Implications of D-instantons:

Hierarchical Non-Perturbative Superpotential Couplings

Ralph Blumenhagen,M.C.,Timo Weigand,hep-th/0609191

original paper

M.C.,Robert Richter,T. Weigand,hep-th/0703028 CFT calculation

R.Blumenhagen,M.C.,D.Lüst,R.Richter,T.Weigand,0707.1871

10 10 5 GUT coupling

R.Blumenhagen,M.C.,R.Richter,T.Weigand,0708.0403

multi-instantons & recombination, `T.Weigand's talk`

M.C.,T.Weigand,0711.0209 first global semi-realistic model

M.C.,P.Langacker, 0803.253 Dirac neutrino masses

M.C.,R.Richter,T.Weigand, 0803.138

multi (non-)BPS instantons, `T.Weigand's talk`

Motivation

• Large classes of perturbatively constructed four-dimensional N=1 supersymmetric vacua
$\longrightarrow$ Major activity w/ Intersecting D6-brane (Type IIA) constructions: semi-realistic, supersymmetric three-family Standard Models and SU(5) GUT models

c.f. Bailin's, Gmeiner's, Honecker's, Tamirgaziu's, Antoniadis's (T-dual)... talks

• Question: How non-perturbative effects modify features of perturbative string vacua ?
$\longrightarrow$ Focus on Type IIA Euclidean D-instanton effects: Euclidean D2 [E2]–instantons; W-superpotential corrections

[In the last part: Type I (IIB) global constructions (magnetized D9 branes) with E1 instantons]

D-brane instantons may generate perturbatively absent couplings with new hierarchical scales, suppressed by $\exp(-{g_s}^{-1})$

$\longrightarrow$Phenomenological implications:

Hierarchical couplings in the supersymmetric Standard Model

- Small Neutrino masses:

(i) seesaw: Majorana masses 10^{11} GeV $< M_M < 10^{15}$ GeV ;
(ii) small Dirac masses 10^{-3} eV

- Hierarchically small μ-terms of order $\mathcal{O}$(TeV)
- Highly suppressed R -parity violating couplings
- Top $\mathcal{O}$ (100 GeV) – bottom $\mathcal{O}$ (10 GeV) mass hierarchy
- Hierarchically suppressed supersymmetry breaking

Extensive past literature on instantons in string theory ...

New string instanton effects in (open string) charged sector:
[Blumenhagen,M.C.,Weigand, hep-th/0609191]
[Ibañez,Uranga hep-th/0609213]

- charged matter coupling corrections

[Florea,Kachru,McGreevy,Saulina 0610003]

- supersymmetry breaking

Further developments: many papers...

Bianchi's, Dudas's , Plauschinn's,
Schmidt-Sommerfeld's, Schellekens's, Verlinde's,
Weigand's,... talks

Outline

1. Motivation - done!
2. Focus on Type IIA: Euclidean D2-brane –focus on superpotential corrections due to O(1) D-instantons

- Zero mode structure
- CFT instanton calculus-just results

3. Specific applications:

- Small neutrino masses

(i) Majorana masses - local construction & explicit CFT calc.
(ii) Dirac neutrino masses due to D-instantons

- Other couplings: $10 10 5_H$ of SU(5) GUT, μ of MSSM, Polonyi-type supersymmetry breaking

$\longrightarrow$ all local set-ups

4. Further developements: First semi-realistic global examples

Model Building with Intersecting D6-branes

Engineering of semi-realistic constructions – Standard Model:

- Non-Abelian gauge symmetry
  $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Chiral matter quarks & leptons
- Family replication 3-copies of fermion families

$\longrightarrow$

Non-abelian symmetry, chiral matter & family replication GEOMETRIC!

- c.f. again, Bailin's, Honecker's, Gmeiner's, Tamirgaziu's, Antoniadis's (T-dual) talks

Status

• $\mathcal{O}(100)$ toroidal orbifold models (geometric phase) with semi-realistic features
-typically suffer from chiral exotics
realistic Yukawa couplings?
-moduli stabilisation ?

• Rational Conformal Field Theory constructions-promising:
`Schellekens's talk`
-models without chiral exotics
-couplings in principle calculated, but hard & hierarchy?
-non-geometric phase–moduli stabilisation?

