String Phenomenology and Dynamical Vacuum Selection

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Classification of the chiral $Z_2 \times Z_2$ heterotic vacua and the origin of Spinor-Vector duality

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1. Introduction

Utilizing the free fermionic construction in heterotic superstring, we obtain a plethora of chiral $N = 1$ SUSY vacua in four space-time dimensions.
I. Antoniadis, C. Bachas, and C. Kounnas, 1987
H. Kawai, D.C. Lewellen, and S.H.-H. Tye, 1987

Many of them are quasi-realistic:

i) Three generations
ii) Correct quantum numbers under $SU(3) \times SU(2) \times U(1)$ of the SM
I. Antoniadis, J. Ellis, J. Hagelin and D.V. Nanopoulos,1989
A.E. Faraggi, D.V. Nanopoulos and K. Yuan, 1990
I. Antoniadis. G.K. Leontaris and J. Rizos, 1990161;
A.E. Faraggi, 1992, G.B. Cleaver, A.E. Faraggi and D.V. Nanopoulos, 1999
G.K. Leontaris and J. Rizos, 1999

Some of the free fermionic models corresponds to $Z_2 \times Z_2$ orbifolds:
C. Kounnas 1995 ; E. Kiritsis, C. Kounnas, P.M. Petropoulos and J. Rizos, 1996,1997
E. Kiritsis and C. Kounnas, 1997 ; A. Gregori, C. Kounnas and J. Rizos, 1999
A. Gregori and C. Kounnas, 1999; A.E. Faraggi, C. Kounnas, S.E.M. Nooij and J. Rizos 2004 ; A.E. Faraggi, C. Kounnas and J. Rizos 2006, 2007

i) Symmetric orbifolds

ii) Asymmetric orbifolds

iii) (Quasi-) Freely acting orbifolds

A special subclass of the free fermionic vacua correspond to symmetric $Z_2 \times Z_2$ (freely acting) orbifold at enhanced symmetry points in the toroidal moduli space.

In this subclass of models the chiral matter spectrum arises from twisted sectors only and thus does not depend on the moduli.

This allows the development of a complete classification of $Z_2 \times Z_2$ symmetric orbifolds via the free fermionic formalism.

The free fermionic construction provides powerful and systematic techniques which facilitate developing a computerized classification algorithm for the twisted matter chiral spectrum.

This fact is of basic importance since it enables a systematic analysis of *all* the models according to the number of spinorial, anti–spinorial and vectorial representations of an underlining $SO(10)$ gauge group, in algebraic formulas.

Our classification allows us to scan a range of over 10^{16} symmetric $Z_2 \times Z_2$ orbifold vacua.

The space of vacua arises from a set of independent generalize GSO projection coefficients $c\begin{bmatrix} b_i \\ b_j \end{bmatrix}$, which correspond a matrix with elements taking values ± 1.

The independent elements of this matrix correspond to the upper block of this matrix. All other elements are fixed by modular invariance and the higher genus factorization of the partition function.

Our classification basis contains 12 vectors. Therefore, the number of independent GGSO projection coefficients is $66 \longrightarrow 2^{66}$ different vacua.

Requiring $N = 1$ space–time supersymmetry reduces the number of independent phases to $55 \longrightarrow 2^{55}$ different vacua.

2. The world-sheet heterotic degrees of freedom; the $SO(10)$ basis sets

- 2 left- and 2 right-moving space time coordinates:

$$\partial X^\mu\,, \qquad \bar\partial X^\mu$$

- 6 compact left- and right-moving internal fermionised coordinates:

$$\partial X^i \equiv y^i\,\omega^i \qquad \bar\partial X^i \equiv \bar y^i\,\bar\omega^i \qquad i=1,\ldots,6$$

- 8 left-moving super-coordinates:

$$S=\{\psi^\mu,\chi^{1,\ldots,6}\}$$

- 32 real or 16 complex right moving 2d-fermions:

$$x=\{\bar\eta^1,\ \bar\eta^2,\ \bar\eta^3,\ \bar\psi^{1,\ldots,5}\},$$
$$z_1=\{\bar\phi^{1,\ldots,4}\}$$
$$z_2=\{\bar\phi^{5,\ldots,8}\}$$

The heterotic string, (in the light-cone gauge), is described in 4D by 20 left–moving and 44 right–moving 2d real fermions.

