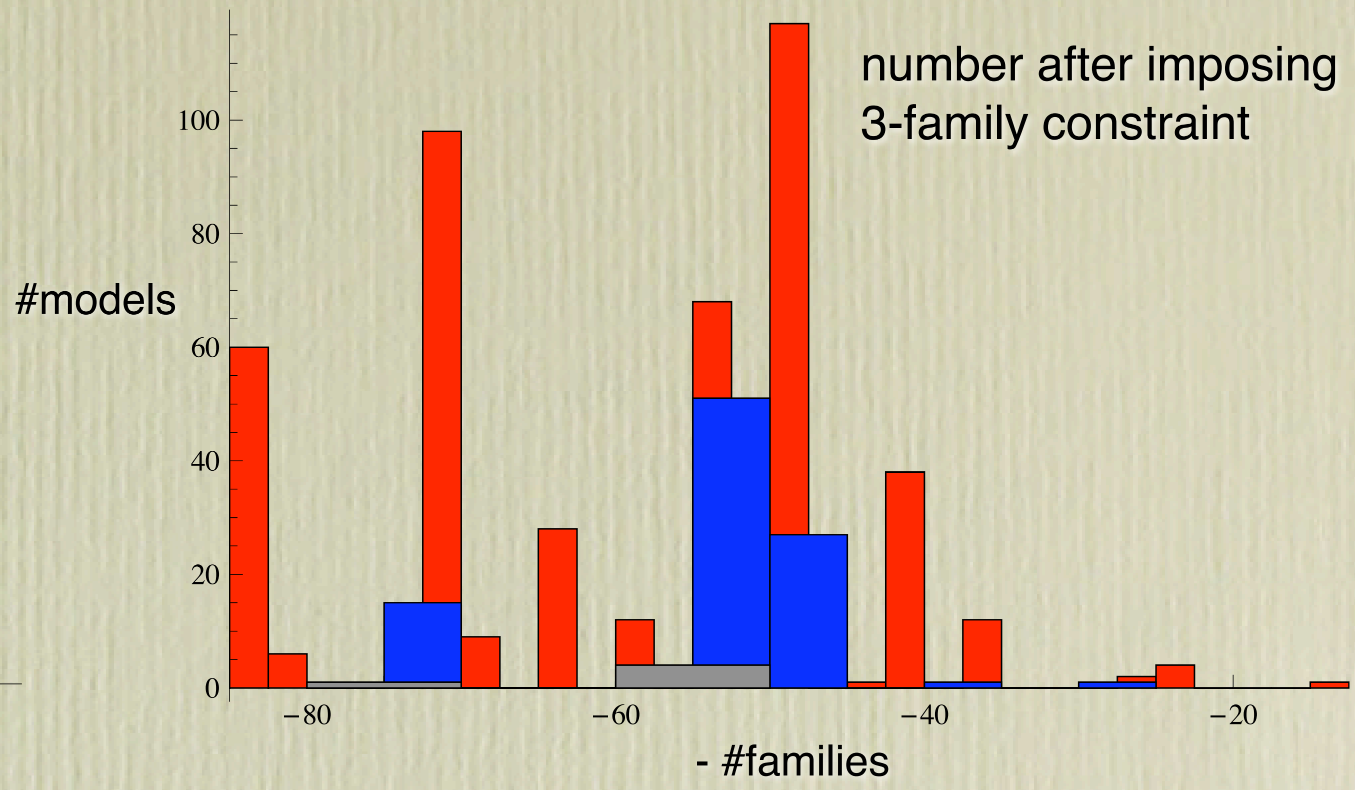


E_6
SO(10)
SU(5)
total
number
#models
1400
1200
1000
800
600
400
200
−100
−80
−60
−40
−20
- #families



E6
SO(10)
SU(5)
total number
#models
1400
1200
1000
800
600
400
200
−100
−80
−60
−40
−20
- #families
number after imposing 3-family constraint
#models
100
80
60
40
20
0
−80
−60
−40
−20
- #families





Spectrum in detail:

