Standard Model Statistics for Intersecting Branes on Z6’

JHEP 0709 (2007) 128, arXiv:0708.2285 [hep-th] & work in progress

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Standard Model Building Approaches

- Heterotic $E_8 \times E_8$ string

  see talks by Gray, Lukas, Manno, Ratz, Trapletti, Zanzi

- Gepner models see talk by Tsulaia
- Free fermionic constructions see talks by Kounnas, Schellekens
- Type II with D-branes at singularities see talk by Verlinde (?)

  F-theory GUTs with branes at singularities

- Magnetised D-branes $\overset{\mathrm{T-dual}}{\Leftrightarrow}$ Intersecting D-branes

  see talks by Antoniadis, Bianchi, Haack, Hebecker, Plauschinn, Quevedo, Schmidt-Sommerfeld —*Bailin, Cvetic, Gmeiner, Timirgaziu, Weigand*

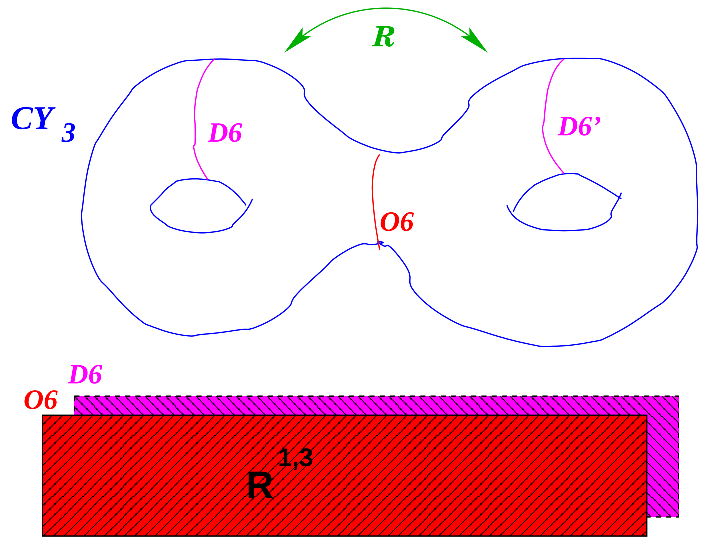
... and lots of people whose name is not on this transparency

Geometry of orbifolds well understood $\Leftrightarrow CY_3$ spaces?

CFT methods provide powerful computational tools

Intersecting D6-Branes

Orientifold of IIA string theory with anti-holomorphic involution $\mathcal{R}$ on the Calabi-Yau 3-fold



- Invariant 3-cycles Π_{O6} are wrapped by O6-planes
- D6$_a$ branes wrap 3-cycles Π_a
- $\mathcal{R}$ images D6$_a$' of D6$_a$ branes wrap Π'_a

Topological constraints:

$\Rightarrow$ RR tadpole cancellation: $\sum_a N_a(\Pi_a + \Pi'_a) = 4\Pi_{O6}$

$\Rightarrow$ K-theory: $\sum_a N_a \Pi_a \circ \Pi_{Sp(2)} = 0$ mod 2

Massless Spectrum

Massless spectrum consists of

- Closed strings: $\mathcal{N}=1$ SUGRA, axion-dilaton mult., $h^-_{1,1}$ complexified Kähler & $h_{2,1}$ complex structure moduli mults., $h^+_{1,1}$ vector mults. ($h^{\pm}_{1,1}$: (anti) invariant cycles under $\mathcal{R}$)
- Open strings: $\prod_a U(N_a)$ gauge groups, sometimes also $SO(2N)$ or $Sp(2N)$ & charged matter

The chiral spectrum is computed from intersection numbers $\Pi_a \circ \Pi_b$ of 3-cycles

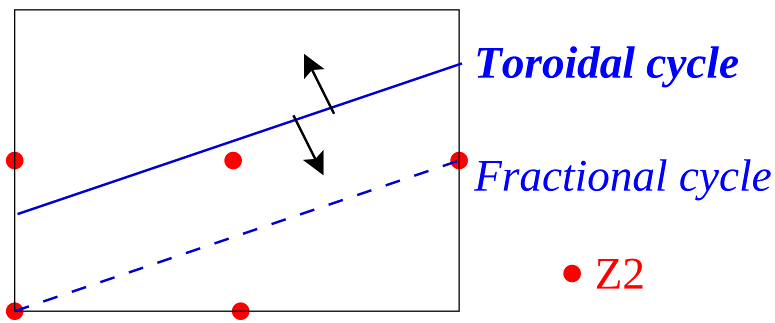
representation	net chirality
$(\mathbf{Anti}_a)$	$\frac{1}{2}\left(\Pi_a \circ \Pi'_a + \Pi_a \circ \Pi_{O6}\right)$
$(\mathbf{Sym}_a)$	$\frac{1}{2}\left(\Pi_a \circ \Pi'_a - \Pi_a \circ \Pi_{O6}\right)$
$(\mathbf{N}_a, \overline{\mathbf{N}}_b)$	$\Pi_a \circ \Pi_b$
$(\mathbf{N}_a, \mathbf{N}_b)$	$\Pi_a \circ \Pi'_b$

Fractional Cycles

Fractional cycles on $T^6/\mathbb{Z}_{2N}$ stuck at $\mathbb{Z}_2$ fixed points on T^4
$\Rightarrow$continuous displacement & Wilson line on T^2 encoded in chiral adjoint
$+$ additional adjoints from orbifold image cycles

$$\Pi^{\text{frac}} = \frac{1}{2}\left(\Pi^{\text{torus}} + \Pi^{\text{ex}}\right) \quad \text{or} \quad \Pi^{\text{rigid}} = \frac{1}{4}\left(\Pi^{\text{torus}} + \sum_{i=1}^{3} \Pi^{\text{ex},(i)}\right)$$

