A (very) light dilaton

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with
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Based on

Distinguishing the Higgs boson from the dilaton at the Large Hadron Collider.
Walter D. Goldberger, BG and Witold Skiba, Phys.Rev.Lett. 100 (2008) 111802

A Very Light Dilaton.
BG and Patipan Uttayarat, JHEP 1107 (2011) 038

Dilatons in Conformal Perturbation Theory

A perfectly reasonable program:

- Assume SM is “embedded” in a CFT
- SM couplings ⇒ CFT perturbations
- CFT spontaneously (and explicitly) broken ⇒ pseudo-dilaton
- Use non-linear realization of spontaneously broken scale invariance
- Determine couplings of dilaton to SM fields:
  - Couples, at leading order, to mass/f where f = dilaton decay constant
  - Much like higgs would couple, but $f \geq v$ so couplings may be suppressed (not a surprise, in SM higgs *is* a pseudo-dilaton).
  - Trace-anomaly mediated couplings: non-decoupling, deviations from SM
- Study phenomenology
  - Reproduces WW, ZZ, bb, widths provided $f \approx v$
  - Alternatively can enhance gg (production) and decrease width by v/f
  - Can easily accommodate larger 2-photon higgs decay rate
  - Fares *no worse than SM higgs* in failure to explain absence of $\tau\tau$ mode
  - Measuring production rates for alternate mechanisms would settle the question

Maybe the only higgs...

and what is observed is a dilaton.

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and what is observed is a dilaton.

Quickly, before experimental question is settled, do some theory

Dilaton self-couplings: Can be determined by Conformal Perturbation Theory

If $$\mathcal{L} = \mathcal{L}_{\mathrm{CFT}} + \sum_n \lambda_n \mathcal{O}_n \,.$$ and $$\gamma_n = d_n - 4$$

perturbation is by small anomalous dimensions or small coupling (for this to make sense, the dimensional parameter is small when compared to the appropriate power of f).

Assume $\langle\chi\rangle = f$ and mass is given, M_σ, then

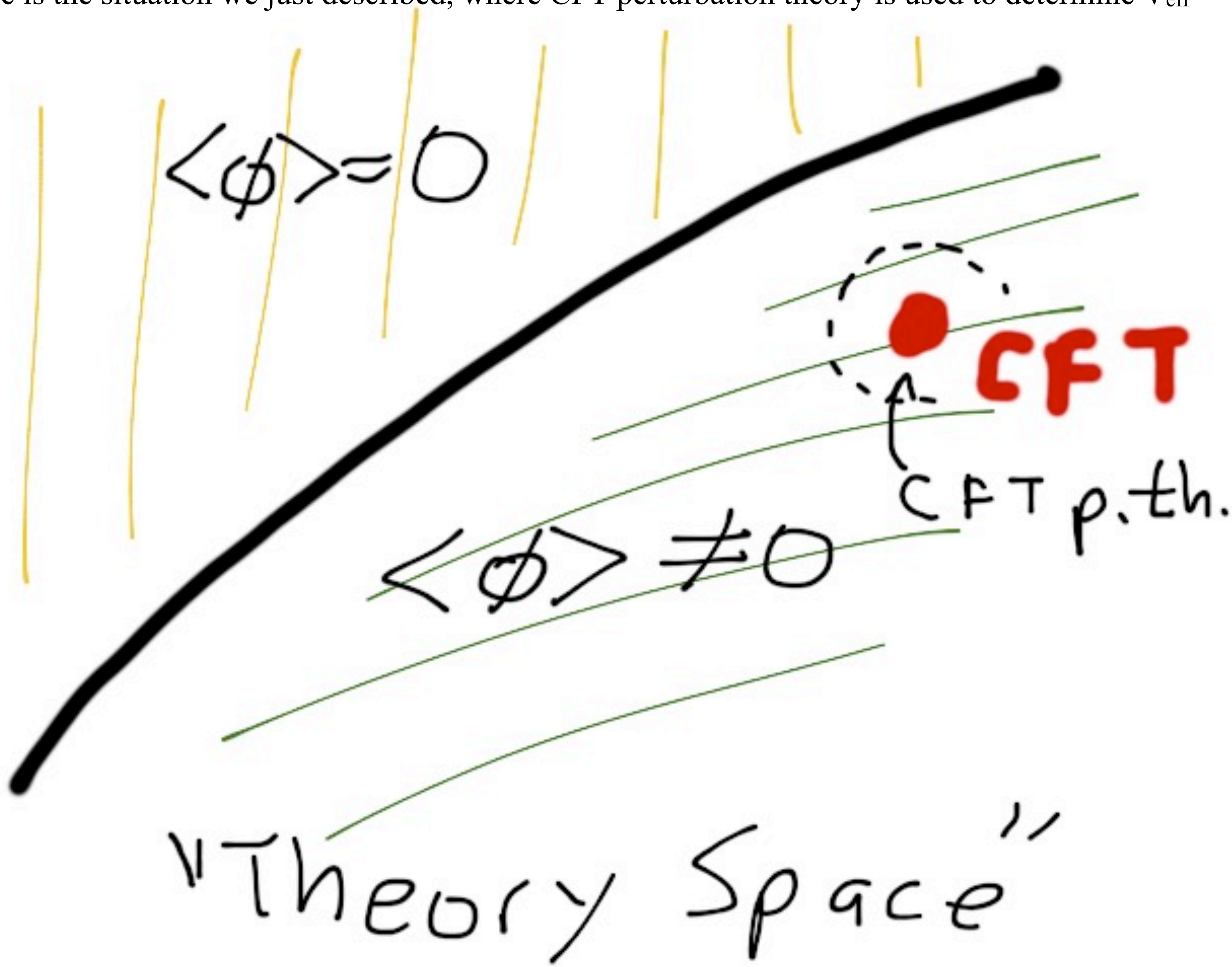
$|\gamma_n| \ll 1$ then $$V_{\mathrm{eff}}(\chi) = \frac{M_\sigma^2}{4f_\sigma^2}\chi^4\left[\ln\left(\frac{\chi}{f_\sigma}\right) - \frac{1}{4}\right] + \mathcal{O}(\gamma^2)\,.$$

or

$|\lambda_n| \ll 1$ then $$V_{\mathrm{eff}}(\chi) = \frac{M_\sigma^2}{f_\sigma^2}\chi^4\sum_n\left\{x_n\left[\frac{1}{4+\gamma_n}\left(\frac{\chi}{f_\sigma}\right)^{\gamma_n} - \frac{1}{4}\right]\right\} + \mathcal{O}(\lambda^2)\,,\quad \textstyle\sum_n \gamma_n x_n = 1.$$

or for one coupling: $$V_{\mathrm{eff}}(\chi) = \frac{M_\sigma^2}{f_\sigma^2\gamma}\chi^4\left[\frac{1}{4+\gamma}\left(\frac{\chi}{f_\sigma}\right)^{\gamma} - \frac{1}{4}\right] + \mathcal{O}(\lambda^2)\,,$$

Let's not dwell on applications. Focus on rationale, underlying dynamics.
Here is the situation we just described, where CFT perturbation theory is used to determine V_{eff}



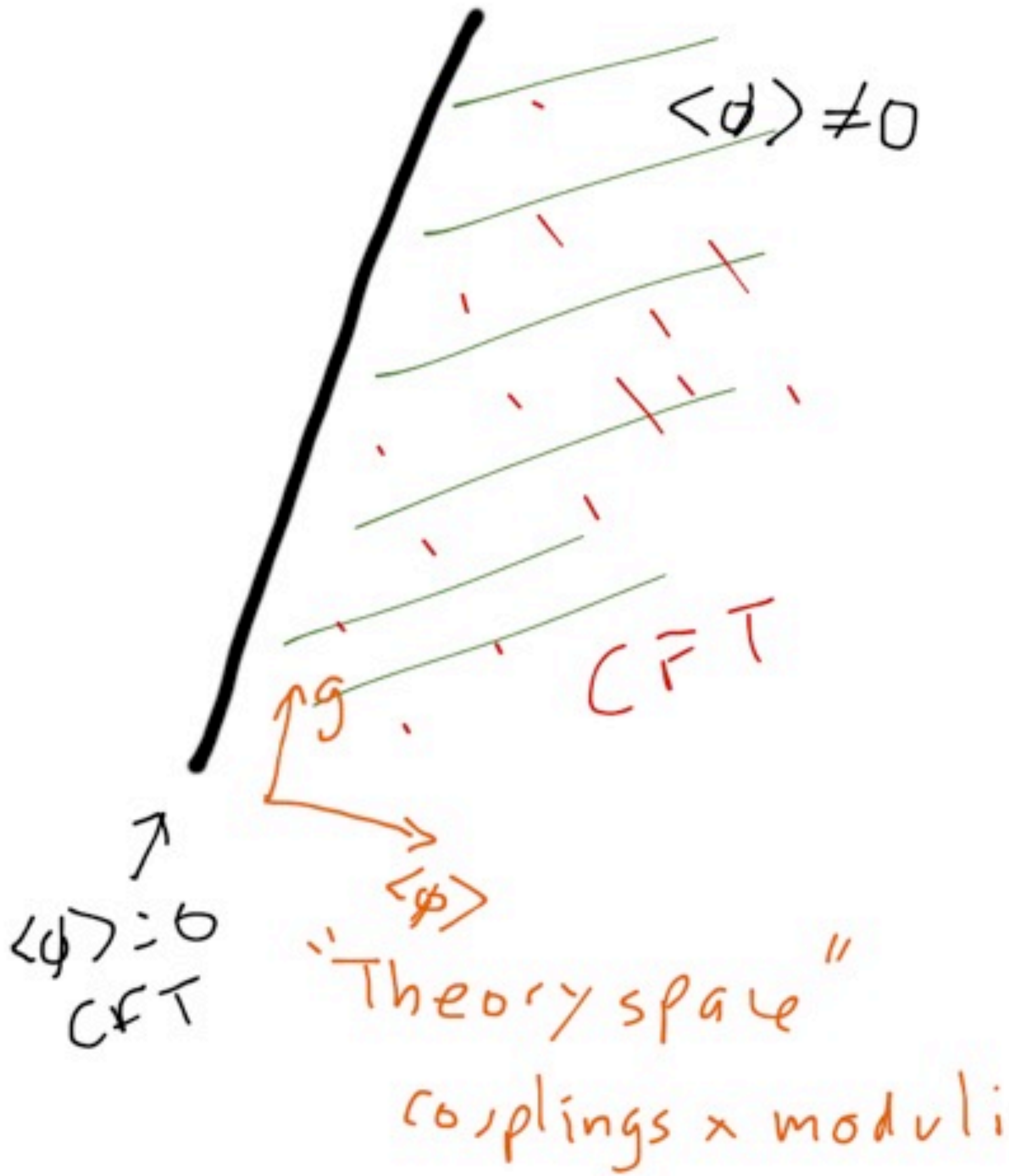
For what kind of model does this happen?