$E_6 : n_{27}$ as above , $n_{\overline{27}} = 0,\ n_1(V) = \mathcal{O}(100)$

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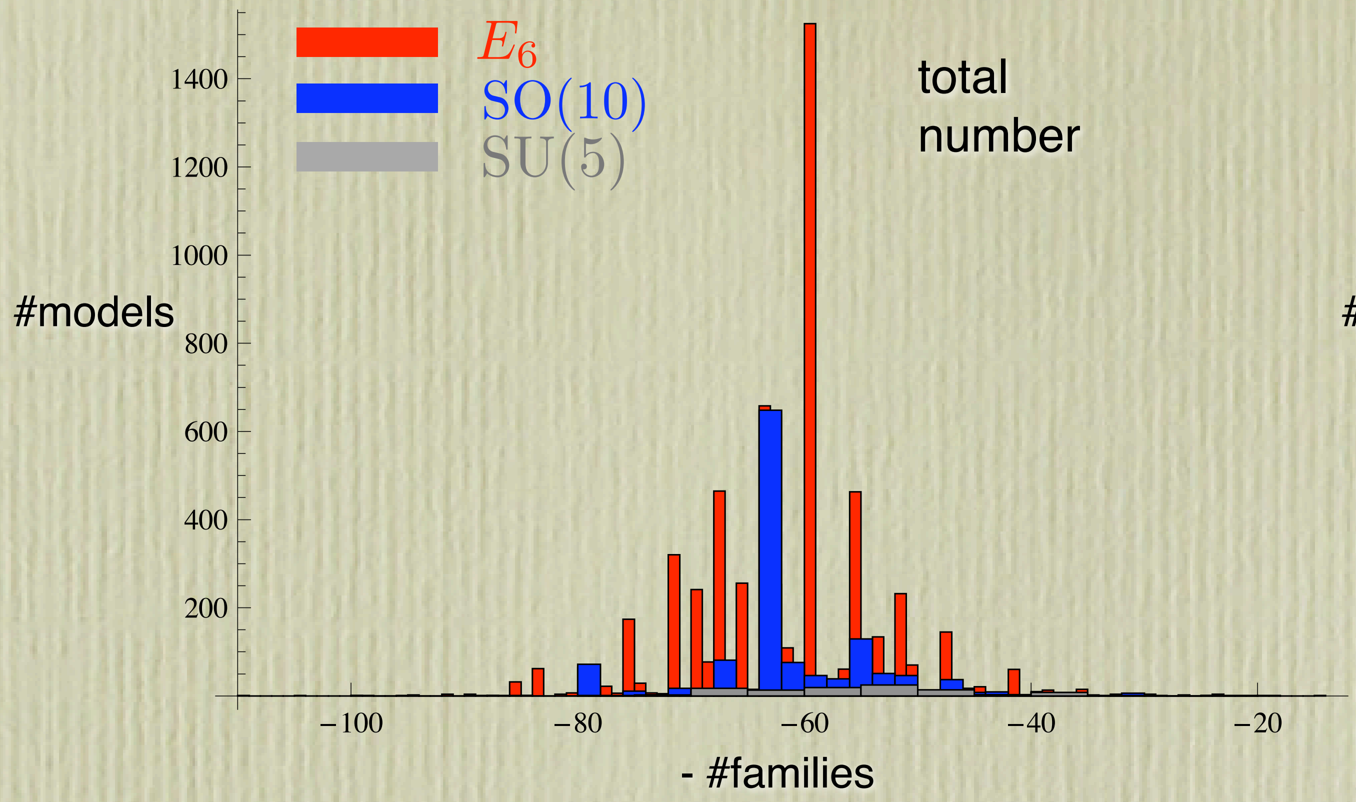