A large number of vacua can be constructed according to the different phases picked up by the 2d fermions $(f_A, A = 1, \ldots, 44)$ when transported along the torus non-contractible loops.

$$f_A \to -e^{i\pi\alpha_i(f_A)}\ f_A,\ \ , A = 1, \ldots, 44\ .$$

Each model corresponds to a particular choice of fermion phases consistent with modular invariance that can be generated by a set of basis vectors $v_i, i = 1, \ldots, n$,

$$v_i = \{\alpha_i(f_1), \alpha_i(f_2), \alpha_i(f_3)) \ldots\}$$

The string spectrum is truncated by a GGSO projection induced by the basis vectors. Different sets of projection coefficients $c\begin{bmatrix} v_i \\ v_j \end{bmatrix} = \pm 1$ consistent with modular invariance give rise to different models.

A string model is defined uniquely by a set of basis vectors $v_i, i = 1, \ldots, N$ and a set of $2^{N(N-1)/2}$ independent projections coefficients $c\binom{v_i}{v_j}, i > j$.

The class of models under investigation, is generated by a set of 12 basis vectors

$$F = \{\psi^\mu, \ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \ \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\}$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\}, \quad i = 1, \ldots, 6$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\}$$
$$z_2 = \{\bar{\phi}^{5,\dots,8}\}$$

Generic N=1 SUSY partition function

$$Z = \oint \frac{d\tau d\bar{\tau}}{(\mathrm{Im}\tau)^2} \frac{\mathrm{Im}\tau^{-1}}{\eta(\tau)^{12}\ \bar{\eta}(\bar{\tau})^{24}} \quad \frac{1}{2}\sum_{(a,b)} \ \frac{1}{2}\sum_{(h_1,g_1)} \ \frac{1}{2}\sum_{(h_2,g_2)}$$

$$\times\ (-1)^{a+b+ab}\, \theta\begin{bmatrix} a \\ b \end{bmatrix}_{\psi^\mu} \theta\begin{bmatrix} a+h_1 \\ b+g_1 \end{bmatrix}_{\chi^{12}} \theta\begin{bmatrix} a+h_2 \\ b+g_2 \end{bmatrix}_{\chi^{34}} \theta\begin{bmatrix} a+h_3 \\ b+g_3 \end{bmatrix}_{\chi^{56}}$$

$$\times \frac{1}{2}\sum_{(\epsilon_1,\zeta_1)} \bar{\theta}\begin{bmatrix} \epsilon_1 \\ \zeta_1 \end{bmatrix}^5_{\bar{\psi}^{12345}} \bar{\theta}\begin{bmatrix} \epsilon_1+h_1 \\ \zeta_1+g_1 \end{bmatrix}_{\bar{\eta}_1} \bar{\theta}\begin{bmatrix} \epsilon_1+h_2 \\ \zeta_1+g_2 \end{bmatrix}_{\bar{\eta}_2} \bar{\theta}\begin{bmatrix} \epsilon_1+h_3 \\ \zeta_1+g_3 \end{bmatrix}_{\bar{\eta}_3}$$

$$\times \frac{1}{2}\sum_{(\epsilon_2,\zeta_2)} \frac{1}{2}\sum_{(H,G)} \bar{\theta}\begin{bmatrix} \epsilon_2 \\ \zeta_2 \end{bmatrix}^4_{\bar{\phi}^{1234}} \ (-)^{HG}\ \bar{\theta}\begin{bmatrix} \epsilon_2+H \\ \zeta_2+G \end{bmatrix}^4_{\bar{\phi}^{5678}}$$

$$\times \ \frac{1}{2^6}\sum_{(\gamma_i,\delta_i)} \Gamma_{6,6}\begin{bmatrix} \gamma_i, & h_I \\ \delta_i, & g_I \end{bmatrix}_{\omega^i,y^i} \ \times\ (-)^{\Phi[(h_I,g_I),\ (\gamma_i,\delta_i),\ (\epsilon_i,\zeta_i),\ (H,G)]}$$