One example we know: Potential with flat directions (moduli space), common in SUSY

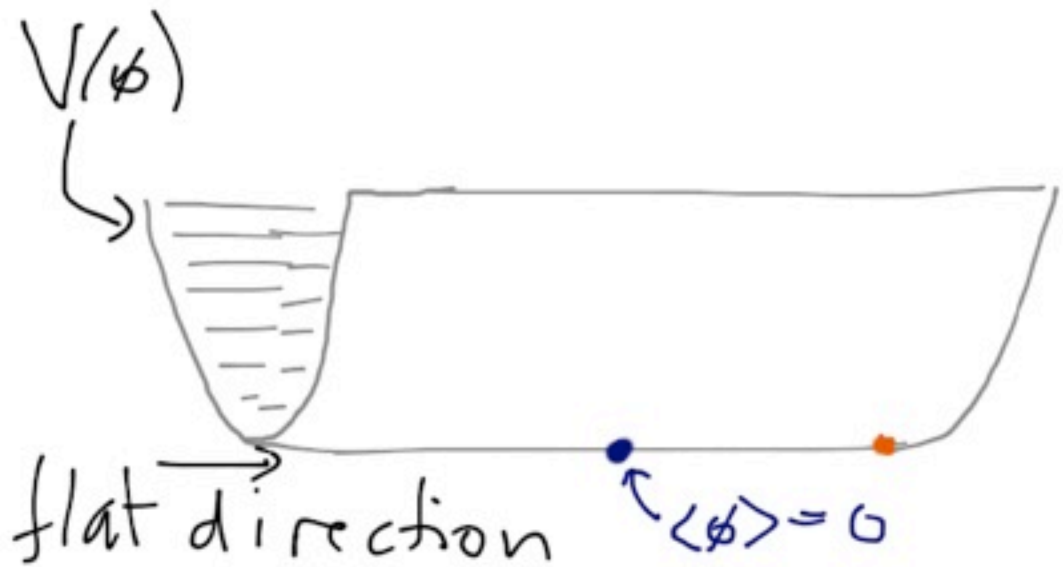
Theory space: includes moduli (as well as coupling constants).

But the picture is different

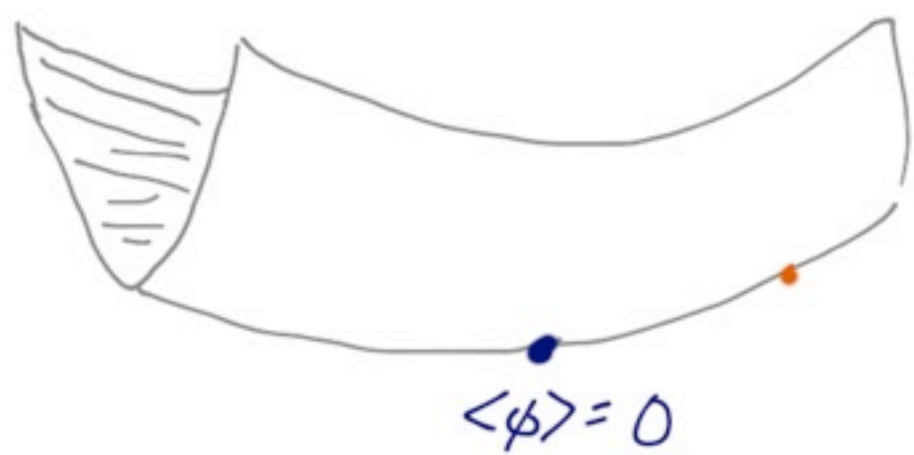
(literally and figuratively)

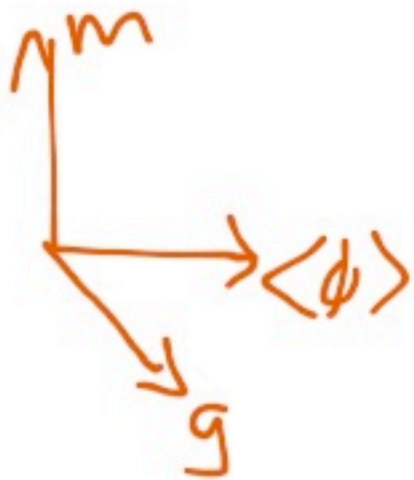


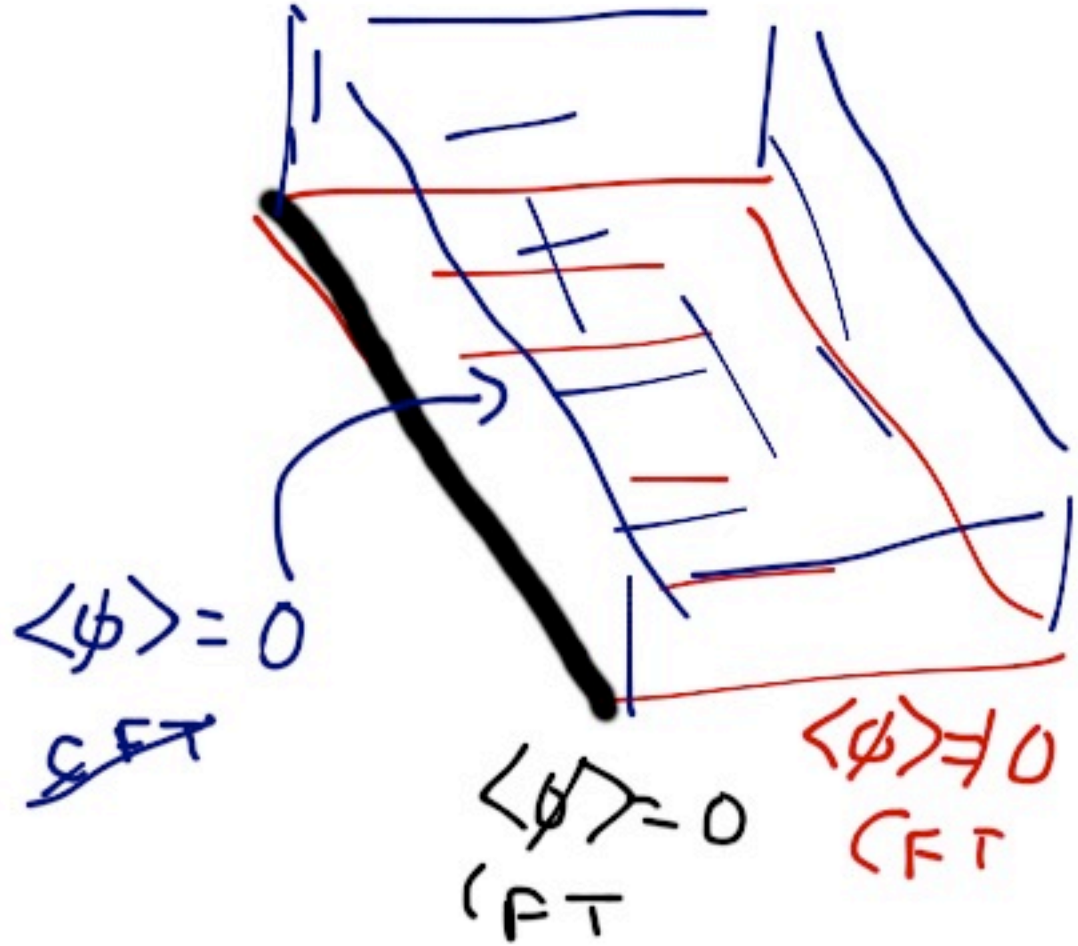
Here couplings are dimensionless



Include relevant deformations (dimensional couplings). Expect generically:







Not clear how to implement the scenario described above.

Nor is it clear how conformal perturbation theory can be used
(other than to verify flow away from IR-FP by relevant deformations).

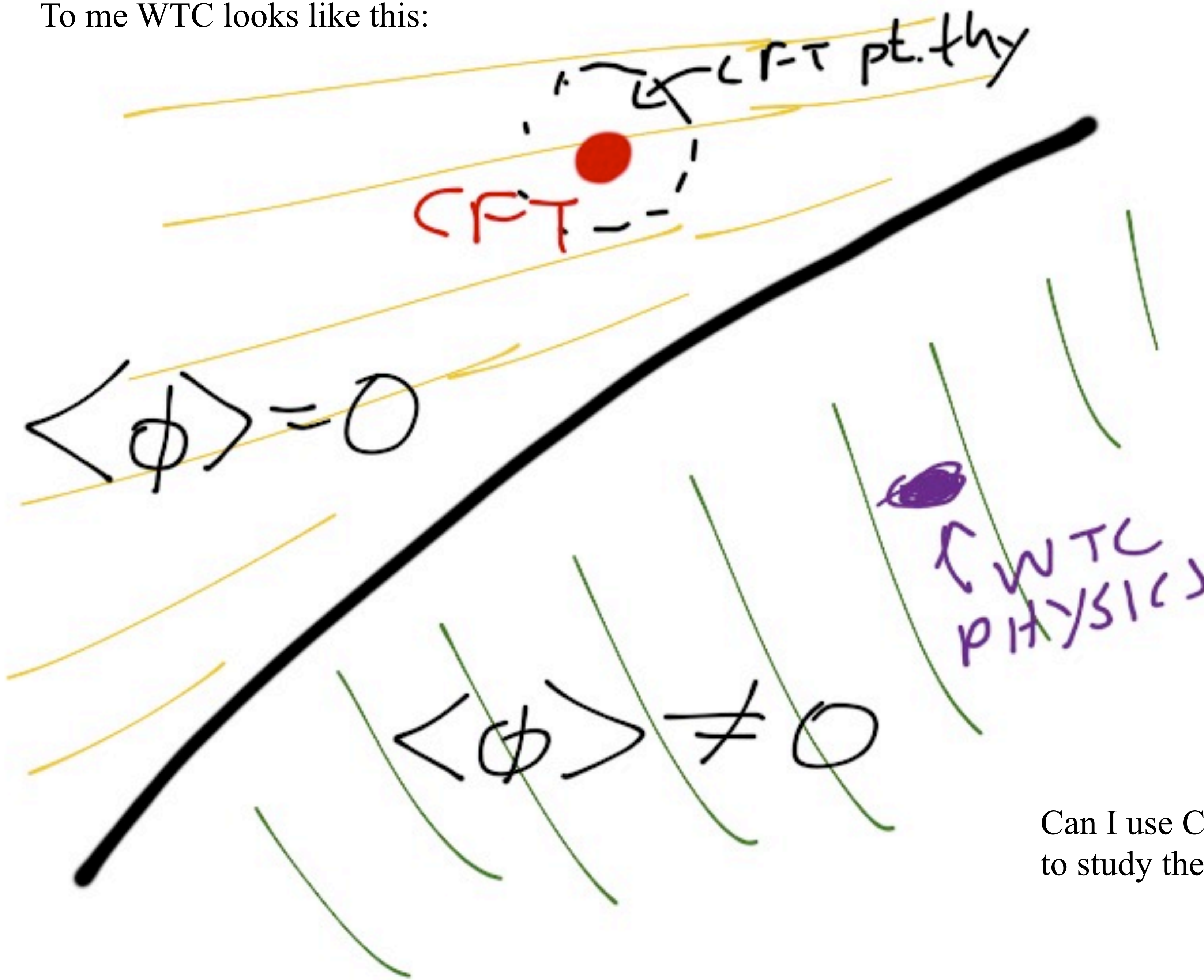
We need a better example.

Moreover...

The real problem is that this does not look to me like Walking Technicolor

which is what I would *really* like to understand *a little* better.

