TWIST AND SH$^{I}_{O}{}^{F}_{U}$T

Massimo BIANCHI

CERN Theory Division
Università di Roma "Tor Vergata" - INFN

Talk at CERN

March 26, 2008

Foreword

- The combined effect of twists and shifts has not been systematically explored in the context of unoriented strings [Vafa, Witten; Angelantonj, Antoniadis, D'Appollonio, Dudas, Sagnotti; ... Camara, Maillard, Pradisi, Dudas; ... Blumenhagen, Schmidt-Sommerfeld]
- Various approaches: Left-Right symmetric asymmetric orbifolds [Narain, Sarmadi, Vafa; ... M.B., Morales, Pradisi], free fermions [Antoniadis, Bachas, Kounnas; Kawai, Lewellen, Tye; ... MB, Sagnotti], covariant lattices [Lerche, Lüst, Schellekens, Warner; ...], ...
- Another twist another shift: (N)MHV amplitudes in $\mathcal{N} = 4$ SYM, BDS and BES conjectures and finiteness of $\mathcal{N} = 8$ supergravity [see *e.g.* Dixon 0803.2475]
- Spinor shifts, recursion relations, generating functions, helicity sums, cut constructability, ...

Plan

- Part I, with Anastasopoulos, Morales, Pradisi (w.i.p.)
  - T-folds with few T's: combining CDMP and DJK models
  - Other means of moduli stabilization *e.g.* (non-)anomalous $U(1)$'s, non-perturbative effects, exotic modular invariants,
  - Magic supergravities [MB, Ferrara; Kounnas, Dolivet, Julia]
- Part II, with Elvang and Freedman (w.i.p.)
  - MHV amplitudes in $\mathcal{N} = 4$ SYM and $\mathcal{N} = 8$ supergravity
  - Operator map, (super)symmetries, generating functions
  - Next-to-MHV amplitudes at tree level
  - Beyond the trees: helicity sums
- Outlook for two

Part I: unoriented twists and shifts

- T-folds with few T's: combining CDMP and DJK models
- Other means of moduli stabilization e.g. (non-)anomalous $U(1)$'s, non-perturbative effects, exotic modular invariants, ...
- Magic supergravities [MB, Ferrara; Kounnas, Dolivet, Julia]

T-folds with few or no T's

CDMP model [Camara, Dudas, Maillard, Pradisi]: standard geometric freely acting orbifold $T^6/Z_2 \times Z_2$, Type I / Heterotic dual pairs
All twisted moduli are massive. Only untwisted moduli T_I, U_I, $I = 1, 2, 3$ (after unoriented projection)
Combine with gaugino condensate(s) in open string sector and/or 3-form fluxes to stabilize dilaton and other moduli.
DJK 'minimal' model [Dolivet, Julia, Kounnas]: non-magic hyper-free model, fermionic construction $G = \psi^\mu \partial X_\mu + \chi^i y^i w^i$. Fermionic sets:
$F, S, \bar{S}, \bar{b}_1, b_1 = \{\psi^\mu, \chi^{1,2}; y^{3,4,5,6}, y^1 w^1 | \bar{y}^5 \bar{w}^5\}$,
$\bar{b}_2 = \{y^6 w^6 | \bar{\psi}^\mu, \bar{\chi}^{3,4}; \bar{y}^{1,2}, \bar{w}^{5,6} \bar{y}^3 \bar{w}^3\}$,
$\bar{b}_3 = \{y^6 w^6 | \bar{\psi}^\mu, \bar{\chi}^{5,6}; \bar{w}^{1,2,3,4} \bar{y}^6 \bar{w}^6\}$
Only dilaton vector-multiplet survives. $\mathcal{N}_R = 0$, $(-)^{F_R}\sigma$ asymmetric freely acting orbifold of $T^6_{SO(12)}$, $y^i w^i \approx$ shifts.

