Update of the running coupling in 10-flavor QCD

N. Yamada (KEK & GUAS)

in collaboration with

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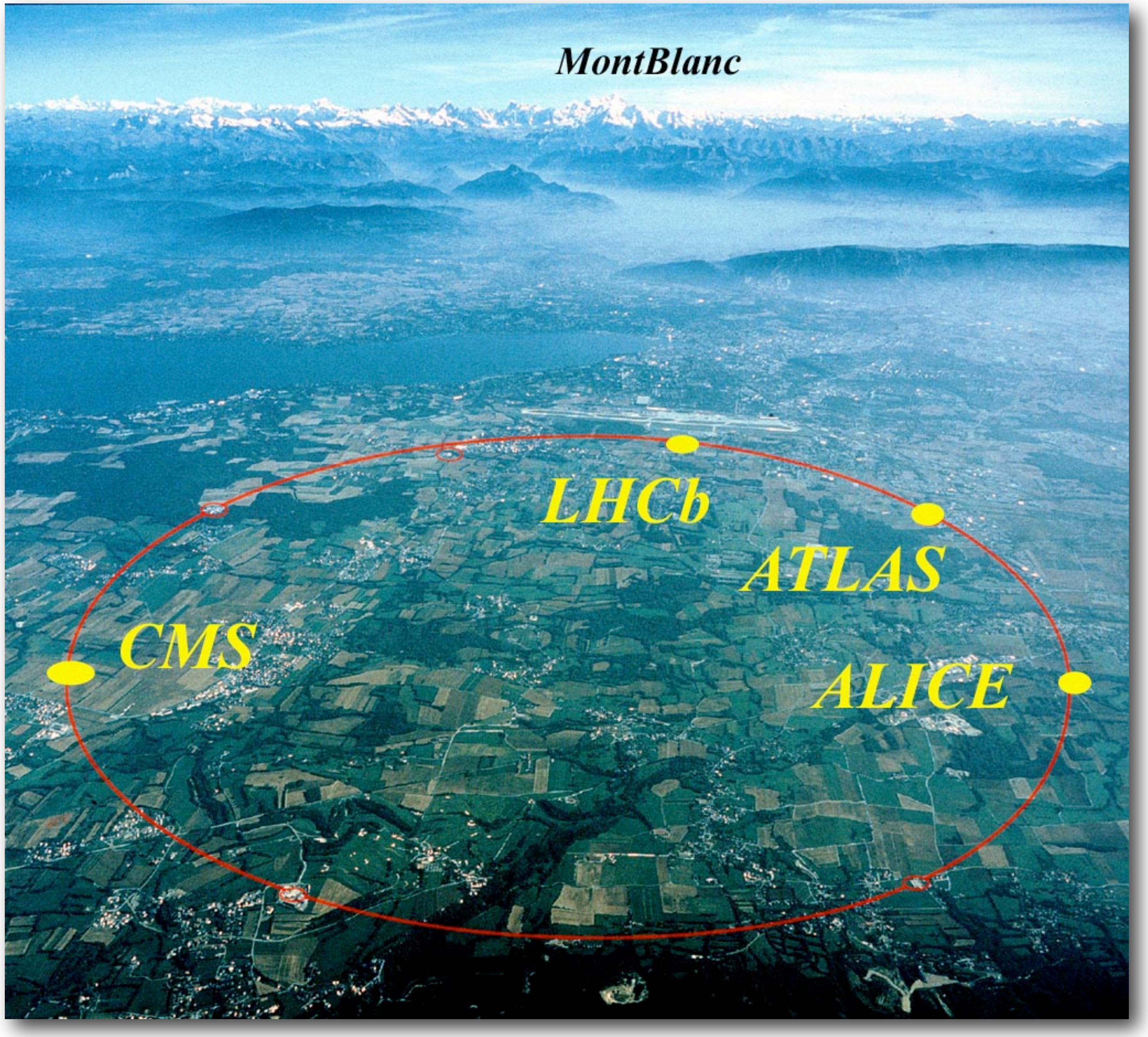
S. Takeda (Columbia)

"Lattice Gauge Theory for LHC Physics", 6-7 November 2009, Boston MA

Contents

Introduction

Going into the LHC era



- Realize particle contents and physics low above the EW scale.
- Confirm Higgs mechanism.
- Among various new physics models, to me the most attractive candidate is ...

Technicolor (TC) [Weinberg(1979), Susskind(1979)]

Attractive alternative to the Higgs sector in the SM

- Strongly interacting vector-like gauge theory

  techni-quarks and techni-gluons instead of quarks and gluons

- "χ-symmetry" is spontaneously broken at Λ_{TC} (like QCD).

  ⇒ Triggers EW symmetry breaking ⇒ W and Z acquire their masses.

- $m_{W^\pm} = g_2 F_{TC} / 2 \Leftrightarrow F_{TC} \sim 250$ GeV

  (F_{TC} : technipion decay constant)

  $\Lambda_{TC} \sim (F_{TC} / f_\pi) \times \Lambda_{QCD} \sim 2600 \times \Lambda_{QCD} \sim O(1)$ TeV

- Elementary scalar is not necessary.

  No "hierarchy problem"

- "Tumbling" might derive the Yukawa hierarchy. [Ultimate desire]

Lattice technique provides the best way to study TC.

Computational Resources

Usually, lattice simulations
are very expensive.
First priority is QCD.