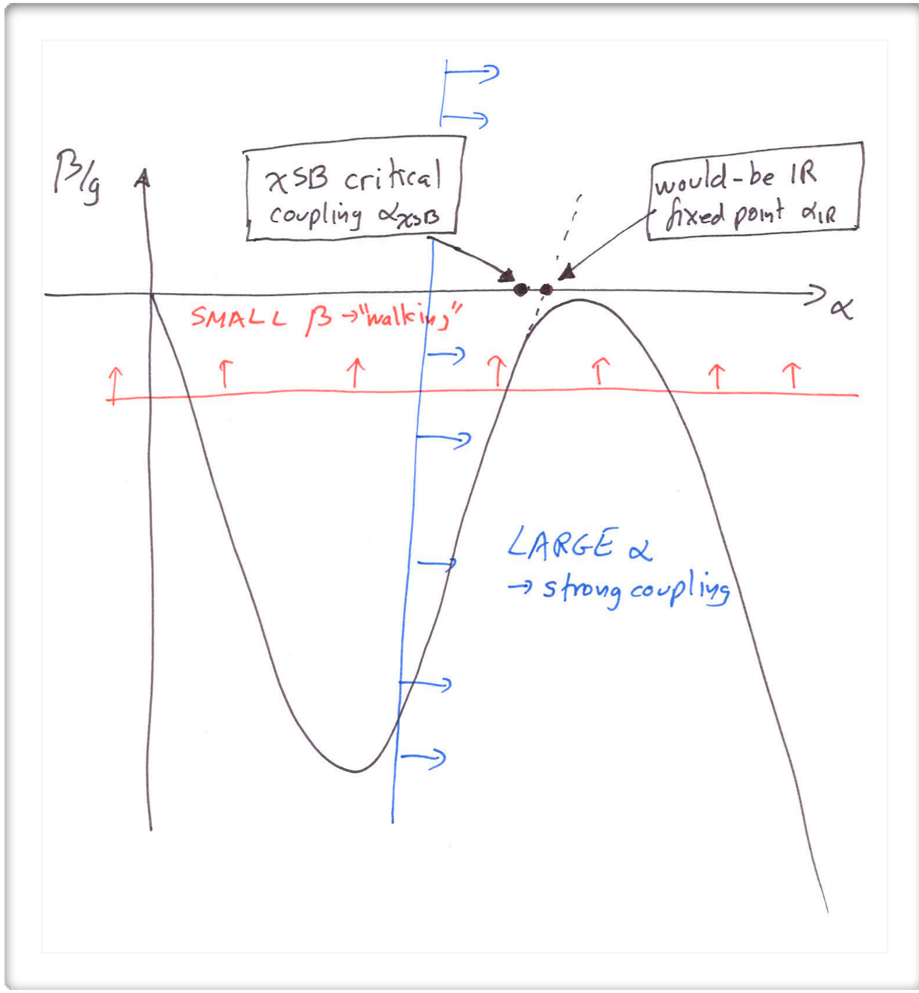
To me WTC looks like this:

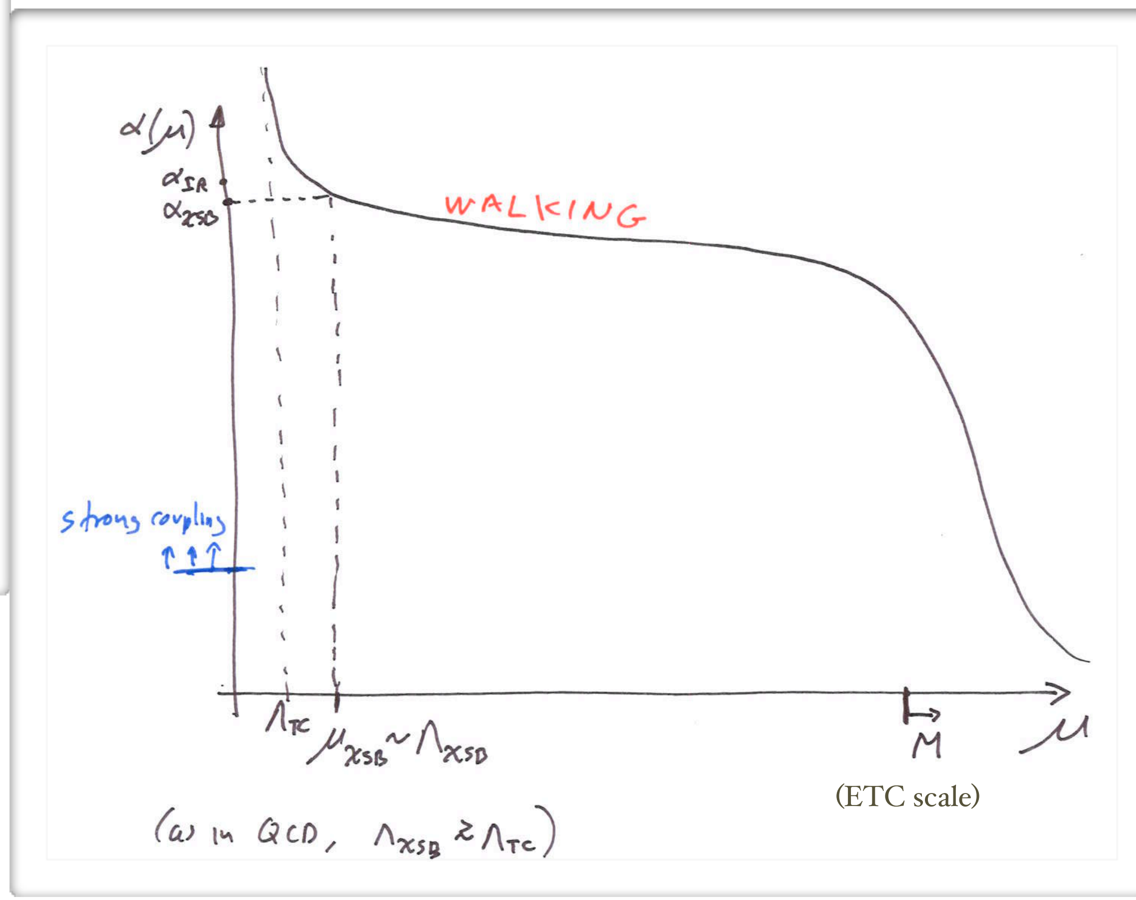


Can I use CFT pert.-thy.
to study the physics here?

Let me explain why I think this is the picture in WTC. Recall for WTC:

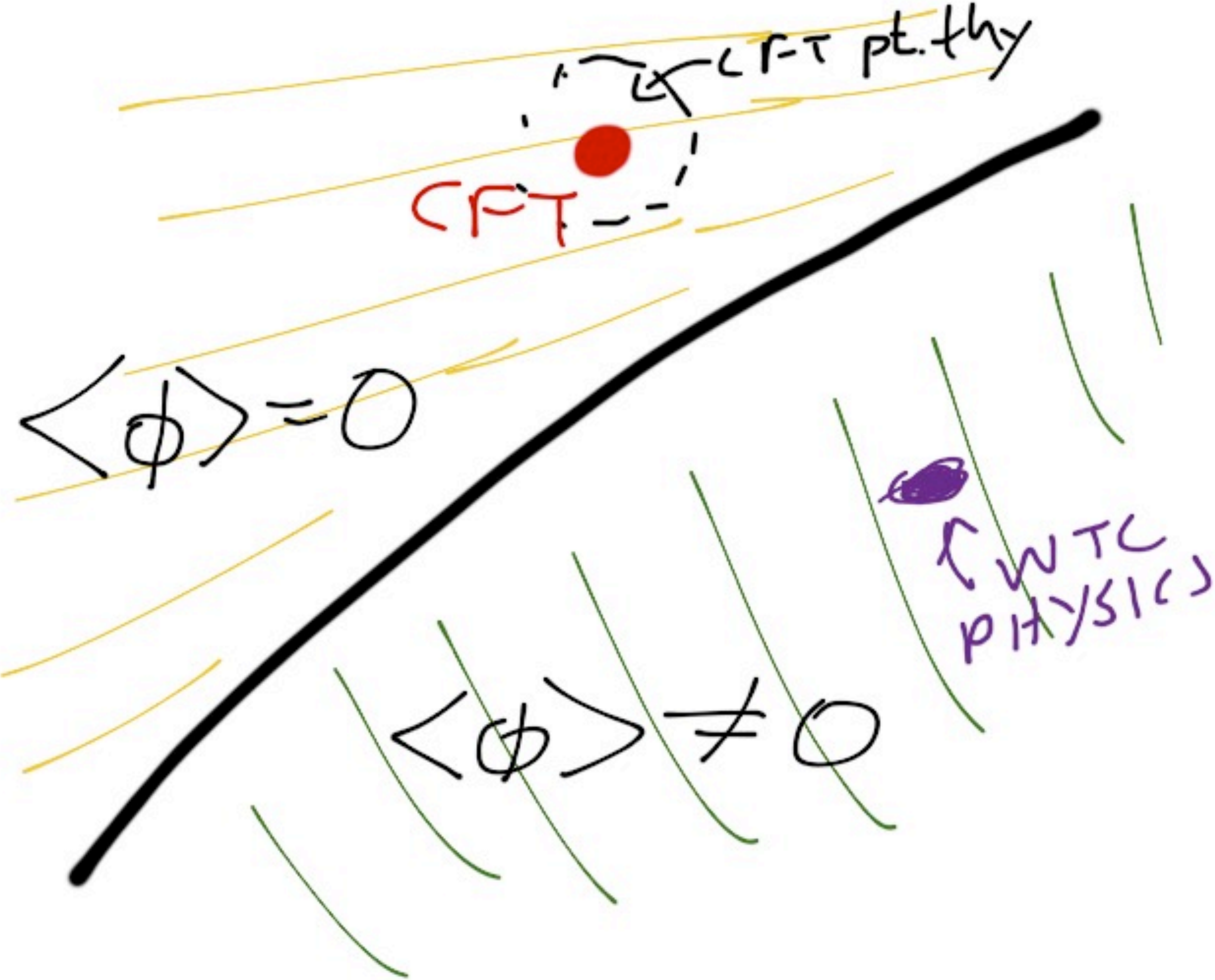
The coupling constant flows toward the "would be" IR fixed-point, α_{IR}.
Close to the fixed-point, the flow is slow and the theory posseses approximate scale symmetry.





Digression: physics does not depend on which point is picked on the RG-trajectory. The statement "as one approaches α_{IR} one reaches first the critical coupling for χSB" is confusing: the symmetry is broken, regardless of where on the RG-trajectory.

Here it is again:



(Approximate) Scale Invariant Theory

WTC framework:

- *strongly interacting* would-be IR fixed-point
- fermion condensate forms
- scale invariance broken spontaneously (condensate) and explicitly ($\beta \neq 0$)
- strongly interacting: difficult to analyze analytically
- Is there a (light) dilaton? The subject of an old, recently rekindled debate

  - Yes (Appelquist, Bai '10)
  - No (Hashimoto, Yamawaki '10; Vecchi '10)

The argument hyper-vulgarized:

small β: explicit breaking is by trace anomaly $T^{\mu}_{\mu} = \frac{\beta}{g} G^2$

η'-like: QCD anomaly, η' not a GB

Will have more to say about this,
but will not review their arguments here.

Having a perturbative toy model with the above properties – an interacting IR-fixed-point and an approximate scale invariance which is broken dynamically – would help *me* understand this better.

