Cataloguing Fermionic Gauge Theories

Lattice Meets Experiments:

Beyond the Standard Model

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Outline

- ❑ Ladder approximation.
- ❑ Supersymmetric conformal window: Seiberg.
- ❑ Supersymmetric conformal window: Three loops in DRbar scheme.
- ❑ Non-SUSY: All-orders beta function.
- ❑ Non-SUSY: Four loops.
- ❑ Multiple representations.
- ❑ Exceptional groups and spinorial representations.

Hunt down the conformal window!

Scheme (in)dependence

- Transformation between two schemes: S and S'

$$\alpha' = \sum_{n=1}^{\infty} h_n \alpha^n, \qquad h_1 = 1 \qquad\qquad \beta'(\alpha') = \frac{\partial \alpha'}{\partial \alpha} \beta(\alpha)$$

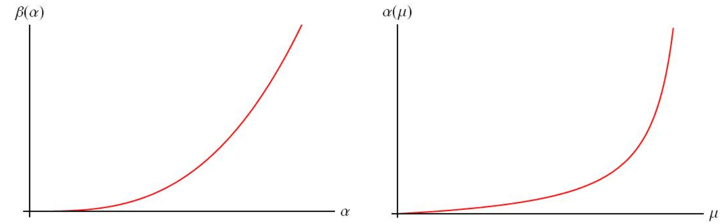
$$m' = m z_m(\alpha) = m \sum_{n=0}^{\infty} l_n \alpha^n, \qquad l_0 = 1 \qquad\qquad \gamma'(\alpha') = \gamma(\alpha) + \frac{\partial \ln z_m(\alpha)}{\partial \alpha} \beta(\alpha)$$

- The first two coefficients of the beta function and the first coefficient of the anomalous dimension are scheme independent.
- The existence of a zero of the beta function is scheme independent.
- The value of the anomalous dimension at a fixed point is scheme independent.

Two loop beta function

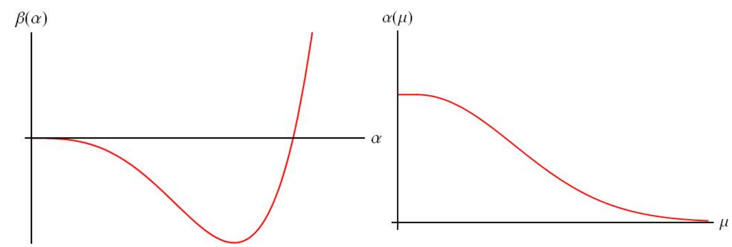
- Non-Asymptotic freedom:

  Large N_f - $\beta_0<0$



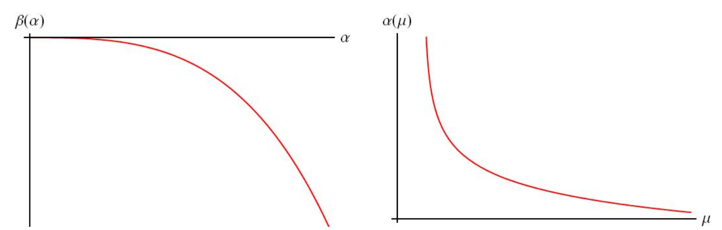
- IR fixed point:

  Intermediate N_f - $\beta_0>0$ and $\beta_1<0$

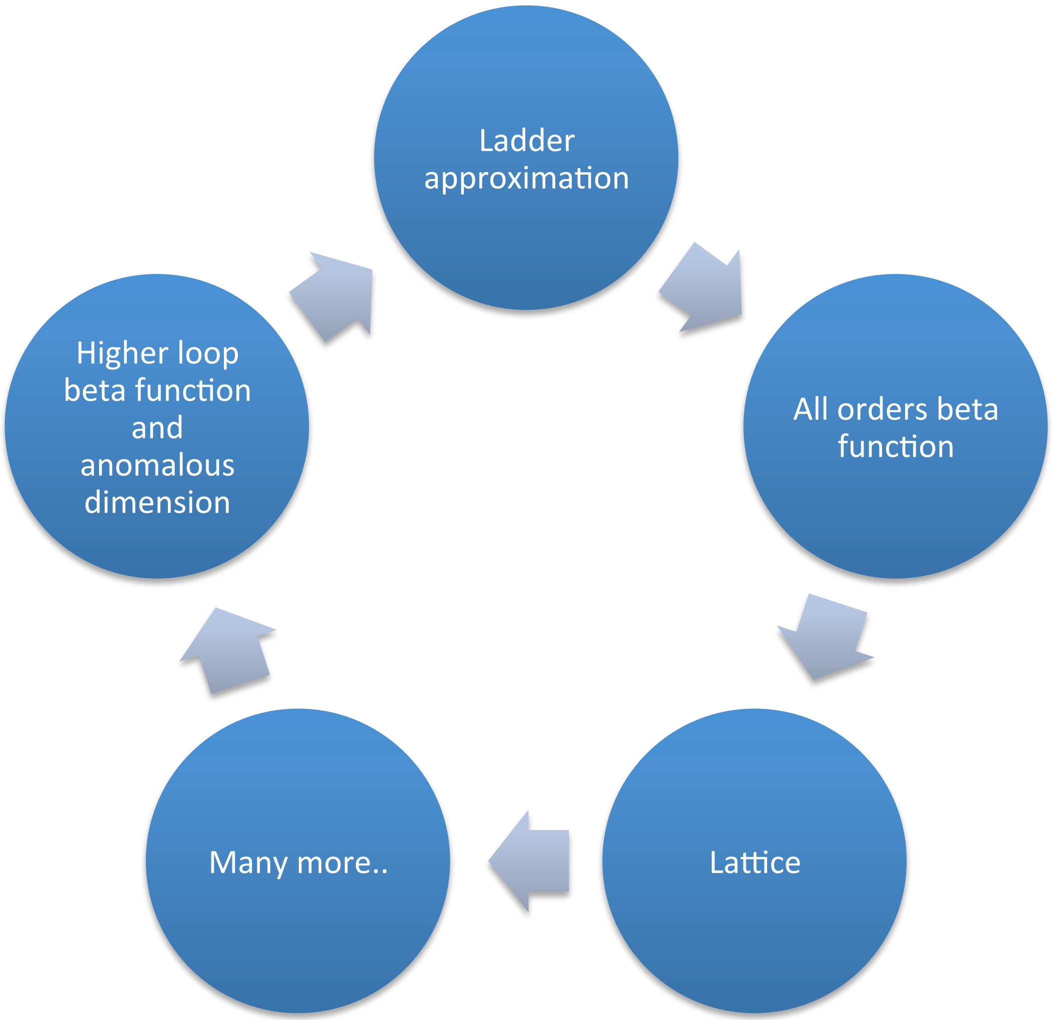


- QCD-like:

  Small N_f - $\beta_0>0$ and $\beta_1>0$



Methods and Techniques



Methods and Techniques

How well do they work??