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are very expensive.
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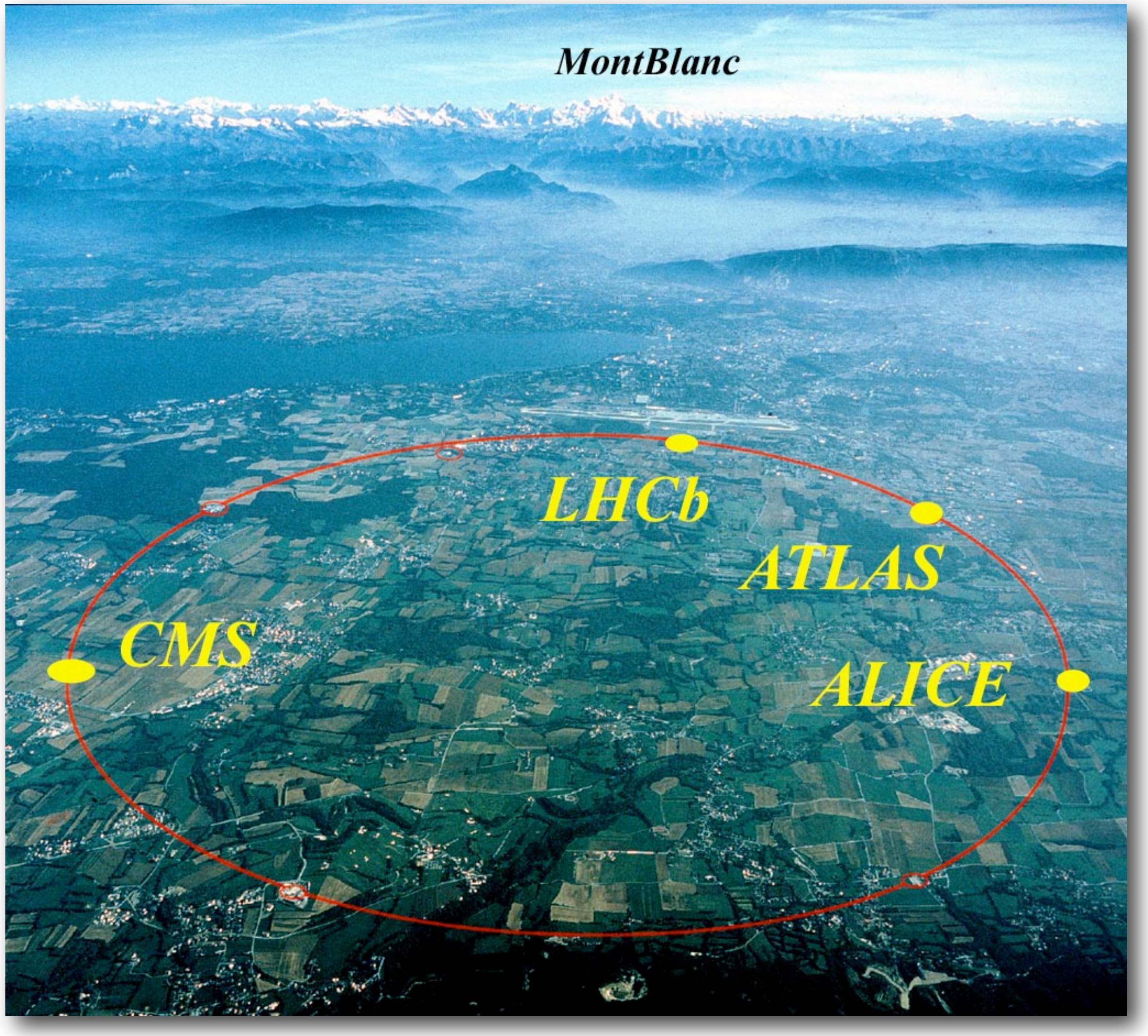
Continuous developments of
Hardware/Software

⬇

Room for non QCD gauge theory


MontBlanc
LHCb
ATLAS
CMS
ALICE

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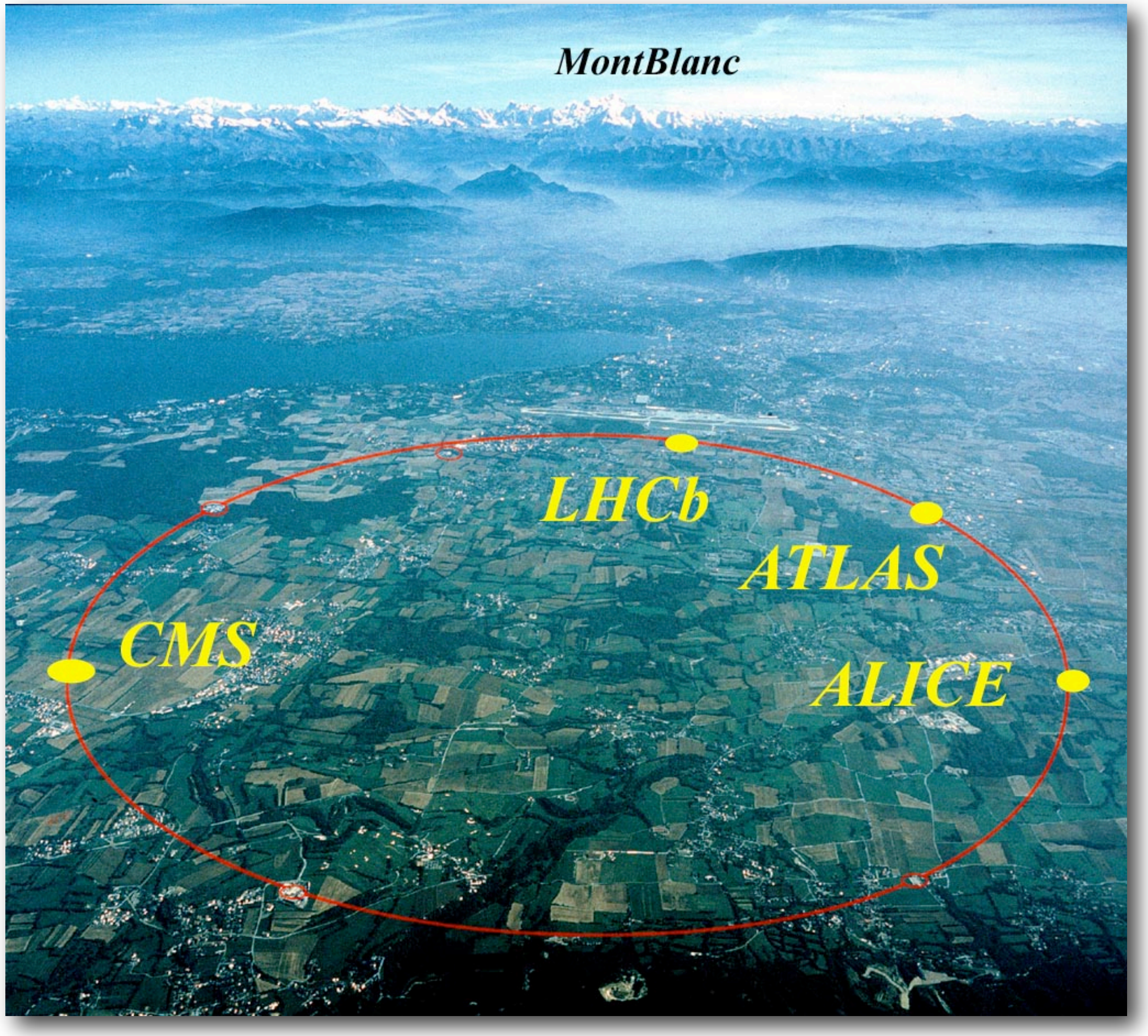