Rigid cycles possible on $T^6/\mathbb{Z}_{2N} \times \mathbb{Z}_{2M}$
$\Rightarrow$only discrete Wilson lines, no adjoint matter
$\Rightarrow$D-instantons see talks by Bianchi, Cvetic, Schmidt-Sommerfeld, Weigand



Full spectrum on orbifolds

Florian Gmeiner, G.H. 0708.2285

Rewrite intersection number on $T^6/\mathbb{Z}_M$ in terms of sectors

$\Pi_a^{\rm torus} \circ \Pi_b^{\rm torus} = -\sum_k I_{a(\theta^k b)}$ $(I_{ab} = \prod_{i=1}^3 (n_a^i m_b^i - m_a^i n_b^i))$, for

$M = 2N$ include $\mathbb{Z}_2$ invariant intersections

$\Pi_a^{\rm ex} \circ \Pi_b^{\rm ex} = -\sum_k I_{a(\theta^k b)}^{\mathbb{Z}_2}$ with relative signs ($\mathbb{Z}_2$ e.v. $+$ Wilson lines)

Chiral + non–chiral massless matter on $T^6/(\mathbb{Z}_{2N} \times \Omega\mathcal{R}$	
$(\mathbf{Adj}_a)$	$1 + \frac{1}{4}\sum_{k=1}^{N-1} \left\| I_{a(\theta^k a)} + I_{a(\theta^k a)}^{\mathbb{Z}_2} \right\|$
$(\mathbf{N}_a, \mathbf{N}_b)$	$\frac{1}{2}\sum_{k=0}^{N-1} \left\| I_{a(\theta^k b')} + I_{a(\theta^k b')}^{\mathbb{Z}_2} \right\|$
$(\mathbf{Anti}_a)$	$\frac{1}{4}\sum_{k=0}^{N-1} \left\| I_{a(\theta^k a')} + I_{a(\theta^k a')}^{\mathbb{Z}_2} + I_a^{\Omega\mathcal{R}\theta^{-k}} + I_a^{\Omega\mathcal{R}\theta^{-k+N}} \right\|$
$(\mathbf{Sym}_a)$	$\frac{1}{4}\sum_{k=0}^{N-1} \left\| I_{a(\theta^k a')} + I_{a(\theta^k a')}^{\mathbb{Z}_2} - I_a^{\Omega\mathcal{R}\theta^{-k}} - I_a^{\Omega\mathcal{R}\theta^{-k+N}} \right\|$

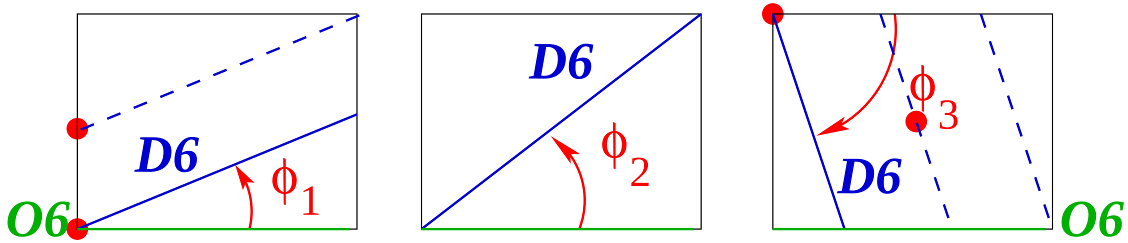
with some modifications for vanishing angles, e.g. $I_{a(\theta^k b)}^0 \to 2$

This can be generalised from fractional to rigid cycles

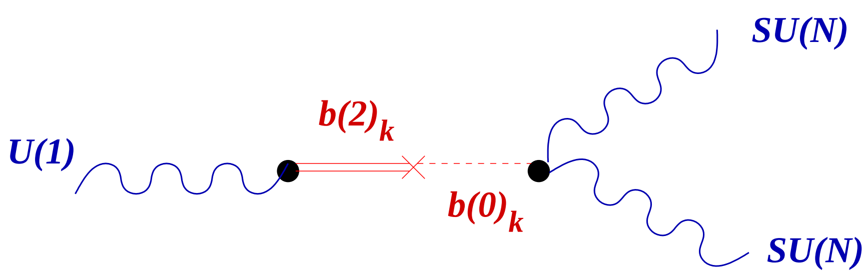
SUSY and Anomalies

Supersymmetry & stability are *not topological*, but moduli dependent: D6-branes have to wrap special Lagrangian 3-cycles – not classified for generic CY_3

On $(T^2)^3$: $\sum_{i=1}^{3} \phi_i = 0$ + $\mathbb{Z}_2$ fixed points hit by torus cycle



Green Schwarz mechanism via Chern-Simons couplings of RR fields, $\int_{\mathbb{R}^{1,3}\times\Pi_a} C_5 \, \mathrm{tr} F_a$ ($\Rightarrow U(1)$ masses) and $\int_{\mathbb{R}^{1,3}\times\Pi_a} C_3 \, \mathrm{tr}\,(F_a \wedge F_a)$