Ladder Approximation

- Two loop fixed-point coupling

$$\alpha_{IR} = -4\pi \frac{\beta_0}{\beta_1}$$

- α_{IR} becomes large as $\beta_1 \rightarrow 0$

- Chiral symmetry breaking could be triggered before α_{IR} is reached.

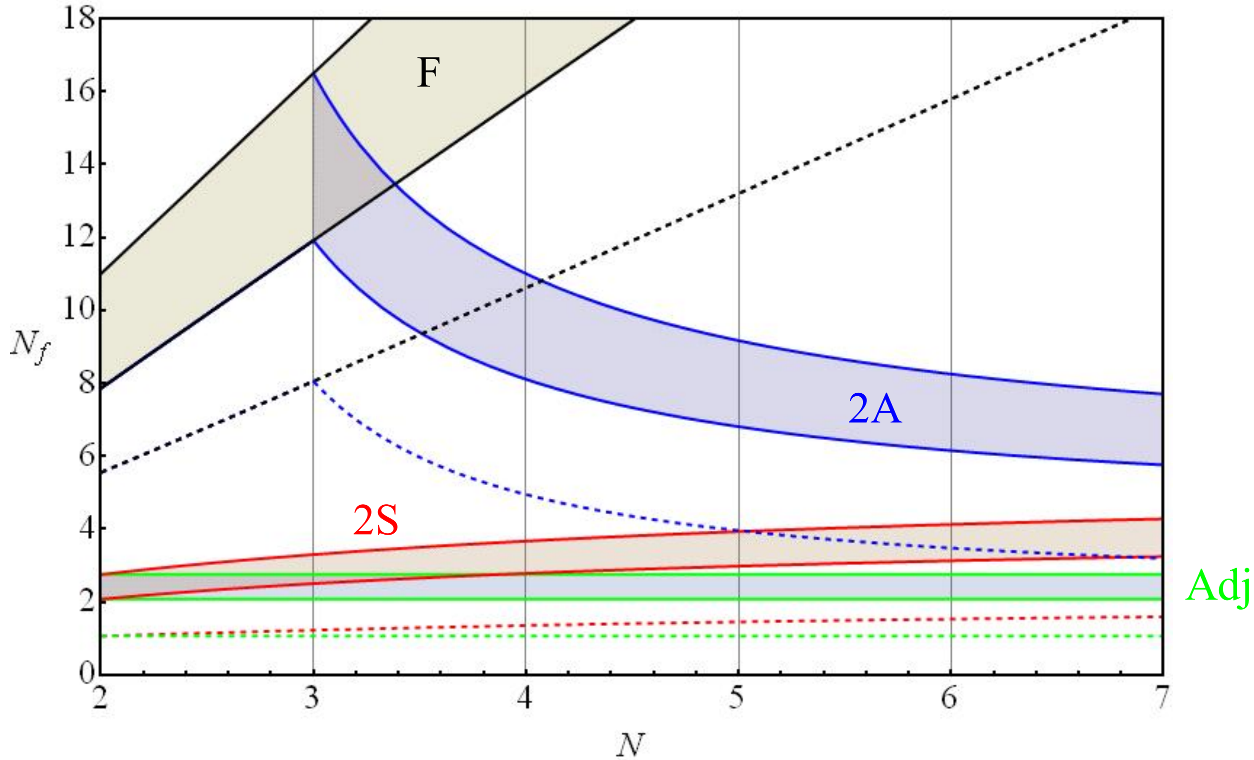
- The gap equation has a solution for the dynamically generated mass when the coupling reaches the value

$$\alpha_c = \frac{\pi}{3C_2(r)}$$

- Critical number of flavors

$$\alpha_c = \alpha_{IR} \qquad \Rightarrow \qquad N_f = \frac{17C_2(G) + 66C_2(r)}{10C_2(G) + 30C_2(r)} \frac{C_2(G)}{T(r)}$$

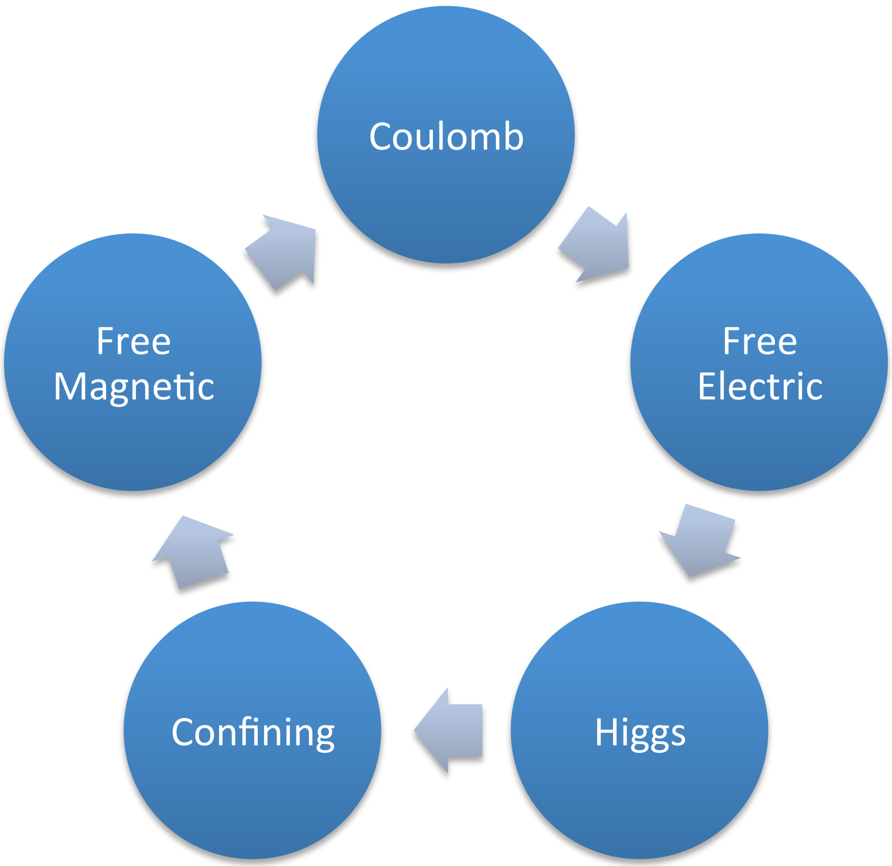
Phase Diagram (ladder approximation)



