SU(3) gauge theories with many massless fermions: methods and mysteries

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Lattice Meets Experiment: Beyond the Standard Model
Boulder, 27 October 2012

arXiv:1207.7162, arXiv:1207.7164 and work in progress
with Anqi Cheng, Anna Hasenfratz and Gregory Petropolous

Thank you!

For your participation

Contributions to the workshop, white paper and community planning

For your patience

As I say a few words about a small portion of our ongoing explorations

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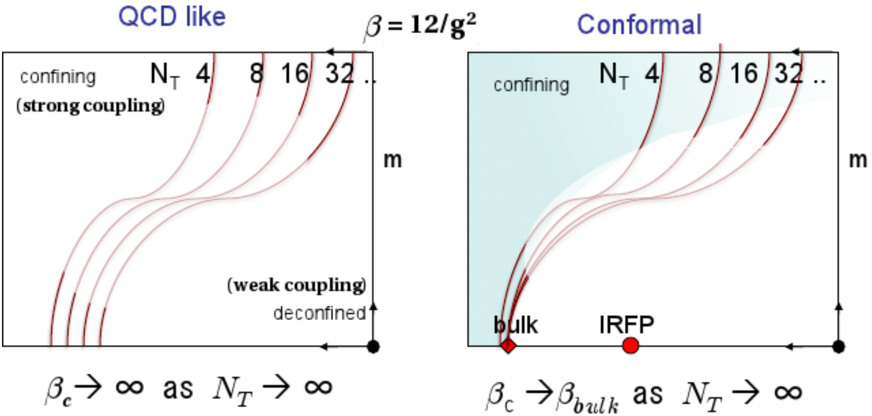
$N_F = 8$ widely believed to be *well below* conformal window

We are currently investigating $N_f = 8$ systems **directly at zero mass**
On this critical surface:

1. Finite-temperature phase transitions don't exhibit QCD-like scaling
2. Eigenvalues provide access to scale-dependence of $\gamma_m(\mu)$
   which also shows clearly non-QCD behavior for $N_F = 8$

Qualitative expectations for the lattice phase diagram

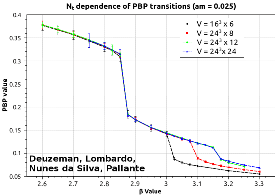
Fermion mass vs. gauge coupling – critical surface is $m \to 0$ chiral limit



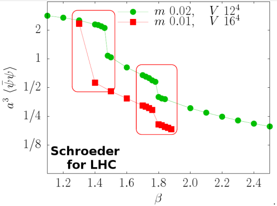
Hope for clear distinction between QCD-like and conformal cases from scaling $\Delta\beta$ of finite-temperature transitions as N_T increases

Of course, it's not that simple

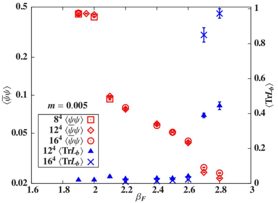
Several groups find novel **intermediate** phase (for $N_F = 12$)



Deuzeman, Lombardo, Nunes da Silva, Pallante



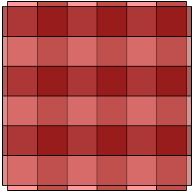
Schroeder for LHC



Suspect intermediate phase has no continuum limit

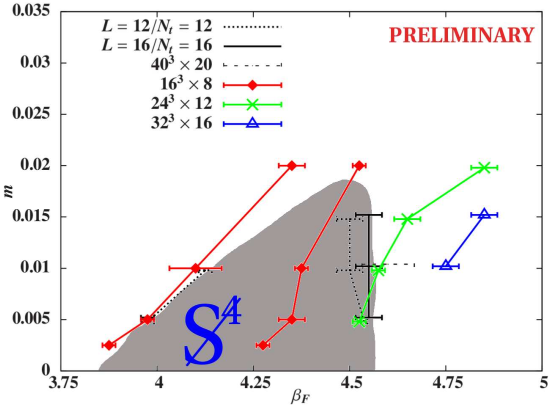
Below I refer to our observation (for $N_F = 8$ and 12)

that it exhibits spontaneous single-site shift symmetry breaking ("$\cancel{S^4}$")



$N_F = 8$ at non-zero mass (as of June)

