

Non-Supersymmetric Seiberg Duality in orientifold QCD and Non-Critical Strings

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D.I., V. Niarchos, arXiv:0705.1240

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- In string theory one expects that only string theories with some degree of spacetime supersymmetry are well-defined
 - ➔ Phenomenological string models need spontaneous or explicit breaking of (global) susy, but at high mass levels the theory is approximately supersymmetric
- These two issues are related by gauge/string correspondences
 - ➔ Are there examples for which they can be addressed altogether?

Outline

- 1 Orientifold QCD, duality and planar equivalence
- 2 Seiberg duality at finite N : a conjecture
- 3 Non-critical type 0' strings
- 4 OQCD in non-critical strings and duality

Seiberg duality in SQCD

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(Seiberg 94)

Seiberg duality in SQCD

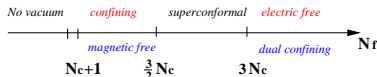
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- Several phases as we vary N_f , with a dual description
- For $\frac{3}{2}N_c < N_f < 3N_c$: *conformal window* \rightarrow interacting IR fixed point for both electric and magnetic Lagrangians



\rightarrow numerous checks thanks to super(conformal) symmetry

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- Planar equivalence provides exact non-perturbative results for (a restricted class of) non-susy gauge theories

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- Similarly, an orientifold "magnetic" theory can be defined for $N_c, N_f \rightarrow \infty$, N_f/N_c fixed
- Predicts a **non-supersymmetric Seiberg duality** in OQCDs in this large N limit

Seiberg duality at finite N : a conjecture

- We propose that (N_c, N_f) OQCD-S/OQCD-AS is dual to a "magnetic theory" with $(N_f - N_c \mp 4, N_f)$ for any N_f and any $N_c > 5$

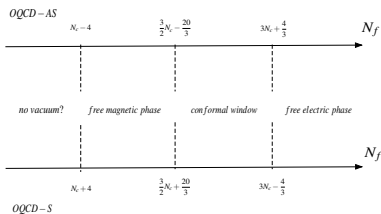
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- 't Hooft anomaly matching is satisfied for any finite N_c, N_f
➔ good consistency check but not powerfull enough
- This duality if true predicts the following phase structure, with a **conformal window** $\frac{3}{2}N_c \pm \frac{20}{3} < N_f < 3N_c \mp \frac{4}{3}$:



➔ more evidence comes from a string theory construction

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- Sagnotti orientifold of type 0B (type 0'B) is tachyon-free but RR tadpole \rightarrow add D9-branes, NSNS tadpole left (Sagnotti 95)

$$\Omega' : |0\rangle_{\text{NS}} \rightarrow -|0\rangle_{\text{NS}} , \quad \psi_r^i \bar{\psi}_r^j |0\rangle_{\text{NS}} \rightarrow \psi_r^j \bar{\psi}_r^i |0\rangle_{\text{NS}}$$

Non-critical type 0'A theories

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$$m^2 = p_\mu p^\mu + p_\phi^2 + \frac{1}{4} + \frac{1}{2}(n_x + w_x)^2 + \dots - \frac{1}{2}$$

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- Crosscap wavefunction can be determined by **modular bootstrap** from the Möbius amplitude:

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- Closed string tachyon is projected out, while no tadpole is generated (RR tadpole is massive) ↪ **perturbatively stable**

OQCD in non-critical strings: framework

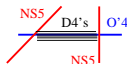
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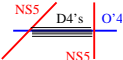
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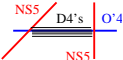
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- RR charge jump of the orientifold across the NS5-branes accounted for by the crosscap state found ➡ extended orientifold (FZZ-like) contains a localized piece (ZZ-like) with opposite RR-charge

Color and flavor branes for QQCD

- D-branes boundary states same as the NSNS part of their type IIA analogues → the latter realize $\mathcal{N} = 1$ SQCD

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- Realize "electric" OQCD-S/OQCD-AS \rightarrow leading order backreaction (holographic β -function) suggests that the gauge theory has a quartic coupling, also for SQCD model:
$$\int d^2\theta Q\tilde{Q}Q\tilde{Q}.$$

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 - Reproduced in the non-critical string context using *brane and crosscap monodromies* under $\mu \rightarrow -\mu$ (Murthy Troost 06)
- ★ In **OQCD**, we found that duality gives a magnetic theory of the same type, with $N_c \rightarrow N_f - N_c \mp 4$
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 ➔ monodromies consistent with backreaction
- Due the orientifold, only $\mu \in \mathbb{R}$ allowed ➔ one goes through strong coupling $\mu = 0$, creating ± 4 extra color branes
 ➔ however no extra stringy dynamics at $\mu = 0$ is expected to affect the gauge theory IR dynamics

Conclusions

- String theories with only bosonic degrees of freedom and full tree-level stability (tachyon- and tadpole- free) are very rare
- On the gauge theory side, few non-susy models inherit properties of susy theories through planar equivalence
- Not surprisingly one can engineer such gauge theories (orientifold QCD) in such string theories (non-critical 0'A)
- The string theory picture strongly supports a Seiberg duality in OQCD at finite N , and predicts an exact conformal window
- A genuine holographic duality pair could be constructed if one knew how to include D-brane and orientifold backreaction properly