$C_3 = b_k^{(0)} \, \omega_k$ + complex structures form complex scalars

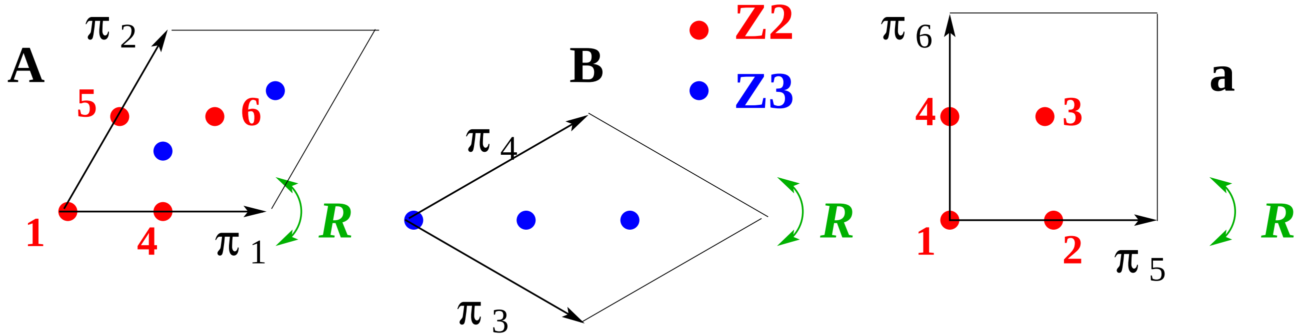
$\Rightarrow$SUSY: # massive U(1)s = # frozen complex structures

The $\mathbb{Z}_6'$ orbifold

Set-up: Bailin, Love '06,

RR.tcc.Solutions & Statistics: F. Gmeiner, G.H. 0708.2285 + work in progress

- Orbifold action $\theta : z^i \to e^{2\pi i v_i} z^i$ with $\vec{v} = 1/6 \cdot (1, 2, -3)$
- Anti-holomorphic involution $\mathcal{R}$ admits two kinds of shapes of tori



- Kähler moduli $(h_{1,1})$: 3 untwisted (volume of each T^2), 12 at θ-fixed points, 12 on θ^2-fixed tori, 8 on θ^3-fixed tori
- Complex structures $(h_{2,1})$: 1 untwisted (shape of T_3), 6 from θ^2 fixed points on $T_1 \times T_2$ times 1-cycle on T_3, 4 at θ^3-fixed points on $T_1 \times T_3$ times 1-cycle on T_2

$T^6/\mathbb{Z}_6'$ - 3-cycles

$\#$ 3-cycles $\equiv b_3 = 2 + 2h_{2,1} = 24$

$\Rightarrow$4 untwisted 3-cycles ρ_i plus 4+4 3-cycles from $\mathbb{Z}_2$ sectors δ_j, $\tilde{\delta}_j$ form 12 dimensional sublattice

$$\rho_1 = \sum_{k=0}^{5} \theta^k(\pi_{135}),\ \rho_2 = \sum_{k=0}^{5} \theta^k(\pi_{235}),\ \rho_3 = \sum_{k=0}^{5} \theta^k(\pi_{136}),\ \rho_4 = \sum_{k=0}^{5} \theta^k(\pi_{236})$$

$\Rightarrow \Pi^{\text{torus}} = \sum_{k=0}^{5} \theta^k \left[\otimes_{i=1}^{3}(n_i\pi_{2i-1} + m_i\pi_{2i})\right] =$
$P\rho_1 + Q\rho_2 + U\rho_3 + V\rho_4$ with $P = Xn_3, Q = Yn_3, U = Xm_3, Y = Ym_3$
and $X = n_1n_2 - m_1m_2, Y = n_1m_2 + m_1n_2 + m_1m_2$

$$\delta_j = \sum_{k=0}^{2} \theta^k(e_{4j} \otimes \pi_3), \quad \tilde{\delta}_j = \sum_{k=0}^{2} \theta^k(e_{4j} \otimes \pi_4), \qquad j = 1\ldots 4$$

$\Rightarrow \Pi^{\text{ex}} = \sum_{j=1}^{4}\left(d_j\delta_j + e_i\tilde{\delta}_j\right)$ with e.g. $d_j = -n_2 - m_2,\ e_j = n_2$

$T^6/\mathbb{Z}_6'$ - RR tadpoles

- Evoid double counting of models by imposing $n_1, n_3, m_3 + bn_3 \geq 0$ and $(n_1, m_1) = (\text{odd}, \text{odd})$
- Fractional cycles have separate RR tadpole & SUSY conditions for torus $+$ exceptional cycles:
  bulk RR tadpole cancellation depends on orientation – for $\mathbf{ABa}$:

$$\sum_a N_a \left(P_a + Q_a\right) = 8, \qquad \sum_a N_a \left(U_a - V_a\right) = 24$$

Each SUSY brane contributes positively (or zero) to each sum $\Rightarrow$ *naive* maximal rank 32
RR tadpoles from $\mathbb{Z}_2$: no O6-plane contribution
$\sum_a N_a \left(d_i^a - e_i^a\right) = 0$ for the $\mathbf{ABa}$ orientation

$T^6/\mathbb{Z}_6'$: K-theory

- K-theory constraint: $\Omega\mathcal{R}$-invariant branes are classified, but not clear which give $SO(2)$ or $Sp(2)$ - take a (maybe too strong) constraint with all as probes, however: net-intersection with model always even, for example on **ABa**: $\Pi_{\text{probe}} = \frac{1}{2}(\rho_1 + \rho_2) \pm \frac{3}{2}(\delta_1 - \tilde{\delta}_1) \pm \frac{3}{2}(\delta_3 - \tilde{\delta}_3)$ leads to the constraint

$$\frac{3}{2} \sum_a N_a \left(U_a + V_a \pm (d_1^a + e_1^a) \pm (d_2^a + e_2^a) \right)$$
$$\overset{\text{RR}-\text{tad.}}{=} 3 \sum_a N_a \left(V_a + d_1^a + d_2^a \right) + 36 \overset{!}{=} 0 \mod 2$$

and subsequent combinatorics of $\{V_a, d_i^a\}$ odd or even depending on (n_i, m_i) odd/even show that no new constraint arises - *independent of bulk SUSY*

