D7-Brane Motion from M-Theory & Obstructions

A. Hebecker (Heidelberg)

in collab. with Andreas Braun & Hagen Triendl

Outline:

- Motivation
- M-theory, F-theory, type IIB
- D7-brane motion and M-theory cycles
- Weak coupling limit
- Physics obstructions to D7-brane motion

Introduction / Motivation

- Moduli stabilization / SUSY breaking / fine tuning of Λ is best understood in type IIB with fluxes

- Derivation of Standard Model is best understood in heterotic $E_8 \times E_8$ & D branes on toric orientifolds (fine tuning of Λ appears difficult because of too small number of perturbatively controlled vacua)

- <u>Desirable</u>: Particle phenomenology & fine tuning of Λ in the same setting

- <u>Our goal</u>: Understand D7 brane dynamics more explicitly (in terms of periods of CY's) to allow for D7 brane model building

- Until recently, relatively little attention has been devoted to D7-brane model building on CY's.
- Important papers include:
  - Jockers, Louis, '04 & '05
  - Gomis, Marchesano, Mateos, '05
  - Watari, Yanagida, '04
- Very recently, some F-theory model building efforts have been published:
  - Donagi, Wijnholt, '0802
  - Beasley, Heckman, Vafa, '0802

  (focussed in part on Great Unification)

Type IIB with D7 branes from M theory ("F theory")

M theory on S^1 with small radius

$\downarrow$

type IIA in $d = 10$; compactify on another small S^1 and use T duality

$\downarrow$

type IIB in $d = 9 \times$ (large S^1)

Thus: M theory on T^2 (with volume $\to 0$)

$=$ type IIB in $d = 10$

The only information left over from T^2 is its compl. structure

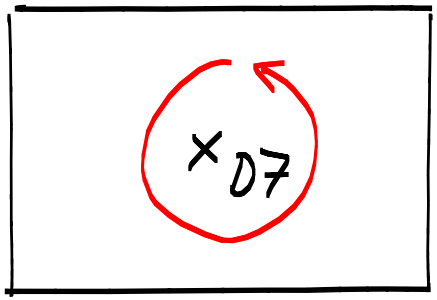
$\Rightarrow$ complex dilaton $\tau(x)$ of type IIB

$$\boxed{\text{type IIB} = \text{compactifications of } 12\,d \text{ theory (F theory) which are torus fibrations}}$$

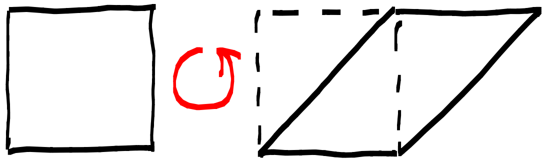
D7 branes in F theory

Recall: $\tau = C_0 + i e^{-\phi}$

D7 branes source C_0 $\Rightarrow$ C_0 goes to $C_0 + 1$ if one "goes around" D7 brane



$\Longrightarrow$



$\tau \to \tau + 1$ for fibre torus

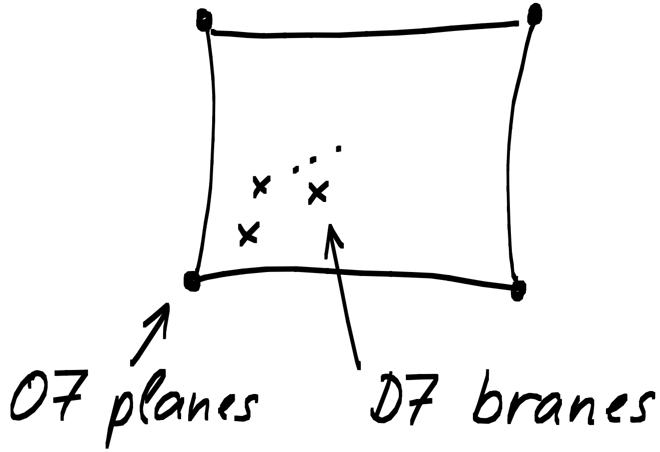
$\Rightarrow$ D7 branes are encoded in non-triviality of torus fibration