$T > 0$ transitions pass through bulk transitions surrounding $\not{S}^4$ phase

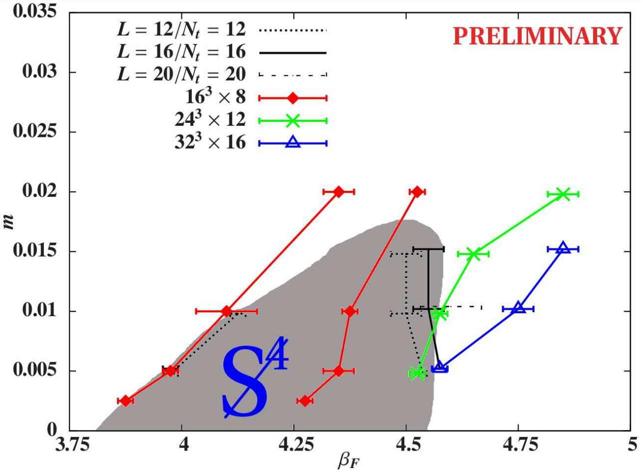


Observe chiral symmetry breaking in systems
between $\not{S}^4$ phase and deconfinement at weak coupling (large β_F)

$\Delta\beta_F$ agrees with two-loop prediction $\Delta\beta_F \approx 0.25$ for $N_T = 12 \rightarrow 16$

$N_F = 8$ at non-zero mass (as of July)

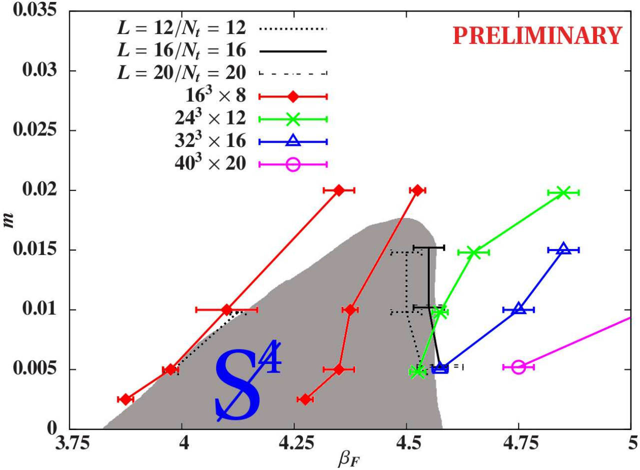
$N_T = 16$ transitions run into the $\not{S}^4$ phase at $m = 0.005$



We lose scaling with $N_T = 12 \rightarrow 16$ as we approach the chiral limit

$N_F = 8$ at non-zero mass

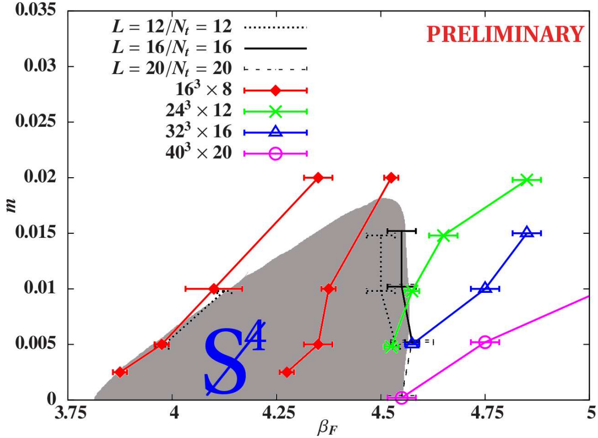
At $m \gtrsim 0.005$, still have scaling with $N_T = 16 \to 20$



Two-loop prediction is now $\Delta\beta_F \approx 0.2$ for $N_T = 16 \to 20$

$N_F = 8$ at **zero** mass

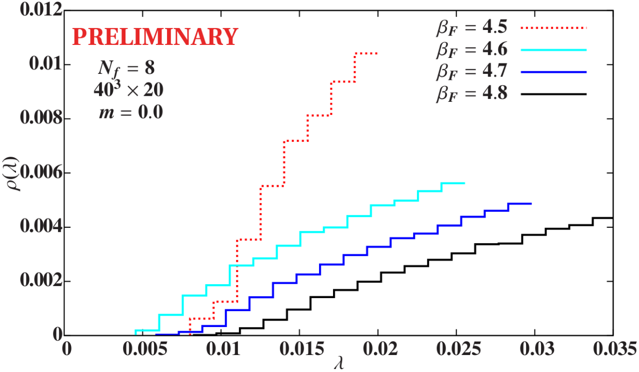
We can study $40^3 \times 20$ (and $24^3 \times 48$) **directly** at $m = 0$
on both sides of transition into $\cancel{S}^4$ phase



