String instantons, fluxes and moduli stabilization

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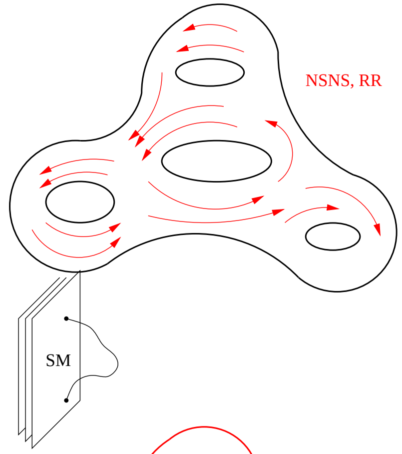
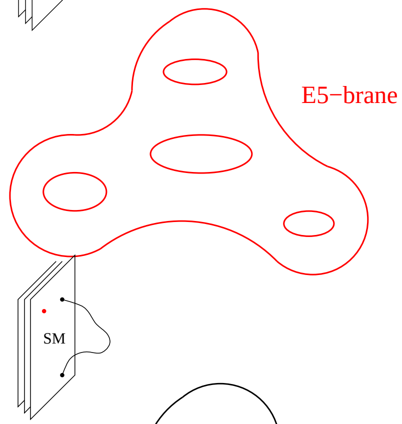
Motivation

- Moduli stabilization:
  - Flux compactifications
    - RR and NSNS fluxes
    - Metric fluxes
    - Non-geometric fluxes



- String instanton effects:
  - QFT instanton effects
    (E5-branes wrapping entire vol.)

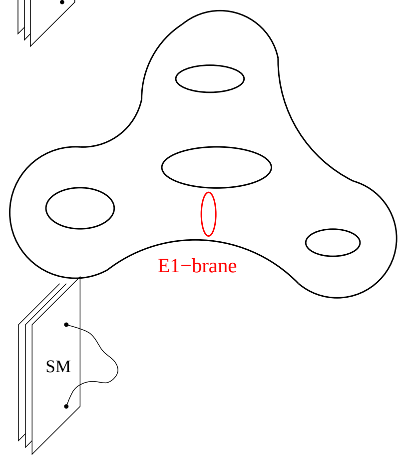
    [ADS superpotential]



  - Stringy instanton effects
    (E1-branes wrapping 2-cycles)

    [mass terms, etc.]

$$S_W \sim e^{\mathcal{D}+\mathcal{A}+\mathcal{M}} \mu^{b_a k_a} \int_{\mathcal{M}} e^{-S_k}$$



Outline

- The freely-acting orbifold
- S-dual pairs
- Threshold corrections
- Instanton effects
- Conclusions

Closely related talks : M. Bianchi, M. Schmidt-Sommerfeld

More on instantonic effects : talks M. Cvetic, T. Weigand...

Freely-acting orbifolds

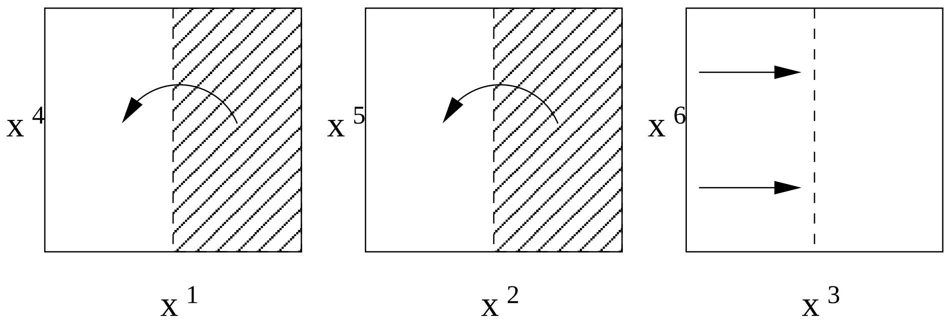
- We are interested in a particular class of twisted tori, given by *flat solvmanifolds*

$$de^a = \frac{1}{2} f^a_{bc} e^b \wedge e^c \,, \quad f^a_{[bc} f^g_{d]a} = 0$$

Example: $de^1 = -e^3 \wedge e^2 \,, \; de^2 = e^3 \wedge e^1 \,, \; de^{3,4,5,6} = 0$