- Ladder approximation: Chiral symmetry breaking is triggered at

$$\gamma \sim 1$$

Supersymmetric QCD



Lessons from SQCD

- Exact NSVZ beta function

$$\beta(\alpha) = -\frac{\alpha^2}{2\pi}\frac{\beta_0 - 2T(r)N_f\gamma(\alpha)}{1-\frac{\alpha}{2\pi}C_2(G)}$$

← NSVZ beta function

$$\gamma(\alpha) = C_2(r)\frac{\alpha}{\pi} + O(\alpha^2)$$

← Anomalous dimension

$$\beta_0 = 3C_2(G) - 2T(r)N_f$$

← First beta function coefficient

- Note: relation between beta function and anomalous dimension.

- At zero of beta funtion

$$\gamma = \frac{3C_2\left(G\right) - 2T\left(r\right)N_f}{2T\left(r\right)N_f}$$

- Conformality requires

$$D(\Phi\tilde{\Phi}) = 2 - \gamma \geq 1$$

- Critical number of flavors

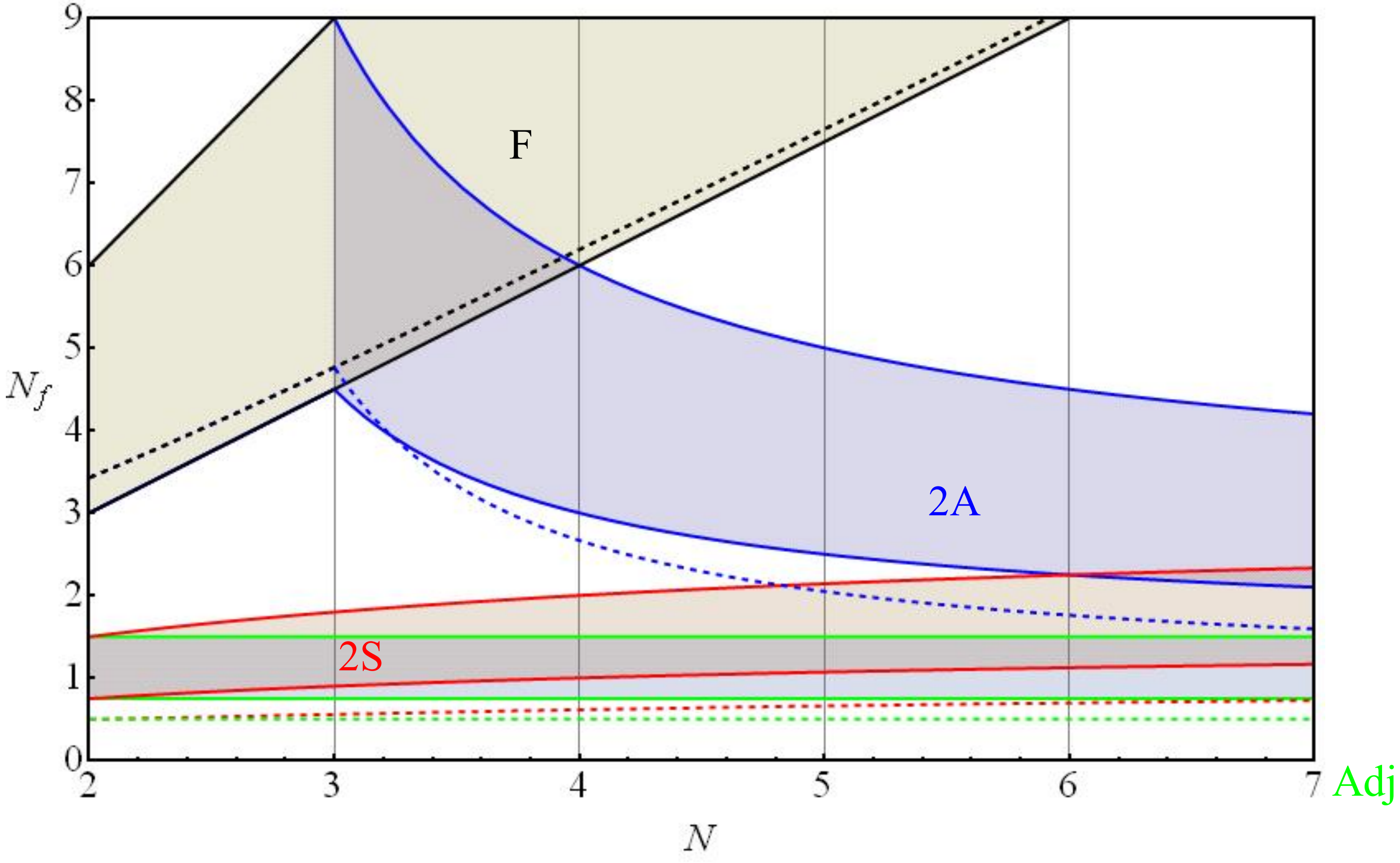
$$N_f^{critical} = \frac{3}{4}\frac{C_2(G)}{T(r)}$$

- Conformal window

Asymptotic freedom

$$\frac{3}{4}\frac{C_2(G)}{T(r)} < N_f^{critical} < \frac{3}{2}\frac{C_2(G)}{T(r)}$$

SUSY conformal window



- Can we match other approximate techniques to the exact results of Seiberg?
- The ladder approximation does not do a good job.

SUSY three loop beta function and anomalous dimension

- Three loop beta function and anomalous dimension are known in DRbar scheme

$$\beta_{3-loop}\left(\alpha\right) = -\beta_0 \frac{\alpha^2}{\left(2\pi\right)^1} - \beta_1 \frac{\alpha^3}{\left(2\pi\right)^2} - \beta_2 \frac{\alpha^4}{\left(2\pi\right)^3} + O(\alpha^5)$$

$$\gamma_{3-loop}\left(\alpha\right) = \gamma_0 \left(\frac{\alpha}{\pi}\right) + \gamma_1 \left(\frac{\alpha}{\pi}\right)^2 + \gamma_2 \left(\frac{\alpha}{\pi}\right)^3 + O(\alpha^4)$$

- Search for an infrared fixed point of the beta function.
- Evaluate the anomalous dimension at the infrared zero of the beta function.