Again lose scaling with N_T, on the $m = 0$ critical surface

Closer look at $40^3 \times 20$ transition with $m = 0$

Eigenvalue densities (histogram) $\rho(\lambda)$ of **massless** Dirac operator
Strong couplings produce smaller λ, until we hit the $\cancel{S^4}$ phase
None of these systems are chirally broken **:** $\langle\overline{\psi}\psi\rangle \propto \rho(0) = 0$



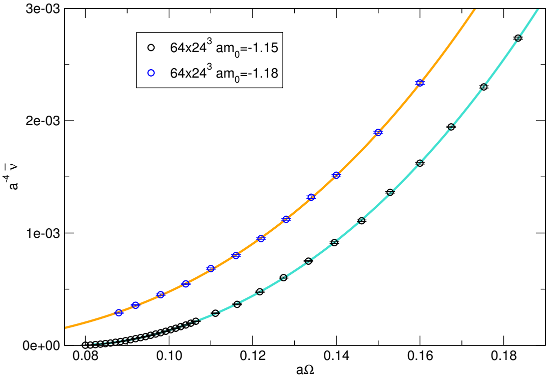
In addition to $\langle\overline{\psi}\psi\rangle$, $\rho(\lambda)$ also related to anomalous dimension γ_m

γ_m from eigenvalue mode number $\nu(\lambda)$

Del Debbio & Zwicky, arXiv:1005.2371

In the chiral limit $\rho(\lambda) \sim \lambda^{\alpha} \quad \Longrightarrow \quad \nu(\lambda) = V \int_{-\lambda}^{\lambda} \rho(\omega) d\omega \sim V \lambda^{1+\alpha}$

RG invariance of **mode number** $\nu(\lambda) \Longrightarrow 1 + \gamma_m = \dfrac{4}{1+\alpha}$



Patella, arXiv:1204.4432

SU(2), $N_F = 2$ adjoint
believed IR-conformal

$\gamma_\star = 0.371(20)$
for fit range $[0.091, 0.18]$

Inspired us to look at $\nu(\lambda)$

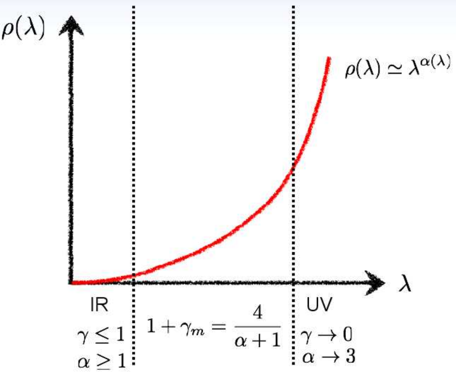
Expectations for eigenvalue mode number analysis

λ defines an energy scale;
fitting $\nu(\lambda) \propto \lambda^{1+\alpha(\lambda)}$ accesses $1 + \gamma_m(\lambda) = \frac{4}{1+\alpha(\lambda)}$ at that scale

For IR-conformal systems:

UV: Asymp. freedom $\Rightarrow \gamma_m(\lambda) \to 0$
corresponding to $\alpha(\lambda) \to 3$

IR: Fixed point $\Longrightarrow \gamma_m(\lambda) \to \gamma_\star$
$\gamma_\star$ scheme-independent,
expect $\gamma_\star \lesssim 1$



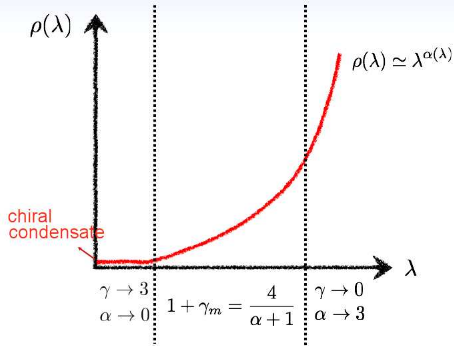
Form of $\rho(\lambda)$ changes from $\rho(\lambda) \propto \lambda^3$ in the UV to $\rho(\lambda) \propto \lambda^{\alpha_\star}$ in the IR

Expectations for eigenvalue mode number analysis

λ defines an energy scale;
fitting $\nu(\lambda) \propto \lambda^{1+\alpha(\lambda)}$ accesses $1 + \gamma_m(\lambda) = \frac{4}{1+\alpha(\lambda)}$ at that scale