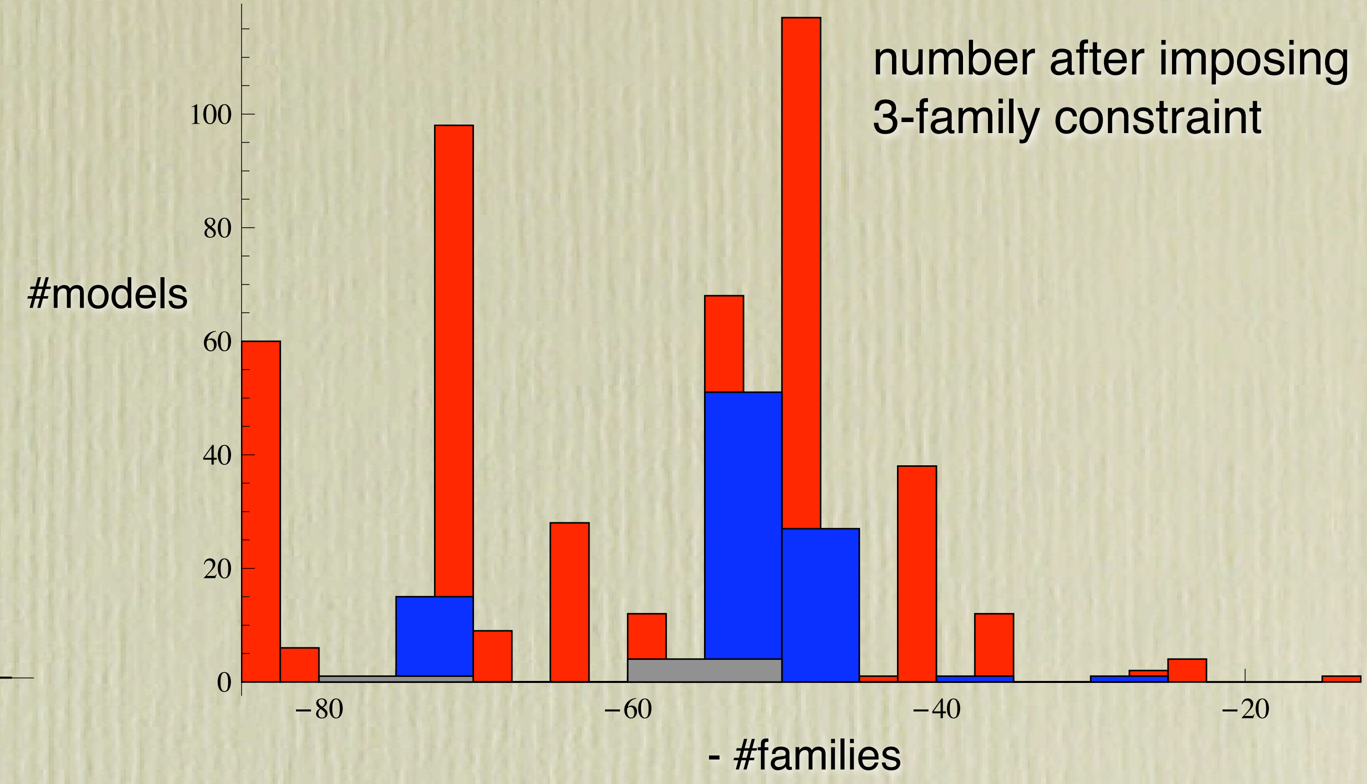

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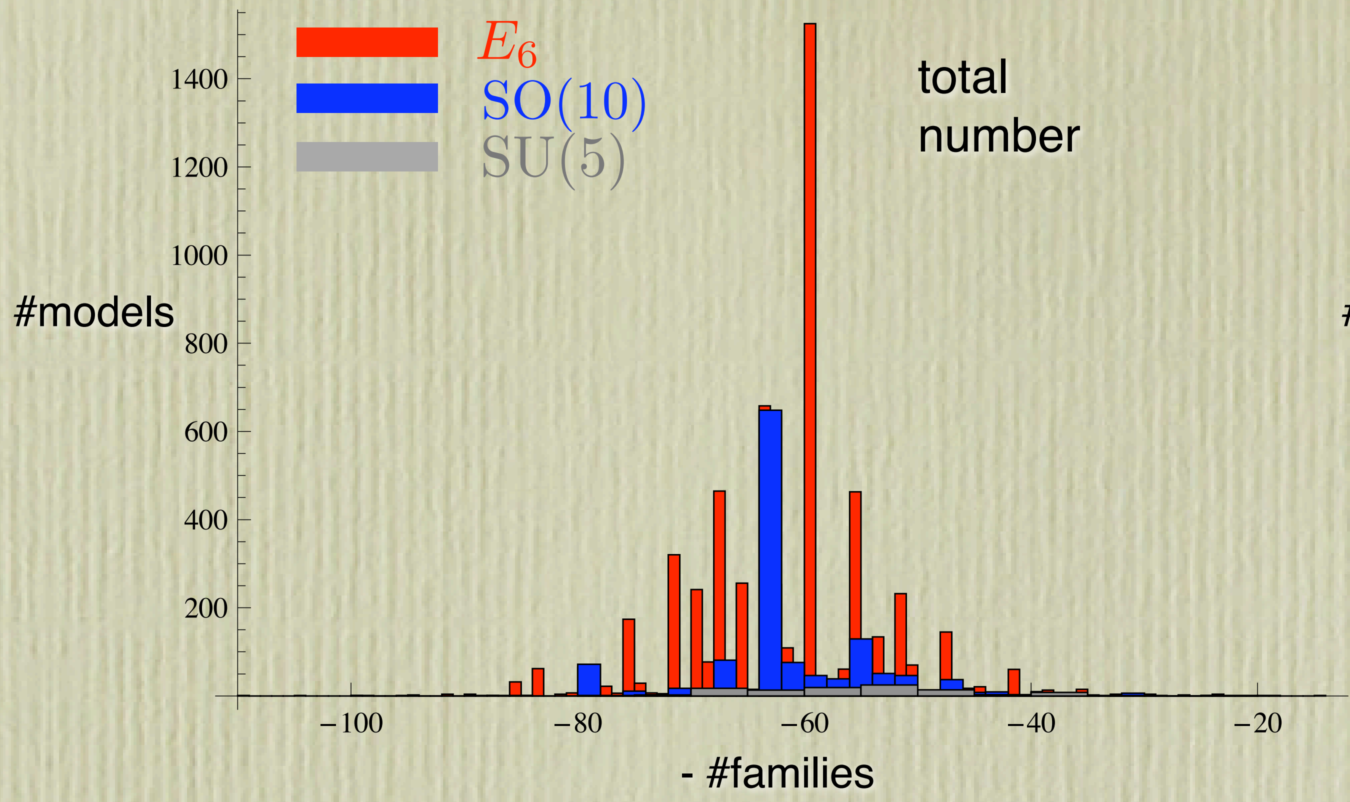