Three loop beta function

- Fundamental representation

$3 < N_f^{Seiberg} < 6$

N	Nf	α_{2l}	α_{3l}
2	4	6.28	2.65
2	5	1.14	0.898

$4.5 < N_f^{Seiberg} < 9$

N	Nf	α_{2l}	α_{3l}
3	5	18.85	3.05
3	6	2.69	1.40
3	7	0.992	0.734
3	8	0.343	0.308

Three loop anomalous dimension

- Fundamental representation

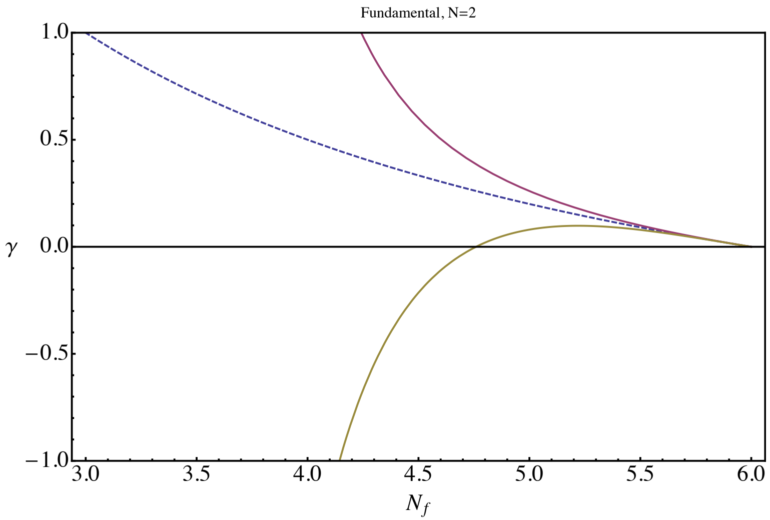
$3 < N_f^{Seiberg} < 6$

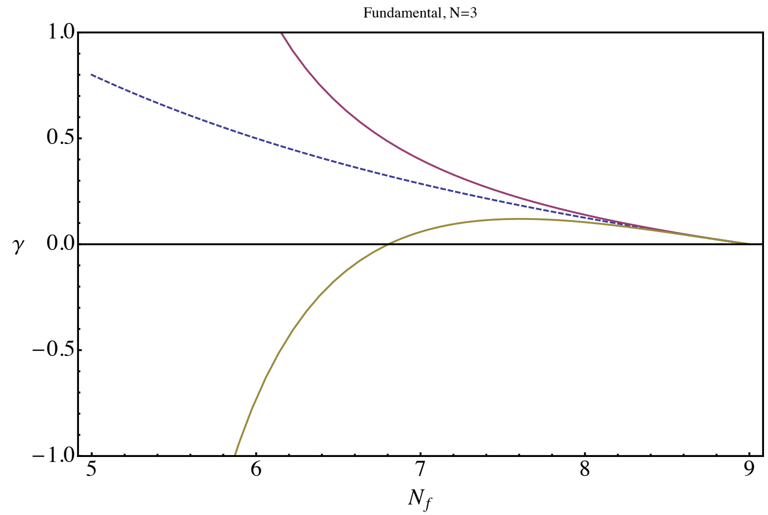
N	Nf	$\gamma_{2l}(\alpha_{2l})$	$\gamma_{3l}(\alpha_{3l})$
2	4	1.875	-1.68
2	5	0.260	0.0802

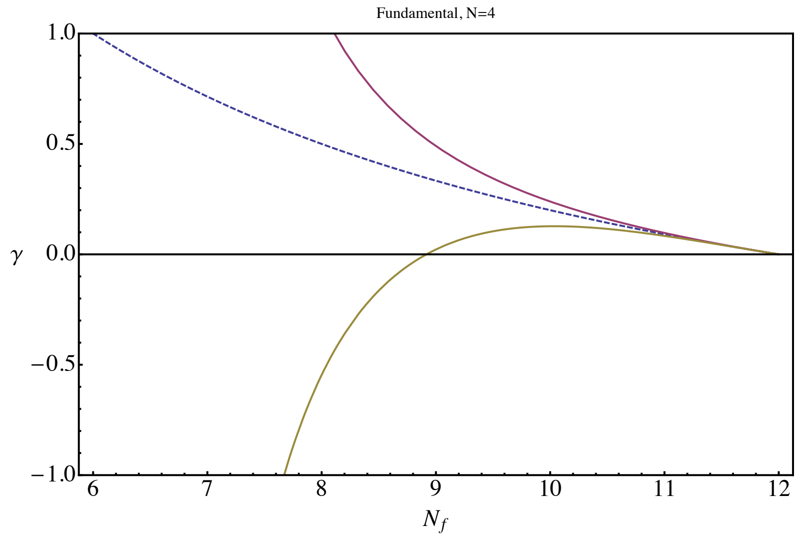
$4.5 < N_f^{Seiberg} < 9$

N	Nf	$\gamma_{2l}(\alpha_{2l})$	$\gamma_{3l}(\alpha_{3l})$
3	6	1.22	-0.730
3	7	0.399	0.0584
3	8	0.139	0.104