Broken dynamically in perturbative case? no scale of SB given *a priori*

Better yet... a perturbative example of walking.

Meaning what?? In perturbative regime couplings *always* walk!!
By perturbative walking I mean running towards an IR fixed point,
slowing down by an arbitrary amount, and then getting ‘detoured’ by SSB

Can it be done??

Why is it non-straightforward?

We have seen $\mathcal{N}=4$ SUSY *SU(N)* bad.

Better attempt(?): Coleman-Weinberg abelian-higgs model.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D^{\mu}\phi^* D_{\mu}\phi - \lambda|\phi|^4$$

- Fine tune mass to zero, classically scale invariant.
- Effective potential develops a minimum away from origin:

$$V = \frac{\lambda}{4!}\varphi_c^{\,4} + \left(\cancel{\frac{5\lambda^2}{1152\pi^2}} + \frac{3e^4}{64\pi^2}\right)\varphi_c^{\,4}\left(\ln\frac{\varphi_c^{\,2}}{M^2} - \frac{25}{6}\right)$$

- Gauge and scale symmetries spontaneously broken.
- Gauge field acquires mass.
- But would-be-dilaton acquires mass too: trace anomaly spoils scaling symmetry.
- So far, so good. But now:

$$M_{\text{dilaton}}/M_{\text{vector}} \sim e^2/16\pi^2$$

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Want: arbitrarily light dilaton without turning off interactions

(note: as Kuti pointed out, Hashimoto and Yamawaki argue that in WTC the analogous ratio is constant).

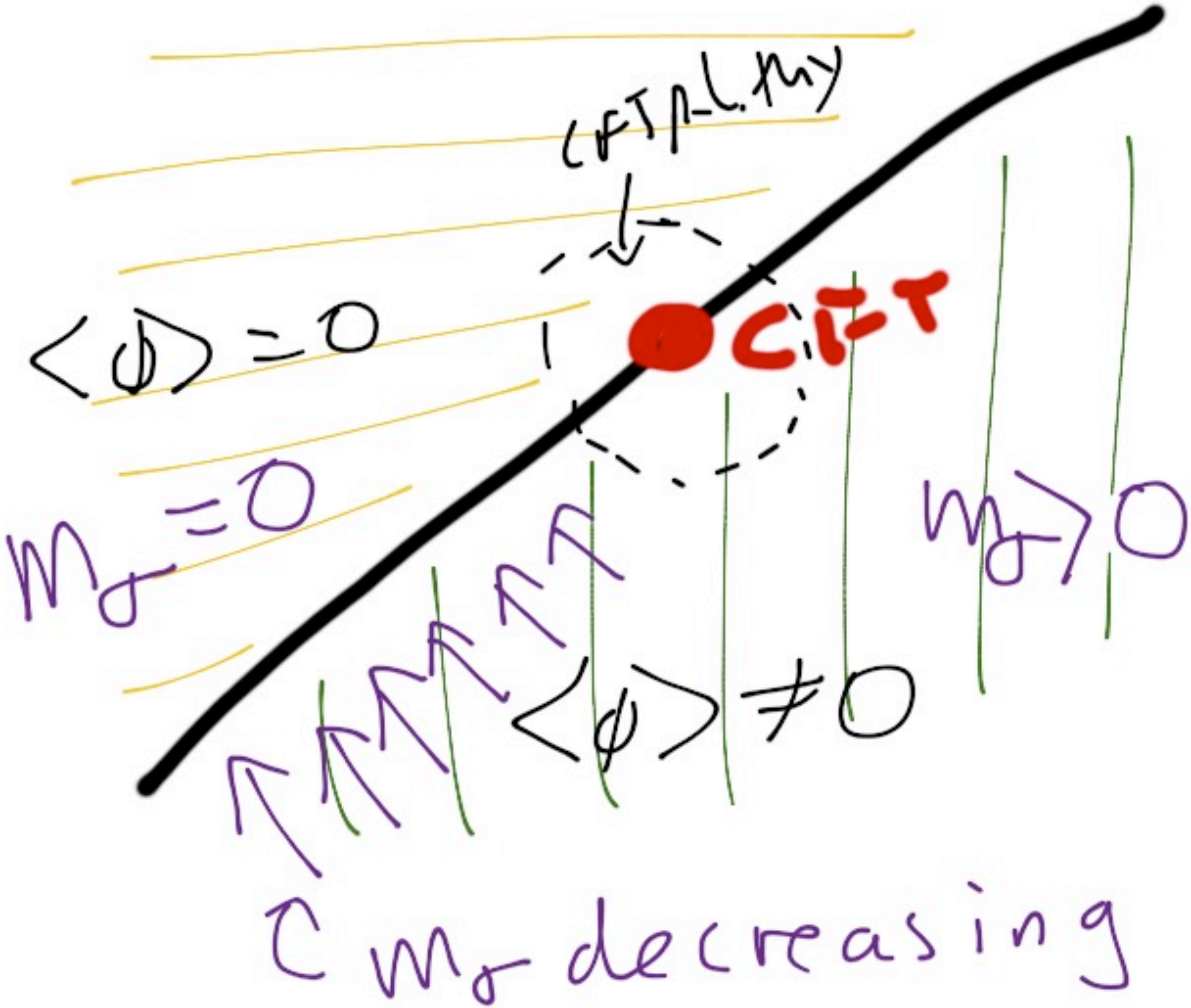
We construct a model which

- ☑ Is perturbative (naively)
- ☑ Has a perturbative IR fixed point (a la Caswell-Banks-Zaks)
- ☑ Spontaneously breaks approximate scale symmetry
- ☑ Dilaton can be made arbitrarily light while still interacting

follow-up by Antipin, Mojaza, Sannino ’11
some qualitative differences; effective
SUSY at low energies...

In the rest of the talk I will explain this model.

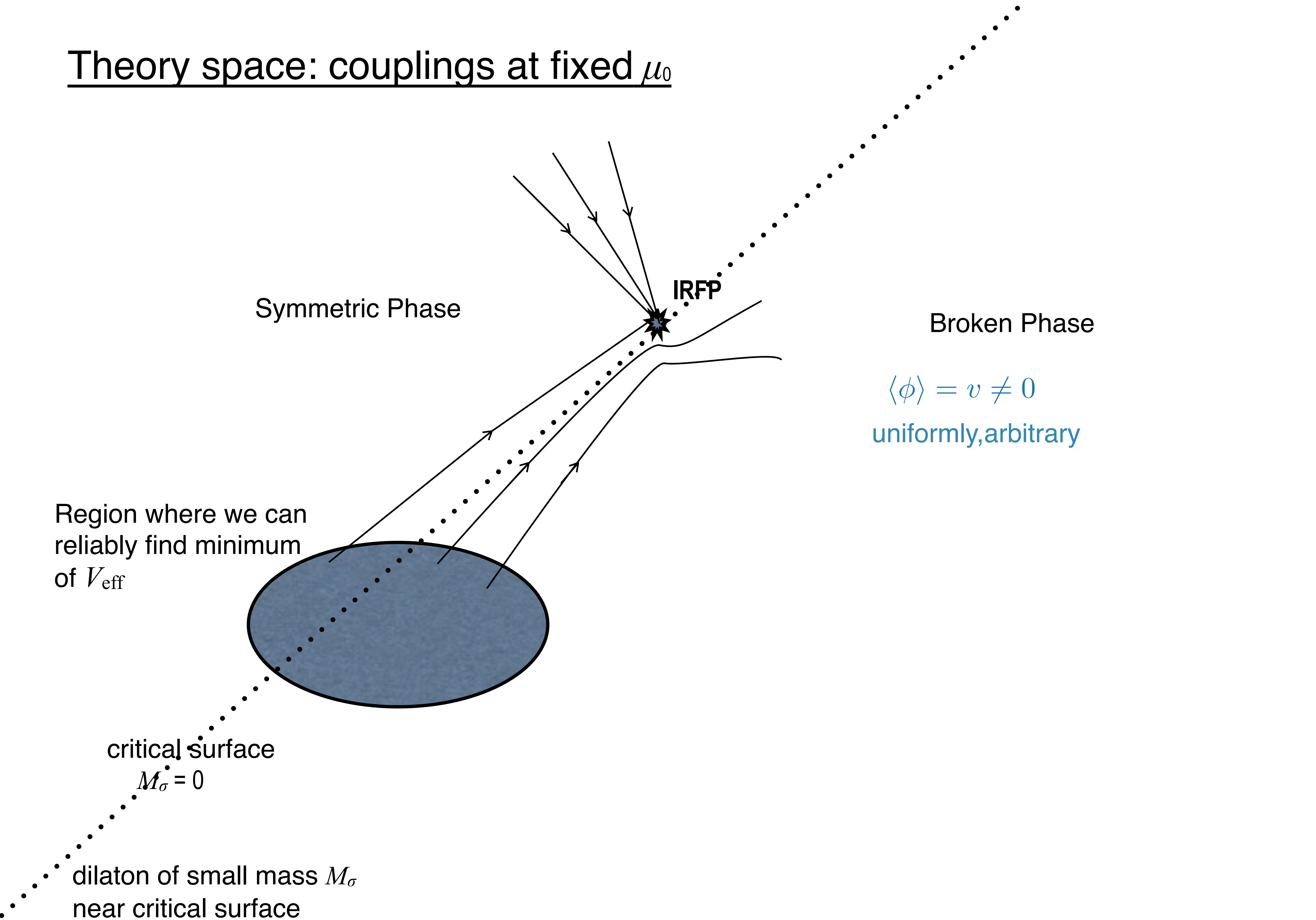
First the big picture: (yes, more pictures!)



CFT pert. thy
$\langle \phi \rangle = 0$
$m_\sigma = 0$
CFT
$m_\sigma > 0$
$\langle \phi \rangle \neq 0$
m_σ decreasing

This time need a better picture...

Theory space: couplings at fixed μ_0



The Model

SU(N) gauge theory with $n_\psi = n_\chi$ fundamental fermions ψ and χ and two scalar singlets ϕ_1 and ϕ_2.

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\, F^{\mu\nu}F_{\mu\nu} + \sum_{j=1}^{n_\chi}\left(\bar{\psi}^j i\not{D}\psi_j + \bar{\chi}^j i\not{D}\chi_j\right) + \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2$$
$$- y_1\left(\bar{\psi}\psi + \bar{\chi}\chi\right)\phi_1 - y_2(\bar{\psi}\chi + \bar{\chi}\psi)\phi_2$$
$$- \frac{1}{24}\lambda_1\phi_1^4 - \frac{1}{24}\lambda_2\phi_2^4 - \frac{1}{4}\lambda_3\phi_1^2\phi_2^2$$

This theory is invariant under discrete $\mathbb{Z}_2$ as well as SU(n_χ) symmetry

$$\begin{aligned} \phi_1, \psi &\to \phi_1, \psi \\ \phi_2, \chi &\to -\phi_2, -\chi \end{aligned} \quad \text{and} \quad \begin{aligned} \psi &\to U\psi \\ \chi &\to U\chi \end{aligned}$$

Nota bene

Masses set to zero (I am not solving the hierarchy problem).
Precisely as with Coleman and Weinberg.
Set them to zero and use dimensional regularization.

Theory has Landau pole. This is a UV issue.
We study the IR properties of the model.
We can take it to be a cut-off theory.