Combining CDMP with DJK

Replace $\bar{b}_3$ with b_2, get a L-R symmetric asymmetric orbifold.
Geometric (freely acting) projections associated to $b_1\bar{b}_1$ and $b_2\bar{b}_2$.
Non geometric (freely acting) projections associated to $b_1, b_2, ...b_1\bar{b}_2$.
All untwisted moduli except dilaton hypermultiplet are projected out ($\mathcal{N}_L = \mathcal{N}_R = 1, \mathcal{N}_{tot} = 2$).
Massless multiplets only from $b_1 b_2$ and $\bar{b}_1\bar{b}_2$ twisted sectors: 2 hypers and 2 vectors.
Unoriented projection produces 1+2+2 chiral multiplets, or 1+2 chiral and 2 vector multiplets.
Combine with (non) anomalous $U(1)$'s and / or other (non)perturbative effects (gaugino condensation, ADS-like superpotentials, ...)

More twist, more shift

Start from Type I / Heterotic on T^4/Z_2. No neutral twisted moduli, only untwisted ones. Compactify on T^2 and project by a freely acting Z_2. Project some of the untwisted moduli, no extra twisted moduli. Another dual pair with $\mathcal{N} = 1$, laboratory for (non)perturbative effects, (non)anomalous $U(1)$'s, ...
In general, consider abstract SCFT with one-loop characters $\mathcal{X}_0 \approx V_2 - S_2 - C_2$ (identity), $\mathcal{X}_i \approx O_2 - S_2$ (massless chiral), $\mathcal{X}_i^c \approx O_2 - C_2$ (massless anti-chiral), $\mathcal{X}_I$ (massive $h_I > 1/2$).
Look for 'exotic' modular invariants of the form

$$|\mathcal{X}_0|^2 + \sum_i [\mathcal{X}_i \bar{\mathcal{X}}_{I(i)} + ...] + ...$$

where $I(i)$ labels massive characters with $h_{I(i)} = 1/2 + n_I$ (if present). All moduli except the dilaton (multiplet) are 'stabilized'. Cannot do any better for perturbative strings in Minkowski space.

Magic $\mathcal{N} = 2$ supergravities

Related to magic square of Freudenthal, Rozenfeld and Tits of the division algebras R, C, H, O [Gunaydin, Sierra, Townsend]

Only octonionic model is not a truncation of $\mathcal{N} = 8$ SG: 27 vector-plets $\mathcal{M}_V = E_{7(-25)}/E_6 \times U(1)$ AND 28 hypers $\mathcal{M}_H = E_{8(-24)}/E_7 \times SU(2)$

Type I description[1]: start from $D = 6$ on T^4/Z_2 with $n_T = 1 + 8$ with 16 $D9$ OR 16 $D5$ (twist and shift!) and reduce to $D = 4$ on T^2 [MB, Ferrara]

Doubly magic $n_H = n_v$, no quantum corrections to geometry (two-derivative effective action)

Potential corrections to higher derivative F-terms (for FHSV: Type II - Heterotic - Type I - Type II' tetrality), black hole entropy ...

Other magic hyper-free supergravities [Dolivet, Julia, Kounnas]

[1] Closely related to Type I FHSV-like model without open strings

Part II: another twist, another shift

- MHV amplitudes in $\mathcal{N} = 4$ SYM and $\mathcal{N} = 8$ supergravity
- Operator map, (super)symmetry, generating functions
- Next-to-MHV amplitudes at tree level
- Beyond the trees: helicity sums

MHV amplitudes in (supersymmetric) gauge theories

Helicity basis $A_\mu \to A^\pm$

$$\langle A^+ A^+ ... A^+ \rangle = 0 \quad , \quad \langle A^- A^+ ... A^+ \rangle = 0$$

to all orders in susy theories, at tree level if non-susy

Parke-Taylor formula (tree level)

$$\langle A^+(1)...A^-(i)...A^-(j)...A^+(n)\rangle = \frac{\langle i,j\rangle^4}{\prod_{k=1}^n \langle k,k+1\rangle}$$

where

$\langle i,j\rangle = u^\alpha(p_i)u_\alpha(p_j) = -\langle j,i\rangle \quad , \quad [i,j] = \bar{u}_{\dot\alpha}(p_i)\bar{u}^{\dot\beta}(p_j) = -[j,i]$

and $p_{\alpha\dot\alpha} = u_\alpha(p)\bar{u}_{\dot\alpha}(p)$ so that $p^2 = 0 = p_{\alpha\dot\alpha}\bar{u}^{\dot\alpha}(p) = p^{\dot\alpha\alpha}u_\alpha(p)$

Basic properties: holomorphy (no [..]'s), no multi-particle poles!