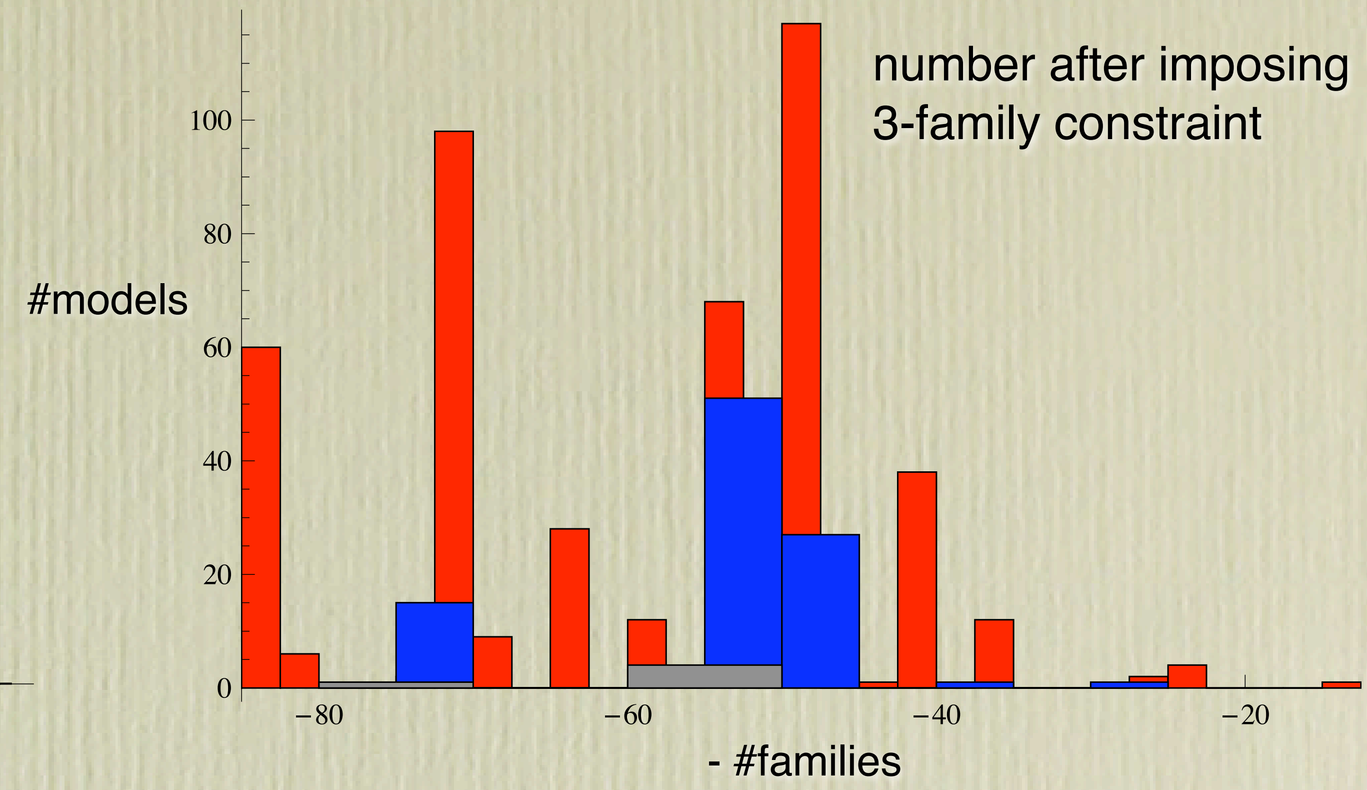
Example for non-genericity: $\mathrm{SO}(10)$ bundle on quintic