$\longrightarrow$ "The devil is in the details"

though more promising models are being constructed...

Specific coupling issues:

• Neutrino masses (if there) -Dirac & of order of charged sector masses

Majorana neutrino masses – absent

• μ-parameter – typically absent

• SU(5) GUT models – absent $10\,10\,5_H$-couplings

[no top (bottom) mass for GUT SU(5) (flipped SU(5))]

• Hierarchical supersymmetry breaking, e.g., à la Polonyi

Perturbative absence of all such couplings due to violation of "anomalous" U(1)

$\longrightarrow$ Turn to non-perturbative effects & non-perturbative violation of anomalous U(1)

Instantons–Heuristics

Probe for non-pert. terms by computing suitable amplitudes in D-instanton background.

Euclidean Dp-brane wrapping internal $(p+1)$-cycle
$\longrightarrow$ for Type IIA relevant objects are Euclidean $D2$-branes (E2-branes), wrapping three-cycles

Rules:

• Instanton sector corresponds to local minimum of (full) string action
$\longrightarrow$ $E2$-brane volume minimizing on internal sLag Ξ
• Integrate over zero modes localized on $E2$
$\longrightarrow$ All fermionic zero modes have to appear for relevant instanton induced couplings exactly once

Massive $U(1)$ and Axion gauging c.f. also Anastasopoulos's talk

Gauge group $\prod_a U(N_a) = \prod_a SU(N_a) \times U(1)_a$

In general $U(1)_a$ becomes massive via Chern-Simons terms:

$$S_{WS} = \sum_a N_a \, \mu_q \int_{\mathbb{R}^{1,3} \times \Pi^a_{q-3}} e^{trF_a} \sum_p C_p$$

CS-coupling induces gauging of global axionic shift symmetry for axion from RR-form C_{p+1} reduced on (p+1)-cycle $\widetilde{\Pi}_{p+1}$:

$$\begin{aligned} A_a &\longrightarrow A_a + d\Lambda_a \\ \int_{\widetilde{\Pi}_{p+1}} C^{(p+1)} &\longrightarrow \int_{\widetilde{\Pi}_{p+1}} C^{(p+1)} + Q_a(\widetilde{\Pi})\Lambda_a \end{aligned}$$

$Q_a(\widetilde{\Pi}_{p+1}) \simeq N_a \int_{\mathcal{M}_6} \delta(\widetilde{\Pi}_{p+1}) \wedge \delta(\Pi^a_{q-3}) \wedge e^{F_a} \;\; (p=2, q=6)$

Effects of axion gauging for D-instantons:
Euclidean D_p-brane on internal $p+1$-cycle Ξ (p=2 E2-instanton)

$$W_{np} \propto e^{-S_{Ep}} = \exp\left[\frac{2\pi}{\ell_s^{p+1}}\left(-\frac{1}{g_s}\int_\Xi \mathrm{Vol}_\Xi + i\int_\Xi C_{p+1}\right)\right]$$

exponential not gauge invariant under $U(1)_a$!
$e^{-S_{Ep}} \longrightarrow e^{i\,Q_a(Ep)\,\Lambda_a}\; e^{-S_{Ep}}$: $Q_a = N_a \int \delta(\Xi)\wedge\delta(\Pi^a_{q-3})\wedge e^{F_a}$

Consequence:
If $Q_a(Ep) \neq 0$ for some a, no terms $W = e^{-S_{Ep}}$ possible, but:

$$W = \prod_i \Phi_i\; e^{-S_{Ep}} \quad \text{with} \quad \sum_i Q(\Phi_i) + Q_a(Ep) = 0 \;\; \forall a$$

Non-perturbative breakdown of global $U(1)$ symmetry possible.

Zero mode structure - Summary

Distinguish 2 types of zero modes:
(i) uncharged (E2-E2 sector)
(ii) charged (E2-D6 sector) under $U(1)_a$:

(i) Zero modes ncharged under $U(1)_a$ for O(1)-instantons:

4 bosonic modes $x^i_E \leftrightarrow$ Poincaré inv. in 4D &
only 2 fermionic modes: θ_α

Ensured for: $E2 = E2'$ (homologically) , $E2$-wrap rigid
3-cycles Ξ, i.e. absence of additional zero modes.