We can integrate the equations as:

$$e^z \equiv e^1 + ie^2 = e^{2\pi i x^3}(dx^1 + idx^2) \,, \quad de^k = dx^k \,, \quad k = 3 \ldots 6$$



$$(x^1, x^2, x^3, x^4, x^5, x^6) \to (-x^1, -x^2, x^3 + \frac{1}{2}, x^4, x^5, x^6)$$

$\mathbb{Z}_2$ freely-acting orbifold

$\mathbb{Z}_2 \times \mathbb{Z}_2$ freely-acting orbifold

- We consider a type I $\mathbb{Z}_2 \times \mathbb{Z}_2$ $\mathcal{N} = 1$ family of models

$$(x^1, x^2, x^3, x^4, x^5, x^6) \xrightarrow{g} (x^1 + 1/2, x^2, -x^3, -x^4, -x^5 + 1/2, -x^6)$$

$$(x^1, x^2, x^3, x^4, x^5, x^6) \xrightarrow{f} (-x^1 + 1/2, -x^2, x^3 + 1/2, x^4, -x^5, -x^6)$$

$$(x^1, x^2, x^3, x^4, x^5, x^6) \xrightarrow{h} (-x^1, -x^2, -x^3 + 1/2, -x^4, x^5 + 1/2, x^6)$$

- *Nice properties:*
    - Flat $\Rightarrow$ **CFT description**
    - No fixed points $\Rightarrow$ twisted sector is massive
    - SUSY spontaneously broken: at large volume $\mathcal{N} = 4$ $\Rightarrow$ adiabatic argument [Vafa,Witten]
    - Chiral adjoints are massive ; there are 3 complex structure, 3 Kähler and 1 axion-dilaton moduli
    - Orbifold action on the CP factors

$$\gamma_g = (I_{n_o}, I_{n_g}, -I_{n_f}, -I_{n_h}) \quad , \quad \gamma_f = (I_{n_o}, -I_{n_g}, I_{n_f}, -I_{n_h}) \, ,$$
$$\gamma_h = (I_{n_o}, -I_{n_g}, -I_{n_f}, I_{n_h}) \, .$$

    - Gauge group is : $SO(n_o) \otimes SO(n_g) \otimes SO(n_h) \otimes SO(n_f)$

- There are chiral multiplets in

$$(\mathbf{n_o}, \mathbf{n_g}, \mathbf{1}, \mathbf{1}) + (\mathbf{n_o}, \mathbf{1}, \mathbf{n_f}, \mathbf{1}) + (\mathbf{n_o}, \mathbf{1}, \mathbf{1}, \mathbf{n_h}) + \\ + (\mathbf{1}, \mathbf{n_g}, \mathbf{n_f}, \mathbf{1}) + (\mathbf{1}, \mathbf{n_g}, \mathbf{1}, \mathbf{n_h}) + (\mathbf{1}, \mathbf{1}, \mathbf{n_f}, \mathbf{n_h})$$

- O9/D9 tadpole conditions : $n_o + n_g + n_h + n_f = 32$.
- There are no O5-planes, due to the free-orbifold action
- Particular case with trivial orbifold action on the CP factors :

  $n_g = n_h = n_f = 0 \Rightarrow$ **pure** $\mathcal{N} = 1$ **SO(32) SYM**
  no matter, asymptotically-free $\longrightarrow$

- We expect nonperturbative (gaugino condensation) effects, producing a potential for the dilaton $W_{np} = ae^{-bS}$.