This is not the theory of everything.
It is a Toy Model.

The fixed-point to leading order in $1/N$ is

$$g_*^2 = 16\pi^2 \frac{2}{75} \frac{\delta}{N}$$

Caswell-Banks-Zaks

$$y_{1*}^2 = y_{2*}^2 = \frac{3}{11} \frac{g_*^2}{N}$$

Drives this

$$\lambda_{1*} = \lambda_{2*} = \lambda_{3*} = \frac{18}{11} \frac{g_*^2}{N} = 6y_{1*}^2$$

Drives this

Minimizing the potential analytically is difficult. But easy to identify some local minima. Focus on minimum which preserves discrete $\mathbb{Z}_2$ (i.e. $\langle\phi_2\rangle = 0$).
The potential reduces to

$$V_{\text{eff}} = \frac{\lambda_1}{24}\phi_1^4 + \frac{(\lambda_1\phi_1^2)^2}{256\pi^2}\left(\ln\frac{\lambda_1\phi_1^2}{2\mu^2} - \frac{3}{2}\right) + \frac{(\lambda_3\phi_1^2)^2}{256\pi^2}\left(\ln\frac{\lambda_3\phi_1^2}{2\mu^2} - \frac{3}{2}\right) - \frac{22N^2y_1^4\phi_1^4}{64\pi^2}\left(\ln\frac{y_1^2\phi_1^2}{\mu^2} - \frac{3}{2}\right)$$

The extremum, $\partial/\partial\phi_1 V_{\text{eff}}(\langle\phi_1\rangle) = 0$, is at

$$-\frac{\lambda_1}{6} = \cancel{\frac{\lambda_1^2}{64\pi^2}}\left(\ln\frac{\lambda_1\langle\phi_1\rangle^2}{2\mu^2} - 1\right) + \frac{\lambda_3^2}{64\pi^2}\left(\ln\frac{\lambda_3\langle\phi_1\rangle^2}{2\mu^2} - 1\right) - \frac{88N^2y_1^4}{64\pi^2}\left(\ln\frac{y_1^2\langle\phi_1\rangle^2}{\mu^2} - 1\right)$$

$$\lambda_1 \sim \frac{\lambda_3^2}{64\pi^2} \sim \frac{88N^2y_1^4}{64\pi^2}$$

(which is why we needed a second scalar)

Vacuum Expectation

λ_1 can be traded with $\langle\phi_1\rangle$ as a free parameter. For consistency, $\frac{\lambda_1}{16\pi^2}\ln\frac{\langle\phi_1\rangle^2}{\mu^2} \ll 1$. Stability of the vev is determined from the eigenvalues of second derivative matrix

$$\frac{\partial^2}{\partial\phi_1^2}V_{\text{eff}}(\langle\phi_1\rangle, 0) = \frac{\lambda_3^2 - 88N^2y_1^4}{32\pi^2}\langle\phi_1\rangle^2$$

$$\frac{\partial^2}{\partial\phi_2^2}V_{\text{eff}}(\langle\phi_1\rangle, 0) = \frac{\lambda_3}{2}\langle\phi_1\rangle^2 + \mathcal{O}(\text{1-loop})$$

Evaluate V_{eff} at $\langle\phi_1\rangle$ yields

$$V_{\text{eff}}(\langle\phi_1\rangle) = -\frac{\lambda_3^2 - 88N^2y_1^4}{512\pi^2}\langle\phi_1\rangle^4$$

Thus when $\varepsilon \equiv \lambda_3^2 - 88N^2y_1^4 \geq 0$, there is a non-trivial minimum.

Pole mass in Broken Phase

The explicit 1-loop pole masses are

$$M_\psi(\mu) = M_\chi(\mu) = y_1 v \left[1 - \frac{g^2}{16\pi^2}\frac{N}{2}\left(3\ln\frac{y_1^2 v^2}{\mu^2} - 4\right)\right]$$

$$\begin{aligned} M_{\phi_1}^2 &= \frac{\lambda_1 v^2}{2} + \frac{3\lambda_1^2 v^2}{64\pi^2}\left(\ln\frac{\lambda_1 v^2}{2\mu^2} - \frac{5}{3} + \frac{2\pi}{3\sqrt{3}}\right) + \frac{3\lambda_3^2 v^2}{64\pi^2}\left(\ln\frac{\lambda_3 v^2}{2\mu^2} - \frac{1}{3} - \frac{2\lambda_1}{3\lambda_3}\right) \\ &\quad + \frac{22N^2 y_1^2}{16\pi^2}\left[y_1^2 v^2 - \frac{\lambda_1 v^2}{12} - 3\left(y_1^2 v^2 - \frac{\lambda_1 v^2}{12}\right)\left(\ln\frac{y_1^2 v^2}{\mu^2}\right)\right. \\ &\quad \left. - 3\int_0^1 \mathrm{d}x \left(y_1^2 v^2 - \frac{x(1-x)}{2}\lambda_1 v^2\right)\ln\left(1 - x(1-x)\frac{\lambda_1}{2y_1^2}\right)\right] \\ &\simeq \frac{\lambda_3^2 - 88N^2 y_1^4}{32\pi^2} v^2 = \frac{\varepsilon}{32\pi^2} v^2 \end{aligned}$$

Since $v = \langle\phi_1\rangle$, it has the same anomalous dimension as ϕ_1.
Using the anomalous dimension and the β functions, one can verify that the masses are RG invariant at 1-loop.

Decay Constant

Define the decay constant f_σ by

$$\langle 0|\Theta^{\mu\nu}(x)|\sigma\rangle = \frac{f_\sigma}{3}\left(p^\mu p^\nu - g^{\mu\nu}p^2\right)e^{ip\cdot x}$$

where p is the momentum of $|\sigma\rangle$. The form of the right hand side is constrained by conservation of $\Theta^{\mu\nu}$. The factor $1/3$ comes from

$$\langle 0|\partial_\mu \mathcal{D}^\mu|\sigma\rangle = \langle 0|\Theta^\mu_\mu|\sigma\rangle = -f_\sigma M_\sigma^2 e^{ip\cdot x}$$

Note that $\Theta^{\mu\nu} = -1/3 v\partial^\mu\partial^\nu\phi_1 + \cdots$.
Thus to lowest order $f_\sigma = v + \cdots$.
The RG invariant expression is easy to guess

$$f_\sigma = vZ_{\phi_1}^{-1/2}$$

where Z_{ϕ_1} is the wavefunction renormalization factor.

Dilaton Mass

Having determined the decay constant f_σ, the mass of the dilaton can be obtained from the trace anomaly.
To lowest order, the mass is

$$M_\sigma^2 = \frac{\lambda_1^2 + \lambda_3^2 - 88 N^2 y_1^4}{32\pi^2} v^2 = \frac{\varepsilon}{32\pi^2} v^2 = M_{\phi_1}^2$$

where λ_1^2 term is dropped for consistency.
RG invariance of M_σ can be inferred from M_{ϕ_1}.

- Given the vev, we can tune ε to make the dilaton light by comparison

Broken Phase

Recall the theory admits a non-trivial minimum provided

- λ_1 is much smaller than other couplings,
- $\varepsilon \equiv \lambda_3^2 - 88N^2 y_1^4 \geq 0$.

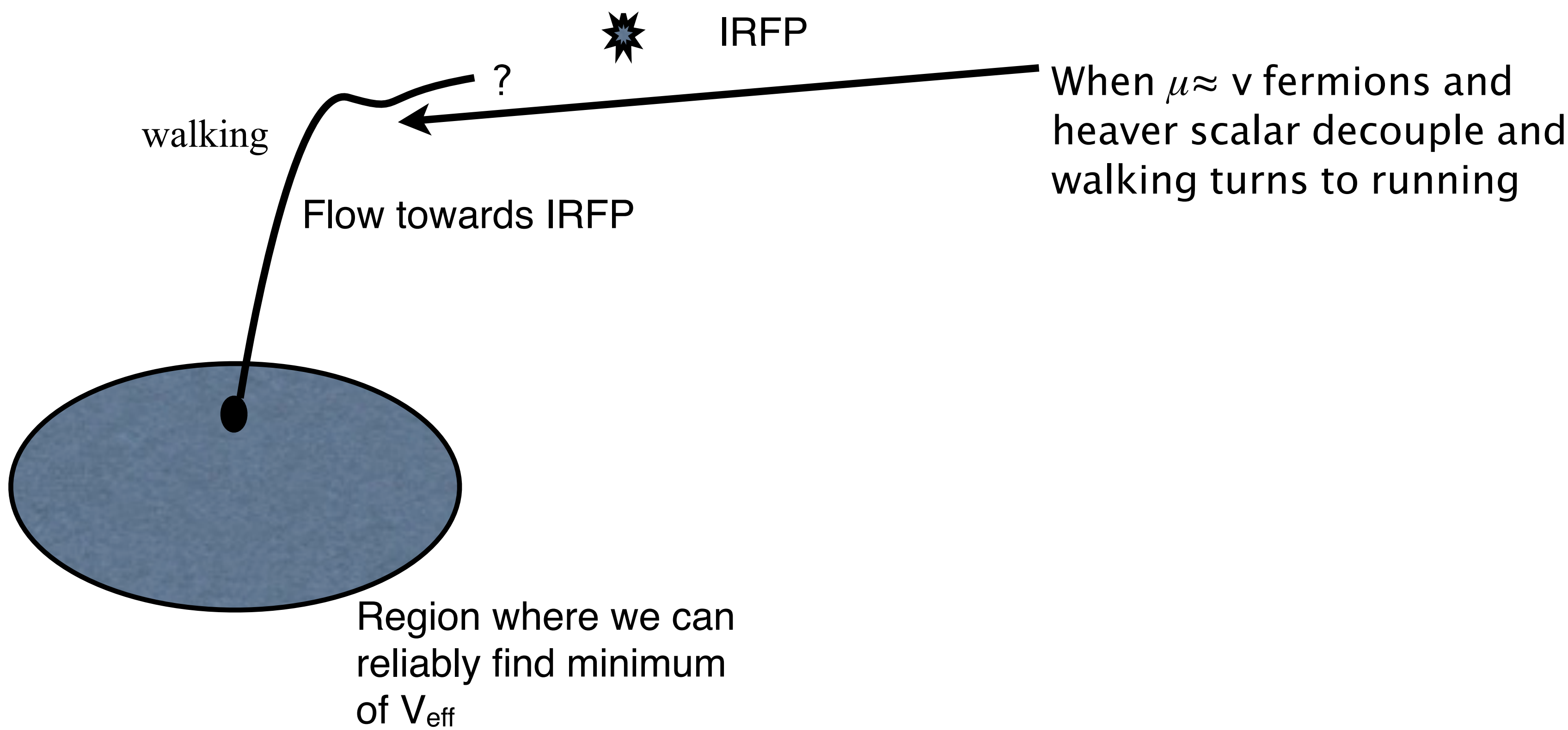
We want to study symmetry breaking close to the IR fixed-point. However, near the fixed-point these conditions are not satisfied.

Use RGE to trace back the RG trajectory to large RG time where perturbative analysis of effective potential yields a non-trivial minimum.

(convention: RG time here grows from IR to UV)

Alternatively, define the theory at scale μ_0 where perturbative analysis yields a non-trivial minimum. Moreover, if the vev is well below μ_0, RG flows will get close to the fixed-point before the massive particles decouple.