$\rightarrow$ yields correct superpot. measure: $\int d^4 x_E \, d^2\theta$

Other (multi-)instanton superpotential contributions
`c.f. Weigand's talk`

(ii) Charged Zero modes (strings between $E2$ and $D6_a$):

$\longrightarrow$ Localized at the each intersection of $E2$ and $D6_a$ branes:

One fermionic zero mode λ_a per inters. Geometric!

zero modes	Reps.	number
$\lambda_{a,I}$	$(-1_E, \Box_a)$	$I = 1, \ldots, [\Xi \cap \Pi_a]^+$
$\overline{\lambda}_{a,I}$	$(1_E, \overline{\Box}_a)$	$I = 1, \ldots, [\Xi \cap \Pi_a]^-$
$\lambda_{a',I}$	$(-1_E, \overline{\Box}_a)$	$I = 1, \ldots, [\Xi \cap \Pi'_a]^+$
$\overline{\lambda}_{a',I}$	$(1_E, \Box_a)$	$I = 1, \ldots, [\Xi \cap \Pi'_a]^-$

total $U(1)_a$ charge of all zero modes:

$Q_a(E2) = N_a \, \Xi \circ (\Pi_a - \Pi'_a)$

in agreement with $e^{-S_{E2}} \longrightarrow e^{i\, Q_a(E2)\, \Lambda_a} \; e^{-S_{E2}}$

Contributions to Matter Couplings

In presence of disk-level couplings to matter fields of type $S = \int_{\Xi} \lambda_a \, \Phi_{ab} \overline{\lambda}_b$ in instanton effective action

$$\int d^4x d^2\theta d\lambda_a d\overline{\lambda}_b e^{-S_{cl} + \int_{\Xi} \lambda_a \, \Phi_{ab} \overline{\lambda}_b} \rightsquigarrow \phi_{ab} e^{-S_{cl.}}$$

Details of calculus, including prescription for loop contribution in [Blumenhagen,M.C.,Weigand hep-th/0609191]

Instanton calculus - Summary

[Blumenhagen,M.C.,Weigand hep-th/0609191]

$$\langle \Phi_{a_1,b_1}(p_1)\cdot \ldots \cdot \Phi_{a_M,b_M}(p_M)\rangle_{Ep-\text{inst}} =$$

$$= \int d^4\tilde{x}\, d^2\tilde{\theta} \sum_{\text{conf.}} \prod_a \big(\prod_i d\widetilde{\lambda}_a^i\big) \big(\prod_i d\widetilde{\overline{\lambda}}_a^i\big)$$

$$\exp(-S_{Ep}) \times \exp\left(Z_0'\right)$$

$$\times \langle \widehat{\Phi}_{a_1,b_1}[\vec{x}_1]\rangle^{\text{tree}}_{\lambda_{a_1},\overline{\lambda}_{b_1}} \cdot \ldots \cdot \langle \widehat{\Phi}_{a_L,b_L}[\vec{x}_L]\rangle^{\text{tree}}_{\lambda_{a_L},\overline{\lambda}_{b_L}} \times$$

$$\prod_k \langle \widehat{\Phi}_{c_k,c_k}[\vec{x}_k]\rangle^{\text{loop}}_{A(Ep,Dq_{c_k})}$$

Phenomenological Implications:

Superpotential Couplings

One can generate perturbatively forbidden matter couplings:

I. Majorana masses for right-handed neutrinos
$\longrightarrow$ SM constructions with neutrino Dirac masses
$H^+ \, L_L \, (N_R)^c$. if present, of order of charged sector masses