MHV 'vertices' [Chacazo, Svrcek, Witten]

MHV amplitudes in $\mathcal{N} = 4$ SYM

Up to color-ordering (non-cyclic permutations) 15 kinds of (susy related) MHV amplitudes [Georgiou, Glover, Khoze]

$$\langle A^- A^- A^+ ... A^+ \rangle \quad , \quad \langle \lambda^- \lambda^+ A^- A^+ ... A^+ \rangle \quad , \quad ...$$

$$\langle \phi\phi\phi\phi A^+ ... A^+ \rangle \quad , \quad ... \langle \lambda_1^+ ... \lambda_8^+ A^+ ... A^+ \rangle \quad .$$

where 15 counts partitions of $8 = \sum_{i=1}^{n} \nu_i$ with $\nu_i \leq 4$

Generating function [Nair]

$$\mathcal{F}_{MHV,n}^{tree,\mathcal{N}=4}(\eta_A^i) = \frac{\mathcal{A}_{MHV,n}^{tree,gauge}(i^-, j^-)}{\langle i, j \rangle^4} \delta^8 \left(\sum_i u^\alpha(p_i) \eta_A^i \right)$$

prefactor from 2-D fermion contractions ($J^a = \psi t^a \psi$) on $S^2_{\vec{p}/E}$ (?)

$\delta^8 \left(\sum_i u^\alpha(p_i) \eta_A^i \right) = \int d^8\theta e^{\theta_\alpha^A \sum_i u^\alpha(p_i) \eta_A^i}$, 1/2-BPS (?)

Further developments

- Perturbative $\mathcal{N} = 4$ SYM $\approx$ topological strings on super-twistor space $CP^{3|4}$ [Witten]
- Recursion relations from 2-spinor shift:

$$|1\rangle \to |1\rangle + z|2\rangle, 2\rangle \to |2\rangle, |1] \to |1], |2] \to |2] - z|1]$$

  $\mathcal{A}_n^{tree}(z)$ rational function of z with simple poles only
  If $A_n^{tree}(z) \to 0$ as $z \to \infty$, use Cauchy's theorem: residues $\approx$ splits into MHV [Britto, Cachazo, Feng, Witten; ...]
- MHV vertices from 'canonical' (super)field re-definition and (non)MHV (loop) amplitudes [Chacazo, Svrcek, Witten; Mansfield; Gorsky, Rosly; Bedford, Brandhuber, Spence, Travaglini; ...]
- Gravity from $(\text{gauge})^2$ [Kawai, Lewellen, Tye; Berends, Giele, Kuijf; Elvang, Freedman] unexpected cancellations and finiteness of $\mathcal{N} = 8$ [Bern, Dixon, Roiban, Bjerrum-Bohr, Dunbar, Kosower, Howe, Stelle] 'No-triangle' hypothesis

Gravity = $(\text{gauge})^2$?

$[\mathcal{N}=4\ SYM]_L \otimes [\mathcal{N}=4\ SYM]_R \to [\mathcal{N}=8\ SG]$ 16 distinct particle states in each $\mathcal{N}=4$ SYM factor and 256 states in $\mathcal{N}=8$ SG

$SU(4)_L \otimes SU(4)_R$ R-symmetry promoted to $SU(8)$ of $\mathcal{N}=8$

Supersymmetry Ward identities $a(p) \leftrightarrow A(p) \otimes \tilde{A}(p)$

$$[Q^a, a] \leftrightarrow [Q^a, A \otimes \tilde{A}] \equiv [Q^a, A] \otimes \tilde{A} \quad [Q^r, a] \leftrightarrow [Q^r, A \otimes \tilde{A}] \equiv A \otimes [Q^r, \tilde{A}]$$