For chirally broken systems:
UV: Asymp. freedom $\Rightarrow \gamma_m(\lambda) \to 0$
corresponding to $\alpha(\lambda) \to 3$

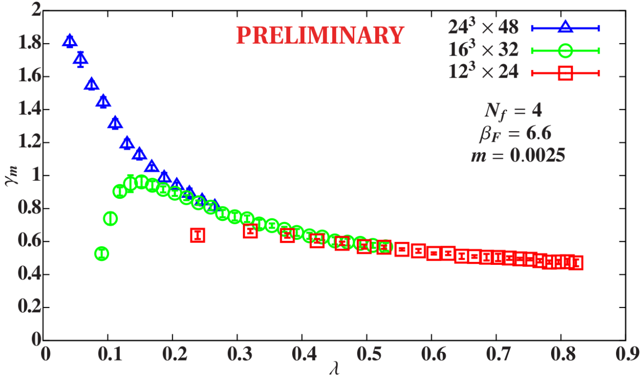
IR: $\langle\overline{\psi}\psi\rangle \propto \rho(0) > 0 \Longrightarrow \alpha(\lambda) \to 0$
would produce "$\gamma_m(\lambda) \to 3$"
but $\rho(\lambda)$ no longer $\sim \lambda^\alpha$



On the lattice we proceed by fitting $\nu(\lambda) \propto \lambda^{1+\alpha}$ in a limited range of λ

$N_F = 4$ runs rapidly : combine several volumes

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in a limited range of λ to find $1 + \gamma_m(\lambda) = \dfrac{4}{1 + \alpha(\lambda)}$



1000 eigenvalues on each volume

Nearby points use overlapping fit ranges

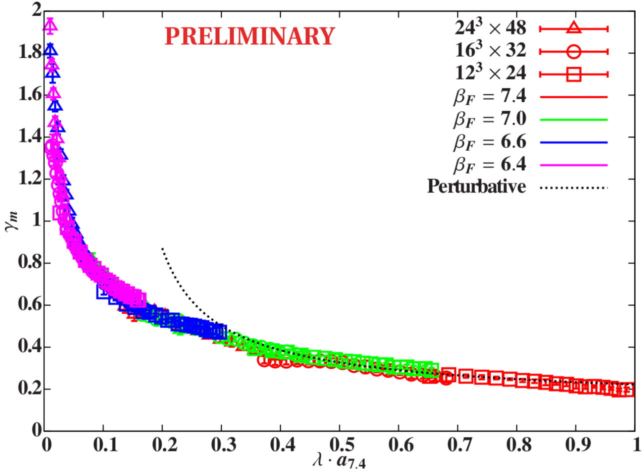
$24^4 \times 48$ system is chirally broken

Focus on overlapping regions where different volumes agree instead of studying finite-volume scaling behavior

Combine multiple couplings and volumes for $N_F = 4$

Rescale $\lambda \to \left(\frac{a_{7.4}}{a}\right)^{1+\gamma_m} \lambda$ to plot in terms of single (smallest) lattice spacing

Match to one-loop perturbation theory at large $\lambda \cdot a_{7.4}$



Relative lattice spacings estimated from Wilson flow & MCRG matching:

$a_{6.6} \approx 2a_{7.4}$

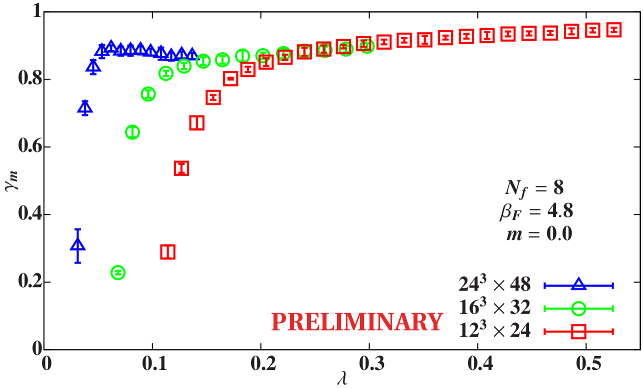
$a_{6.4} \approx 2a_{7.0}$

$a_{6.4} \approx 1.3a_{6.6}$

Finite-volume "tails" omitted

$N_F = 8$ behaves very differently than $N_F = 4$

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in a limited range of λ to find $1 + \gamma_m(\lambda) = \dfrac{4}{1 + \alpha(\lambda)}$