Theory parameter space: couplings at fixed μ_0



Symmetric Phase

For a point in parameter space where $\varepsilon < 0$

- $V_{eff}(\langle\phi_1\rangle)$ becomes positive and the non-trivial minimum disappears,
- the effective potential seems to be unbounded from below along ϕ_1 direction for large ϕ_1.

The second point threatens the validity of the model.

However, at large ϕ_1 perturbative analysis breaks down.
Can extend the range of perturbativity using the improved effective potential which effectively re-sumslarge logarithms.

$$V_{eff}^{\text{imp}} = \frac{1}{24}\bar{\lambda}_1(t)e^{-4\int_0^{t'}\gamma_{\phi_1}\mathrm{d}t'}\phi_1^4$$

Here $t = \ln\phi_1/\mu_0$. This form is valid as long as $\bar{\lambda}_1(t)$ is perturbative.

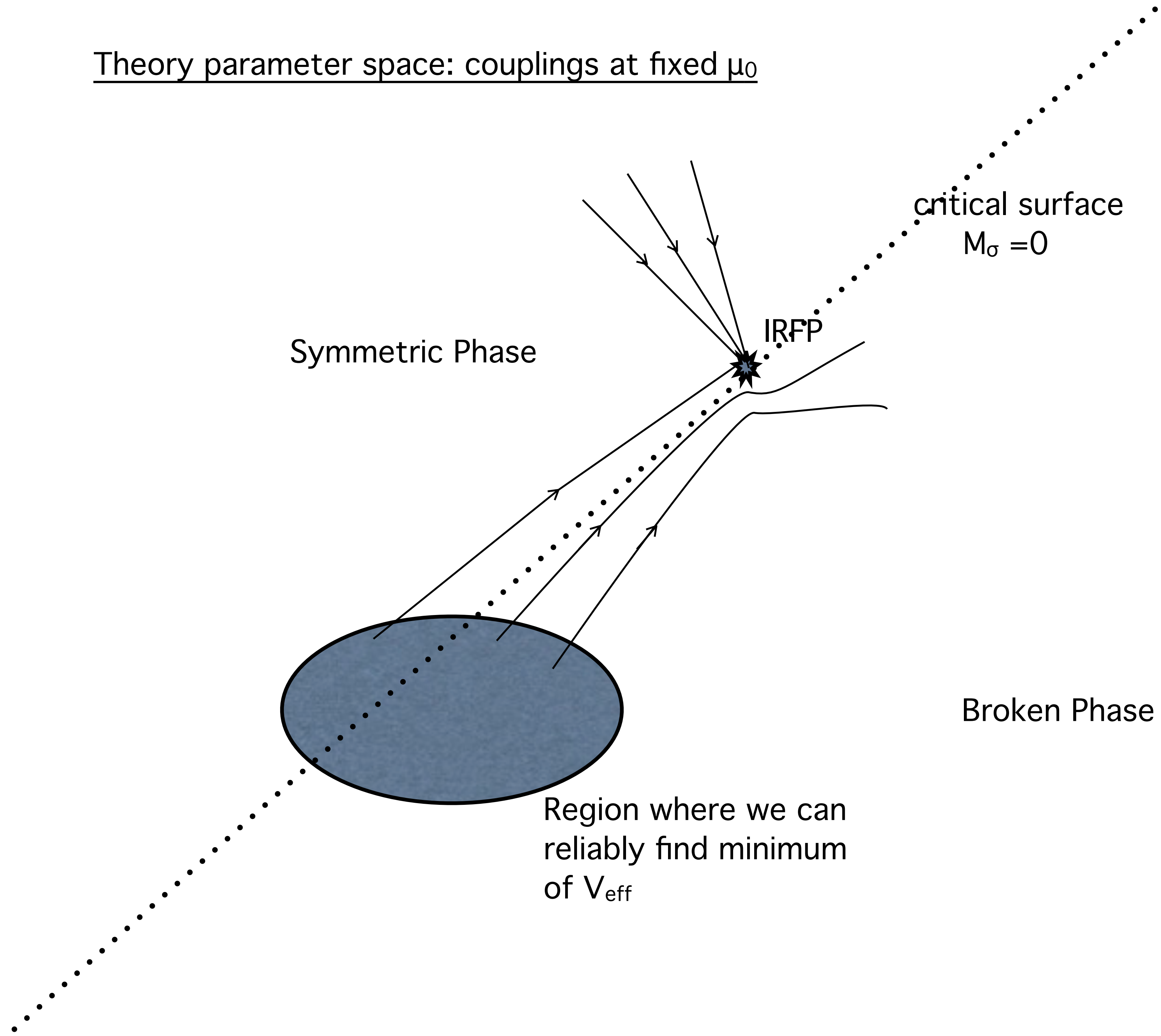
Symmetric Phase (cont.)

For points in parameter space closed to the IR fixed-point, gauge coupling drives the Yukawa coupling to 0 in the UV.

Thus in the far UV, the β-function for λ_1 has a Landau pole.

- The effective potential is bounded from below because $\bar{\lambda}_1(t) > 0$ for large ϕ_1.

Theory parameter space: couplings at fixed μ_0



Final remarks

Is conformal perturbation theory applicable in this model? Recall GGS had

$$V_{\text{eff}}(\chi) = \frac{M_\sigma^2}{4f_\sigma^2}\chi^4 \left[\ln\left(\frac{\chi}{f_\sigma}\right) - \frac{1}{4}\right] + \mathcal{O}(\gamma^2).$$

To compare our model with GGS, we view our model as $\mathcal{L}(g) = \mathcal{L}(g_*) + (\mathcal{L}(g) - \mathcal{L}(g_*))$. In our model the dilaton field is identified with ϕ_1 and the anomalous dimensions are small.

Our effective potential for ϕ_1 turns out to be exactly the same as GGS.

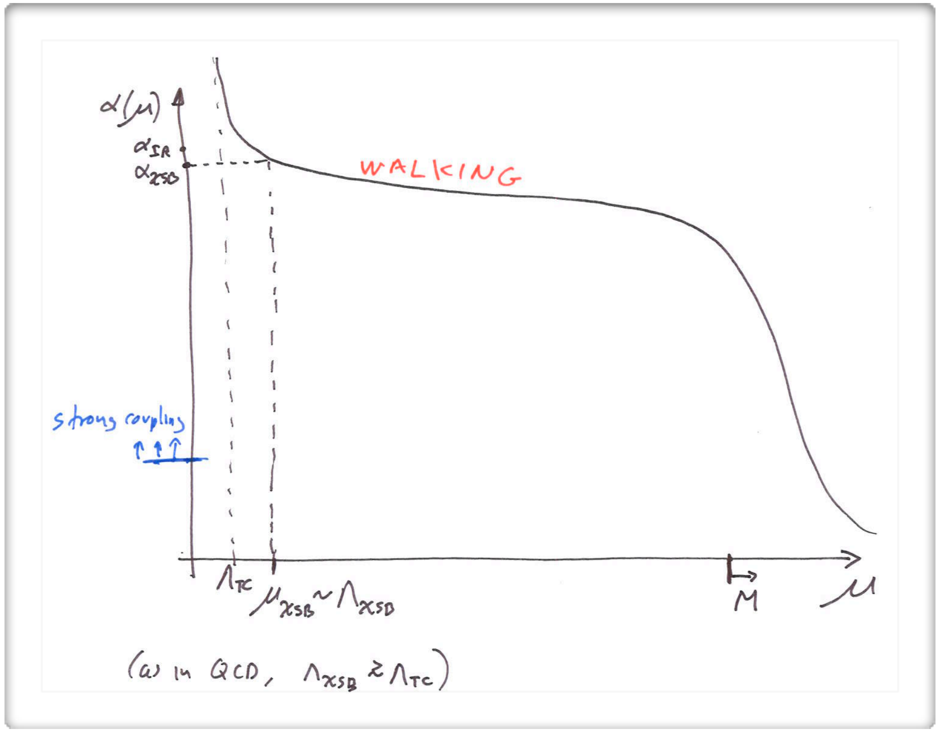
What about lessons for WTC?

In our model eventually glueballs form and dilaton mass expected to be no smaller than about glueball mass

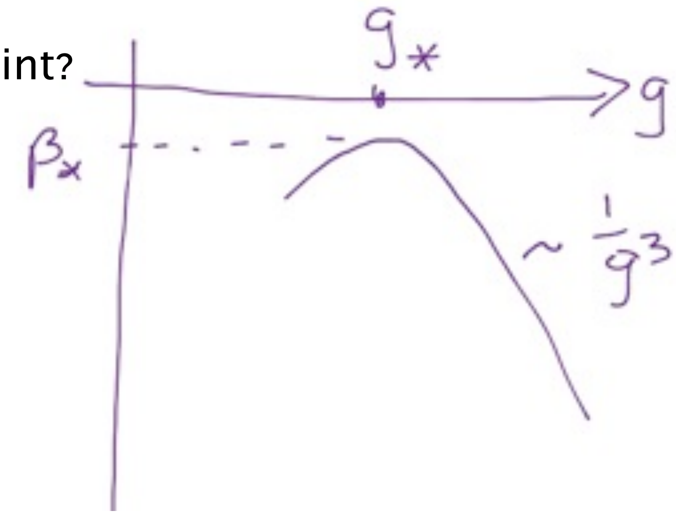
$$\Lambda \simeq v e^{-\frac{32\pi^2}{b_0 g_*^2}} \ll v$$

But in WTC g_* is large. And CFT perturbation theory questionable.

Is there a separation $\Lambda_{\chi SB} \ll \Lambda_{TC}$?



Better approximation to running from close to fixed point?



$$\Lambda \approx v \exp\left(-\int_{g_*}^{\infty} \frac{dg}{\beta(g)}\right)$$

Model beta function:

$$\beta(g) \approx \beta_* - \frac{b_{\text{eff}}}{16\pi^2}(g-g_*)^3$$

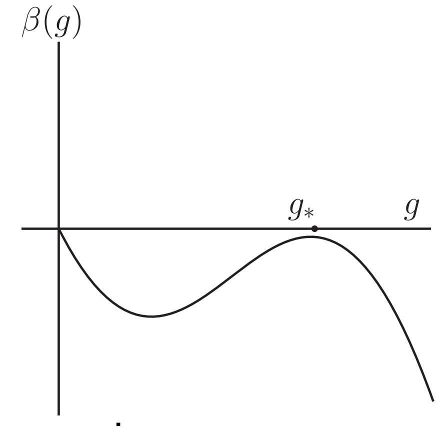
Then

$$\Lambda \approx v \exp\left(-\left(\frac{2^7\pi^5}{3^{\frac{9}{2}} b_{\text{eff}}\beta_*^2}\right)^{\frac{1}{3}}\right)$$

It seems that there can be a separation between scales and a light dilaton provided

$$\beta_* \ll 1$$

One more (really, really, the last) word on Dilaton in WTC?



AB say:

$$M_\sigma^2 \simeq \frac{s(\alpha_* - \alpha_c)}{\alpha_c}\Lambda^2 \simeq \frac{N_f^c - N_f}{N_f^c}\Lambda^2,$$

First equation: In our model the critical coupling is a critical surface, the IRFP is on critical surface, 0=0, correct but not interesting and not what is intended

Second equation: $(N^c - N)/N$ plays role of ε, measures distance to critical surface, and equation is qualitatively correct!