$$0 \to V \to \mathcal{O}_X^{\oplus 2}(2) \oplus \mathcal{O}_X^{\oplus 4}(1) \xrightarrow{g} \mathcal{O}_X^{\oplus 2}(4) \to 0\ . \qquad g = \begin{pmatrix} 4x_3^2 & 9x_0^2 + x_2^2 & 8x_2^3 & 2x_3^3 & 4x_1^3 & 9x_1^3 \\ x_0^2 + 10x_2^2 & x_1^2 & 9x_2^3 & 7x_3^3 & 9x_1^3 + x_2^3 & x_1^3 + 7x_4^3 \end{pmatrix}$$





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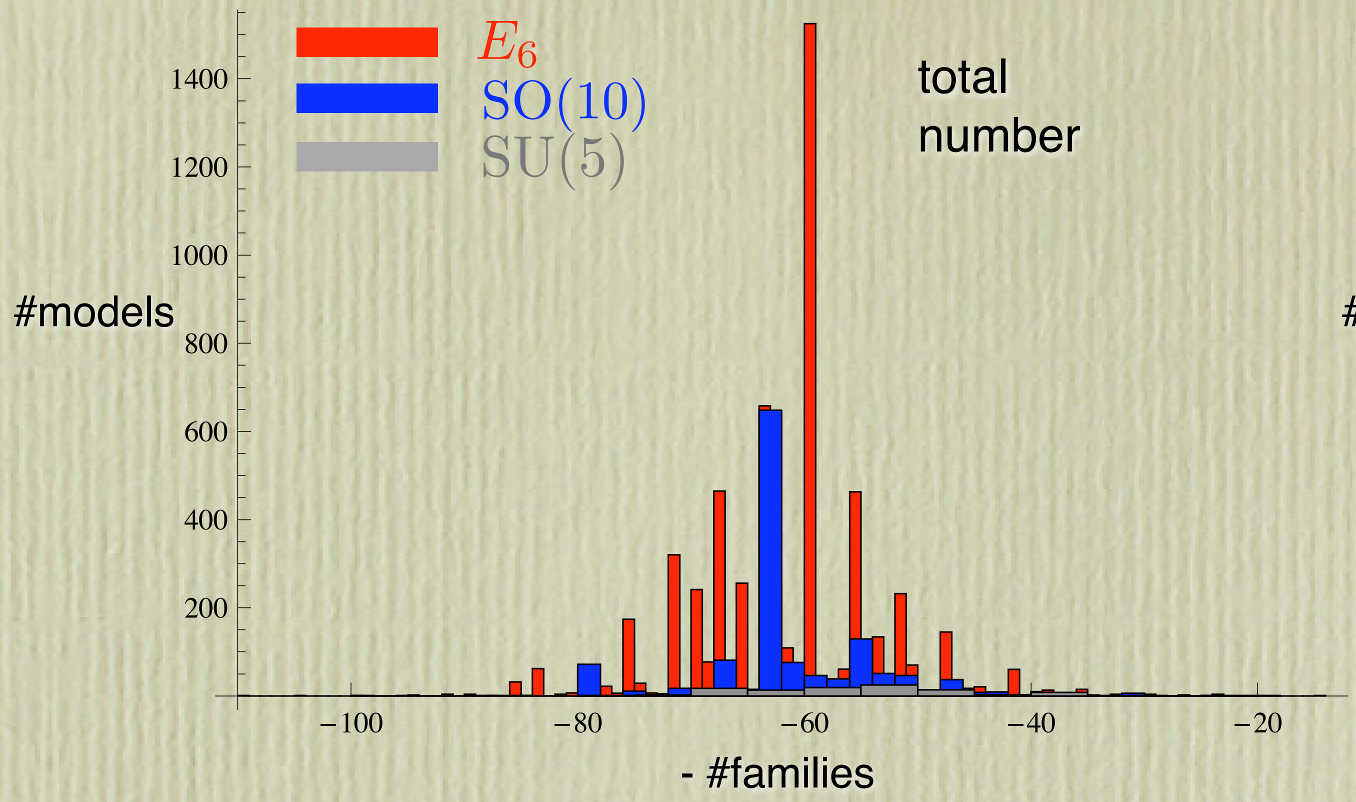
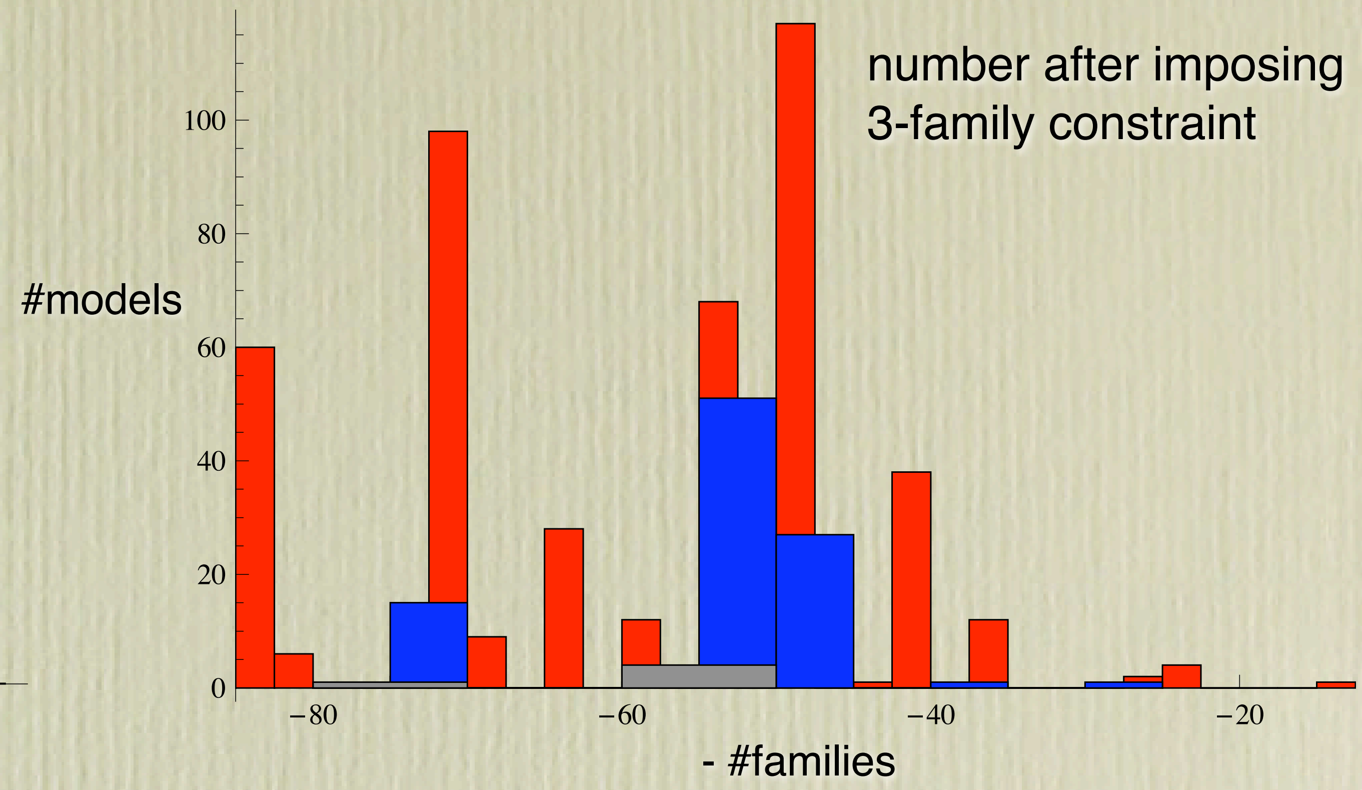
With Macaulay: $n_{16} = 90 \,, \qquad n_{\overline{16}} = 0 \,, \qquad n_{10} = 13 \,, \qquad n_1 = 277$