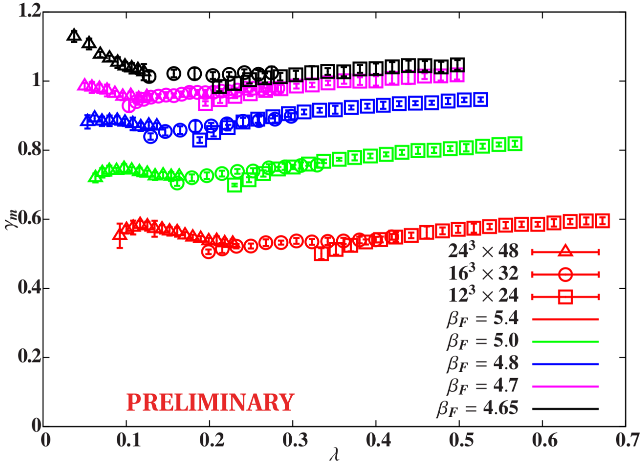


1000 eigenvalues on each volume

$m = 0$, all have $\rho(0) = 0$

In overlapping regions γ_m roughly independent of λ at fixed coupling β_F
No sign of asymptotic freedom – may be slight increase for larger λ

Cannot combine multiple couplings for $N_F = 8$



Relative
lattice spacings
not yet estimated
(not rescaling λ)

Finite-volume
“tails” omitted

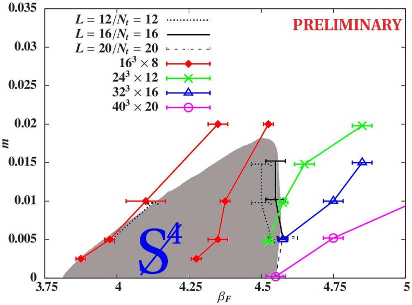
γ_m remains roughly independent of λ in overlapping regions,
increases for stronger couplings until $\cancel{S^4}$ phase at $\beta \lesssim 4.65$
Clear contrast with $N_F = 4$

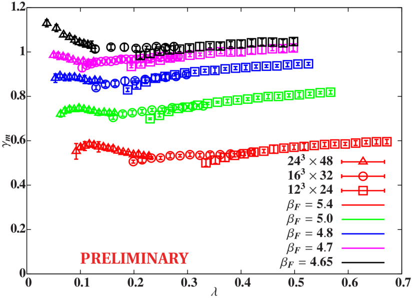
Recapitulation

$N_F = 8$ seems interesting, eigenvalues seem promising

Finite-temperature transitions on the $m = 0$ critical surface
show no scaling up to $N_T = 20$

$\gamma_m(\mu)$ accessible from eigenvalue mode number
Clear contrast between $N_F = 4$ and 8, latter more sensitive to β than λ





Thank you!

Thank you!

Collaborators

Anqi Cheng, Anna Hasenfratz, Gregory Petropolous

Funding and computing resources

DEPARTMENT OF ENERGY · UNITED STATES OF AMERICA

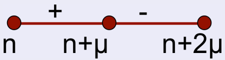
USQCD

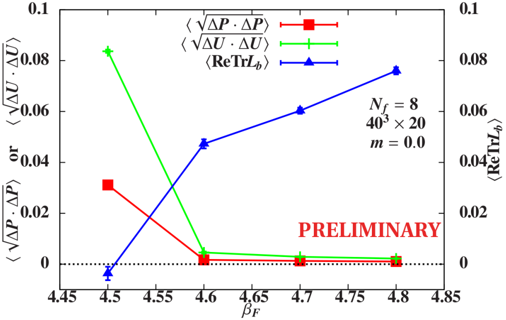
Backup: $\not{S}^4$ order parameters

Differences of plaquettes $\square$ or links $\overline{\chi}U\chi$

$$\Delta P_\mu = \langle \mathrm{ReTr}\,\square_{n,\mu} - \mathrm{ReTr}\,\square_{n+\mu,\mu}\rangle_{n_\mu\ \text{even}}$$