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Time to study TC from the first principles!

Problems in traditional TC

- FCNC & m_q
- S-parameter
- ...

Walking Technicolor (WTC)

Holdom(1981), Yamawaki, Bando, Matsumoto(1986), Appelquist, Karabali, Wijewardhana(1986), Akiba, Yanagida(1986), Bando, Morozumi, So, Yamawaki(1987)

At $\mu < M_{\rm ETC}$, $\dfrac{g_{ETC}^2}{M_{ETC}^2} C(\mu) (\bar{F}F)(\bar{f}f)$ & $\dfrac{g_{\rm ETC}^2}{M_{\rm ETC}^2} (\bar{f}' \Gamma_\mu f)(\bar{f}' \Gamma_\mu f)$

quark/lepton mass FCNC

$$C(\mu) = \exp\left(\int_\mu^{M_{ETC}} d\mu' \frac{\gamma(\mu')}{\mu'} \right) = \exp\left(\int_{g^2(\mu)}^{g^2(M_{ETC})} dg'^2 \frac{\gamma(g'^2)}{\beta(g'^2)} \right)$$

C(μ) >>1 may resolve **the FCNC & m_q problem**!

This occurs if $g_{\rm TC}^2$ walks over a wide range of the energy scale at a relatively large value of $g_{\rm TC}^2$ ($\beta(g_{\rm TC}^2)\approx 0$).

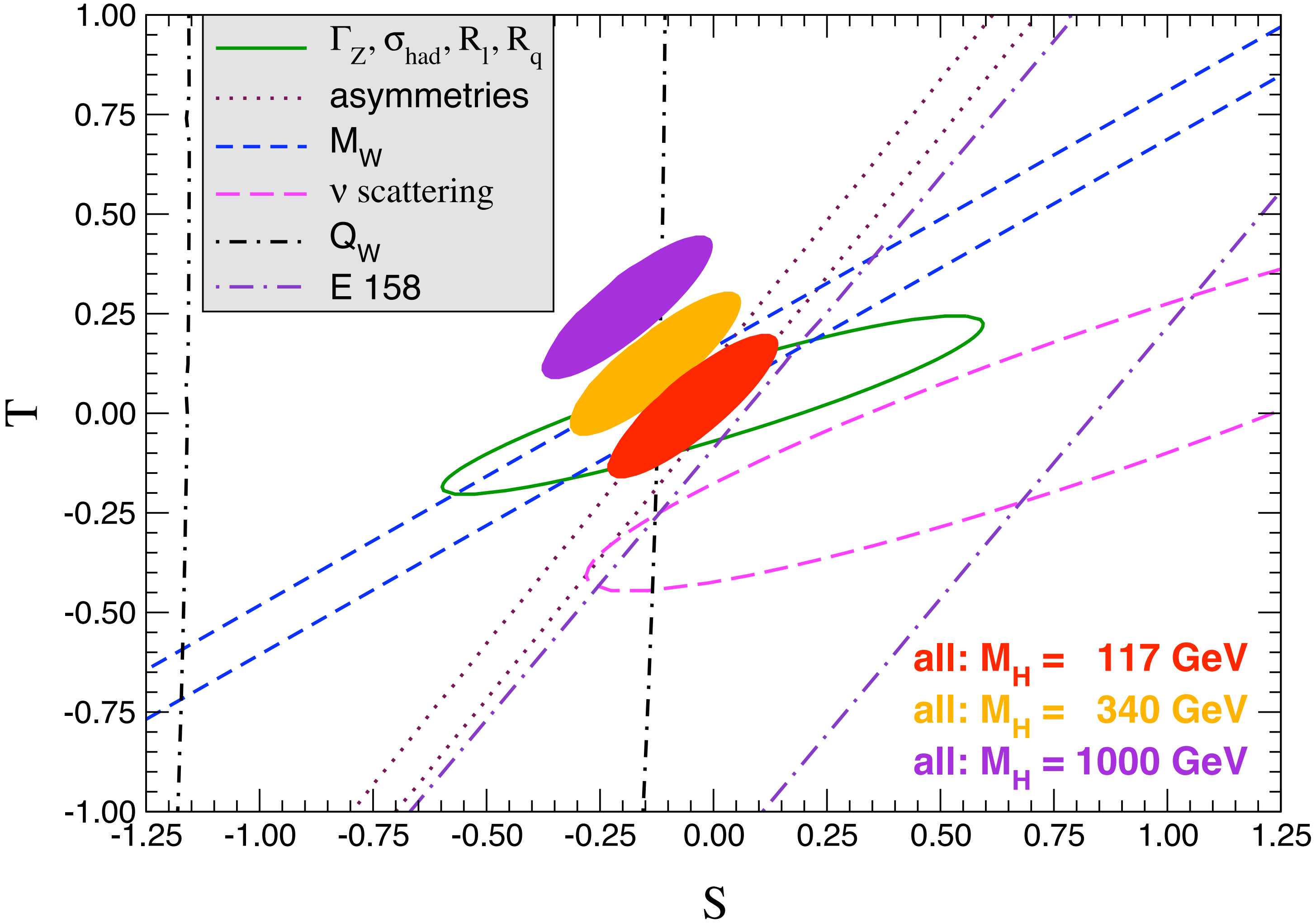
➡ Find such theories!