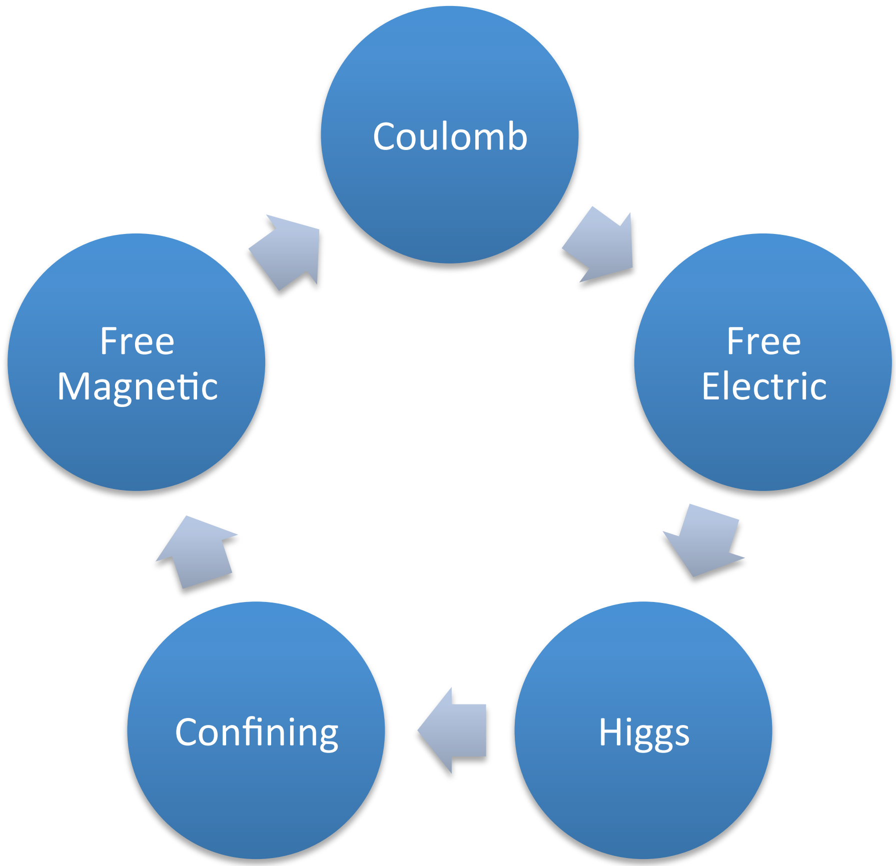
SUSY anomalous dimension vs. number of flavors







QCD



All-orders Beta Function

$$\beta(\alpha) = -\frac{\alpha^2}{2\pi}\frac{a + bN_f\gamma(\alpha)}{1 - \frac{\alpha}{2\pi}c}$$

$$\gamma(\alpha) = 3C_2(r)\frac{\alpha}{2\pi} + O(\alpha^2)$$

Anomalous dimension

- Matching to two loop beta function

$$a = \beta_0$$

$$\beta_0 c + 3C_2(r)N_f b = \beta_1$$

- Has same form as NSVZ.

All-orders Beta Function

- Assume b and c do not depend on the number of flavors

$$b = \frac{\beta_1\left(\bar{N}_f\right)}{3C_2(r)\bar{N}_f}$$

$$c = \frac{\beta_1^{YM}}{\beta_0^{YM}}$$

- With $\bar{N}_f$ being the critical number of flavors for which asymptotic freedom is lost

$$\beta(\alpha) = \frac{\alpha^2}{2\pi} \frac{\beta_0 + \frac{\beta_1\left(\bar{N}_f\right)}{3C_2(r)\bar{N}_f} N_f \gamma(\alpha)}{1 - \frac{\alpha}{2\pi}\frac{\beta_1^{YM}}{\beta_0^{YM}}}$$

- With these coefficents the anomalous dimension appearing in $\beta(\alpha)$at a Banks-Zaks fixed point coincides with the perturbative value.

Bounds on Conformal Window

- Analysis similar to SUSY case:

  – At zero of beta function

$$\gamma = -\frac{\beta_0\left(N_f\right)\bar{N}_f}{\beta_1\left(\bar{N}_f\right)N_f} 3C_2\left(r\right)$$

  – Conformality requires

$$D(\tilde{\psi}\psi) = 3 - \gamma \geq 1 \qquad \Rightarrow \qquad \gamma \leq 2$$