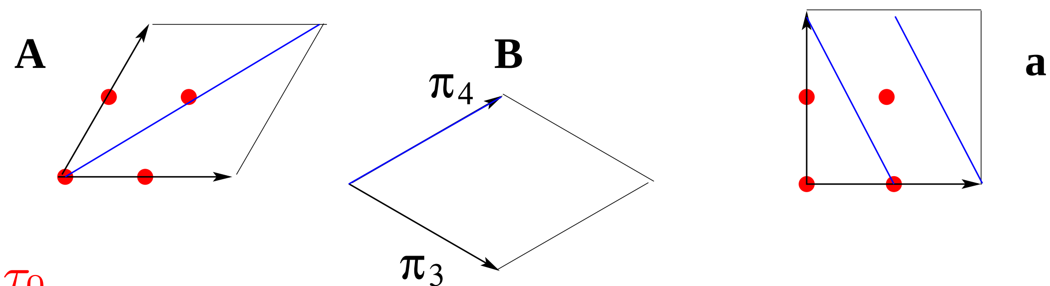
$T^6/\mathbb{Z}_6'$: SUSY

- SUSY for branes on **ABa**: toroidal per brane

R_1, R_2: radii on T_3

$$\frac{R_1}{\sqrt{3}R_2}(P-Q)-(U+V)=0 \quad (P+Q)-\frac{R_2}{\sqrt{3}R_1}(V-U)>0$$

SUSY of $\mathbb{Z}_2$ sector: only exceptional cycles through which the toroidal cycle passes occur. There are three signs: $\mathbb{Z}_2$ eigenvalue & two Wilson lines on $T_1 \times T_3$



$$\Pi^{\mathrm{ex}} = (-1)^{\tau_0} \times$$

$$\times \left(e_{11} + (-1)^{\tau_1} e_{61} + (-1)^{\tau_3} e_{12} + (-1)^{\tau_1+\tau_3} e_{62}\right) \otimes (n_2\pi_3 + m_2\pi_4)$$

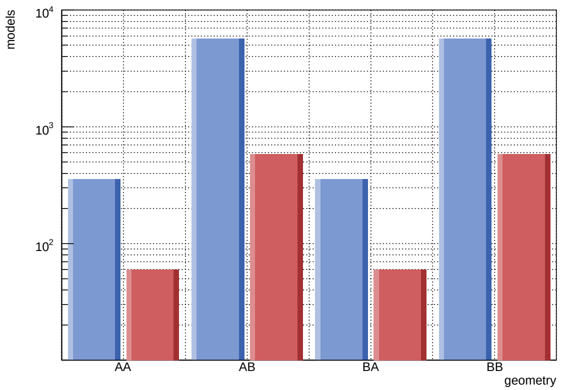
$$+ \text{two } \theta - \text{images}$$

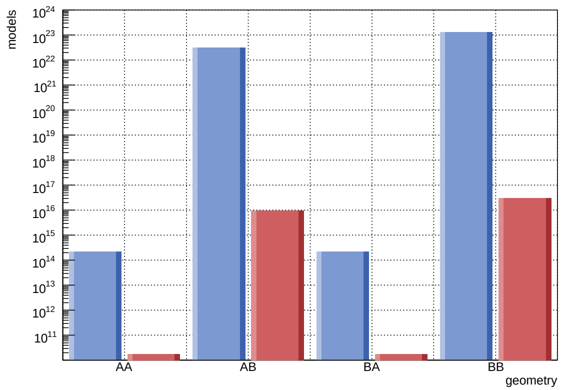
Results: SUSY & RR tadpoles

Intersection pattern of 12 dim. sublattice: $I^{\mathrm{bulk}} = \begin{pmatrix} 0 & 2A \\ 2A & 0 \end{pmatrix}$

and $I^{\mathbb{Z}_2} = \mathrm{diag}(2\varepsilon, \ldots, 2\varepsilon)$ with $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ and $\varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ rich enough to allow for 3-generation models

Large number of SUSY solutions $\mathcal{O}(10^4)$ for the toroidal RR tadpoles depends on geometry: $\mathbf{ABa} \simeq \mathbf{BBa}$ preferred



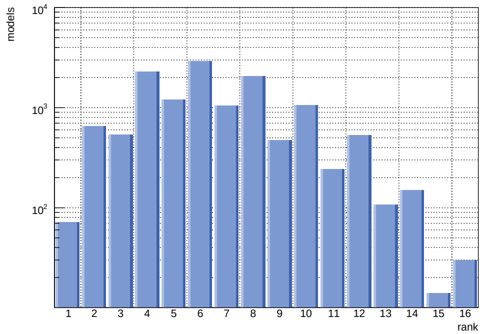


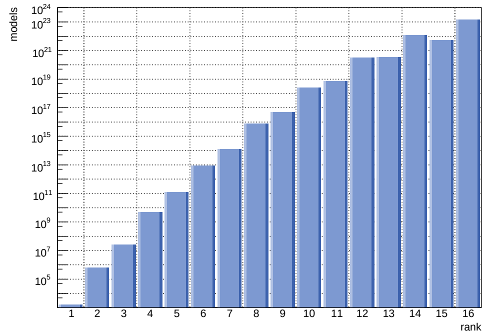
Taking into account the $\mathbb{Z}_2$ part leads to $\mathcal{O}(10^{23})$ SUSY RR tadpole solutions, with 3×10^{22} on $\mathbf{ABa}$ and 10^{23} on $\mathbf{BBa}$