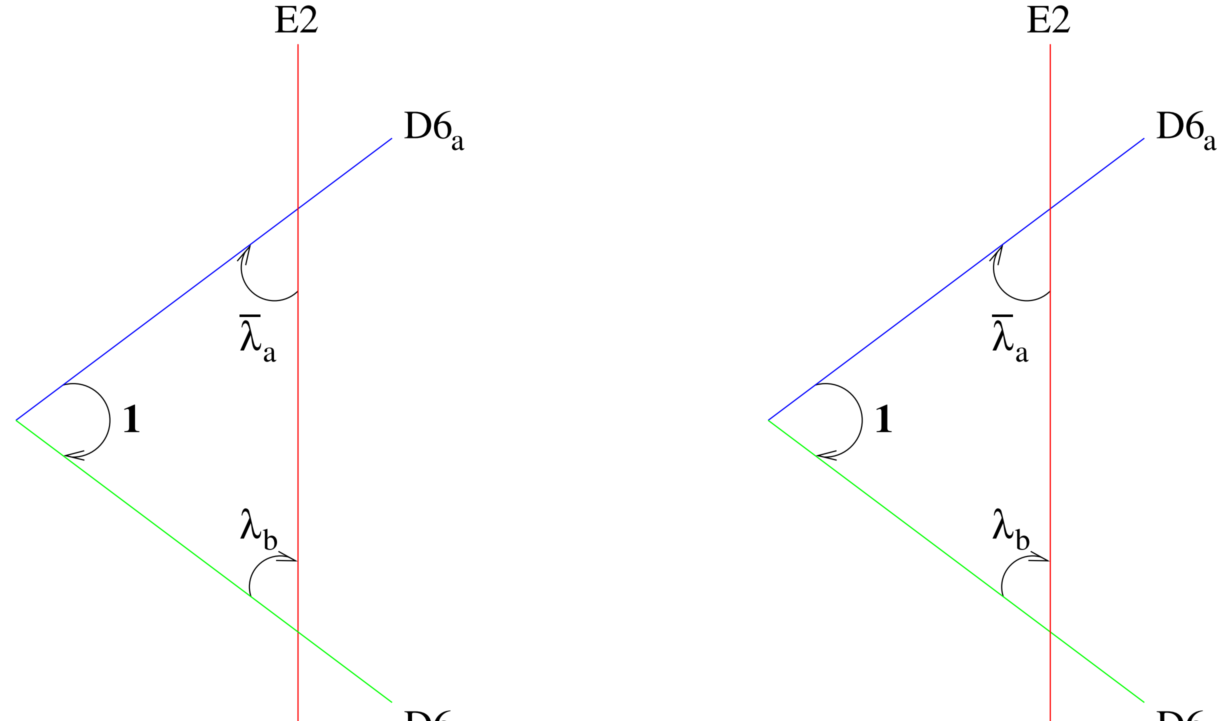
Majorana masses $M_m \, (N_R)^c \, (N_R)^c$ perturbatively forbidden

N_R^c – SM singlets, typically charged under anomalous $U(1)_a \times U(1)_b$, say as $(1, -1)$.

Non-pert. coupling possible if CY_3 possesses rigid 3-cycle Ξ with zero mode structure:

$$\Xi \cap \Pi_{SM} = \Xi \cap \Pi_a = \Xi \cap \Pi'_b = 0$$
$$[\Xi \cap \Pi'_a]^- = 2, \ [\Xi \cap \Pi'_a]^+ = 0, \ [\Xi \cap \Pi_b]^+ = 2, \ [\Xi \cap \Pi_b]^- = 0.$$

Four – $\bar{\lambda}_a^{1,2}$ and $\lambda_b^{1,2}$ absorbed via the two disk diagrams:



$\longrightarrow$ Non-pert. Majorana coupling:

$W_m = M_m\,(N_R)^c\,(N_R)^c$ with $M_m = x\,M_s\,e^{-\frac{2\pi}{\ell_s^3\,g_s}\mathrm{Vol}_{E2}}$

Use $\frac{1}{\alpha_{\mathrm{GUT}}} = \frac{1}{\ell_s^3\,g_s}\mathrm{Vol}_{D6} \longrightarrow M_m = x\,M_s\,e^{-\frac{2\pi}{\alpha_{\mathrm{GUT}}}\frac{\mathrm{Vol}_{E2}}{\mathrm{Vol}_{D6}}}$

For seesaw mechanism need $10^{11}\mathrm{GeV} < M_m < 10^{15}\mathrm{GeV}$

Possible within natural regime for

$0.4 \cdot R_{D6} > R_{E2} > 0.2 \cdot R_{D6}$ (w/ x=$\mathcal{O}$ (1)).