The End

Supplementary Slides

Numerical

$N = 20$, $n_f = 11/2\,N$, $\delta = 0.2$,
$g(\mu_0) = \frac{4}{9} g_*$, $y_1(\mu_0) = 0.32 y_{1*}$, $y_2(\mu_0) = \frac{1}{5} y_{2*}$,
$\lambda_1(\mu_0) = \frac{1}{30}\lambda_{2*}$, $\lambda_2(\mu_0) = 3\lambda_{2*}$, $\lambda_3(\mu_0) = 5.2\lambda_{3*}$.
These condition corresponds to $\varepsilon \gtrsim 0$.
The vev is at

$$\ln \frac{\langle \phi_1 \rangle}{\mu_0} \simeq -29$$

and the spectrum

$$\frac{M_{\psi,\chi}}{v} \simeq 8.5 \times 10^{-3}, \quad \frac{M_{\phi_1}}{v} \simeq 7.9 \times 10^{-4}, \quad \frac{M_{\phi_2}}{v} \simeq 9.5 \times 10^{-2}.$$

Fractional correction to the effective potential from higher order terms are approximated to be

$$\left| \frac{N g_*^2}{16\pi^2} \ln \left(\frac{y_1^2 v^2}{\mu^2} \right) \right| \simeq 0.2.$$

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So we start from region were Veff has competition between tree level and 1-loop

and the spectrum

$$\frac{M_{\psi,\chi}}{v} \simeq 8.5\times 10^{-3},\quad \frac{M_{\phi_1}}{v} \simeq 7.9\times 10^{-4},\quad \frac{M_{\phi_2}}{v} \simeq 9.5\times 10^{-2}.$$

Fractional correction to the effective potential from higher order terms are approximated to be

$$\left|\frac{Ng_*^2}{16\pi^2}\ln\left(\frac{y_1^2v^2}{\mu^2}\right)\right| \simeq 0.2.$$

Numerical

Will change this to $0.45 y_{1*}$ for symmetric phase

$N = 20$, $n_f = 11/2\, N$, $\delta = 0.2$,
$g(\mu_0) = \frac{4}{9} g_*$, $y_1(\mu_0) = 0.32 y_{1*}$, $y_2(\mu_0) = \frac{1}{5} y_{2*}$,
$\lambda_1(\mu_0) = \frac{1}{30}\lambda_{2*}$, $\lambda_2(\mu_0) = 3\lambda_{2*}$, $\lambda_3(\mu_0) = 5.2\lambda_{3*}$.
These condition corresponds to $\varepsilon \gtrsim 0$.
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Numerical: Couplings Evolution

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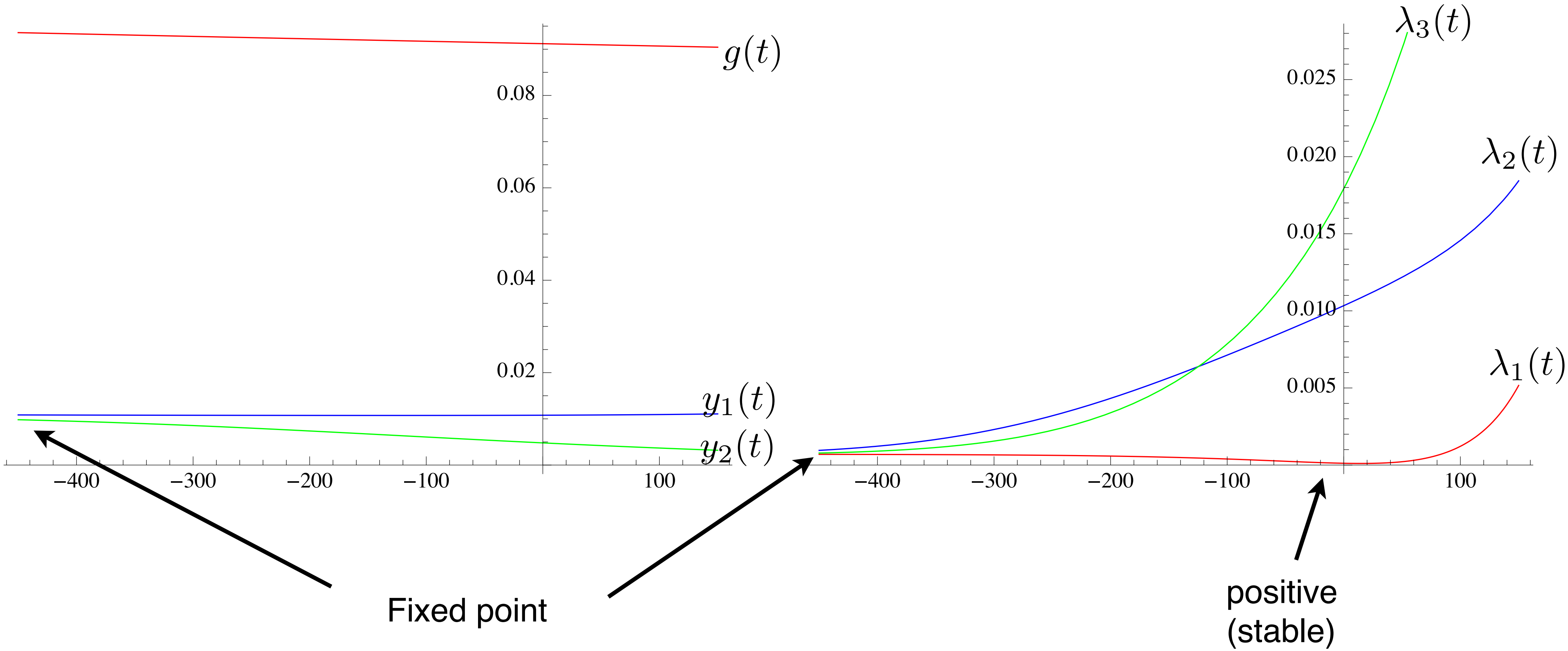
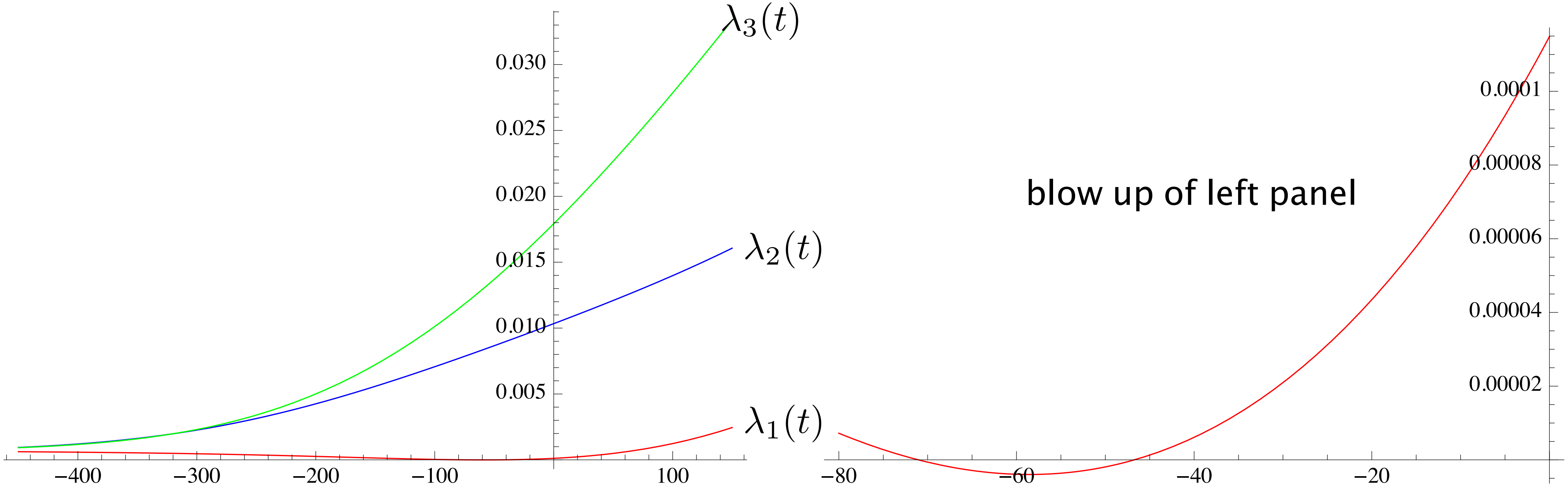
$g(\mu_0) = \frac{4}{9} g_*$, $y_1(\mu_0) = 0.45 y_{1*}$, $y_2(\mu_0) = \frac{1}{5} y_{2*}$,
$\lambda_1(\mu_0) = \frac{1}{30}\lambda_{1*}$, $\lambda_2(\mu_0) = 3\lambda_{2*}$, $\lambda_3(\mu_0) = 5.2\lambda_{3*}$. These condition corresponds to $\varepsilon < 0$. Symmetric phase



Numerical: Broken Phase

$y_1(\mu_0) = 0.32 y_{1*}$. This corresponds to a positive ε. Broken phase



The coupling λ_1 becomes negative during the flow.
This agrees with our expectation from the improved effective potential.

Gauge coupling walks. Eventually runs again. Endgame: glueballs.

Notes on: Dilatons in Conformal Perturbation Theory

- Assume SM is "embedded" in a CFT
- SM couplings ⇒ CFT perturbations
- CFT spontaneously broken ⇒ pseudo-dilaton

Effects of scale invariance: write lagrangian in basis of scale-eigenoperators

$$\mathcal{L} = \sum_i g_i(\mu)\mathcal{O}_i(x), \qquad \text{with } [\mathcal{O}_i] = d_i$$

where
$$\begin{aligned} \mathcal{O}_i(x) &\rightarrow e^{\lambda d_i}\mathcal{O}_i(e^\lambda x), \\ \mu &\rightarrow e^{-\lambda}\mu, \end{aligned} \qquad \text{under} \qquad x^\mu \rightarrow e^\lambda x^\mu.$$

Then
$$\partial_\mu S^\mu = T^\mu{}_\mu = \sum_i g_i(\mu)(d_i - 4)\mathcal{O}_i(x) + \sum_i \beta_i(g)\frac{\partial}{\partial g_i}\mathcal{L}$$

Dilaton as conformal compensator: $\chi(x) \to e^{\lambda}\chi(e^{\lambda}x)$

Then simply replace above $g_i(\mu) \to g_i\left(\mu\frac{\chi}{f}\right)\left(\frac{\chi}{f}\right)^{4-d_i}$

where $f = \langle\chi\rangle$ = order parameter for scale symmetry breaking

and $\chi(x) = fe^{\sigma(x)/f}$ with σ = dilaton (GB of spontaneously broken scale symmetry)

Then expanding about $\bar{\chi}(x) = \chi(x) - f$.

$$\mathcal{L}_\chi = \frac{1}{2}\partial_\mu\bar{\chi}\partial^\mu\bar{\chi} + \frac{\bar{\chi}}{f}T^\mu{}_\mu + \cdots$$