Generating functions $\mathcal{G}^{\mathcal{N}=8}_{MHV} \approx \mathcal{F}^{\mathcal{N}=4,L}_{MHV} \mathcal{F}^{\mathcal{N}=4,R}_{MHV}$

Complicated $\mathcal{N}=8$ SG trees from simpler tree amplitudes of $\mathcal{N}=4$ (generalize quadratic KLT relations)

Operator map

b_+	$=$	$B_+ \tilde{B}_+$	b^-	$=$	$B^- \tilde{B}^-$
f_+^a	$=$	$F_+^a \tilde{B}_+$	f_a^-	$=$	$F_a^- \tilde{B}^-$
f_+^r	$=$	$B_+ \tilde{F}_+^r$	f_r^-	$=$	$B^- \tilde{F}_r^-$
b_+^{ab}	$=$	$B_+^{ab} \tilde{B}_+$	b_{ab}^-	$=$	$B_{ab}^- \tilde{B}^-$
b_+^{ar}	$=$	$F_+^a \tilde{F}_+^r$	b_{ar}^-	$=$	$-F_a^- \tilde{F}_r^-$
b_+^{rs}	$=$	$B_+ \tilde{B}_+^{rs}$	b_{rs}^-	$=$	$B^- \tilde{B}_{rs}^-$
f_+^{abc}	$=$	$\alpha_4\, \epsilon^{abcd}\, F_d^- \tilde{B}_+$	f_{abc}^-	$=$	$-\alpha_4\, \epsilon_{abcd}\, F_+^d \tilde{B}^-$
f_+^{abr}	$=$	$B_+^{ab} \tilde{F}_+^r$	f_{abr}^-	$=$	$B_{ab}^- \tilde{F}_r^-$
f_+^{ars}	$=$	$F_+^a \tilde{B}_+^{rs}$	f_{ars}^-	$=$	$F_a^- \tilde{B}_{rs}^-$
f_+^{rst}	$=$	$\tilde{\alpha}_4\, \epsilon^{rstu}\, B_+ \tilde{F}_u^-$	f_{rst}^-	$=$	$-\tilde{\alpha}_4\, \epsilon_{rstu}\, B^- \tilde{F}_+^u$

Example / test of the map

Two scalar MHV amplitude : three distinct decompositions of the $SU(8)$ indices

First all $SU(8)$ indices in one $SU(4)$, $b_{abcd} \leftrightarrow \epsilon_{abcd} B^- \otimes \tilde{B}_+$,
$\langle b^- \, b^-_{abcd} \, b_+^{efgh} \, b_+ \ldots b_+ \rangle \longrightarrow$

$$\alpha_4^2 \, \epsilon_{abcd} \, \epsilon^{efgh} \Big\langle B^- \; B^+ \; B^- \; B_+ ... B_+ \Big\rangle \Big\langle \tilde{B}^- \; \tilde{B}^- \; \tilde{B}_+ \tilde{B}_+ ... \tilde{B}_+ \Big\rangle$$

$$= 4! \, \delta^{efgh}_{abcd} \, \frac{\langle 13 \rangle^4}{\langle 12 \rangle^4} \Big\langle B^- \; B^- \; B_+ \; B_+ ... B_+ \Big\rangle \Big\langle \tilde{B}^- \; \tilde{B}^- \; \tilde{B}_+ \; \tilde{B}_+ ... \tilde{B}_+ \Big\rangle$$