In the fermionic formulation the h_I–twisted and γ_i–shifted $\Gamma_{6,6}\begin{bmatrix}\gamma_i, & h_I\\ \delta_i, & g_I\end{bmatrix}_{\omega^i,y^i}$ lattice, take the following form $\left(h_3 = -h_1 - h_2,\ g_3 = -g_1 - g_2 \right)$:

$$\Gamma_{6,6}\begin{bmatrix}\gamma_i, & h_I\\ \delta_i, & g_I\end{bmatrix}_{\omega^i,y^i} \equiv$$

$$\times\ \theta\begin{bmatrix}\gamma_1\\ \delta_1\end{bmatrix}^{\frac{1}{2}}_{\omega^1} \bar\theta\begin{bmatrix}\gamma_1\\ \delta_1\end{bmatrix}^{\frac{1}{2}}_{\bar\omega^1} \theta\begin{bmatrix}\gamma_1+h_1\\ \delta_1+g_1\end{bmatrix}^{\frac{1}{2}}_{y^1} \bar\theta\begin{bmatrix}\gamma_1+h_1\\ \delta_1+g_1\end{bmatrix}^{\frac{1}{2}}_{\bar y^1} \times\ \theta\begin{bmatrix}\gamma_2\\ \delta_2\end{bmatrix}^{\frac{1}{2}}_{\omega^2} \bar\theta\begin{bmatrix}\gamma_2\\ \delta_2\end{bmatrix}^{\frac{1}{2}}_{\bar\omega^2} \theta\begin{bmatrix}\gamma_2+h_1\\ \delta_1+g_1\end{bmatrix}^{\frac{1}{2}}_{y^2} \bar\theta\begin{bmatrix}\gamma_2+h_1\\ \delta_2+g_1\end{bmatrix}^{\frac{1}{2}}_{\bar y^2}$$

$$\times\ \theta\begin{bmatrix}\gamma_3\\ \delta_3\end{bmatrix}^{\frac{1}{2}}_{\omega^3} \bar\theta\begin{bmatrix}\gamma_3\\ \delta_3\end{bmatrix}^{\frac{1}{2}}_{\bar\omega^3} \theta\begin{bmatrix}\gamma_3+h_2\\ \delta_3+g_2\end{bmatrix}^{\frac{1}{2}}_{y^3} \bar\theta\begin{bmatrix}\gamma_3+h_2\\ \delta_3+g_2\end{bmatrix}^{\frac{1}{2}}_{\bar y^3} \times\ \theta\begin{bmatrix}\gamma_4\\ \delta_4\end{bmatrix}^{\frac{1}{2}}_{\omega^4} \bar\theta\begin{bmatrix}\gamma_4\\ \delta_4\end{bmatrix}^{\frac{1}{2}}_{\bar\omega^4} \theta\begin{bmatrix}\gamma_4+h_2\\ \delta_4+g_2\end{bmatrix}^{\frac{1}{2}}_{y^4} \bar\theta\begin{bmatrix}\gamma_4+h_2\\ \delta_4+g_2\end{bmatrix}^{\frac{1}{2}}_{\bar y^4}$$

$$\times\ \theta\begin{bmatrix}\gamma_5\\ \delta_5\end{bmatrix}^{\frac{1}{2}}_{\omega^5} \bar\theta\begin{bmatrix}\gamma_5\\ \delta_5\end{bmatrix}^{\frac{1}{2}}_{\bar\omega^5} \theta\begin{bmatrix}\gamma_5+h_3\\ \delta_5+g_3\end{bmatrix}^{\frac{1}{2}}_{y^5} \bar\theta\begin{bmatrix}\gamma_5+h_3\\ \delta_5+g_3\end{bmatrix}^{\frac{1}{2}}_{\bar y^5} \times\ \theta\begin{bmatrix}\gamma_6\\ \delta_6\end{bmatrix}^{\frac{1}{2}}_{\omega^6} \bar\theta\begin{bmatrix}\gamma_6\\ \delta_2\end{bmatrix}^{\frac{1}{2}}_{\bar\omega^6} \theta\begin{bmatrix}\gamma_6+h_3\\ \delta_6+g_3\end{bmatrix}^{\frac{1}{2}}_{y^6} \bar\theta\begin{bmatrix}\gamma_6+h_3\\ \delta_6+g_3\end{bmatrix}^{\frac{1}{2}}_{\bar y^6}$$