$\mathbb{Z}_2 \times \mathbb{Z}_2$ freely-acting orbifold

Torus amplitude

$$T = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3 |\eta|^4} \frac{1}{4} \Bigg[|\tau_{oo} + \tau_{og} + \tau_{oh} + \tau_{of}|^2 \Lambda_1 \Lambda_2 \Lambda_3 + |\tau_{oo} + \tau_{og} - \tau_{oh} - \tau_{of}|^2 (-1)^{m_1} \Lambda_1 \left| \frac{4\eta^2}{\vartheta_2^2} \right|^2$$

$$+ |\tau_{oo} - \tau_{og} + \tau_{oh} - \tau_{of}|^2 (-1)^{m_3} \Lambda_3 \left| \frac{4\eta^2}{\vartheta_2^2} \right|^2 + |\tau_{oo} - \tau_{og} - \tau_{oh} + \tau_{of}|^2 (-1)^{m_2} \Lambda_2 \left| \frac{4\eta^2}{\vartheta_2^2} \right|^2 +$$

$$+ |\tau_{go} + \tau_{gg} + \tau_{gh} + \tau_{gf}|^2 \Lambda_1^{n_1 + \frac{1}{2}} \left| \frac{4\eta^2}{\vartheta_4^2} \right|^2 + |\tau_{go} + \tau_{gg} - \tau_{gh} - \tau_{gf}|^2 (-1)^{m_1} \Lambda_1^{n_1 + \frac{1}{2}} \left| \frac{4\eta^2}{\vartheta_3^2} \right|^2 +$$

$$+ |\tau_{ho} + \tau_{hg} + \tau_{hh} + \tau_{hf}|^2 \Lambda_3^{n_3 + \frac{1}{2}} \left| \frac{4\eta^2}{\vartheta_4^2} \right|^2 + |\tau_{ho} - \tau_{hg} + \tau_{hh} - \tau_{hf}|^2 (-1)^{m_3} \Lambda_3^{n_3 + \frac{1}{2}} \left| \frac{4\eta^2}{\vartheta_3^2} \right|^2 +$$

$$+ |\tau_{fo} + \tau_{fg} + \tau_{fh} + \tau_{ff}|^2 \Lambda_2^{n_2 + \frac{1}{2}} \left| \frac{4\eta^2}{\vartheta_4^2} \right|^2 + |\tau_{fo} - \tau_{fg} - \tau_{fh} + \tau_{ff}|^2 (-1)^{m_2} \Lambda_2^{n_2 + \frac{1}{2}} \left| \frac{4\eta^2}{\vartheta_3^2} \right|^2 \Bigg]$$

Klein-bottle amplitude

$$K = \int_0^\infty \frac{dt}{t^3 \eta^2} \frac{1}{8} (\tau_{oo} + \tau_{og} + \tau_{oh} + \tau_{of})$$

$$\{ P_1 P_2 P_3 \; + \; (-1)^{m_1} P_1 W_2 W_3 \; + \; W_1 (-1)^{m_2} P_2 W_3 \; + \; W_1 W_2 (-1)^{m_3} P_3 \}$$

Other models

- **Racetrack:**

  Add a “massive” Wilson line along x^2 to the $SO(p) \times SO(32-p)$ model $\Rightarrow SO(p) \times SO(32-p)$ racetrack model.

$$A_2 = (e^{2\pi i \mathbf{a}}) \quad , \qquad \mathbf{a} = (\mathbf{0}_p, \mathbf{1/2}_{32-p}) \; .$$

  $SO(p)$ and $SO(32-p)$ live in different orientifold fixed points. Massless states are pure SYM multiplets. Massive level contains in particular a $(\mathbf{p}, \mathbf{q})$ vector multiplet.
  Tree-level gauge kinetic functions are $f_{SO(p)} = f_{SO(32-p)} = S$.
  Gaugino condensation then generates