Next split $SU(8)$ indices, $\langle b^- \, b_{abcr} \, b^{efgs} \, b_+ \ldots b_+ \rangle \longrightarrow$

$$(-1) \, \alpha_4^2 \, \epsilon_{abcd} \, \epsilon^{efgh} \Big\langle B^- \; F_+^d \; F_h^- \; B_+ ... B_+ \Big\rangle \Big\langle \tilde{B}^- \; \tilde{F}_r^- \; \tilde{F}_+^s \; \tilde{B}_+ ... \tilde{B}_+ \Big\rangle$$

$$= (-)^2 \epsilon_{abcd} \, \epsilon^{efgh} \delta^h_d \, \frac{\langle 12 \rangle}{\langle 13 \rangle} \, \Big\langle B^- \; B_+ \; B^- \; B_+ ... B_+ \Big\rangle \, \delta^s_r \, \frac{\langle 13 \rangle}{\langle 12 \rangle} \Big\langle \tilde{B}^- \; \tilde{B}^- \; \tilde{B}_+ \; \tilde{B}_+ ... \tilde{B}_+ \Big\rangle$$

$$= 3! \, \delta^s_r \, \delta^{efg}_{abc} \, \frac{\langle 13 \rangle^4}{\langle 12 \rangle^4} \, \Big\langle B^- \; B^- \; B_+ \; B_+ ... B_+ \Big\rangle \Big\langle \tilde{B}^- \; \tilde{B}^- \; \tilde{B}_+ \tilde{B}_+ ... \tilde{B}_+ \Big\rangle \, .$$

The third distinct split of the scalar $SU(8)$ indices OK

Generating function for $\mathcal{N} = 8$ SG MHV amplitudes

$$\mathcal{G}^{\mathcal{N}=8}_{MHV,n}(\eta_{iA}) = \frac{\mathcal{M}^{grav}_{MHV,n}(1^-, 2^-, ...)}{\langle 1\, 2\rangle^8} \mathcal{F}^{\mathcal{N}=4,L}_{MHV,n}(\eta_{ia}) \times \mathcal{F}^{\mathcal{N}=4,R}_{MHV,n}(\eta_{ir})$$

where $A = 1, ...8,\ a, b, \dots = 1, 2, 3, 4,\ r, s, \dots = 5, 6, 7, 8$ and n-graviton MHV amplitude [KLT; Berends, Giele, Kuijf; Elvang, Freedman]

$$\mathcal{M}^{grav}_{MHV} = \sum_{\mathcal{P}(i_4,...,i_n)} \frac{\langle 1\, 2\rangle\langle i_3\, i_4\rangle}{\langle 1\, i_3\rangle\langle 2\, i_4\rangle} s_{1 i_n} \left(\prod_{s=4}^{n-1} \beta_s\right) \mathcal{A}^{gauge}_{MHV}(1^-, 2^-, i_3^+, \dots, i_n^+)^2$$

with $\beta_s = -\frac{\langle i_s\, i_{s+1}\rangle}{\langle 2\, i_{s+1}\rangle} \langle 2|\, i_3 + i_4 + \dots + i_{s-1} |i_s]$

Crucial fact $\mathcal{M}^{grav}_{MHV,n}(1^-, 2^-, ...)/\langle 1\, 2\rangle^8$ totally symmetric

Properties of MHV generating function

- Neat connection between $\mathcal{N}=8$ and $\mathcal{N}=4$ amplitudes
- Bookkeeping for spin and R-symmetry factors. Manifest $SU(8)$ symmetry
- Practical tool for calculating MHV and next-to-MHV amplitudes.
- Associate with each state a differential operator: $h^{+} \to 1$, $\psi^{+A} \to D_{\eta}^{A}$, ... $h^{-} \to D_{\eta}^{8}$
- Tantalizing analogy with zero-mode counting in instanton calculus and with light-cone superfield [Brink, Ramond, ..., Kovacs, ...]
  $\mathcal{A}_{\mathcal{N}=4} = 1/\partial_{+}A^{-} + 1/\partial_{+}\lambda_{A}^{-}\hat{\theta}^{A} + \phi_{AB}\hat{\theta}^{A}\hat{\theta}^{B} + \lambda^{+A}(\hat{\theta}^{3})_{A} + A^{+}\hat{\theta}^{4}$
- To calculate amplitudes, apply differential operators to $\mathcal{G}_n$
- Number of MHV amplitudes in $\mathcal{N}=8$ supergravity = partitions of 16 with $n_{max}=8$. That's 186
- Susy Ward identities have unique solutions for MHV amplitudes, $\mathcal{G}_n$ satisfies WI's hence $\mathcal{G}_n$ is the correct, unique generating function