The generic partition function Z is modular invariant:

$$\tau \longrightarrow \tau + 1 :$$

$$\left[{a \atop b}\right] \longrightarrow \left[{a \atop b+a+1}\right], \quad \left[{\gamma_i \atop \delta_i}\right] \longrightarrow \left[{\gamma_i \atop \delta_i+\gamma_i+1}\right], \quad \left[{\epsilon_i \atop \zeta_i}\right] \longrightarrow \left[{\epsilon_i \atop \zeta_i+\epsilon_i+1}\right], \quad \left[{h_I \atop g_I}\right] \longrightarrow \left[{h_I \atop g_I+h_I}\right], \quad \left[{H \atop G}\right] \longrightarrow \left[{H \atop G+H}\right]$$

$$\tau \longrightarrow \frac{-1}{\tau} :$$

$$\left[{a \atop b}\right] \longrightarrow \left[{b \atop a}\right], \quad \left[{\gamma_i \atop \delta_i}\right] \longrightarrow \left[{\delta_i \atop \gamma_i}\right], \quad \left[{\epsilon_i \atop \zeta_i}\right] \longrightarrow \left[{\zeta_i \atop \epsilon_i}\right], \quad \left[{h_I \atop g_I}\right] \longrightarrow \left[{g_I \atop h_I}\right], \quad \left[{H \atop G}\right] \longrightarrow \left[{G \atop H}\right]$$

Provided that the phase $(-)^{\Phi}$ remain invariant:

$$(-)^{\Phi[(h_I,g_I),\ (\gamma_i,\delta_i),\ (\epsilon_i,\zeta_i),\ (H,G)]} \quad \xrightarrow[\tau \to \frac{-1}{\tau}]{\tau \to \tau+1} \quad (-)^{\Phi[(h_I,g_I),\ (\gamma_i,\delta_i),\ (\epsilon_i,\zeta_i),\ (H,G)]}$$

There are in total 2^{55} independent choices. Some of those are the following:

$$(-)^{\Phi} = 1, \quad (-)^{\gamma_i g_I + \delta_i h_I + h_I g_I}, \quad (-)^{H g_I + G h_I}, \quad (-)^{\gamma_1\delta_2+\gamma_2\delta_3+\gamma_3\delta_1+\delta_1\gamma_2+\delta_2\gamma_3+\delta_3\gamma_1}, \ldots$$

Some comments:

- The vectors F, S generate an $N = 4$ supersymmetric model, with $SO(44)$ gauge symmetry.

- The vectors e_i, $i = 1, \ldots, 6$ give rise to all possible symmetric shifts of the six internal fermionized coordinates ($\partial X^i = y^i \omega^i, \bar{\partial} X^i = \bar{y}^i \bar{\omega}^i$). Their addition breaks the $SO(44)$ gauge group, but preserves $N = 4$ supersymmetry.

- The vectors b_1 and b_2 defines the $Z_2 \times Z_2$ orbifold twists, which breaks $N = 4$ to $N = 1$ supersymmetry, and defines the $U(1)^3 \times SO(10)$ gauge symmetry.

- The z_1 and z_2 vectors give rise to the $SO(8) \times SO(8)$ gauge group.

- The above choice of basis vectors is the most general which is compatible with a $SO(10)$ Kac–Moody level one algebra.

• The requirement of $N=1$ SUSY implies the absence of the arguments (a,b) in the phase factor Φ.

• The vector bosons from the untwisted sector generate an

$$SO(10)\times U(1)^3\times SO(8)^2$$

gauge symmetry. Depending on the choices of the projection coefficients, extra gauge bosons may arise from the x sector

$$x = F + S + \sum_{i=1}^{6} e_i + z_1 + z_2 = \{\bar{\eta}^{123}, \bar{\psi}^{12345}\} \ ,$$

In that case the $SO(10)\times U(1)^3$ enhanced to $E_6\times U(1)^2$.

• Other gauge enhancements :

$$SO(8)\times SO(8)\rightarrow SO(16)\rightarrow E_8, \quad SO(10)\times SO(8)\rightarrow SO(18),$$
$$SO(10)\times SO(8)\times SO(8)\rightarrow SO(26), \quad SO(8)\times U(1)\rightarrow SO(10)$$

3. Results

A) Statistical analysis of 10^{10} models with (2,2) and (2,0) 2d-superconformal:

B) Exact results for all 2^{44} models with (2,0) 2d-superconformal

Our results (A+B) analysis revealed a bell shape distribution according to the net number of chiral families

- The 15% of the models have three net chiral families.