$$W_{np} \;=\; A_p^{(k)} e^{-a_p S} + A_q^{(l)} e^{-a_q S} \; .$$

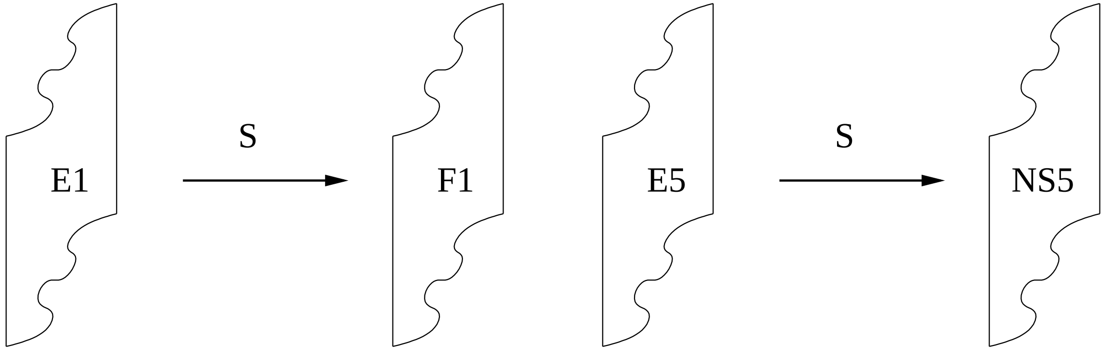
- **Unitary gauge groups :**

  There is an orbifold action on the CP that produces unitary gauge groups

$$I_N = n + \bar{n} + m + \bar{m} \quad , \quad g_N = n + \bar{n} - m - \bar{m} \; ,$$
$$f_N = i(n - \bar{n} + m - \bar{m}) \quad , \quad h_N = i(n - \bar{n} - m + \bar{m}) \; .$$

S-duality

Dictionary: type I $\rightarrow$ SO(32) heterotic



- Adiabatic argument: identification of the orbifold action in the large radius limit (KK modes)
- Modular invariance: fixes the action on the winding modes:

$$X \rightarrow X + \pi R \implies \begin{cases} X_L \rightarrow X_L + \frac{\pi R}{2} + \frac{\pi \alpha'}{2R} \\ X_R \rightarrow X_R + \frac{\pi R}{2} - \frac{\pi \alpha'}{2R} \end{cases}$$

[different Chan-Paton embeddings correspond to different actions in the winding modes]

Use S-duality to compute string instanton effects in type I side

Toroidal vacua : Bachas, Kiritsis et al

Orbifolds : CDMP, Bianchi-Morales, Blumenhagen-Schmidt-Sommerfeld

Heterotic $SO(32)$ model

$$T = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3 \eta^2 \bar{\eta}^2} \frac{1}{4} \Big[(\tau_{oo} + \tau_{og} + \tau_{oh} + \tau_{of}) \Lambda_1 \Lambda_2 \Lambda_3 +$$

$$(\tau_{oo} + \tau_{og} - \tau_{oh} - \tau_{of})(-1)^{m_1+n_1} \Lambda_1 \left| \frac{4\eta^2}{\vartheta_2^2} \right|^2 + (\tau_{oo} - \tau_{og} + \tau_{oh} - \tau_{of})(-1)^{m_3+n_3} \Lambda_3 \left| \frac{4\eta^2}{\vartheta_2^2} \right|^2 +$$

$$(\tau_{oo} - \tau_{og} - \tau_{oh} + \tau_{of})(-1)^{m_2+n_2} \Lambda_2 \left| \frac{4\eta^2}{\vartheta_2^2} \right|^2 + (\tau_{go} + \tau_{gg} + \tau_{gh} + \tau_{gf}) \Lambda_1^{m_1+\frac{1}{2}, n_1+\frac{1}{2}} \left| \frac{4\eta^2}{\vartheta_4^2} \right|^2 +$$

$$(\tau_{ho} + \tau_{hg} + \tau_{hh} + \tau_{hf}) \Lambda_3^{m_3+\frac{1}{2}, n_3+\frac{1}{2}} \left| \frac{4\eta^2}{\vartheta_4^2} \right|^2 + (\tau_{fo} + \tau_{fg} + \tau_{fh} + \tau_{ff}) \Lambda_2^{m_2+\frac{1}{2}, n_2+\frac{1}{2}} \left| \frac{4\eta^2}{\vartheta_4^2} \right|^2$$

$$(\tau_{go} + \tau_{gg} - \tau_{gh} - \tau_{gf})(-1)^{m_1+n_1} \Lambda_1^{m_1+\frac{1}{2}, n_1+\frac{1}{2}} \left| \frac{4\eta^2}{\vartheta_3^2} \right|^2 -$$

$$-(\tau_{ho} - \tau_{hg} + \tau_{hh} - \tau_{hf})(-1)^{m_3+n_3} \Lambda_3^{m_3+\frac{1}{2}, n_3+\frac{1}{2}} \left| \frac{4\eta^2}{\vartheta_3^2} \right|^2 -$$

$$-(\tau_{fo} - \tau_{fg} - \tau_{fh} + \tau_{ff})(-1)^{m_2+n_2} \Lambda_2^{m_2+\frac{1}{2}, n_2+\frac{1}{2}} \left| \frac{4\eta^2}{\vartheta_3^2} \right|^2 \Big] \times (\overline{O}_{32} + \overline{S}_{32})$$