S-parameter

- S-parameter is estimated to be large in simple QCD-like TC.
- However, since in WTC dynamics might be drastically changed, naive estimation will not work.
- Need lattice calculations!

cf.
JLQCD Collaboration, PRL101, 242001 (2008);
RBC and UKQCD, arXiv:0909.4931 [hep-lat]

Methodology has been established.



Candidates of WTC

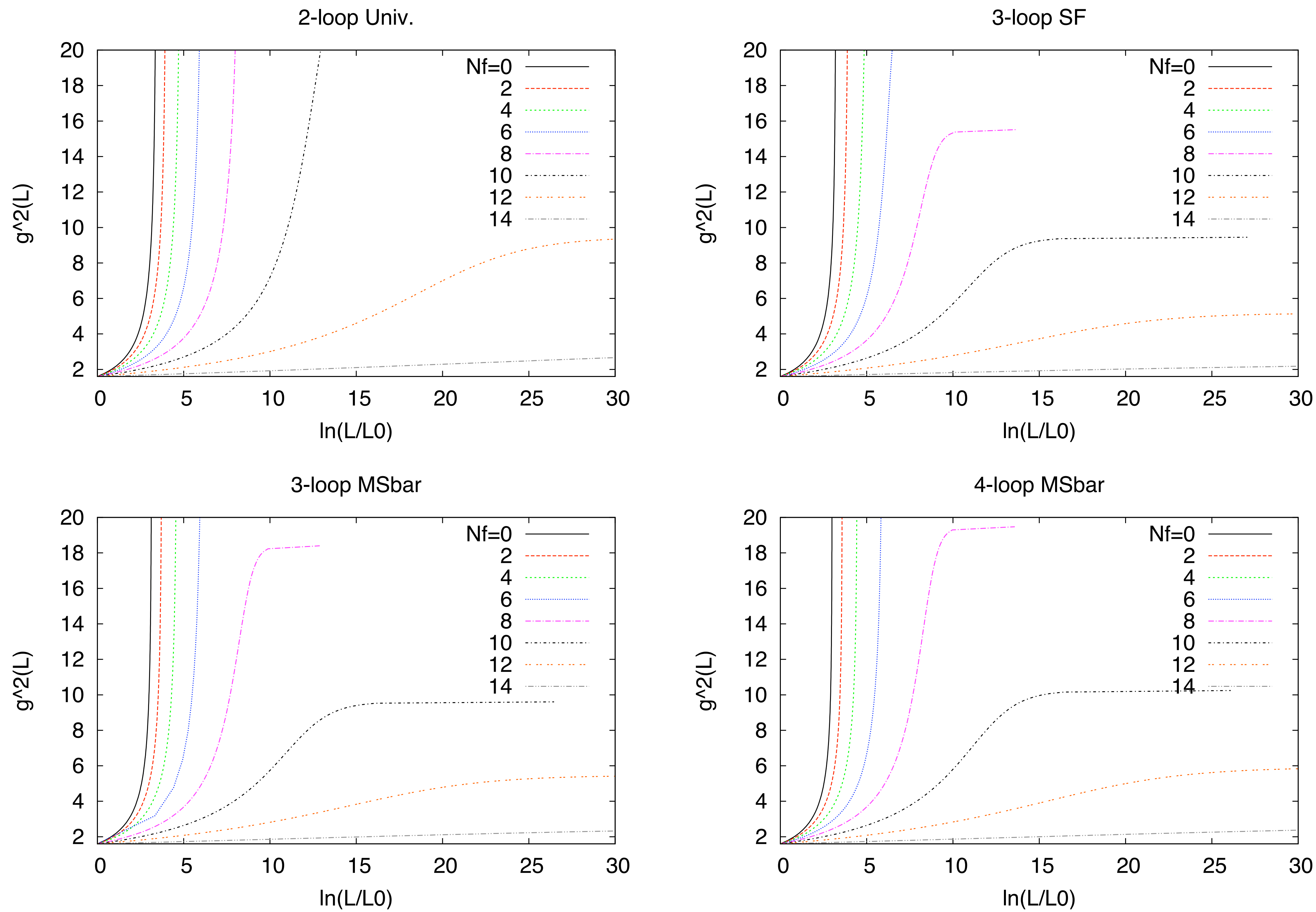
So far, three SU(Nc) gauge theories have been intensively studied on the lattice.

	N_c	N_f	Rep.
Large N_f QCD	3	6~16	fund.
Sextet QCD	3	2	sextet
Two-color adjoint QCD (Minimal WTC)	2	2	adjoint

This work:
Running coupling of N_c=3 & N_f=10 gauge theory, *i.e.* 10-flavor QCD

Review of Perturbative Analysis

SU(3) gauge theory with N_f fundamental Dirac fermions:



N. Yamada

Review of Perturbative Analysis

SU(3) gauge theory with N_f fundamental Dirac fermions:

Perturbative IRFP for SU(3) gauge theory with fermions in fund. rep.