- Mirror symmetry under the exchange of spinorials , and the anti-spinorials of $SO(10)$

$$S \quad \leftrightarrow \quad \bar{S}$$

- Vector-Spinor duality symmetry. Additional symmetry in the distribution, under exchange of vectorial, and spinorial plus anti–spinorial, representations of $SO(10)$.

$$V \quad \leftrightarrow \quad (S + \bar{S})$$

The vector-spinor duality symmetry is evident when the $SO(10)$ is enhanced to E_6, in which case $\#(16+\overline{16}) = \#(10)$ since the 27 and $\bar{2}7$ contains both the spinorials and vectorials of the $SO(10)$

$$27 \rightarrow 16+10+1, \qquad \overline{27} \rightarrow \overline{16}+10+1$$

Thanks to the algebraic form of the GGSO projections in the fermionic formulation, we were able to demonstrate that the $V \leftrightarrow S_t$ duality persists in all SO(10) vacua.

We further show the existence of self–dual vacua in which $\#(16+\bar{1}6) = \#(10)$, but in which the $SO(10)$ symmetry is not enhanced to E_6

Furthermore, we find that the $V \leftrightarrow S_t$ duality holds separately for each of the three twisted planes of the $Z_2 \times Z_2$ orbifolds.

This precise observation, let us to conjecture that the origin of $V \leftrightarrow S_t$ duality relies in $N = 2$ string vacua.

4. The origin of $V \leftrightarrow S_t$ duality

• The $S, \overline{S}$ and V representations of $SO(10)$ comes from the three twisted $N = 2$ sectors

$$(h_1 = 0,\ h_2 = 1,\ h_3 = -1),\ \ (h_1 = 1,\ h_2 = 0,\ h_3 = -1),\ \ (h_1 = 1,\ h_2 = -1,\ h_3 = 0)$$

The relevant part of the partition function in the first plane $(h_1 = 0, h_2 = h_3 = h = 1)$

$$\dots\ \bar{\theta}\begin{bmatrix}\epsilon_1\\ \zeta_1\end{bmatrix}^4_{\bar{\psi}^{1234}} \bar{\theta}\begin{bmatrix}\epsilon_1\\ \zeta_1\end{bmatrix}_{\bar{\psi}^5} \bar{\theta}\begin{bmatrix}\epsilon_1\\ \zeta_1\end{bmatrix}_{\bar{\eta}_1} \bar{\theta}\begin{bmatrix}\epsilon_1+h\\ \zeta_1+g\end{bmatrix}_{\bar{\eta}_2} \bar{\theta}\begin{bmatrix}\epsilon_1+h\\ \zeta_1+g\end{bmatrix}_{\bar{\eta}_3} \dots \Gamma_{2,2}\begin{bmatrix}t_i\\ s_i\end{bmatrix}$$

where $\Gamma_{2,2}\begin{bmatrix}t_i\\ s_i\end{bmatrix}$ is the shifted lattice of the first plane

$$\Gamma_{2,2}\begin{bmatrix}t_i\\ s_i\end{bmatrix} = \frac{1}{4}\sum_{(\gamma_i,\delta_i)} \theta\begin{bmatrix}\gamma_1\\ \delta_1\end{bmatrix}_{\omega^1,y^1} \bar{\theta}\begin{bmatrix}\gamma_1\\ \delta_1\end{bmatrix}_{\bar{\omega}^1,\bar{y}^1} \theta\begin{bmatrix}\gamma_2\\ \delta_2\end{bmatrix}_{\omega^2,y^2} \bar{\theta}\begin{bmatrix}\gamma_2\\ \delta_2\end{bmatrix}_{\bar{\omega}^2,\bar{y}^2} (-)^{s_i\gamma_i+t_i\delta_i+s_it_i}$$

- The S or $\overline{S}$ representation arise when $\epsilon_1 = 1$

- The V representation arise when $\epsilon_1 + h = 1$

Four possibilities to couple the lattice characters (t_i, s_i) to (ϵ_i, ζ_i), (h, g) :

- Inserting 1 $\longrightarrow$ (2,2) superconformal, $SO(10) \rightarrow E_6$, $[S_t]= [V]$

- Inserting $(-)^{sh+tg}$ $\longrightarrow$ freely acting orbifold, $[S_t]= [V]=0$