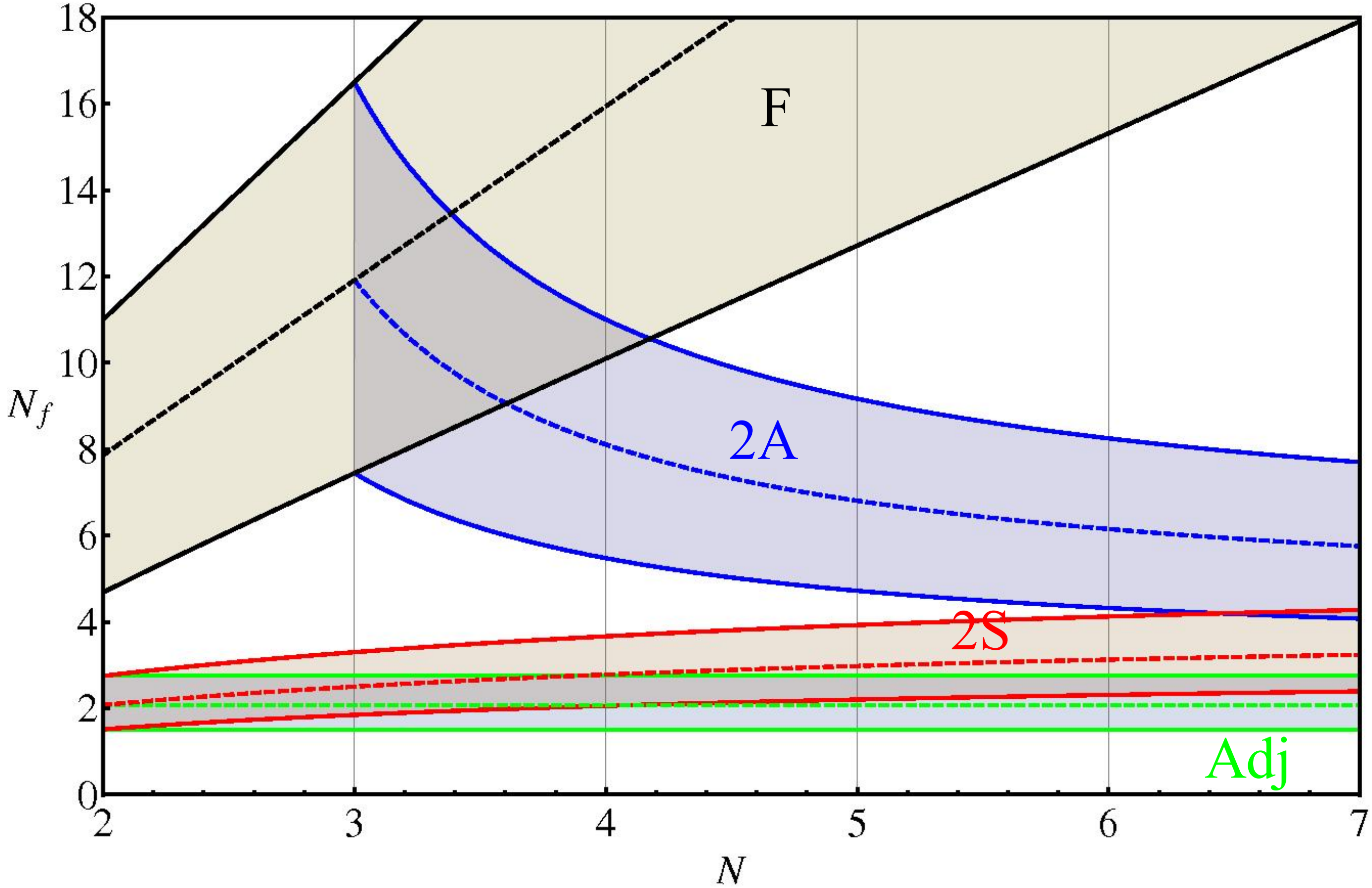
  – Ladder approximation

$$\gamma = 1$$

  – Critical number of flavors

$$N_f = \frac{121 C_2(G) C_2(r)}{2\left(7C_2(G) + 33C_2(r)\right)T\left(r\right)}$$

Phase Diagram



Four loop beta function and anomalous dimension

- Four loop beta function and anomalous dimension are known in MSbar scheme

$$\beta_{4-loop}(\alpha) = -\beta_0 \frac{\alpha^2}{(2\pi)^1} - \beta_1 \frac{\alpha^3}{(2\pi)^2} - \beta_2 \frac{\alpha^4}{(2\pi)^3} - \beta_3 \frac{\alpha^5}{(2\pi)^4} + O(\alpha^6)$$

$$\gamma_{4-loop}(\alpha) = \gamma_0 \left(\frac{\alpha}{\pi}\right) + \gamma_1 \left(\frac{\alpha}{\pi}\right)^2 + \gamma_2 \left(\frac{\alpha}{\pi}\right)^3 + \gamma_3 \left(\frac{\alpha}{\pi}\right)^4 + O(\alpha^5)$$

- New group invariants enter at the four loop level.

- Search for an infrared fixed point of the beta function.
- Evaluate the anomalous dimension at the infrared zero of the beta function

Four loop beta function

- Fundamental representation

N	N_f	α_{2l}	α_{3l}	α_{4l}
2	8	1.26	0.688	0.760
2	9	0.595	0.418	0.444
2	10	0.231	0.196	0.200

N	N_f	α_{2l}	α_{3l}	α_{4l}
3	10	2.21	0.764	0.815
3	11	1.23	0.578	0.626
3	12	0.754	0.435	0.470
3	13	0.468	0.317	0.337

Four loop beta function

- Adjoint representation

N	N_f	α_{2l}	α_{3l}	α_{4l}
2	2	0.628	0.459	0.493
3	2	0.419	0.306	0.323
4	2	0.314	0.2295	0.241

- 2-indexed symmetric representation

N	N_f	α_{2l}	α_{3l}	α_{4l}
3	2	0.842	0.500	0.522
3	3	0.085	0.079	0.080

Four loop anomalous dimension

- Fundamental representation

N	N_f	$\gamma_{2l}(\alpha_{2l})$	$\gamma_{3l}(\alpha_{3l})$	$\gamma_{4l}(\alpha_{4l})$
2	8	0.752	0.272	0.204
2	9	0.275	0.161	0.157
2	10	0.0910	0.0738	0.0748

N	N_f	$\gamma_{2l}(\alpha_{2l})$	$\gamma_{3l}(\alpha_{3l})$	$\gamma_{4l}(\alpha_{4l})$
3	11	1.61	0.439	0.250
3	12	0.773	0.312	0.253
3	13	0.404	0.220	0.210

Four loop anomalous dimension

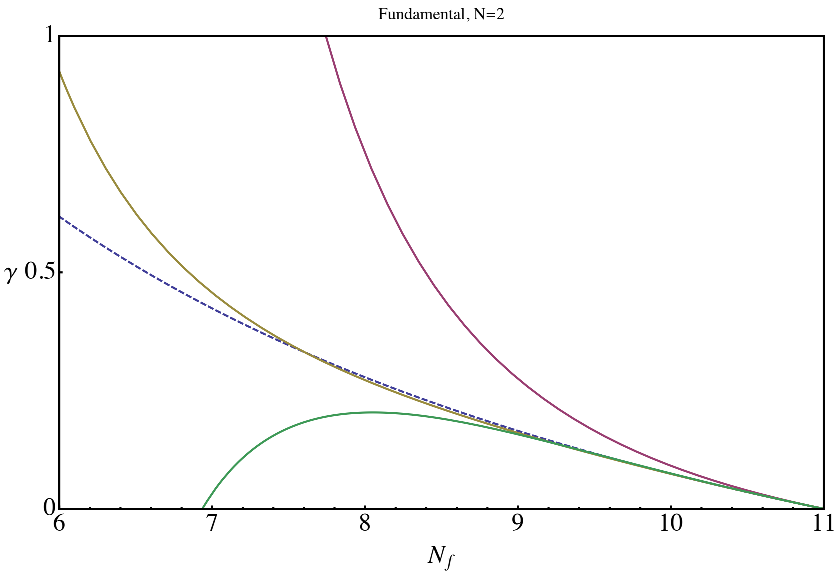
- Adjoint representation

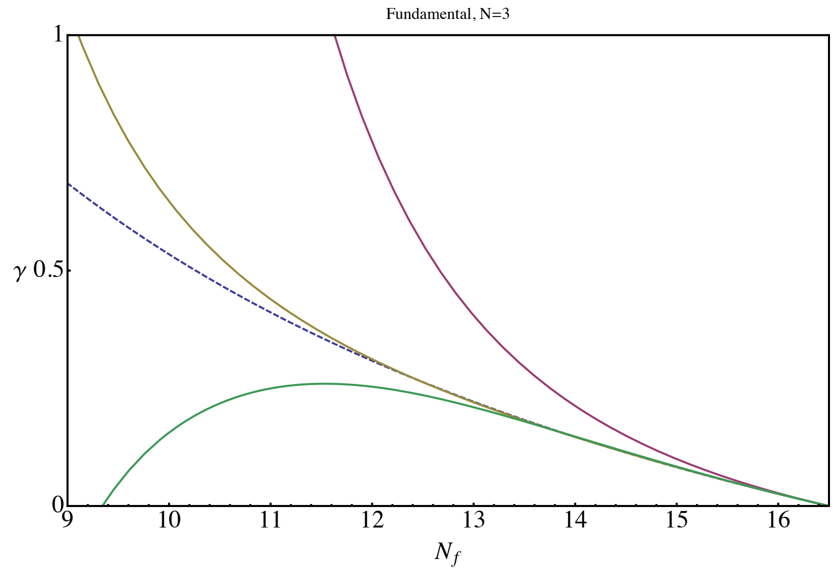
N	N_f	$\gamma_{2l}(\alpha_{2l})$	$\gamma_{3l}(\alpha_{3l})$	$\gamma_{4l}(\alpha_{4l})$
2	2	0.820	0.543	0.500
3	2	0.820	0.543	0.523
4	2	0.820	0.543	0.532