N_f	4	6	8	10	12	14	16
2-loop universal				27.74	9.47	3.49	0.52
3-loop SF	43.36	23.75	15.52	9.45	5.18	2.43	0.47
3-loop $\overline{\mathrm{MS}}$		159.92	18.40	9.60	5.46	2.70	0.50
4-loop $\overline{\mathrm{MS}}$			19.47	10.24	5.91	2.81	0.50

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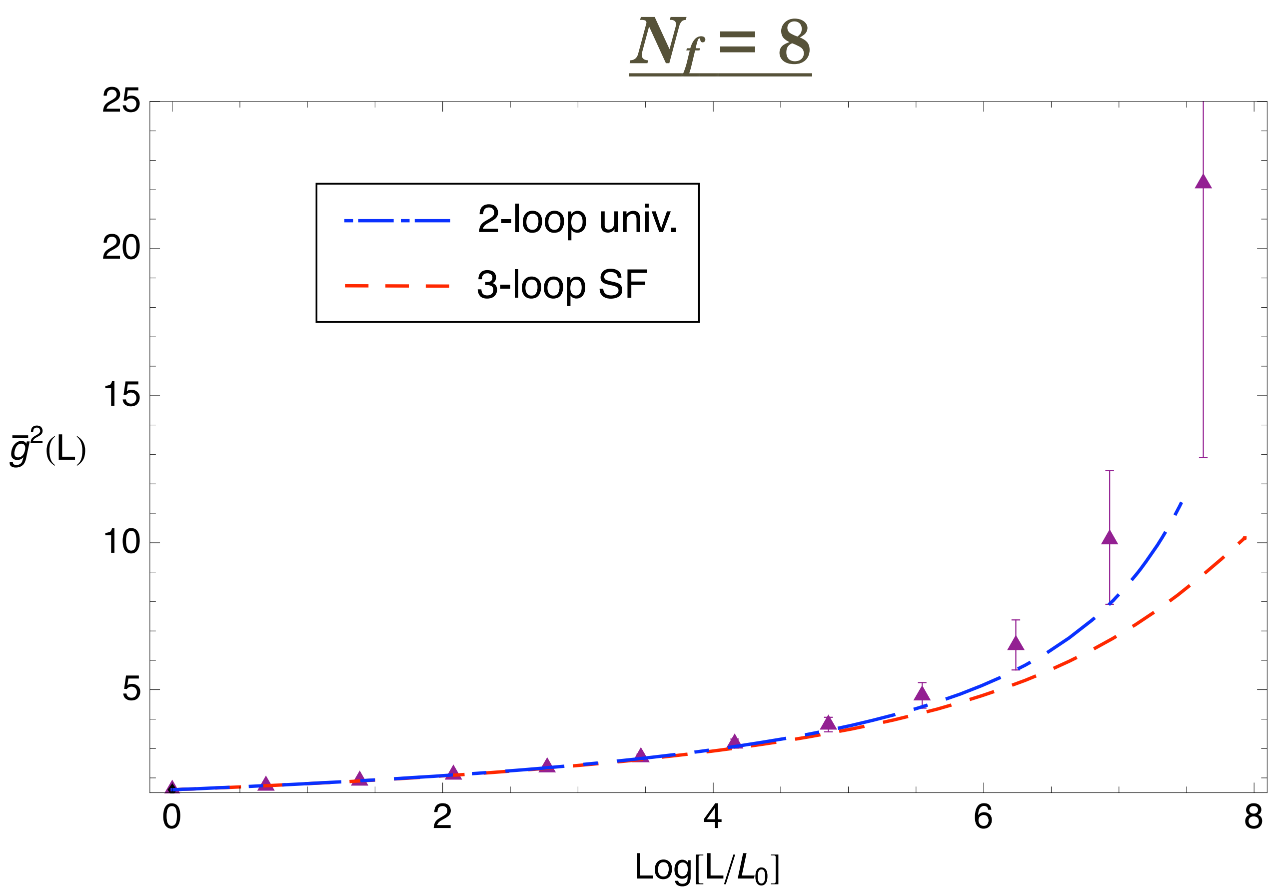
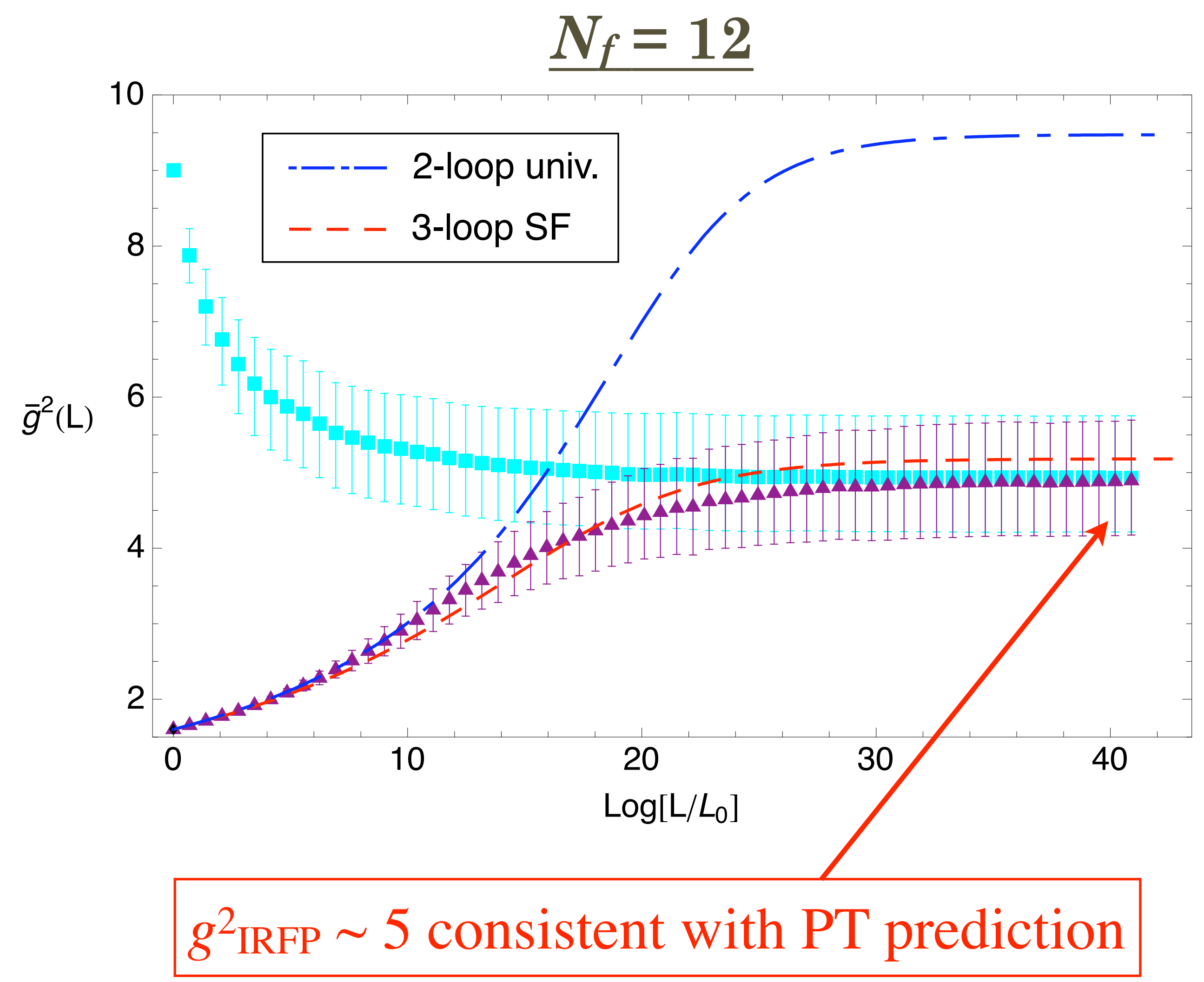
- Schwinger–Dyson analysis suggests that SχSB occurs when $g_{\mathrm{crit}}^2 \sim \pi^2$.
- PT analysis suggests that N_f =8~12 are interesting!