$\longrightarrow$ Concrete local realization on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$

[M. C., Robert Richter, Timo Weigand, hep-th/0703028]

Local 4-family supersymmetric 3-stack GUT model:
$U(5)_c \times U(1)_a \times U(1)_b$ w/ O(1) E2-instanton

Result: $< \nu^A \nu^B >_{E2_i} = \frac{2\pi}{g_s} \mathcal{V}_{E2} \overrightarrow{v}^T \mathcal{M} \overrightarrow{v} \; (2\pi)^4 \delta^4(k^A + k^B)$

$$\mathcal{M} = x\, M_s\, e^{-\frac{2\pi}{\alpha_{\rm GUT}} \frac{8}{57}} \begin{pmatrix} A_i & 0 & B_i & 0 \\ 0 & C_i & 0 & D_i \\ B_i & 0 & E_i & 0 \\ 0 & D_i & 0 & F_i \end{pmatrix} ,$$

$x =$

$$\frac{(4\pi)^{3/2}\pi^2}{16} \left[\Gamma_{1-\theta^1_{ab},1-\theta^1_{E2a},1+\theta^1_{E2b}} \prod_{I=2}^3 \Gamma_{-\theta^I_{ab},-\theta^I_{E2a},1+\theta^I_{E2b}}\right]^{\frac{1}{2}} \; e^{Z'}$$

$e^{Z'}$–one-loop contribution [Akerblom et al. arXiv:0705.236[he-th]]

Overall exponential suppression scale fixed by SUSY:

$$\frac{\mathrm{Vol}_{E2}}{\mathrm{Vol}_{\Pi_c}} = \Big(\prod_I \frac{(n^I_{E2})^2 + (\widetilde{m}^I_{E2})^2 U_I^2}{(n^I_c)^2 + (\widetilde{m}^I_c)^2 U_I^2}\Big)^{1/2} = \frac{8}{57}$$

• Sum up contributions from all 64 factorizable E2-instantons, taking leading contribution (smallest triangle) $\longrightarrow M_M \simeq 10^{10} GeV$
for triangles of order string scale (as required)

• Together with perturbative Dirac masses of $\mathcal{O}$(TeV) (due to Yukawa couplings of the type $\overline{\mathbf{5}}\,\mathbf{5}_H\,\mathbf{1}$)

$\longrightarrow$ Can engineer small neutrino masses of $\mathcal{O}$(1eV) via see-saw mechanism.

Further applications:

- μ -term $\mu H^+ H^-$ forbidden perturbatively

$\longrightarrow$ can well be generated by $E2$-instantons

$\longrightarrow$ appropriate volume ratio may yield $\mu \simeq \mathcal{O}(\text{TeV})$

- R-parity violating couplings in the MSSM can be induced

- GUT $SU(5)$ suffer from absence of pert. Yukawa couplings

$\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H$, where $\mathbf{10}$ from (a,a')-intersection

Can be generated by $E2$-instantons

[R.Blumenhagen,M.C.,D. Lüst,R.Richter,T.Weigand,0707.1871]

$\longrightarrow$ Opens the door for (flipped) SU(5) phenomenology

Another Implication: Small Dirac Neutrino Masses

M.C.,P.Langacker, 0803.253

Starting point: due to U(1) charges of 5, $\bar{5}$, N_R, within $U(5)_a \times U(1)_b \times U(1)_c$ GUT, perturbative Dirac neutrino Yukawa couplings $\bar{5}\ 5_H\ N_R$ absent

Specifically, non-zero intersection nos:

$$[\Pi_a \cap \Pi_b']^- \neq 0, \quad [\Pi_b \cap \Pi_c']^- \neq 0, \quad [\Pi_a \cap \Pi_c']^+ \neq 0$$