- Inserting $(-)^{s\epsilon_1+t\zeta_1}$ $\longrightarrow$ (2,0) superconformal, *only* V

- Inserting $(-)^{s(\epsilon_1+h)+t(\zeta_1+g)}$ $\longrightarrow$ (2,0) superconformal, *only* $S, \overline{S}$

Starting from the self-dual configurations $\left[1\ ,(-)^{sh+tg}\right]$ and then perform an x-map

$$x = \{\psi^{12345}, \eta^{123}\}$$

we obtain the two other cases

$$\left[1\ ,\ (-)^{sh+tg}\right] \ \longrightarrow\ (-)^{s\epsilon_1+t\zeta_1}\left[1\ ,\ (-)^{sh+tg}\ \right] = \left[(-)^{s\epsilon_1+t\zeta_1}\ ,\ (-)^{s(\epsilon_1+h)+t(\zeta_1+g)}\right]$$

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* The $V \leftrightarrow S_t$ duality emerges from the initial $(2,2)$ super-conformal symmetry.

*

* $\longrightarrow$ the triality symmetry of $V, S, \overline{S}$ representations of $SO(8)$, $[\psi^5, \eta^1, \eta^2, \eta^3]$

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* The different choices of GGSO coefficients break (spontaneously) $(2,2) \rightarrow (2,0)$

* eliminating from the massless spectrum either

*

* V , or S_t or even both V and S_t

****

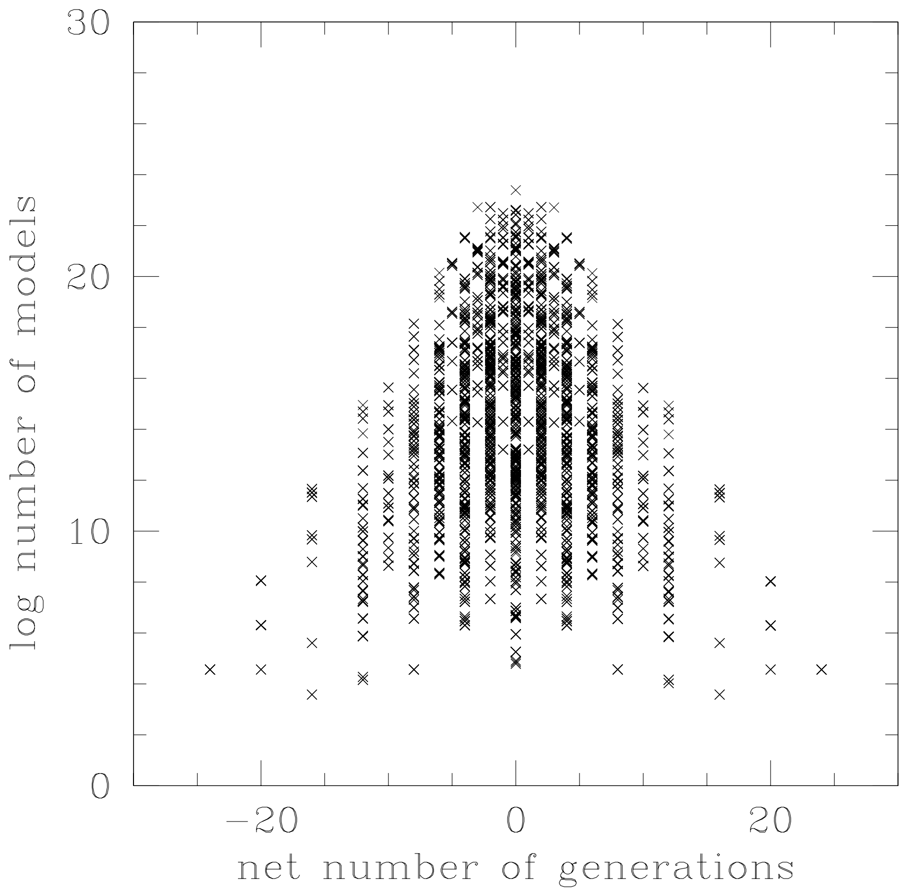

Figure 1:

Scater plot of log of the number of models vs the net number of chiral families

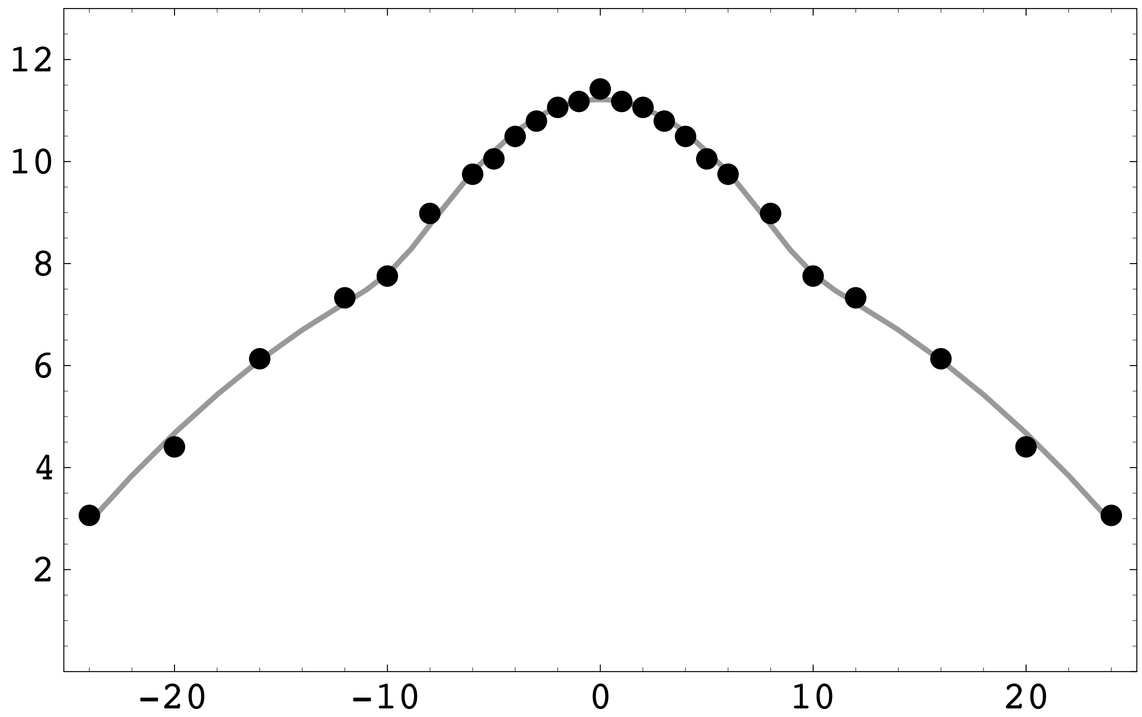

Figure 2:

Total number of models versus net chirality.
The fit corresponds to the sum of Gaussians :

$F = Ae^{-ax^2} + Be^{-\frac{ax^2}{4}}$, with $A = 1.64 \times 10^{11}$, $B = 4.39 \times 10^{8}$ and $a = 9.13 \times 10^{-2}$