8- & 12-flavor QCD with SF

Appelquist, Fleming, Neil, PRL100:171607, 2008; PRD79:076010, 2009





Conclusion: N_f=12 is conformal while N_f=8 is not.

⇒ N_f=12 is too large and N_f=8 is too small for WTC

Analysis method

trick to take the "continuum limit"

Simulation parameters

- **Schrödinger Functional method**
  Luscher, Weisz, Wolff, NPB(1991), and many papers
- **Wilson plaquette gauge + Wilson fermions**
  No $O(a)$ improvement at all
- Box size : L/a = (4), **6**, **8**, **12**, (16)
- $\beta = 4.4 \sim 24.0$
- # of trajectories 5 k ~ 200 k
- Standard HMC algorithm with some improvements in solver

Some remarks

- Wilson fermion breaks χ-symmetry
  $\Rightarrow$ need to tune $\varkappa$ to m_q=0
  Very demanding but we did
- Continuum limit is taken by putting two reasonable assumptions
  Validity will be checked in future.
- Why Wilson?
  SW clover →bulk phase transition.
- Calculation is still in progress
  All results are preliminary.

How to get a running

Step. 1

Calculate the SF coupling on $l_1=L_1/a$ at various g_0^2.
Then make an interpolating function by fitting data.

$$g_{l_1}^2 = g_{l_1}^2(g_0^2)$$

Repeat this with $l_2=L_2/a$, $l_1'=L_1'/a$. $l_2'=L_2'/a$.

$$g_{l_2}^2 = g_{l_2}^2(g_0^2),\ g_{l_1'}^2 = g_{l_1'}^2(g_0^2),\ g_{l_2'}^2 = g_{l_2'}^2(g_0^2)$$

(In this analysis, $l_2=l_1'$ or $l_2=l_2'$.)