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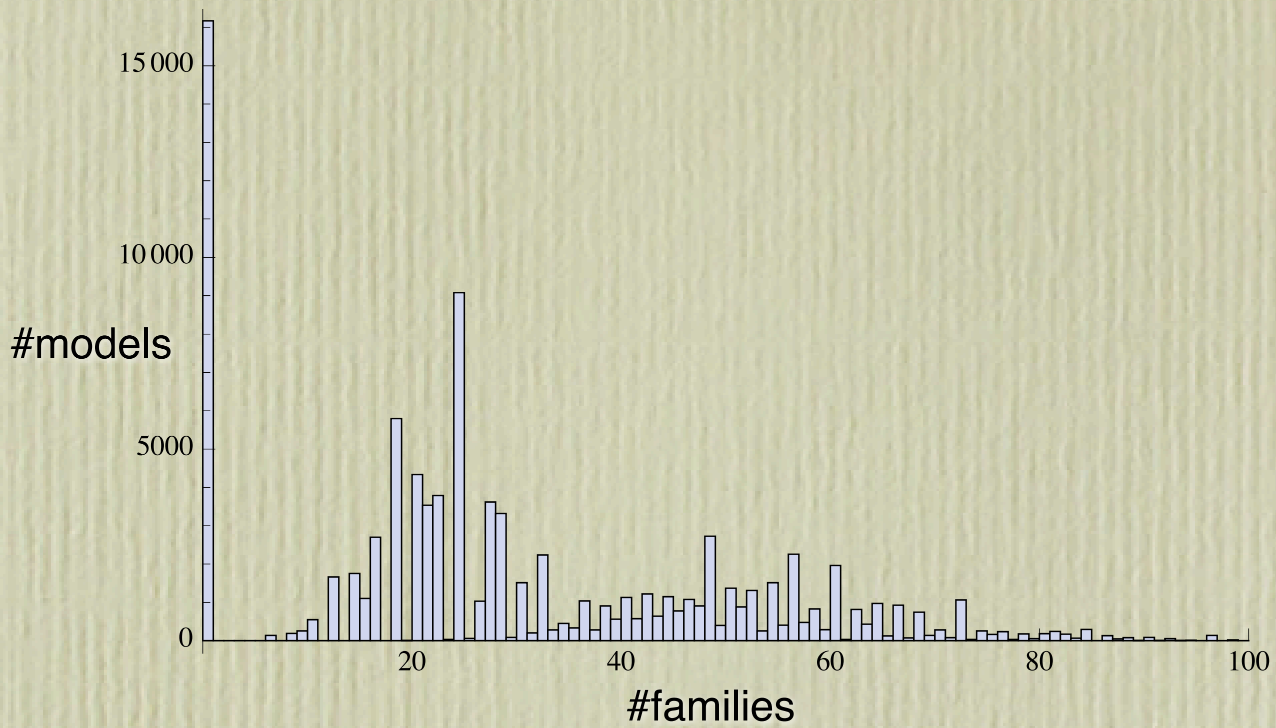
Leads to appr. 100000 rank 3 bundles .