Threshold corrections

Knowledge of threshold corrections to gauge couplings important for :

- check of S-duality
- one-loop heterotic threshold corrections $\longrightarrow$ nonperturbative E1 instanton corrections on the type I side
- Nonperturbative (E5) gauge instantons /gaugino condensation effects on type I are corrected by E1 instantons $\longrightarrow$ modification of moduli potentials and therefore moduli stabilization.

Threshold corrections

The 1-loop heterotic threshold corrections are generically written as

$$\frac{4\pi^2}{g_a^2}\Big|_{\text{1-loop}} = \frac{4\pi^2}{g_a^2}\Big|_{\text{tree}} + \int_{\mathcal{F}} \frac{d^2\tau}{4\tau_2}\,\mathcal{B}_a(\tau)$$

with $\mathcal{B}_a(\tau \to 0) \;\to\; b_a$.

- Type I : we use the background field method:[Bachas, Fabre ; Antoniadis,Bachas, E.D.]

$$F_{23} = BQ \quad\Rightarrow\quad \Lambda(B) = \Lambda_0 + \frac{1}{2}\left(\frac{B}{2\pi}\right)^2 \int_0^\infty \frac{dt}{4t}\,\mathcal{B}_a(t) + \ldots$$

type I: $$\int \frac{dt}{4t}\,\mathcal{B}_{SO(32)}(t) = \frac{15}{2}\sum_{i=1}^{3}\left(\pi \mathrm{Re}\,U_i + \log\left[(\mathrm{Re}\,U_i)(\mathrm{Re}\,T_i)\mu^2\left|\frac{\vartheta_4}{\eta^3}(2iU_i)\right|^{-2}\right]\right)$$

heterotic: $$\int \frac{d^2\tau}{4\tau_2}\,\mathcal{B}_{SO(32)}(\tau) = -\frac{1}{96}\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2}\sum_{i=1}^{3}\Big[(-1)^{m_i+n_i}\hat{Z}_i\bar{\vartheta}_3^2\bar{\vartheta}_4^2 -$$

$$-\hat{Z}_i^{m_i+\frac{1}{2},n_i+\frac{1}{2}}\bar{\vartheta}_2^2\bar{\vartheta}_3^2 + (-1)^{m_i+n_i}\hat{Z}_i^{m_i+\frac{1}{2},n_i+\frac{1}{2}}\bar{\vartheta}_2^2\bar{\vartheta}_4^2\Big]\,\frac{\bar{E}_4(\bar{\hat{E}}_2\bar{E}_4 - \bar{E}_6)}{\bar{\eta}^{24}}$$

Threshold corrections

- It is possible to extract the holomorphic gauge couplings f_a

[Kaplunovsky,Louis]

$$\frac{4\pi^2}{g_a^2(\mu^2)} = \text{Re } f_a + \frac{b_a}{4}\log\frac{M_P^2}{\mu^2} + \frac{c_a}{4}K$$

and the non-perturbative scale of the asymptotically-free theory

$$\Lambda_a = M_P e^{-\frac{2f_a}{|b_a|}}$$

- Gauge kinetic function:

type I: $$f_{SO(32)} = S - 15\sum_{i=1}^{3}\log\frac{\vartheta_4}{e^{\pi U_i/2}\eta^3}(2iU_i)$$

heterotic: $$f_{SO(32)} = S - 15\sum_{i=1}^{3}\left[\log\frac{\vartheta_4}{e^{\pi U_i/2}\eta^3}(2iU_i) - 2\sum_{n=1}^{\infty}(-1)^n\log(1-e^{-2\pi nT_i})\right] + \ldots$$

confirms nonperturbative corrections of [Blumenhagen, Lust and coll.]