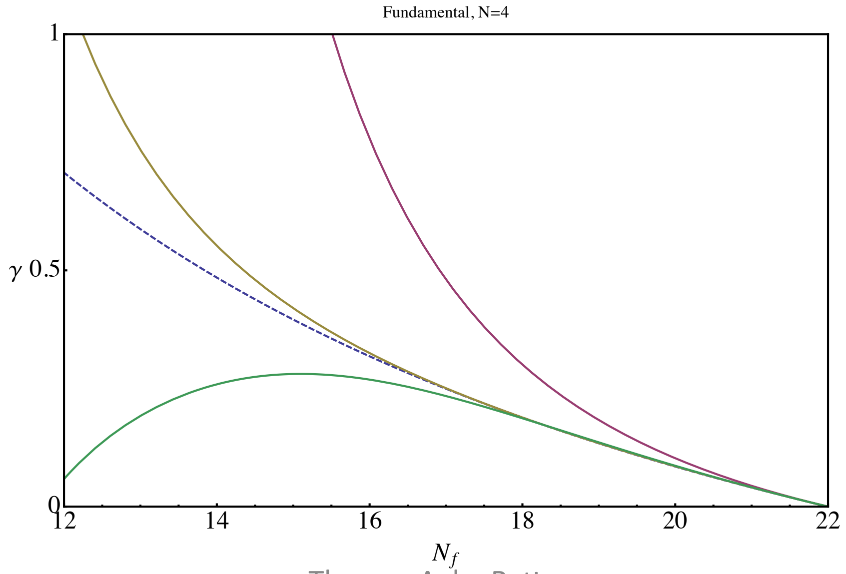
- 2-indexed symmetric representation

N	N_f	$\gamma_{2l}(\alpha_{2l})$	$\gamma_{3l}(\alpha_{3l})$	$\gamma_{4l}(\alpha_{4l})$
3	2	2.44	1.28	1.12
3	3	0.144	0.133	0.133

Anomalous dimension vs. number of flavors







Special Theories

- N=3 and the fundamental representation
  - ❑ N_f =8 breaks chiral symmetry $\gamma_{\beta-\text{function}} = 0.87 \quad \gamma_{4-loop} = NA$
  - ❑ N_f =12 ?? $\gamma_{\beta-\text{function}} = 0.31 \quad \gamma_{4-loop} = 0.253$

- N=2 and the adjoint representation
  - ❑ N_f =2 is conformal $\gamma_{\beta-\text{function}} = 0.46 \quad \gamma_{4-loop} = 0.500$

- N=3 and the 2-indexed symmetric representation
  - ❑ N_f =2 ?? $\gamma_{\beta-\text{function}} = 0.83 \quad \gamma_{4-loop} = 1.12$

Multiple Representations

- Kuti: Theory space is enormous.
- Construct walking technicolor models with multiple representations: Smallest possible naive S parameter + smallest number of additional fermions.
- Other famous examples:

1) SU(5) grand unified theory: $\overline{5}+10$

2) MSSM: Adj +F

Beta Function – Multiple Representations

$$\beta(\alpha) = -\frac{\alpha^2}{2\pi}\frac{a + \sum_{i=1}^{k} b(r_i) N_f(r_i)\gamma_i(\alpha)}{1 - \frac{\alpha}{2\pi}c}$$

$$\gamma_i(\alpha) = 3C_2(r_i)\frac{\alpha}{2\pi} + O(\alpha^2)$$

Anomalous dimensions

- Consider the zero of the beta function. Bound the conformal house by bounding the values of the anomalous dimensions.

Ladder Approximation – Multiple Representations

- There is a critical coupling associated to the triggering of each of the condensates

$$\alpha_c(r_1) = \frac{\pi}{3C_2(r_1)} \qquad \alpha_c(r_2) = \frac{\pi}{3C_2(r_2)}$$

- Assume: $C_2(r_1) > C_2(r_2)$ Conformal House: $\alpha_c(r_1) = \alpha_{IR}(r_1, r_2)$