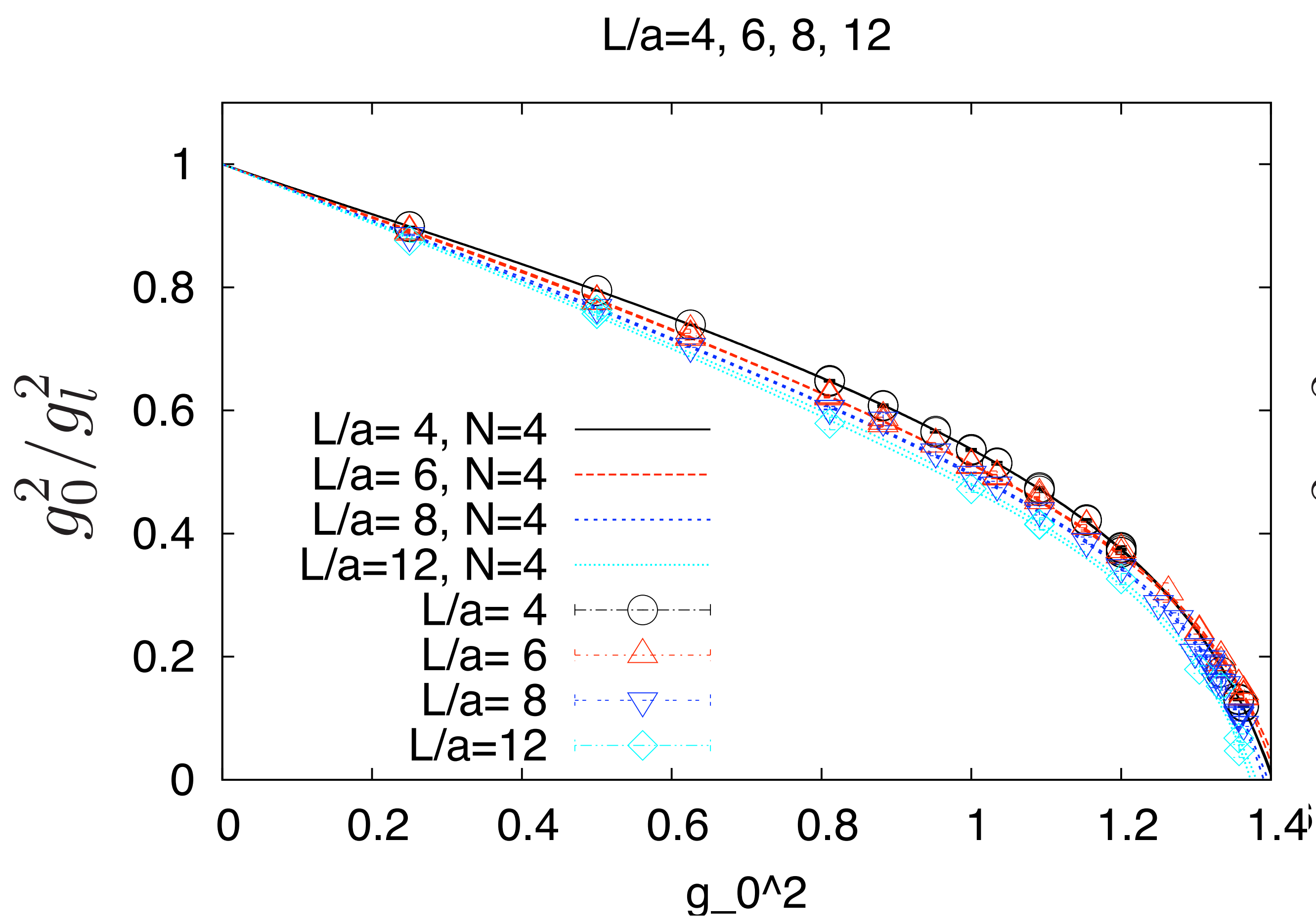
Fit to interpolating function

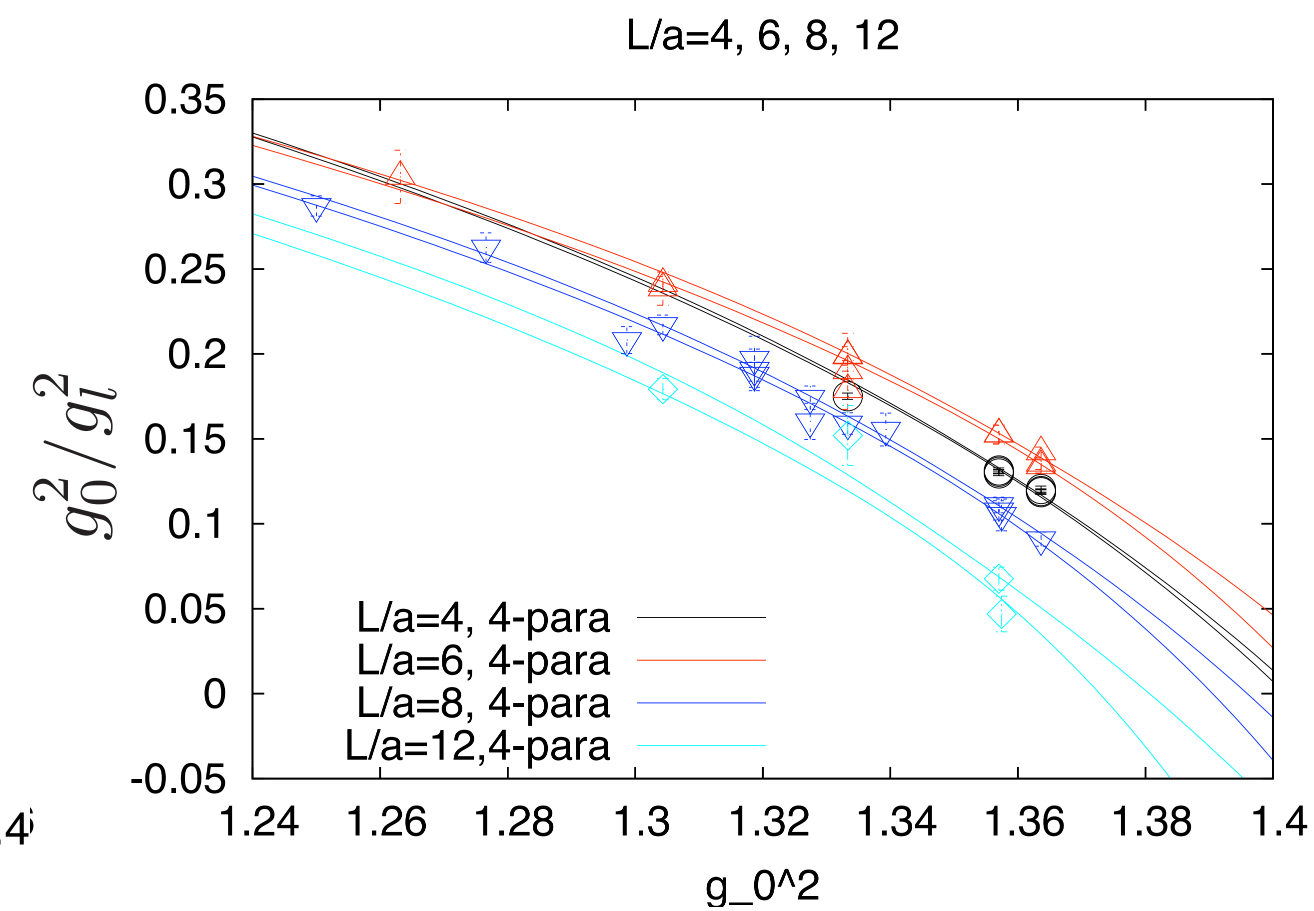
Functional form:

$$\frac{g_0^2}{{g_{L/a}^{\text{fit}}}^2} = \frac{1 - a_{L/a,1}\, g_0^4}{1 + p_1\, g_0^2 + \sum_{n=2}^{N} a_{L/a,n} \times {g_0^2}^n}$$

p_1 : fixed to the know value

N : fixed to 4 by monitoring χ^2/dof





This form describes the whole data very well.

How to get a running

Step. 2

Choose an initial value of running, which implicitly sets the initial scale, L_0. $u_\mathrm{i} = \alpha(L_0)$.

On l_1 and l'_1 lattices, find g_0^{*2} satisfying

$$u_i = g_{l_1}^2(g_0^{*,2}) \Rightarrow g_0^{*,2}(u_i, l_1)$$

$$u_i = g_{l'_1}^2(g_0^{*,2}) \Rightarrow g_0^{*,2}(u_i, l'_1)$$

Then estimate

$$g_{l_2}^2(g_0^{*,2}) = g_{l_2}^2(u_i, l_1, l_2) = u_{i+1}(u_i, r, l_1)$$

$$g_{l'_2}^2(g_0^{*,2}) = g_{l'_2}^2(u_i, l'_1, l'_2) = u_{i+1}(u_i, r', l'_1)$$

where $r = l_2/l_1$, $r' = l_2'/l_1'$.