Results: Probabilities

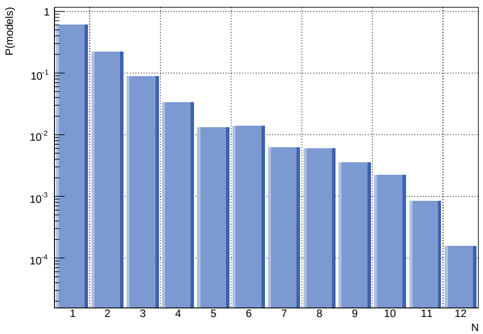
Scaling behaviour of solutions of toroidal *(left)* and complete *(right)* solutions: *The set of SUSY solutions is complete!*

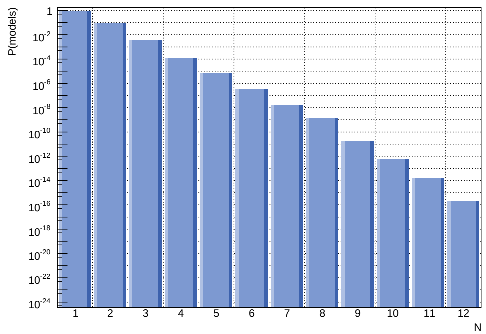
(a) total rank





(b) Probability $\mathcal{N}$ to find a single gauge factor of rank N





For toroidal/fractional part $\mathcal{N}(N) \approx \sum_{k=1}^{T+1-N} \frac{T^4}{N^2}(n_e)^k =$ $= \frac{T^4}{N^2} f_{N,T}$ with $f_{N,T} = (T+1-N)$ or $(n_e)^{T+1-N}$ with the effective O6-plane charge T in RR tadpoles fits with Douglas, Taylor '06

Standard Models I

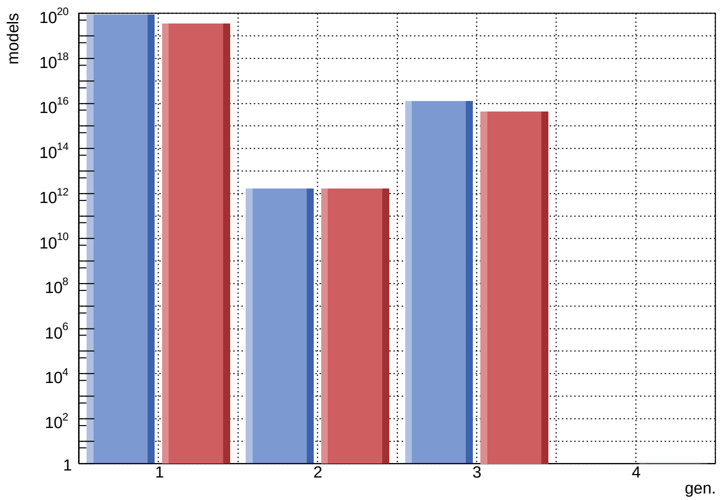
Ansatz: $U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d$ with three different choices of hyper charge (two with u_R in $\mathbf{Anti}_a$), or $U(3)_a \times Sp(2)_b \times U(1)_c \times U(1)_d$

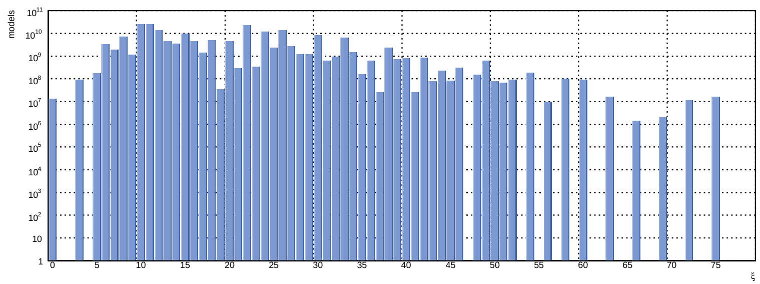
On $T^6/\mathbb{Z}_6'$: only one type with n SUSY generations and RR tadpoles canceled, for $n = 3$ only on **ABa** and **BBa**

$U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d$	
particle	n
Q_L	$\chi^{ab} + \chi^{ab'}$
u_R	$\chi^{a'c} + \chi^{a'd}$
d_R	$\chi^{a'c'} + \chi^{a'd'} + \chi^{\mathbf{Anti}_a}$
L	$\chi^{bc} + \chi^{bd} + \chi^{b'c} + \chi^{b'd}$
e_R	$\chi^{cd'} + \chi^{\mathbf{Sym}_c} + \chi^{\mathbf{Sym}_d}$
$Q_Y = \frac{1}{6}Q_a + \frac{1}{2}Q_c + \frac{1}{2}Q_d$	

Standard Models II

$SU(3) \times SU(2) \times U(1)_Y$: 3 generation models without chiral exotics possible: $\mathcal{O}(10^{15})$ models with massless hyper charge - ($\mathcal{O}(10^{16})$ with massive $U(1)_Y$)