SUSY Ward identities for MHV amplitudes

Two kinds of operators [Grisaru, Pendleton, Van Nieuwenhuizen]
$[Q^a, \alpha] = [p, \epsilon]\beta$, $[Q^a, \beta] = 0$, $[\tilde{Q}_a, \beta] = \langle \epsilon p \rangle \alpha$, and $[\tilde{Q}_a, \alpha] = 0$
Two-component spinors, two independent constraints on the amplitudes per each set $|\epsilon\rangle$, $[\epsilon|$.
In the generating function approach, introduce SUSY generators:
$Q^A = \sum_i [i| D_{\eta^i_A}$ and $\tilde{Q}_A = \sum_i |i\rangle \eta^i_A$ so that $\{Q^A, \tilde{Q}_A\} = \delta^A_B P$
(with $P = \sum_i |i\rangle [i|$)
Obviously $Q^A \mathcal{G} = 0$ ($P = 0$) and $\tilde{Q}_A \mathcal{G} = 0$ ($\mathcal{G} \approx \delta(\tilde{Q}_A)$
Concrete WI's $\tilde{\epsilon}^A \tilde{Q}_A \mathcal{D}^{(17)} \mathcal{G} = 0$ and $\epsilon_A Q^A \mathcal{D}^{(15)} \mathcal{G} = 0$
As in $\mathcal{N} = 4$ SYM, susy generator produces correct algebra on differential operators and hence correct SUSY Ward identities

Applications beyond tree level: Intermediate state sum

1-loop MHV amplitude in $\mathcal{N}=4$ SYM from tree level
For 2-particle cuts, dropping dynamical prefactors, compute
$D_1^{(4)} D_2^{(4)} \delta^{(8)}(I_a)\delta^{(8)}(J_a)$, where $D_l^{(4)} = \prod_{a=1}^4 \partial/\partial\eta_{la}$, $l=1,2$, and
$I_a = |-l_1\rangle\eta_{1a} + |l_2\rangle\eta_{2a} + |i\rangle\eta_{ia} + \sum_m |m\rangle\eta_{ma}$,
$J_a = |l_1\rangle\eta_{1a} + |-l_2\rangle\eta_{2a} + |j\rangle\eta_{ja} + \sum_n |n\rangle\eta_{na}$
$D_1^{(4)} D_2^{(4)}$ acts on η_{1a} and η_{2a} in both I_a and J_a arguments of δ-functions in generating functions for 'left' and 'right' amplitudes.
Each intermediate state comes from a particular split of the action of $D_1^{(4)} D_2^{(4)}$ on I_a and J_a. Opposite helicities the two ends.
Use Schouten's identity $\langle i l_1\rangle\langle k l_2\rangle - \langle j l_1\rangle\langle i l_2\rangle = \langle ij\rangle\langle l_1 l_2\rangle$ to simplify
to $D_1^{(4)} D_2^{(4)}(D_i\delta^{(8)}(I_a) D_j\delta^{(8)}(J_a)) = \langle ij\rangle^4\langle l_1 l_2\rangle^4$
3-particle cuts can be handled similarly

Beyond MHV: Next-to-MHV (NMHV) amplitudes

Genuine NMHV start at 6-pts *e.g.* $\langle A^- A^- A^- A^+ A^+ A^+ \rangle$, while $\langle A^- A^- A^- A^+ A^+ \rangle$ is anti-MHV.

Counting NMHV amplitudes

In $\mathcal{N} = 4$ SYM number of NMHV amplitudes is number of partitions of 12 with $n_{max} = 4$. That's 34.

In $\mathcal{N} = 8$ SG number of NMHV amplitudes is number of partitions of 24 with $n_{max} = 8$. That's 919 !!

1/4 BPS ?

In $\mathcal{N} = 4$ SYM, NMHV amplitudes can be calculated using the 'MHV vertex method' [Cachazo, Svrcek, Witten; Georgiu, Glover, Khoze; ...]