Note:
- We assume $\mathrm{Im}\,\tau \to \infty$ almost everywhere
- There are also monodromy points at which $T^2 \to$ "$-T^2$" (O7-planes)

Explicit analysis of simplest example

- F-theory on K3 corresponds to type IIB on $T^2/\mathbb{Z}_2$ with 4 O7 planes & 16 D7 branes:

(cf. Görlich, Kachru, Tripathy, Trivedi; Lüst, Mayr, Reffert, Stieberger)



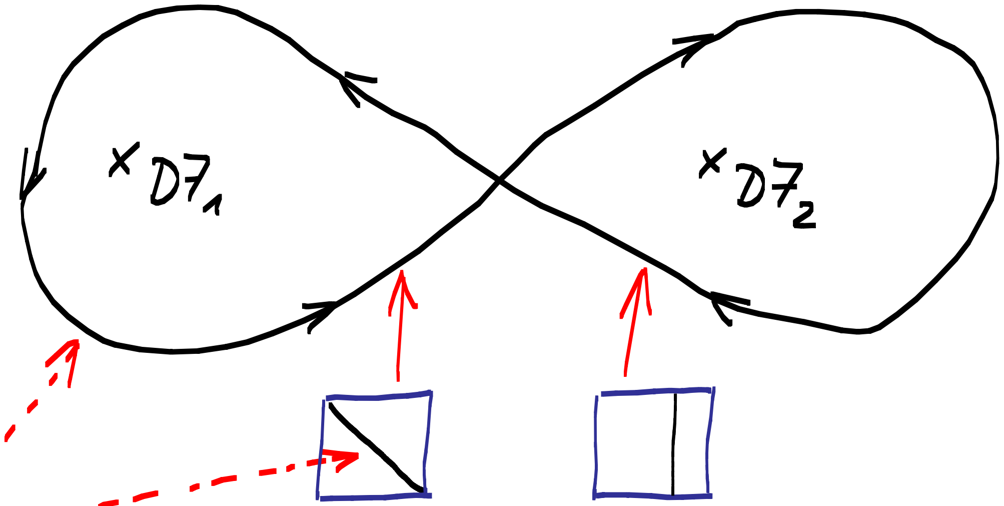
"pillow geometry"

- over every point of this $S^2 = \mathbb{CP}^1$ there is a T^2 with complex structure τ

(= K3 as elliptic fibration with base $\mathbb{CP}^1$)

We want to identify K3-cycles in "moving D-brane picture"

Basic Building Block:

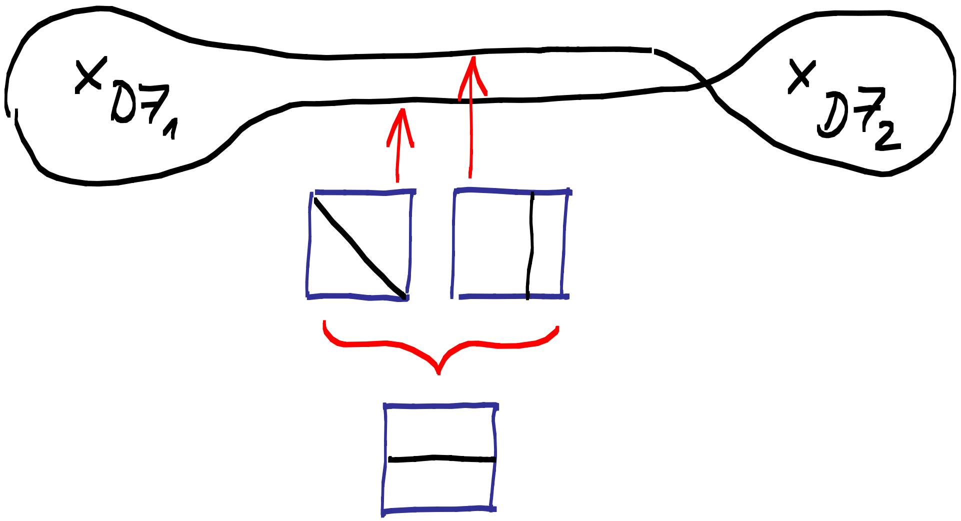


These are the two 1-dim. legs of a non-trivial 2-cycle of K3

fibre torus

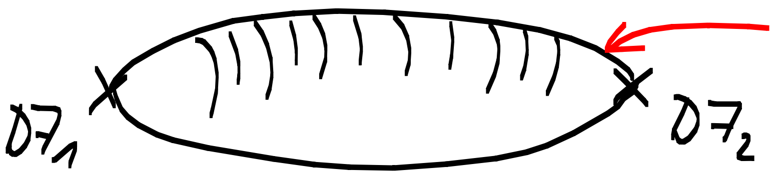
(we have redrawn the torus after the monodromy $\Rightarrow$ change of cycle)

Further deformation of this cycle:

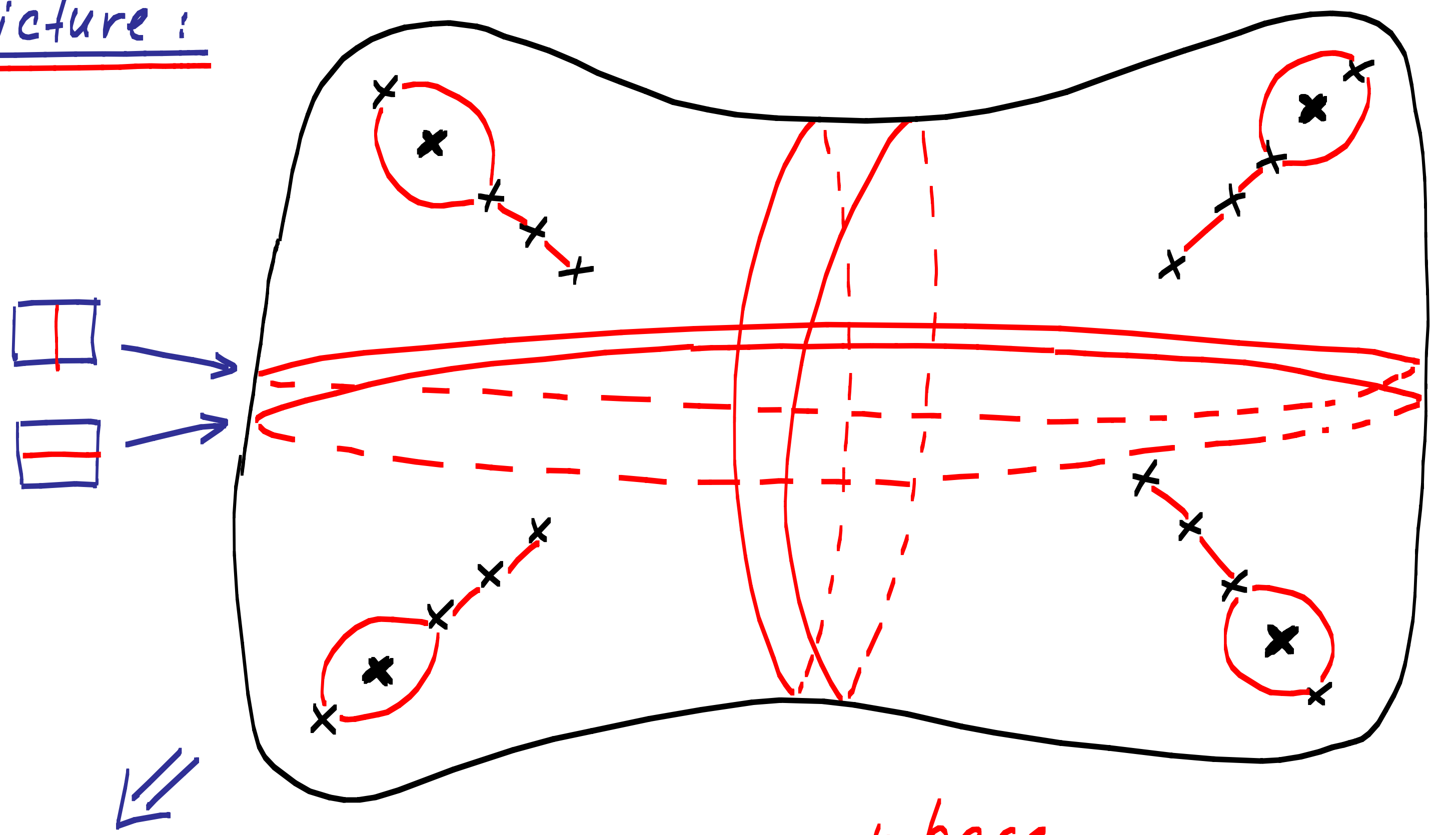


$\Rightarrow$ natural expectation:

The cycle is an S^2 wrapping the T^2 horizontally between the branes. At each brane, the horizontal extension of T^2 shrinks to zero & the cycle "ends":



Final picture:



22 cycles of K3

We have explicitly translated the sizes (periods) of these cycles into Ω of K3 and into D-brane positions on the pillow.

- The above picture only holds in the weak coupling limit

$$\tau \to i\infty .$$

(A. Sen, '97)

(Otherwise the gradient of τ strongly deforms the CY and the O-plane / D-brane picture is lost.)

- To understand this, we need the Weierstrass description of T^2-fibrations:

$$y^2 = x^3 + f \cdot x + g$$

torus in $\mathbb{C}P^2$

functions on base space, specifying shape of torus at every point (actually: sections in bundle L^4 & L^6 with $c_1(L) = c_1(B)$)

The shape of the torus at every point of the base is now given by

$$j(\tau) \sim \frac{f^3}{4f^3 + 27g^2}$$

Modular fct., mapping fund. domain of τ to the complex plain

<u>Weak coupling limit:</u>

$$f = c\eta - 3h^2$$
$$g = h(c\eta - 2h^2) + c^2\chi$$

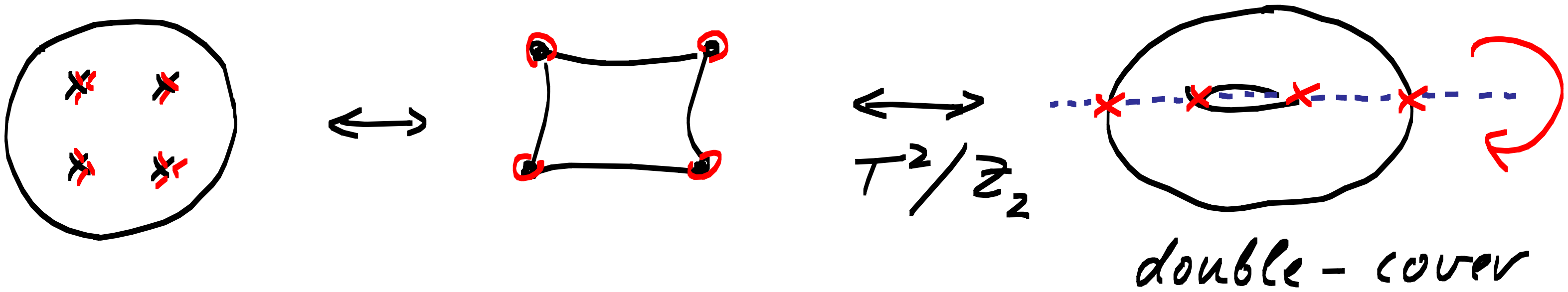
with $c \to 0$

$$\Rightarrow \quad j(\tau) \sim \frac{(c\eta - 3h^2)^3}{c^2 \underbrace{h^2}_{\text{vanishes at O-plane positions}} \underbrace{(\eta^2 + 12h\chi)}_{\text{vanishes at D-brane positions}}} \longrightarrow \infty \text{ (almost everywhere)}$$