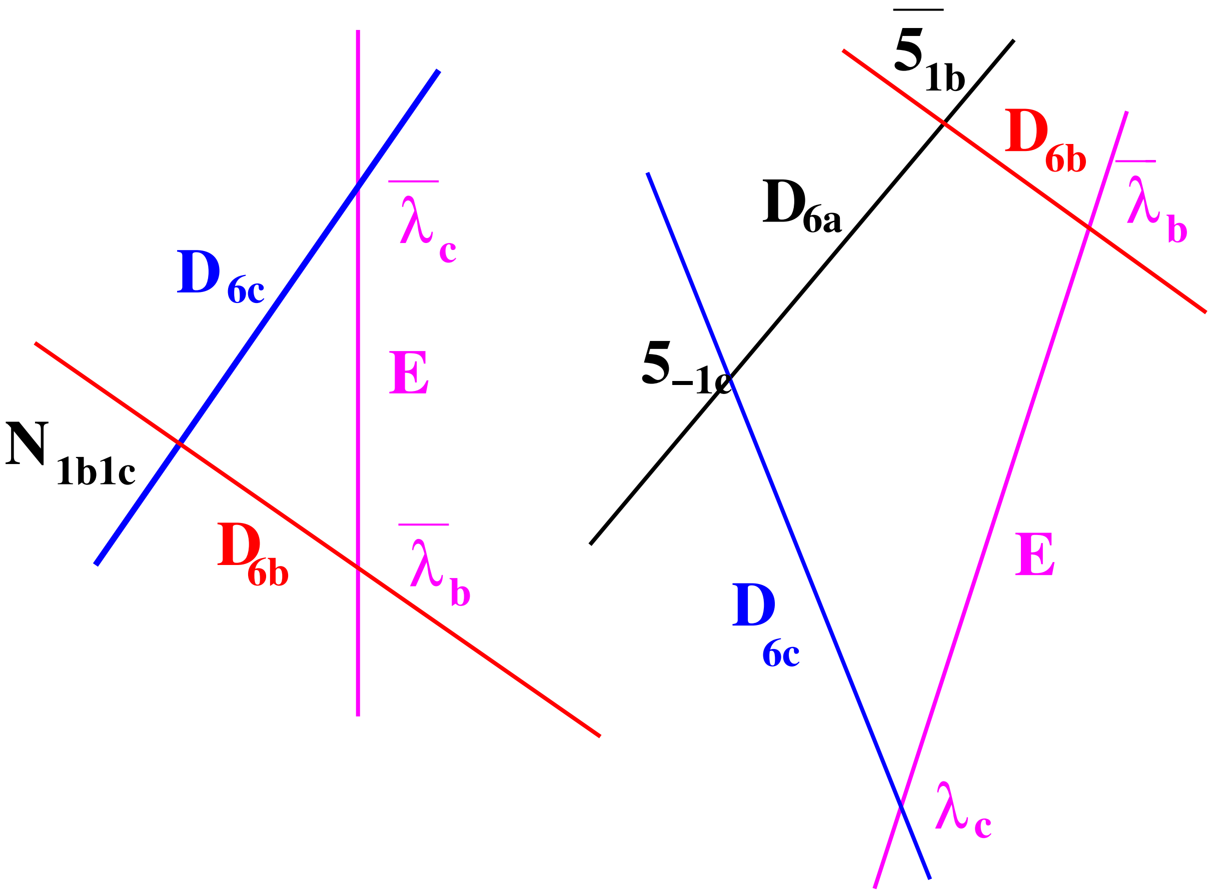
produce spectrum: $5_{(1_a,1_b,0)}$, $\bar{5}_{(-1_a,0,-1_c)}$, $N_{R\ (0,1_b,1_c)}$ - Dirac coupling absent

Instanton intersection nos.:

$$[\Xi \cap \Pi_a] = 0, \quad [\Xi \cap \Pi_b]^- = 2 \quad [\Xi \cap \Pi_c]^{-\ N=2} = 1$$

i.e. 2 $\overline{\lambda}_b$ with $(-1_b, 1_E)$ & one vector pair: $\lambda_c + \overline{\lambda}_c$ with $(-1_c, 1_E) + (1_c, -1_E)$

Fermionic zero modes contribute to disk diagrams:



Generating non-perturbative Dirac neutrino couplings.