Multiple Representations

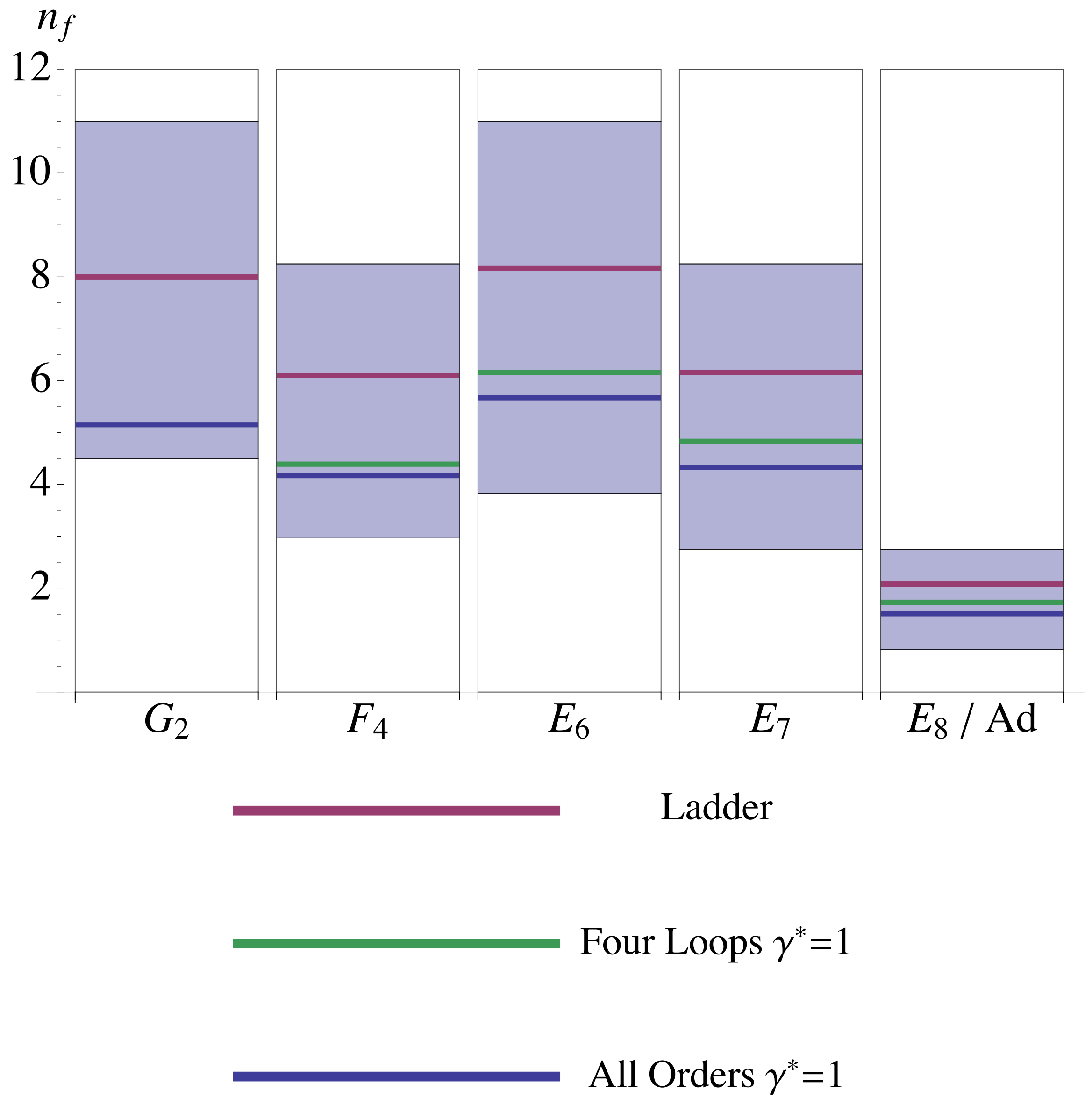
- Gauge charges:

	$SU(2)_{TC}$	$SU(2)_L$	$U(1)_Y$
$(U_L, D_L)^T$	2	2	0
U_R	$\bar{2}$	1	½
D_R	$\bar{2}$	1	-½
λ^f	Adj	1	0

- Global symmetry:

$$SU(4) \times SU(2) \times U(1) \quad \rightarrow \quad Sp(4) \times SO(2) \times Z_2$$

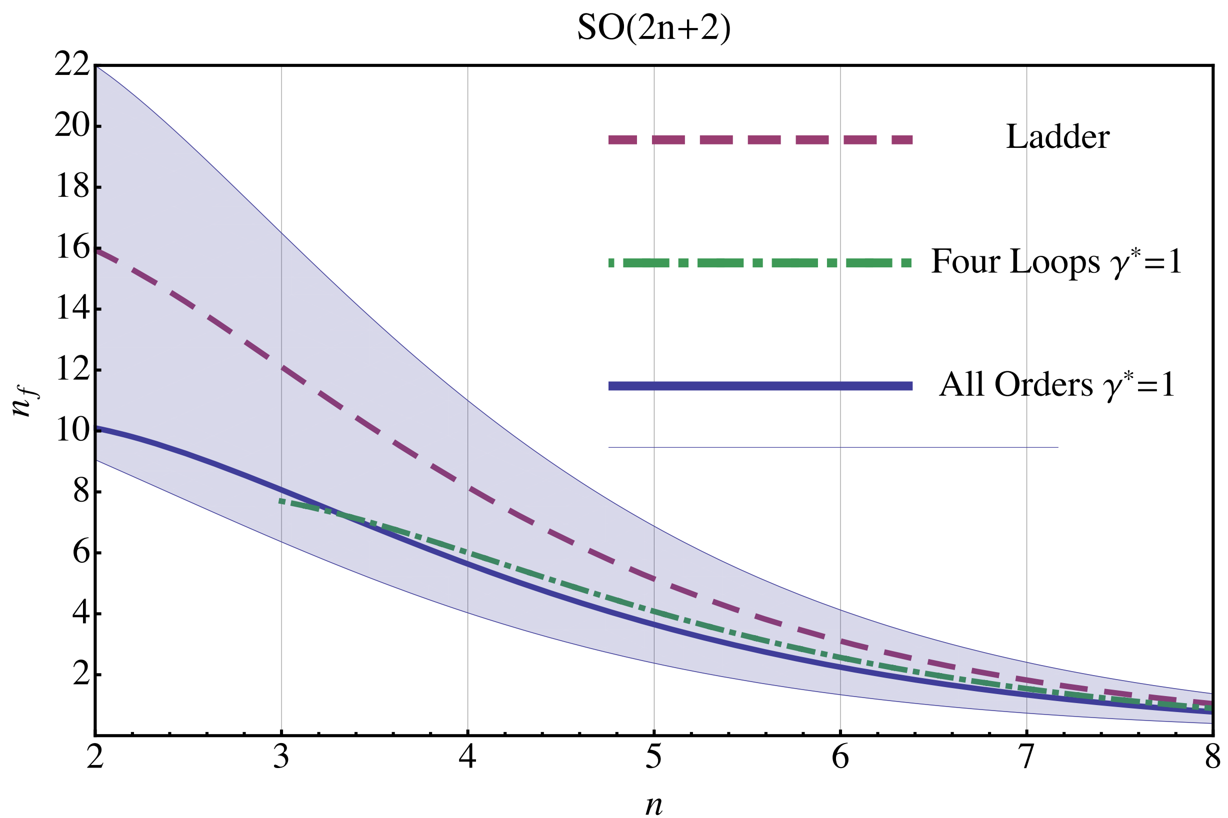
Exceptional Groups (Fundamental)



Exceptional Groups (Adjoint)

Adj	G_2	F_4	E_6	E_7	E_8
Asym Free	2.75	2.75	2.75	2.75	2.75
4 loops	1.70	1.72	1.73	1.73	1.73
All orders	1.51	1.51	1.51	1.51	1.51
Ladder	2.08	2.08	2.08	2.08	2.08

Spinorial Representations



- Similiar for N odd.

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