EW sector: strongly interacting higgs sector (or higgsless models), below $\Lambda_{EW} \sim 4\pi v \simeq 1$ TeV

$$\mathcal{L}_{\chi EW} = -\frac{1}{4}(B_{\mu\nu})^2 - \frac{1}{2}\mathrm{tr}W^2_{\mu\nu} + \frac{1}{4}v^2\mathrm{tr}D_\mu U^\dagger D^\mu U + \cdots;$$

with U a unimodular 2 by 2 matrix, $D_\mu U = \partial_\mu U + ig_1 B_\mu U\frac{\tau_3}{2} - ig_2\vec{W}_\mu\cdot\frac{\vec{\tau}}{2}U$

At tree level

$$\mathcal{L}_{\chi,SM} = \left(\frac{2\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2}\right)\left[m_W^2 W^+_\mu W^{-\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu\right] + \frac{\bar{\chi}}{f}\sum_\psi m_\psi\bar{\psi}\psi$$

much like the SM's higgs (but f a free parameter, $f \geq v$)

Dilaton self-couplings (and conformal perturbation theory)

If exact scale invariance, dilaton self-interactions

$$\mathcal{L}_\chi = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{c_4}{(4\pi\chi)^4}\left(\partial_\mu\chi\partial^\mu\chi\right)^2 + \cdots$$

$c_4 \sim \mathcal{O}(1)$
depends on details
of underlying CFT

Now break symmetry with scaling eigenoperators,

$\mathcal{L}_{CFT} \to \mathcal{L}_{CFT} + \lambda_\mathcal{O}\mathcal{O}(x)$ $\quad\quad \Delta_\mathcal{O} \neq 4$

then $$V(\chi) = \chi^4 \sum_{n=0}^{\infty} c_n(\Delta_\mathcal{O}) \left(\frac{\chi}{f}\right)^{n(\Delta_\mathcal{O}-4)}$$

From this we derive effective potentail by placing conditions on cn
(i) That it has $\langle\chi\rangle = f$
(ii) That it gives mass M_σ

$\overline{\text{MS}}$ β Functions

For large N with $n_\chi = 11N/4\,(1-\delta/11)$, the leading terms are

$$(16\pi^2)\frac{\partial g}{\partial t} = -\frac{\delta N}{3}g^3 + \frac{25N^2}{2}\frac{g^5}{16\pi^2}$$

$$(16\pi^2)\frac{\partial y_1}{\partial t} = 4y_1y_2^2 + 11N^2y_1^3 - 3Ng^2y_1$$

$$(16\pi^2)\frac{\partial y_2}{\partial t} = 3y_1^2y_2 + 11N^2y_2^3 - 3Ng^2y_2$$

$$(16\pi^2)\frac{\partial \lambda_1}{\partial t} = 3\lambda_1^2 + 3\lambda_3^2 + 44N^2\lambda_1y_1^2 - 264N^2y_1^4$$

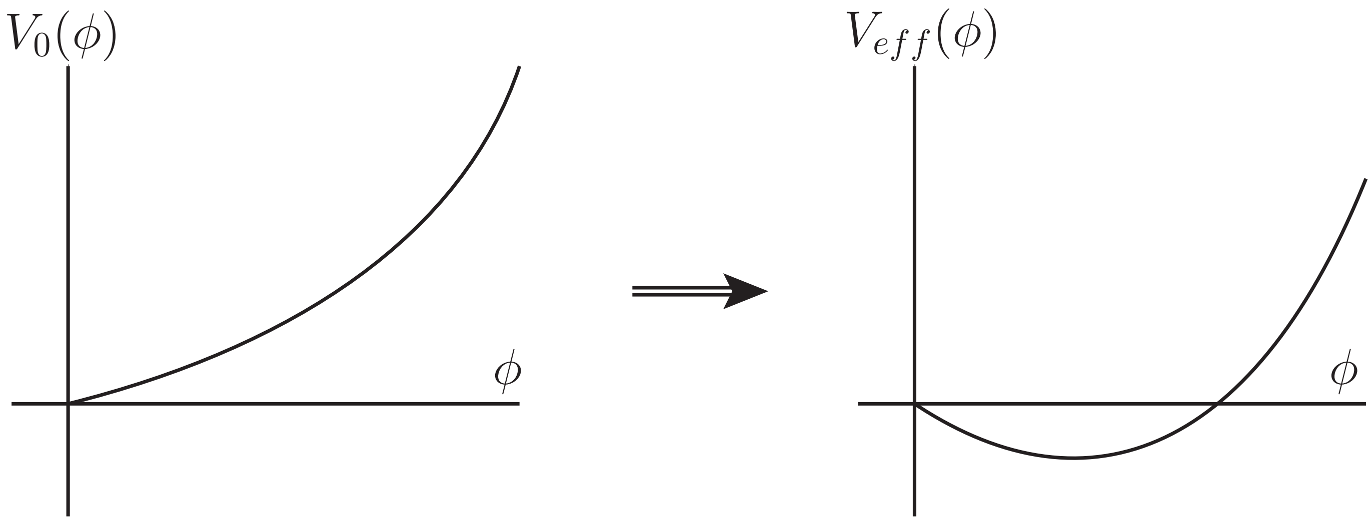
$$(16\pi^2)\frac{\partial \lambda_2}{\partial t} = 3\lambda_2^2 + 3\lambda_3^2 + 44N^2\lambda_2y_2^2 - 264N^2y_2^4$$

$$(16\pi^2)\frac{\partial \lambda_3}{\partial t} = \lambda_1\lambda_3 + \lambda_2\lambda_3 + 4\lambda_3^2 + 22N^2\lambda_3y_1^2 + 22N^2\lambda_3y_2^2 - 264N^2y_1^2y_2^2$$

Effective Potential

At tree-level, $\langle \phi_i \rangle = 0$ and all the particle are massless. The theory flows to the IR fixed-point.

However, quantum effect could drastically change the structure of the vacuum (Coleman, Weinberg '73)



The non-trivial vev gives mass to both fermions and scalars and alters the RG trajectory.

Effective Potential (cont.)

The effective potential in $\overline{\text{MS}}$ is

$$
\begin{aligned}
V_{\text{eff}} = & -\frac{1}{24}\lambda_1\phi_1^4 - \frac{1}{24}\lambda_2\phi_2^4 - \frac{1}{4}\lambda_3\phi_1^2\phi_2^2 \\
& - \frac{11N^2M_{f+}^4}{(64\pi^2)}\left(\ln\frac{M_{f+}^2}{2\mu^2} - \frac{3}{2}\right) - \frac{11N^2M_{f-}^4}{(64\pi^2)}\left(\ln\frac{M_{f-}^2}{2\mu^2} - \frac{3}{2}\right) \\
& + \frac{M_{s+}^4}{(64\pi^2)}\left(\ln\frac{M_{s+}^2}{\mu^2} - \frac{3}{2}\right) + \frac{M_{s-}^4}{(64\pi^2)}\left(\ln\frac{M_{s-}^2}{2\mu^2} - \frac{3}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
M_{f\pm} &= y_1\phi_1 \pm y_2\phi_2, \\
M_{s\pm}^2 &= \frac{(\lambda_1+\lambda_3)\phi_1^2 + (\lambda_2+\lambda_3)\phi_2^2}{4} \\
&\quad \pm \frac{\sqrt{(\lambda_1-\lambda_3)^2\phi_1^4 + (\lambda_2-\lambda_3)^2\phi_2^4 - 2(\lambda_1\lambda_2 - \lambda_1\lambda_3 - \lambda_2\lambda_3 - 7\lambda_3^2)\phi_1^2\phi_2^2}}{4}
\end{aligned}
$$

Role of ϕ_2

Note that ϕ_2 never enters any calculations above.
Moreover, one can get an attractive IR fixed–point with just one scalar singlet.
This raises the question: what is the purpose of the second singlet?

Role of ϕ_2

Note that ϕ_2 never enters any calculations above. Moreover, one can get an attractive IR fixed–point with just one scalar singlet.
This raises the question: what is the purpose of the second singlet?

- It allows us to introduce more couplings, in particular the cross–coupling λ_3.
- Without the second singlet, the extremum found by perturbative analysis would have been the maximum.
    - The scalar potential appears to be unbounded from below.
    - Possible to have non–trivial minimum at higher scale which is inaccessible to perturbative analysis.

Dilaton: The particle (state)

We may look for the dilaton state, σ, by using the following generic criteria:

- spinless state
- couples strongly/linearly to the energy-momentum tensor
- lightest such state

Clearly ϕ_1 satisfies all of the above.

- It is the only state whose mass starts at 1-loop (modulo gauge fields)
- It is the only state which couples linearly to the energy-momentum tensor when expanded about $\phi_1 = v, \ \phi_2 = 0$

Thus we identify σ with a single particle state created by ϕ_1.

Dilatation Current

The dilatation current, $\mathcal{D}^{\mu}$, is constructed from the improved energy-moementum tensor, $\Theta^{\mu\nu}$, of Callan, Coleman and Jackiw.

$$\mathcal{D}^{\mu} = x_{\nu}\Theta^{\mu\nu}$$

$$\Theta^{\mu\nu} = -F^{a\mu\lambda}F^{a\nu}_{\lambda} + \frac{1}{2}\bar{\chi}i(\gamma^{\mu}D^{\nu} + \gamma^{\nu}D^{\mu})\chi + \frac{1}{2}\bar{\psi}i(\gamma^{\mu}D^{\nu} + \gamma^{\nu}D^{\mu})\psi$$
$$+ \partial^{\mu}\phi_i\partial^{\nu}\phi_i - g^{\mu\nu}\mathcal{L} - \frac{1}{2}\kappa(\partial^{\mu}\partial^{\nu} - g^{\mu\nu}\partial^2)\phi_i^2$$

κ is the improvement term. It is a total derivative.
The CCJ improved tensor is the one with $\kappa = 1/3$.

- The improvement term does not change the charges constructed from $\Theta^{\mu\nu}$.
- The matrix elements of $\Theta^{\mu\nu}$arefinite, it doesn't get renormalized.

Trace Anomaly

The divergence of the dilatation current is the trace of the improved energy-momentum tensor.
Classically Θ_μ^μ vanishes for theory without any dimensional couplings.
Quantum effects make Θ_μ^μ non-zero, this is known as trace anomaly.
For the theory under consideration

$$\Theta_\mu^\mu = \gamma_{\phi_1}\phi_1\partial^2\phi_1 + (4\gamma_{\phi_1}\lambda_1 - \beta_{\lambda_1})\frac{\phi_1^4}{24} + \dots$$

Terms involving other fields are omitted.

Terms proportional to γ_{ϕ_1} are usually omitted.
They cancel when EOM is applied but can contribute to off-shell matrix element and Green functions.
Also these terms are needed to make the trace RG-invariant.

Why is it non-straightforward? Try the obvious stuff first:

SSB in CFT?

- Take CFT with moduli space (common in SCFT). Say, for definiteness:
  $\mathcal{N} = 4$ SUSY $SU(N)$ ➠ flat directions ➠ expand about a point away from origin

- EFT = $SU(N) \to SU(N-k) \times SU(k) \times U(1)$.

- But EFT has $\mathcal{N} = 4$ SUSY unbroken, "Dilaton" is exactly massless together with partners ➠ no mass gap

- Perturbations: flow into ??? (possibly another CFT, interacting), fate of "dilaton?"

- More generally: want to study CFT perturbed by classically marginal deformation AND want to understand phase structure (vacuum)