Mean number of chiral exotics is computed from

v: visible sector — h: hidden

$$\zeta \equiv \sum_{v,h} \left| \chi^{vh} - \chi^{v'h} \right|$$

$\Rightarrow$there can still be an *excess of Higgs* candidates

Complex structures for SM

Complex structure values ϱ on $\mathbf{ABa}$ for n generations:

n	ϱ	#models	n	ϱ	#models	n	ϱ	#models
1	$1/2$	$8.7 \cdot 10^{18}$	2	$1/5$	$2.5 \cdot 10^{11}$	3	$1/2$	$9.7 \cdot 10^{9}$
	$5/2$	$3.4 \cdot 10^{13}$					$1/4$	$9.6 \cdot 10^{6}$
	$7/4$	$2.7 \cdot 10^{6}$					$1/6$	$1.2 \cdot 10^{14}$
							$3/2$	$4.9 \cdot 10^{14}$
							$9/4$	$4.9 \cdot 10^{7}$

On $\mathbf{BBa}$: frequencies by $\mathcal{O}(10)$ larger with $\varrho \to 3/(4\varrho)$

Bailin & Love's possible solution with $(\chi^{ab}, \chi^{ab'}) = (\underline{2, 1})$: 1/4 on $\mathbf{ABa}$ (3 on $\mathbf{BBa}$) — the one with the *smallest fequency*

SUSY SM Example

Example with SM sector with complex structure $\varrho = 1/2$ and $SU(3)\times SU(2)\times U(1)_{Y\,(\times U(1)^2_{\text{massless}}\times U(1)^2_{\text{massive}})}$ $(\chi^{ab},\chi^{ab'})=(0,3)$:

- Chiral spectrum contains abundance of Higgs candidates

$$3\times\Big[(\mathbf{3},\mathbf{2})_{1/6}+(\bar{\mathbf{3}},\mathbf{1})_{1/3}+(\bar{\mathbf{3}},\mathbf{1})_{-2/3}+5\times(\mathbf{1},\mathbf{2})_{-1/2}$$
$$+4\times(\mathbf{1},\mathbf{2})_{1/2}+(\mathbf{1},\mathbf{1})_{1}+(\mathbf{1},\mathbf{1})_{0}\Big]$$

- Adjoints: $2\times(\mathbf{8},\mathbf{1})_0+10\times(\mathbf{1},\mathbf{3})_0+36\times(\mathbf{1},\mathbf{1})_0$
- Non-chiral matter:

$$\Big[(\mathbf{3},\mathbf{2})_{1/6}+6\times(\mathbf{3},\mathbf{1})_{-1/3}+3\times(\mathbf{3},\mathbf{1})_{2/3}+4\times(\mathbf{1},\mathbf{2})_{-1/2}$$
$$+8\times(\mathbf{1},\mathbf{2})_0+4\times(\mathbf{1},\mathbf{1}_2)_0+6\times(\mathbf{1},\mathbf{3}_2)_0+4\times(\mathbf{1},\mathbf{1})_0$$
$$+6\times(\mathbf{1},\mathbf{1})_{1/2}+4\times(\mathbf{1},\mathbf{1})_1\;+c.c.\Big]$$

Standard Models III

Suppression factors w.r.t. the total # of solutions on $T^6/\mathbb{Z}_6'$:

- 0.4 from $U(1)_Y$ massless
- 7.3×10^{-4} for $U(3) \times U(2)/Sp(2) \times U(1)$ and n generations
- 2.6×10^{-8} for $n = 3$ generations

$\Rightarrow$very similar to $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2 \sim 10^{-9}$

Gmeiner, Blumenhagen, Honecker, Lüst, Weigand '05

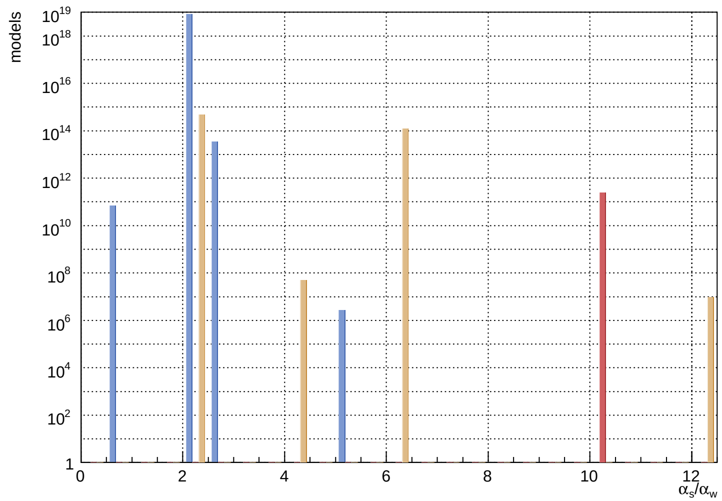
$\Rightarrow$different from $T^6/\mathbb{Z}_6 \sim 10^{-22}$ Gmeiner, Lüst, Stein '07

Standard Models IV

Gauge couplings

$$\frac{1}{\alpha_{a,\text{string}-\text{tree}}} = \frac{4\pi}{g^2_{a,\text{string}-\text{tree}}} \sim \frac{M_{\text{Planck}}}{M_{\text{string}}} \cdot \text{Vol}(D6_a)$$

$\Rightarrow$ratios independent of scales: $\alpha_s/\alpha_w = \text{Vol}(D6_b)/\text{Vol}(D6_a)$