Local construction [(ν_2, n_2)integ.;c.s.moduli$\chi_1 = \chi_2 = 1/\sqrt{5},\ \chi_3 = n_2\sqrt{5}/2$]:

nos	(n_a^1, m_a^1)	(n_a^2, m_a^2)	(n_a^3, m_a^3)
5_a	$(\nu_2, 2\nu_2/n_2)$	$(1,1)$	$(0,-1)$
1_b	$(n_2, 2)$	$(-1,2)$	$(-1,1)$
1_c	$(-1,0)$	$(1,1)$	$(-1,1)$
1_E	$(1,0)$	$(0,1)$	$(0,-1)$

Spectrum [(ν_2, n_2)=(1,1), (3,3)-4-family]:

sector	number	$U(5)_a \times U(1)_b \times U(1)_c$
(a, a')	$4(\nu_2 - 2\frac{\nu_2}{n_2})$	$\mathbf{10}_{(2,0,0)} + \overline{\mathbf{15}}_{(-2,0,0)}$
(a, b')	$16\nu_2$	$\mathbf{5}_{(1,1,0)}$
(a, c')	$16\frac{\nu_2}{n_2}$	$\overline{\mathbf{5}}_{(-1,0,-1)}$
(b, c')	16	$\mathbf{1}_{(0,1,1)}$

Non-pertrubative Dirac neutrino Yukawa coupling:

$$Y = \exp(-S_{inst}) = x \exp(-\frac{2\pi}{\alpha_{GUT}} \frac{Vol_{E2}}{Vol_{D6a}})$$

$\frac{Vol_{E2}}{Vol_{D6a}} = (6\nu_2)^{-1}$ for the specific local construction.
Taking: $\nu_2 = 1$ and $\alpha_{GUT} \sim \{25^{-1}, 30^{-1}\}$ and VEV of the Higgs doublet ~ 100 GeV, in turn yields neutrino Dirac masses in the range

$$m_{Dirac} \sim \ \{2\,10^{-3}, 0.4\} \ \ \mathrm{eV}$$

Reasonable regime for the allowed range of neutrino masses

Summary

• Focus on O(1) E2-brane instantons in Type IIA
General CFT calculus: for generating superpotential disk and one-loop contributions

• Formalism applicable to vacua with exactly solvable CFT:
toroidal orientifolds, RCFT (Gepner Model orientifolds)

• Phenomenological implications:
Hierarchical couplings:
neutrino Majorana masses or small Dirac masses/
μ-terms/Yukawa couplings
$\longrightarrow$ Only explicit local examples exist!

Challenge: search for semi-realistic global models

Type I picture:

Constructions of (semi-realistic) examples on globally defined Calabi-Yau spaces (algebraic geometry):

[M.C.,T.Weigand 0711.0209]

- Elliptically fibered Calabi-Yau spaces X (Example: $\pi : X \rightarrow B = dP_4$)
- Introduce N_a (magnetized) D9-branes via holomorphic stable line bundles E_a (and extensions) - $U(N_a)$
- Stacks of N_i D5-branes wrapping the holomorphic curve Γ_i - $Sp(2N_i)$
- Spectrum: encoded in various cohomology groups (technical: c.f.,Blumenhagen,Honecker,Weigand'05)
- Tadpole cancellation associated w/ D5: $\sum_a N_a \mathrm{ch}_2(E_a) - \sum_i N_i \gamma_i = -c_2(TX)$ w/ D9: $\sum_a N_a c_1(V_a) \in H^2(X, 2\mathbf{Z})$.

Instantons: red $E1$-instantons

wrap rigid $C = \mathbf{P}^1$ curves - O(1)-instantons

Charged zero modes λ in the D9-E1 sector:

state	rep	cohomology
λ_a	$(N_a, 1_E)$	$H^0(\mathbf{P}^1, V_a^\vee(-1)\|_{\mathbf{P}^1})$
$\overline{\lambda}_a$	$(\overline{N}_a, 1_E)$	$H^1(\mathbf{P}^1, V_a^\vee(-1)\|_{\mathbf{P}^1})^*$

For line bundles $V_a = L_a$: $K_{\mathbf{P}^1} = \mathcal{O}(-2)$, and $L_a(-1)|_{\mathbf{P}^1} = \mathcal{O}(x_a - 1)$, w/ $x_a = \int_{\mathbf{P}^1} L_a$.