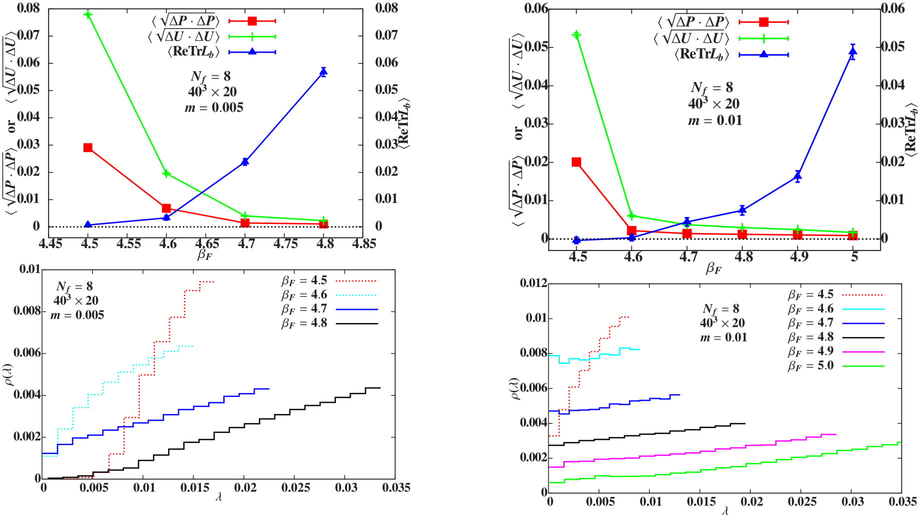
$$\Delta U_\mu = \langle \alpha_{\mu,n}\overline{\chi}_n U_{\mu,n}\chi_{n+\mu} - \alpha_{\mu,n+\mu}\overline{\chi}_{n+\mu} U_{\mu,n+\mu}\chi_{n+2\mu}\rangle_{n_\mu\ \text{even}}$$





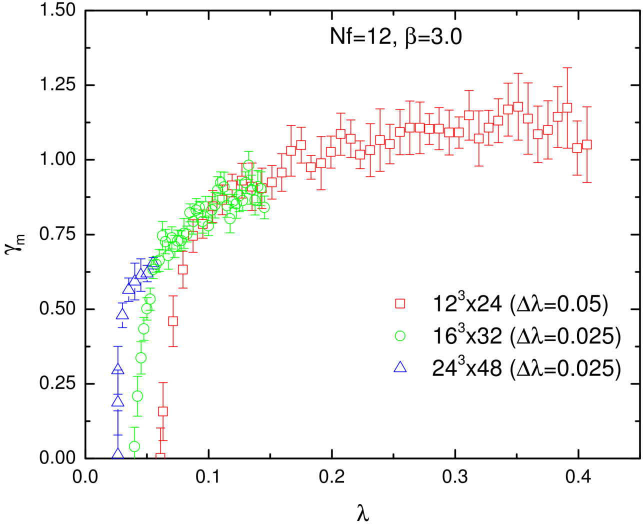
$N_F = 8$ results with $m = 0$ confirm signals in eigenvalue densities

Backup: $N_F = 8$ transitions for $40^3 \times 20$ with $m > 0$



Between transitions, chirally broken systems with $\rho(0) > 0$

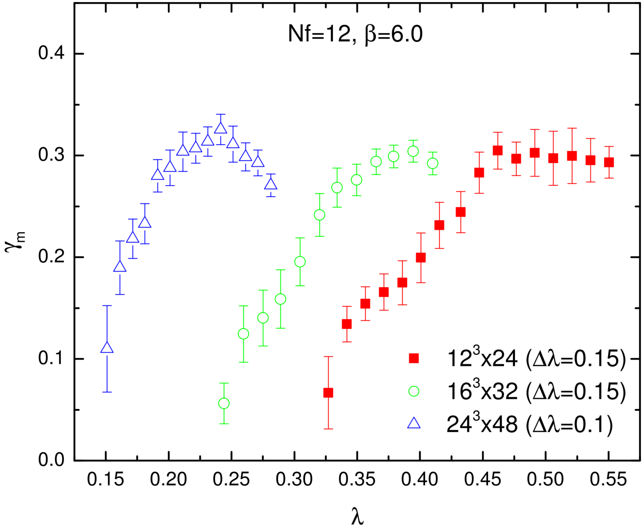
Backup: γ_m from eigenvalues for $N_F = 12$



$m = 0.0025$

At strong coupling (near $\cancel{S^4}$ phase) γ_m clearly increases with λ

Backup: γ_m from eigenvalues for $N_F = 12$



$m = 0.0025$

At weaker coupling, $\gamma_m \approx 0.3$ – better overlap needed