1, 2, 3 generations

1-loop running $b_a/(16\pi^2)\ln(M^2_{\text{string}}/\mu^2)$ with $\varphi^{\mathbf{Adj}_a} \geq 2$ and

$$b_{SU(N_a)} = N_a\left(\varphi^{\mathbf{Adj}_a} - 3\right) + \sum_{b\neq a} \frac{N_b}{2}\left(\varphi^{ab} + \varphi^{ab'}\right) + \frac{N_a - 2}{2}\,\varphi^{\mathbf{Anti}_a} + \frac{N_a + 2}{2}\,\varphi^{\mathbf{Sym}_a}$$

$\Rightarrow$confinement ($b < 0$) very unlikely

Standard Model V

Massless $U(1)$s are linear combinations of $U(1)_i \subset U(N_i)$

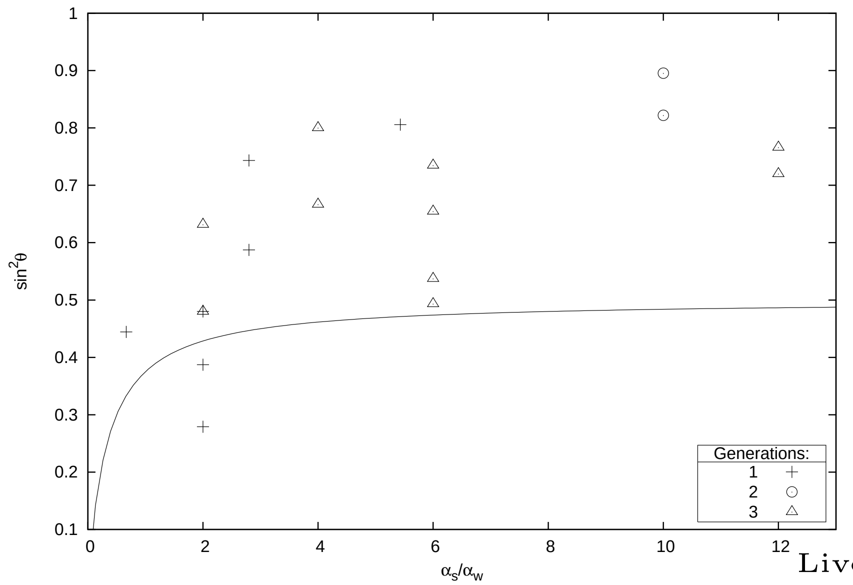
$$U(1)_X = \sum_i x_i U(1)_i \qquad \Rightarrow \qquad \frac{1}{\alpha_X} = \sum_i 2\, N_i\, x_i^2 \frac{1}{\alpha_i}$$

with 1-loop beta function coefficient

$$b_{U(1)_a} = N_a \left(\sum_{b \neq a} N_b \left(\varphi^{ab} + \varphi^{ab'} \right) + 2\,(N_a + 1)\, \varphi^{\mathbf{Sym}_a} + 2\,(N_a - 1)\, \varphi^{\mathbf{Anti}_a} \right) \geq 0$$

$$b_X = \sum_i x_i^2\, b_i + 2 \sum_{i<j} N_i N_j x_i x_j \left(-\varphi^{ij} + \varphi^{ij'} \right)$$

Weak mixing angle $\sin^2\theta_w = \alpha_Y / (\alpha_Y + \alpha_w)$



Standard Model VI

If one assumes an underlying Pati-Salam or $SU(5)$ GUT structure, there is a relation

$$\frac{1}{\alpha_Y} = \frac{2}{3}\frac{1}{\alpha_s} + \frac{1}{\alpha_w} \qquad \text{or} \qquad \frac{1}{\alpha_s} = \frac{1}{\alpha_w} = \frac{3}{5}\frac{1}{\alpha_Y}$$

represented by a line on the previous plot

$T^6/\mathbb{Z}_6'$: no hint for such a relation

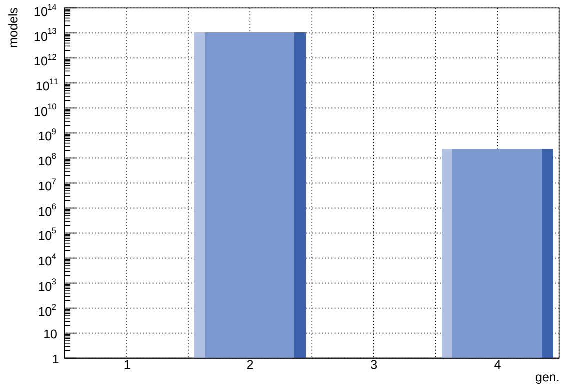
$T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$: 88% of models fit to Pati-Salam relation

$T^6/\mathbb{Z}_6$ example: all bulk cycle have same length $\Rightarrow \alpha_s = \alpha_w$, if fifth stack of branes is included in $U(1)_Y$, the $SU(5)$ relation holds

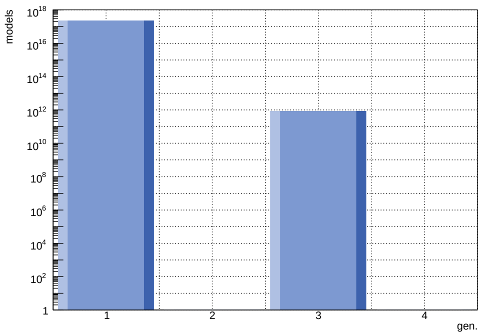
$SU(5)$ and Pati-Salam

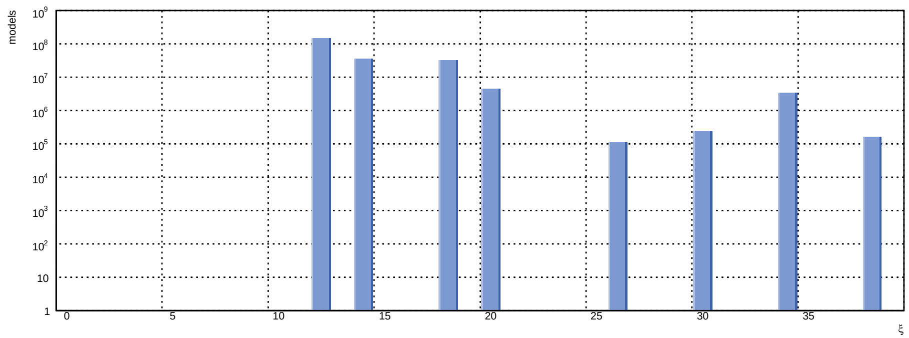
A systematic search gives:

$SU(5)$: only $n = 2, 4$ generations & 1 or 2 chiral symmetrics:



$SU(4) \times SU(2)_L \times SU(2)_R$: $\mathcal{O}(10^{12})$ 3 generation models but $>$10 chiral exotics, however, search incomplete!





Trinification

Ansatz: $U(3)_a \times U(3)_b \times U(3)_c$ with n generations of

$$(\overline{\mathbf{3}}_a, \mathbf{3}_b, \mathbf{1}) + (\mathbf{1}, \overline{\mathbf{3}}_b, \mathbf{3}_c) + (\mathbf{3}_a, \mathbf{1}, \overline{\mathbf{3}}_c)$$

$\Rightarrow$no SUSY $+$ RR tadpole solution without chiral exotics in $(\mathbf{Sym}_a, \mathbf{1}, \mathbf{1})$, $(\mathbf{Anti}_a, \mathbf{1}, \mathbf{1})$ and $(\mathbf{3}_a, \mathbf{3}_b, \mathbf{1}) \ldots$

$\Rightarrow$only $n = 2$ generations appear

Comparison with $T^6/\mathbb{Z}_6$

Formulae G.H., Ott '04; Statistics Gmeiner, Lüst, Stein '07

$T^6/\mathbb{Z}_6$ acts by $\vec{v} = 1/6 \cdot (1, 1, -2)$ with 6 inequivalent orientations of $SU(3)^3$ lattices under $\Omega\mathcal{R}$

- 2 untwisted 3-cycles, 10 twisted 3-cycles at $\mathbb{Z}_2$ fixed points form 12 dim. unimodular basis.
  No 3-cycles from $\mathbb{Z}_3$ subsector!
- SUSY selects one untwisted cycle, O6-plane untwisted
  $\Rightarrow \Pi_a \circ \Pi_{O6} = 0$ leads to $\#$ Anti $= \#$ Sym
  $\Rightarrow$constraints on model building: no $SU(5)$ GUTs possible, all quarks and leptons are bifundamentals
- Bulk RR tadpole cancellation gives *naive* maximal rank 8 for 5 geometries, 12 for 1 geometry
- $U(3) \times U(2) \times U(1)^2$ admits at most a *'hidden'* $U(1)$
  (or $Sp(2)$ or $SO(2)$)
- 2 generation models have chiral exotics, 1 with/without exotics

$T^6/\mathbb{Z}_6$

- 3 generations with additional $U(1)$ (or $Sp(2)$ or $SO(2)$) occurs 5.7×10^6 times, three (H_u, H_d) generations with non-standard Yukawa couplings, only for *one* geometry - *there is only one kind of SM like chiral spectrum!*

	sector	$SU(3)_a \times SU(2)_b$	Q_a	Q_b	Q_c	Q_d	Q_e	Q_{B-L}	Q_Y
Q_L	ab'	$3 \times (\overline{\mathbf{3}}, \mathbf{2})$	-1	-1	0	0	0	$\frac{1}{3}$	$\frac{1}{6}$
U_R	ac	$3 \times (\mathbf{3}, 1)$	1	0	-1	0	0	$-\frac{1}{3}$	$-\frac{2}{3}$
D_R	ac'	$3 \times (\mathbf{3}, 1)$	1	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$
L	bd'	$3 \times (1, \mathbf{2})$	0	1	0	1	0	-1	$-\frac{1}{2}$
E_R	cd	$3 \times (1, 1)$	0	0	1	-1	0	1	1
N_R	cd'	$3 \times (1, 1)$	0	0	-1	-1	0	1	0
H_d	be	$3 \times (1, \mathbf{2})$	0	1	0	0	-1	0	$-\frac{1}{2}$
H_u	be'	$3 \times (1, \mathbf{2})$	0	1	0	0	1	0	$\frac{1}{2}$

- Total # SUSY models *estimated* 3.4×10^{28} — by $\sim 10^5$ larger than $T^6/\mathbb{Z}_6'$ — $\Rightarrow$ SM probability with 1.7×10^{-22} — much smaller than for $T^6/\mathbb{Z}_6'$, $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ — *but distribution of bulk & fractional solutions similar to* $T^6/\mathbb{Z}_6'$

Conclusions

- Geometric intuition for intersecting D6-branes
- Orbifold models: complete spectra computable
- $T^6/\mathbb{Z}_6'$ particularly fertile for SM spectra: $\mathcal{O}(10^{15})$
- 3 generations suppressed by $\sim 10^{-8}$
- SM without chiral exotics exist, PS not fully explored

Open questions

- SM examples without excess of Higgs candidates?
- Results beyond massless spectra: How are interactions for *fractional* brane computed in CFT?
- Realistic values of gauge couplings at low energy??
- SUSY breaking, cosmological constant ...?
- Other orbifolds even more fertile?