Simplest example: base $\mathbb{CP}^1$; L - polynomials of degree 2

h - section of L^2, i.e. polynomial of degree 4

$\Rightarrow$ 4 O-planes at zeroes of h (i.e. at generic positions)



$\underline{\eta^2 + 12 h \chi}$ – η - section of L^4, i.e. polyn. of degree 8

χ - section of L^6, – " – 12

apparently non-generic polynomial of degree 16 ;

zeroes ($\triangleq$ D7-branes) can nevertheless move arbitrarily

Next-to-simplest example: base $\mathbb{CP}^1 \times \mathbb{CP}^1$,
L - polyn.s of degree (2,2)

$\eta^2 + 12 h \chi$ - η - polynomial of degree (8,8)
- χ - — " — (12,12)

with h fixed, one can check that this polynomial has 224 complex degrees of freedom ($\hat{=}$ indep. coefficients)

By contrast:

a generic polynomial of degree (16,16) has 288 degrees of freedom.

The difference: $64 = \frac{1}{2} \times$ (# of intersections between D7-branes & O7-planes)

Comment: We know from Sen's analysis that 224 is correct since we are dealing with the well-studied "Bianchi-Sagnotti-Gimon-Polchinski model".

Following Jockers & Louis (based on earlier work by Lerche et al.), we can also count the degrees of freedom of a freely moving (fully recombined) D7-brane:

[It is given by $h^{(1,0)}_-$ - the 1-cycles of the D7-brane-double-cover in the double-cover-CY, odd under the orientifold projection.]

The result: 288 - agrees with the degrees of freedom of a generic polynomials of degree (16,16)

We conclude: D7-branes can not be considered as freely moving holomorphic submanifolds.

Question: What goes wrong with the weak-coupling ($\hat{=}$ no-backreaction) picture developed previously?

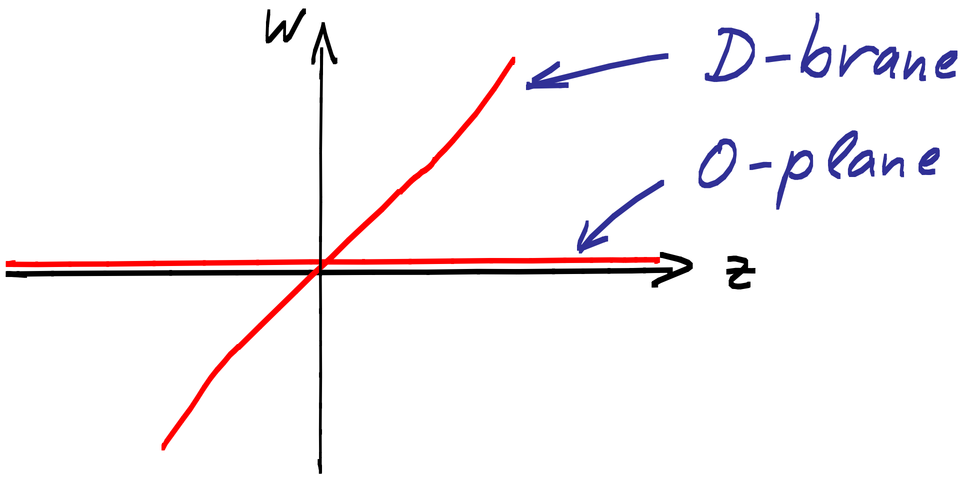
Answer: $\tau \to i\infty$ can only be realized away from the O7-planes