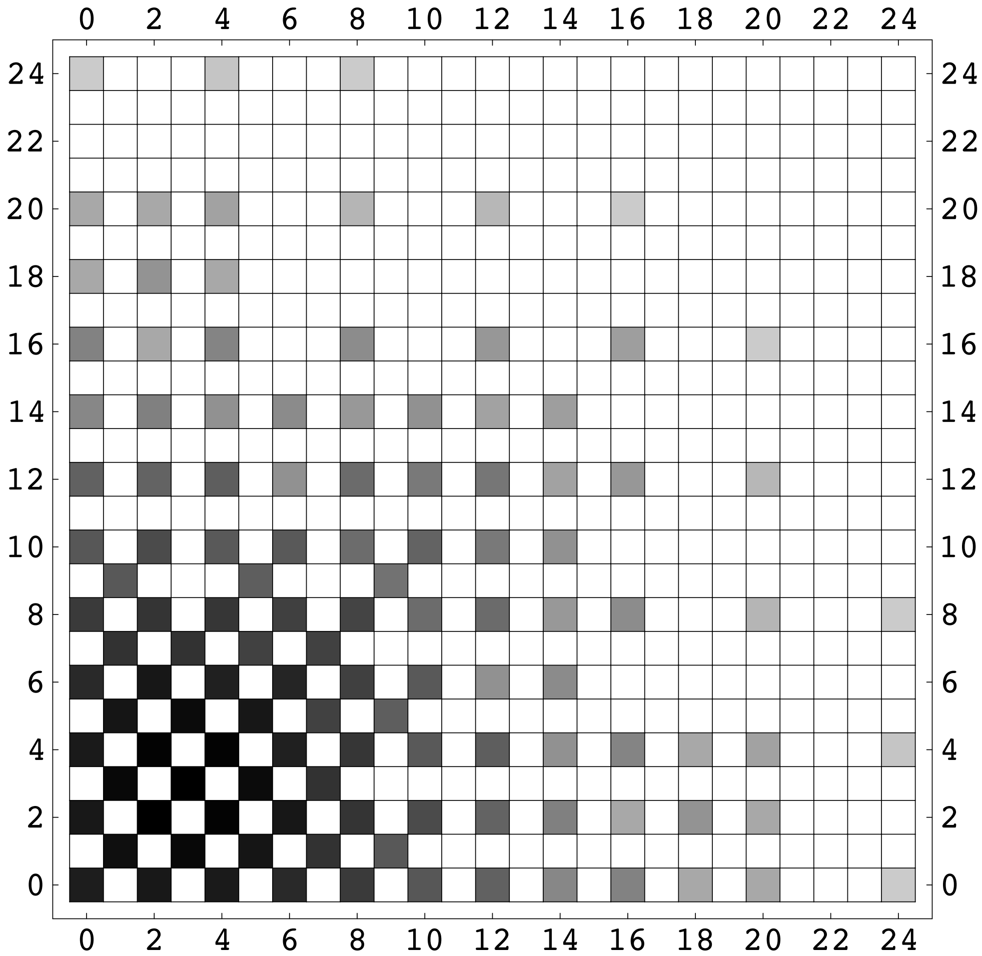

Figure 3:

Density plot of the number of models versus the number of vectors and spinors plus anti–spinors.

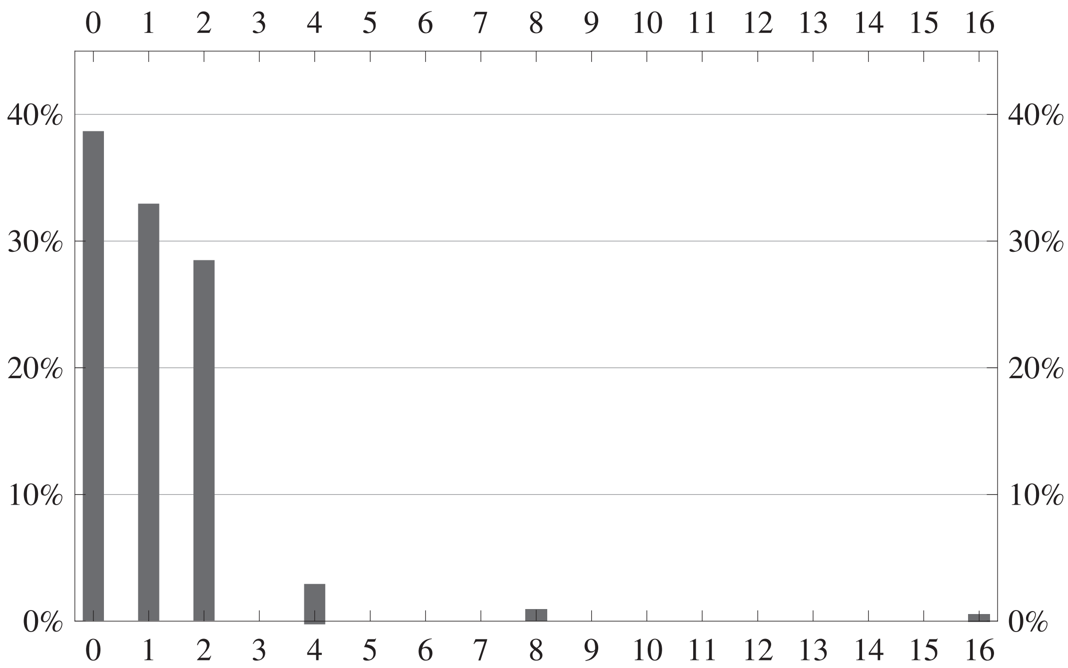

Figure 4:

Percentage of models versus the number of $N = 2$ $SO(12)$ spinorials/vectorials