$$W_{np} = \prod_{i=1}^{3}\left[e^{-\pi U_i/2}\frac{\vartheta_4}{\eta^3}(2iU_i)\prod_{n=1}^{\infty}\left(\frac{1-e^{-4\pi(n+\frac{1}{2})T_i}}{1-e^{-4\pi nT_i}}\right)^2\right]\times\ldots\times e^{-S/15}$$

Euclidean brane instantons

- E5-brane instantons (QFT instantons):

  - Neutral sector:

$$A^{(0)}_{E5-E5} + M^{(0)}_{E5-O9} = \frac{p(p-1)}{2}\,(\overbrace{V_4O_2O_2O_2}^{a_\mu} - \overbrace{C_4C_2C_2C_2}^{M^{\alpha,---}}) \;-\; \frac{p(p+1)}{2}\,\overbrace{S_4S_2S_2S_2}^{\lambda^{\dot\alpha,---}}$$

  - Charged sector:

$$A^{(0)}_{E5-D9} = Np(\overbrace{S_4O_2O_2O_2}^{\omega_\alpha} - \overbrace{O_4C_2C_2C_2}^{\mu^A})$$

Euclidean brane instantons

- E5-brane instantons (QFT instantons):

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  - Charged sector:

$$A^{(0)}_{E5-D9} = Np(\overbrace{S_4O_2O_2O_2}^{\omega_\alpha} - \overbrace{O_4C_2C_2C_2}^{\mu^A})$$

- $E1_i$-brane instantons (stringy instantons):

- Doublets: $\frac{1}{\sqrt{2}}\,[\,|0,0,\pi R_5/2,0\rangle + |\pi R_3,0,3\pi R_5/2,0\rangle\,]$

  - Neutral sector:

$$A^{(0)}_{E1-E1} + M^{(0)}_{E1-O9} = \frac{Q(Q+1)}{2}\,(\overbrace{V_4O_2O_4}^{x^\mu} - \overbrace{C_4C_2S_4}^{\Theta^{\dot{\alpha},-,a}}) + \frac{Q(Q-1)}{2}\,(\overbrace{O_4V_2O_4}^{y^i} - \overbrace{S_4S_2S_4}^{\Theta^{\alpha,+,a}})$$

  - Charged sector:

$$A^{(0)}_{E1-D9} = NQ(-\overbrace{O_4S_2O_4}^{\Psi^+})$$

- Quartets: $\frac{1}{2}\,[\,|0,0,0,0\rangle + |0,0,\pi R_5,0\rangle + |\pi R_3,0,0,0\rangle + |\pi R_3,0,\pi R_5,0\rangle\,]$

A potential puzzle

Consider for ex. $SO(p) \times SO(32-p)$ dual typeI-heterotic pairs.

- On type I side, instanton spectra have the same pattern for all even p.

- On the heterotic side, the freely-acting orbifold operations are constrained by modular invariance. In the twisted sectors, the masses of the lattice states (m,n) are shifted as

$$(m,n) \rightarrow (m+s_1, n+s_2) \, , \quad \text{where}$$

- $p = 8 \, (mod \, 8) \rightarrow s_1 = s_2 = 1/2$
- $p = 4 \, (mod \, 8) \rightarrow s_1 = 1/2, s_2 = 0$

  - Despite apparences, heterotic perturbative threshold corrections differ only by some phases for the two cases.
  - They should be interpreted as E1 corrections on type I side. Interpretation of the different phases ? (Pablo Camara, E.D ;work in progress)

Conclusions

- We considered a class of *compact* twisted tori models on which to build type I/heterotic S-dual pairs.
- These are a perfect laboratory on which to make explicit computations of non-perturbative string effects.
- E.g. $E1$-instanton corrections to gauge couplings and the ADS superpotential.
- Interesting subtleties from the S-duality on instantonic corrections.
- Interesting possibilities for moduli stabilization, if more fluxes.
- Important to construct semi-realistic models and work out various effects of euclidian D-brane effects for generating mass hierarchies,supersymmetry breaking, moduli stabilization, etc