Additional zero modes from the D5-E1 counted by the extension groups $Ext_X(j_*\mathcal{O}|_{\Gamma_i}, i_*\mathcal{O}|_C)$: vanish Γ_i and C do not intersect

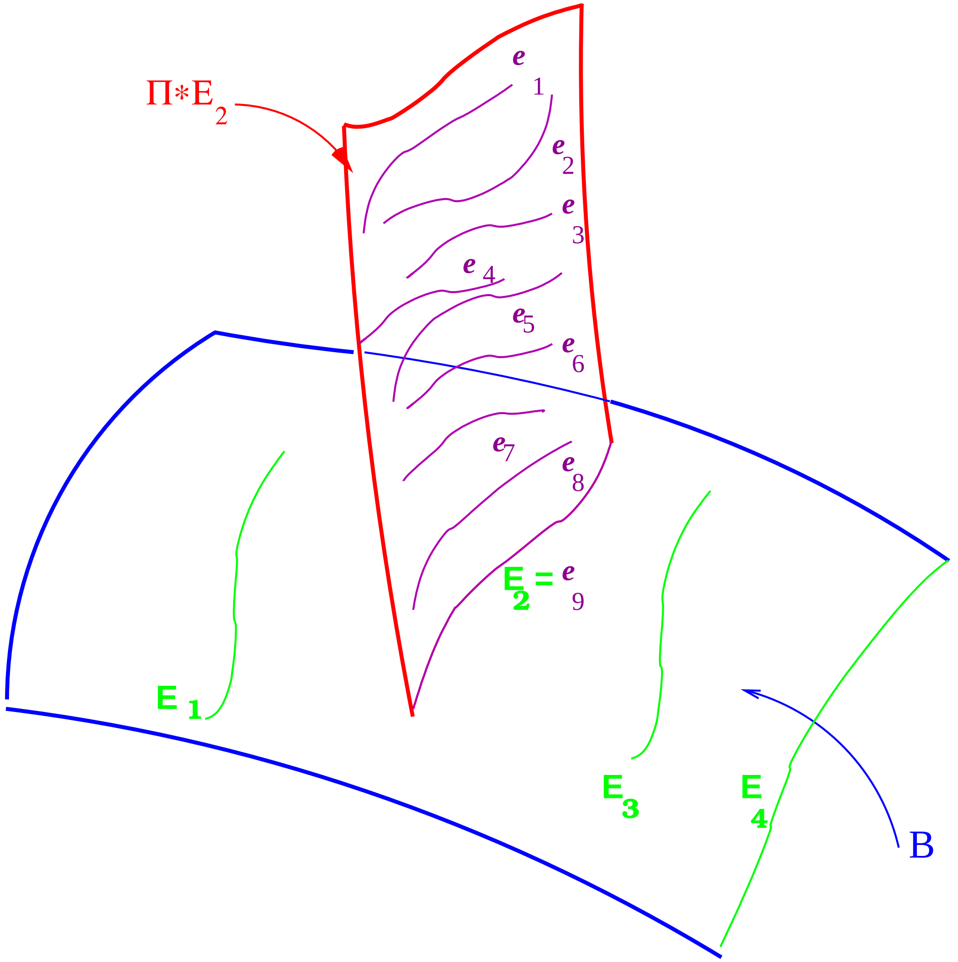


Figure 1: The dP_9 surface $\pi^* E_4$ inside the fibration $\pi : X \to B = dP_4$

• Global four-family $U(5)_a \times U(1)_b \times U(1)_c$ models with Majorana masses
(& examples Polonyi-type terms responsible for supersymmetry breaking)

Bundle	N	$c_1(L) = q\sigma + \pi^*(\zeta)$
L_a	5	$\pi^*(-2E_3)$
L_b	1	$2\sigma + \pi^*(-2l - 2E_1 + 3E_2 + 2E_3 + 2E_4)$
L_c	1	$-2\sigma + \pi^*(2l - E_2 - 2E_3 - 2E_4)$

• Non-perturbative couplings engineered to have correct hierarchy

$\longrightarrow$

Work in progress on globally defined three-family models with vector bundles

`M.C.,T.Weigand work in progress`