[near the (naked!) O7-planes, $\mathrm{Im}\,\tau$ is small
$\Rightarrow$ strong-coupling effects]

Note: This is also suggested by the "missing degrees of freedom" $= \frac{1}{2} \times$(# of intersections)

Intersection points in more detail:

- Consider $\eta^2 + 12hX = 0$ in 2 complex dimensions z, w (as for $\mathbb{C}P^1 \times \mathbb{C}P^1$).
- Let $h = w$, i.e. the O-plane is at $w = 0$.
- Let the intersection point be at $z = w = 0$:



(We will now show that such a "generic" intersection is impossible.)

- Note: $\eta^2 + 12hX = 0$ & $h = 0$ at $z = w = 0$ implies $\eta = 0$.

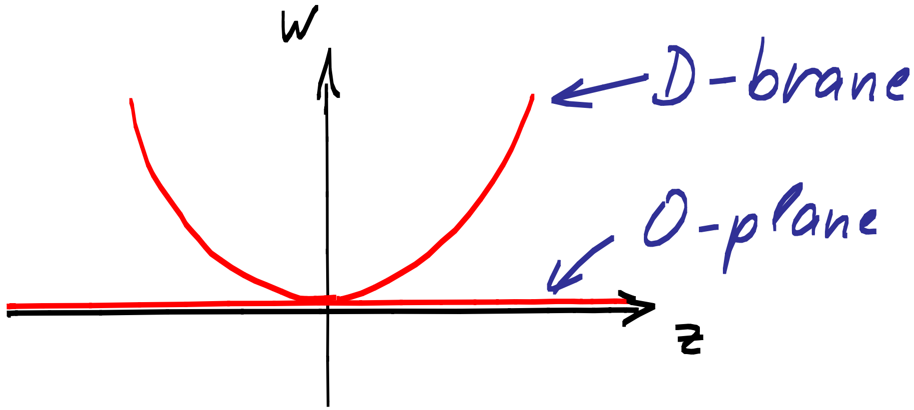
- Thus, expanding η & X at $z = w = 0$ gives:

$$X(z,w) = n_0 + n_1 z + n_2 w + \cdots$$
$$\eta(z,w) = \qquad m_1 z + m_2 w + \cdots$$

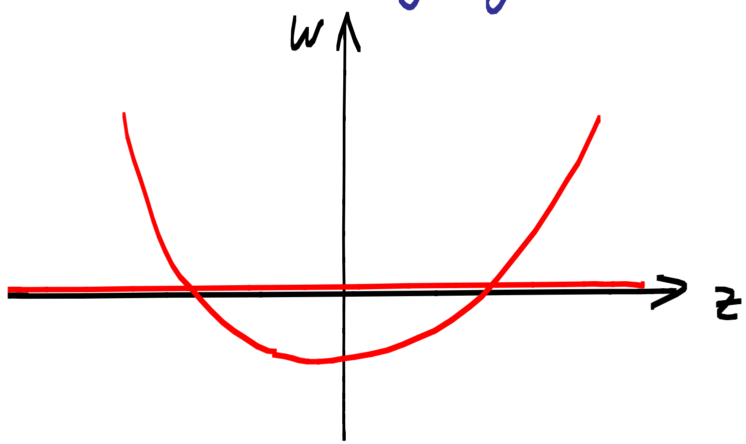
$$\Rightarrow \quad \eta^2 + 12hX = \underline{m_1^2 z^2 + 12 n_0 w + \cdots = 0}$$

(all other terms are subdominant, e.g. w^2 & zw are subdominant w.r.t. w)

- $az^2 + bw = 0$ describes (the complex version of a parabola "touching" the O-plane with its vertex:



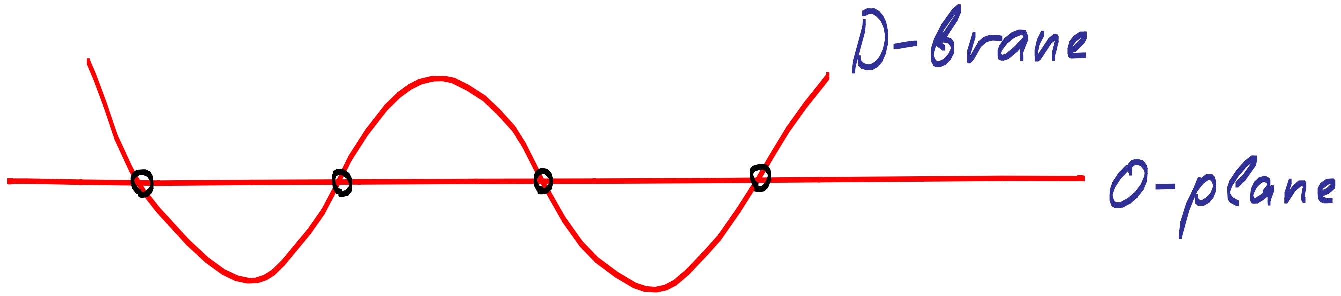
This can be viewed as a special case (realized in type IIB) of the following generic case (not allowed in type IIB):



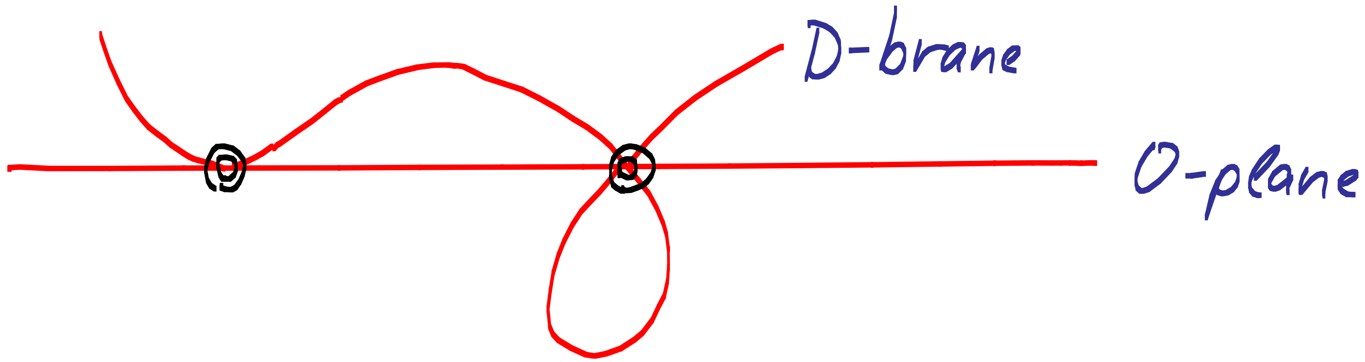
We see: in type IIB, any O-plane/D-brane intersection point is a double intersect. pt.

Counting of degrees of freedom:

generic holomorphic submanifold:



weakly coupled type IIB orientifold



parameterizing the D-brane by its intersections + extra freedom, we see that the # of degrees of freedom is reduced by

$$\boxed{1/2 \times (\#\text{ of intersection points})}$$

Summary (1)

- In a simple example, we have seen how to obtain an explicit description of D7-brane motion through periods of integral M-theory cycles.

To do:

- Fix positions of branes and hence gauge symm. using M-theory fluxes
- Extend analysis to less trivial examples (CY 3- and 4-folds)

(Work in progress with C. Lüdeling, R. Valandro, A. Braun, H. Triendl)

Summary (2)

- The picture of O7-planes & moving D7-branes only makes sense in the weak coupling limit
- In this limit, the D7-brane does not simply move as a generic holomorphic submanifold
- The physics obstructions

  (as opposed to certain mathematical obstructions known in complex geometry)

  responsible for this effect are:

  D7-branes always intersect O7-planes in double-intersection points