15 000
10 000
#models
5000
0
20
40
60
80
100
#families


15 000
10 000
#models
5000
0
20
40
60
80
100
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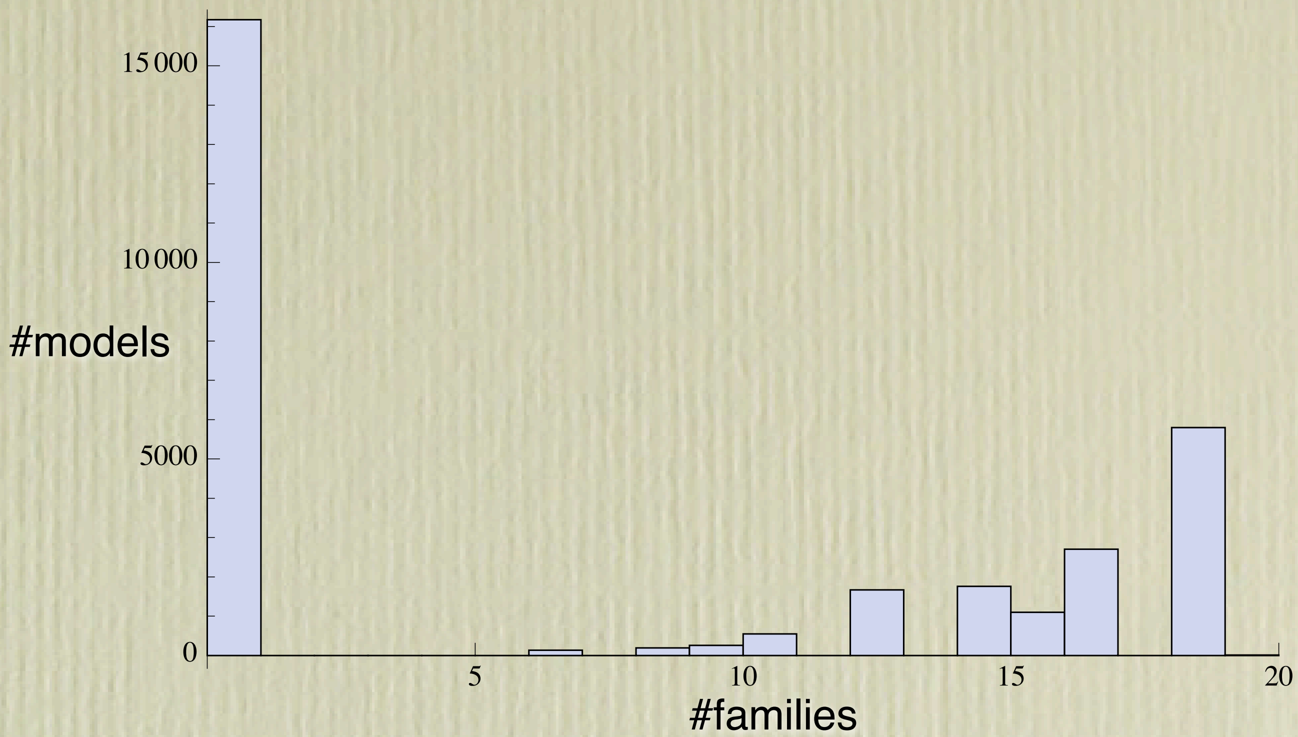
15 000
10 000
#models
5000
0
5
10
15
20
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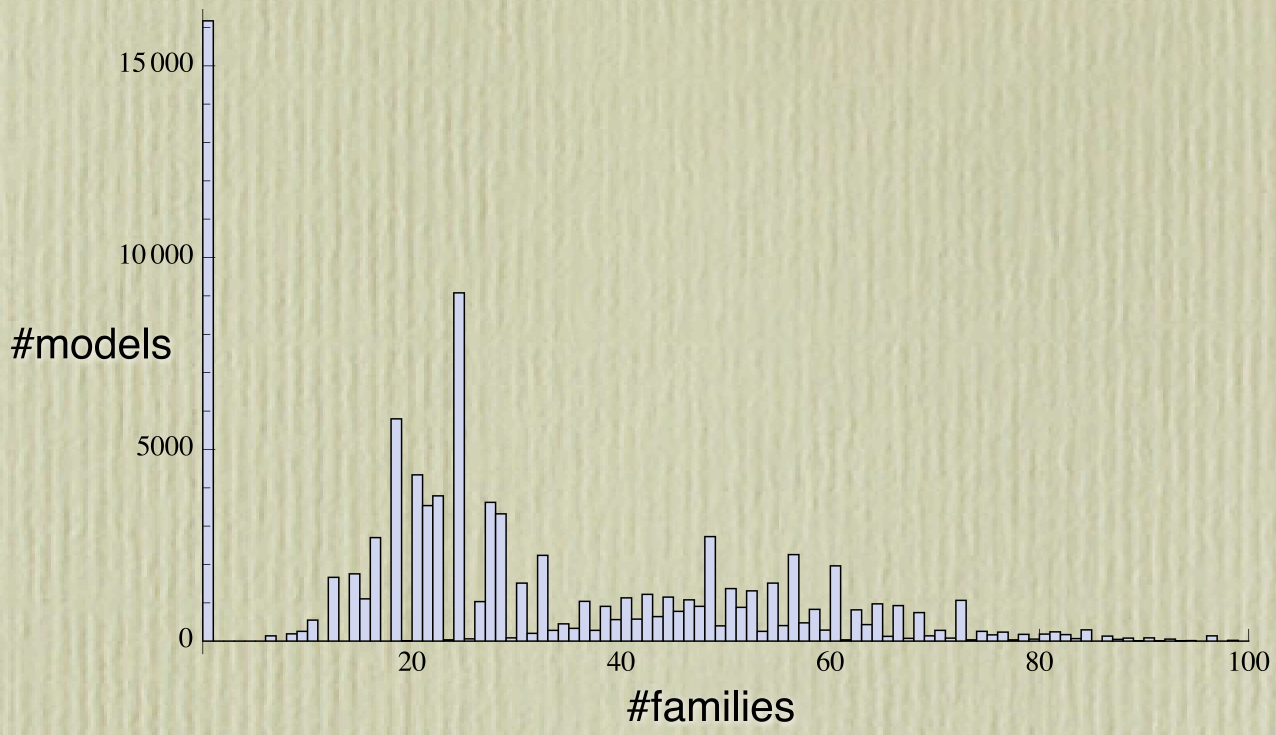

Number of models with #families | 3 and Euler number | 3 : 17255

Number of such models with #families <= 20 : 6982

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- Analyse discrete symmetries and introduce Wilson lines. This can be done systematically (e.g. toric symmetries $X^i \to e^{2\pi i q_i/n} X^i$). Alternatively, U(n) bundles: twisting $L \times V$ is stable if V is.

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Thanks!