NMHV generating functions in $\mathcal{N}=4$ SYM

Generating function for individual 'diagrams' *e.g.* for external lines $i+1$ and j adjacent to internal line I on the left and $j+1$ and i adjacent on the right

$$\mathcal{F}^{(i+1,j|j+1,i)}_{NMHV,n} = \Big(\prod_{i=1}^{n}\langle i\,(i+1)\rangle\Big)^{-1}\delta^{(8)}\Big(\sum_{i=1}^{n}|i\rangle\eta_{ia}\Big)\prod_{a=1}^{4}\sum_{k\epsilon I}\langle P_I\,k\rangle\eta_{ka}\frac{1}{D_I}$$

$$\frac{1}{D_I} = \frac{1}{P_I^2}\,\frac{\langle j(j+1)\rangle\langle i(i+1)\rangle}{\langle j\,P_I\rangle\langle (j+1)P_I\rangle\langle i\,P_I\rangle\langle (i+1)P_I\rangle}\,.$$

$1/D_I$ correct off-shell continuation of Parke-Taylor vertex factors.
SUSY WI's OK,
Derive $\mathcal{F}^{(i+1,j|j+1,i)}_{NMHV,n}$ via 3-line shift recursion relations: split into sum of 'MHV vertex diagrams' requires that $\mathcal{A}_{NMHV}(z)\rightarrow 0$ for $z\longrightarrow\infty$ [Risager]
Not rigorously proven for full $\mathcal{N}=4$ SYM theory ... but no counter examples.

NMHV generating functions in $\mathcal{N} = 8$ SG

In $\mathcal{N} = 8$ SG, similar set-up BUT we have found counter examples to $\mathcal{M}^{\mathcal{N}=8}_{n,NMHV}(z) \to 0$ for $z \to \infty$.
Typically, for amplitudes involving scalars, $\mathcal{M}_{n,NMHV}(z) \to \mathcal{O}(1)$ for $z \to \infty$
Possible way out: summing all diagrams $\mathcal{M}_{n,NMHV}(z) \to 0$, needed for recursion relations to work in NMHV sector, and beyond.
Low-energy theorems: 70 scalars of $\mathcal{N} = 8$ SG non-linear sigma model on $E(7,7)/SU(8)$. Exact Goldstone bosons. Low-energy theorems à la Adler and Weinberg. Trivial one soft-pion limit in MHV sector ($q_\phi = 4$) and NMHV sector (?). Interesting two soft pions limits.

Recent developments in the planar limit

- MHV amplitudes and light-like Wilson loops

$$\frac{\mathcal{A}^{full,SYM}_{MHV,n}(p_i,h_i)}{\mathcal{A}^{tree,SYM}_{MHV,n}(p_i,h_i)} = W(\lambda;x_i)|_{x_i - x_{i+1} = p_i}$$

  independent of helicities / species, tested up to 6-pts at 2-loops [Brandhuber, Heslop, Travaglini; Drummond, Henn, Korchemsky, Sokactchev; BDKRSVV; ...]
- Exponentiation of 4-pt amplitudes, BDS conjecture [Bern, Dixon, Smironov]

$$\mathcal{A}^{finite}_{4pt}(s,t) = C\exp\left\{\frac{1}{8}\gamma_{cusp}(\lambda)\log^2(s/t)\right\}$$

  strong coupling AdS/CFT test [Alday, Maldacena], integrability prediction for cusp anomalous dimension [Beisert, Eden, Staudacher]
- Dual conformal invariance of scalar boxes $x \leftrightarrow p$

Outlooks

Part I:
Twists and shifts can conveniently combine with other mechanisms, *e.g.* open and closed string fluxes, non-anomalous $U(1)$'s, instanton effects, ... of moduli stabilization. Explicit computations are feasible.
Model with only 2 twisted moduli.
Part II:
Generating functions techniques are instrumental to (dis)prove $(\mathcal{N}=8) \leftrightarrow (\mathcal{N}=4)_L \otimes (\mathcal{N}=4)_R$ at tree level in (N)MHV sector and beyond
Need a better understanding of spinor shifts, ...