How to get a running

Step. 3

$$g^2_{l_2}(g_0^{*,2}) = g^2_{l_2}(u_i, l_1, l_2) = u_{i+1}(u_i, r, l_1)$$
$$g^2_{l'_2}(g_0^{*,2}) = g^2_{l'_2}(u_i, l'_1, l'_2) = u_{i+1}(u_i, r', l'_1)$$

Take the continuum limit of u_{i+1} to obtain $u^{SF}_{i+1}(u_i, r)$ [explained later].

$u^{SF}_{i+1}(u_i, r) = \alpha(rL_0)$

Repeating the above gives you
$\alpha(L_0)$, $\alpha(rL_0)$, $\alpha(r^2L_0)$, ..., $\Rightarrow$"running".

“Continuum limit”

Two assumptions:

- Discretization error is dominated by linear.
- $B^{\mathrm{SF}}(u,r) = \frac{\ln(r)}{\ln(r')} B^{\mathrm{SF}}(u,r')$

Discrete beta function:
$$B^{l_1 l_2}(u,r) = \frac{1}{g^2_{l_1 l_2}(u,r)} - \frac{1}{u}$$

Y. Shamir, B. Svetitsky and T. DeGrand,
PRD78(2008)031502

$$B^{\mathrm{SF}}(u,r) = \frac{1}{g^2_{\mathrm{SF}}(u,r)} - \frac{1}{u}$$

where $r = l_2/l_1$ and $l_i = L_i/a$

"Continuum limit"

Two assumptions:

- Discretization error is dominated by linear.
- $B^{\rm SF}(u,r) = \frac{\ln(r)}{\ln(r')} B^{\rm SF}(u,r')$

$$
\begin{aligned}
\frac{1}{u_1^{l_1 l_2}|_{u_0}} - \frac{1}{u_1^{\rm SF}|_{u_0}} &= \left(\frac{1}{u_1^{l_1 l_2}|_{u_0}} - \frac{1}{u_0}\right) - \left(\frac{1}{u_1^{\rm SF}|_{u_0}} - \frac{1}{u_0}\right) \\
&= B^{l_1 l_2}(u_0, r) - B^{\rm SF}(u_0, r) \\
&= e(u_0)\left(\frac{1}{l_1} - \frac{1}{r l_1}\right) + O(a^2),
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{u_1^{l'_1 l'_2}|_{u_0}} - \frac{1}{u_1^{\rm SF}|_{u_0}} &= B^{l'_1 l'_2}(u_0, r') - B^{\rm SF}(u_0, r') \\
&= e(u_0)\left(\frac{1}{l'_1} - \frac{1}{r' l'_1}\right) + O(a^2)
\end{aligned}
$$

“Continuum limit”

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- Discretization error is dominated by linear.
- $B^{\mathrm{SF}}(u,r) = \frac{\ln(r)}{\ln(r')} B^{\mathrm{SF}}(u,r')$

$$B^{l_1 l_2}(u_0,r) = \underline{B^{\mathrm{SF}}(u_0,r)} + e(u_0)\left(\frac{1}{l_1} - \frac{1}{r l_1}\right) + O(a^2)$$

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$$\begin{aligned}\frac{\ln(r)}{\ln(r')} B^{l'_1 l'_2}(u_0,r') &= \frac{\ln(r)}{\ln(r')} B^{\mathrm{SF}}(u_0,r') + \frac{\ln(r)}{\ln(r')} e(u_0)\left(\frac{1}{l'_1} - \frac{1}{r' l'_1}\right) + O(a^2) \\ &\approx \underline{B^{\mathrm{SF}}(u_0,r)} + e(u_0)\frac{\ln(r)}{\ln(r')}\left(\frac{1}{l'_1} - \frac{1}{r' l'_1}\right).\end{aligned}$$

“Continuum limit”

Two assumptions:

- Discretization error is dominated by linear.
- $B^{\rm SF}(u,r) = \frac{\ln(r)}{\ln(r')} B^{\rm SF}(u,r')$

Perturbative analysis seems to support this approximation holds very well.

$$
\begin{aligned}
\delta B(u_0,r,r') &= B^{\rm SF}(u_0,r) - \frac{\ln(r)}{\ln(r')} B^{\rm SF}(u_0,r') \\
&= {u_0}^2 \ln(r) \ln\left(\frac{r}{r'}\right) \Bigg[-\frac{1}{2} b_1 b_2 + u_0 \Big\{ -\frac{1}{3} {b_1}^2 b_2 \ln(rr') - \Big(b_1 b_3 + \frac{1}{2}{b_2}^2\Big)\Big\} \\
&\quad + {u_0}^2 \frac{1}{12}\Big\{ -3{b_1}^3 b_2 \big(\ln^2(r) + \ln(r)\ln(r') + \ln^2(r')\big) - 12{b_1}^2 b_3 \ln(rr') \\
&\qquad -10 b_1 {b_2}^2 \ln(rr') - 18 b_1 b_4 - 18 b_2 b_3 \Big\} \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
b_1 &= 0.0548823, \\
b_2 &= -0.00197834, \\
b_3^{\overline{\rm MS}} &= -0.000388922, \\
b_4^{\overline{\rm MS}} &= 5.75135 \times 10^{-6}
\end{aligned}
$$

$$
\begin{aligned}
\delta B(u_0, 8/6, 12/8) &= -1.8395\times 10^{-6} u_0^2 - 7.03597\times 10^{-7} u_0^3 + 3.0832310^{-8} u_0^4 + \cdots \\
\delta B(u_0, 12/8, 12/6) &= -6.33242\times 10^{-6} u_0^2 - 2.51606\times 10^{-6} u_0^3 + 5.384\times 10^{-8} u_0^4 + \cdots
\end{aligned}
$$

"Continuum limit"

Two assumptions:

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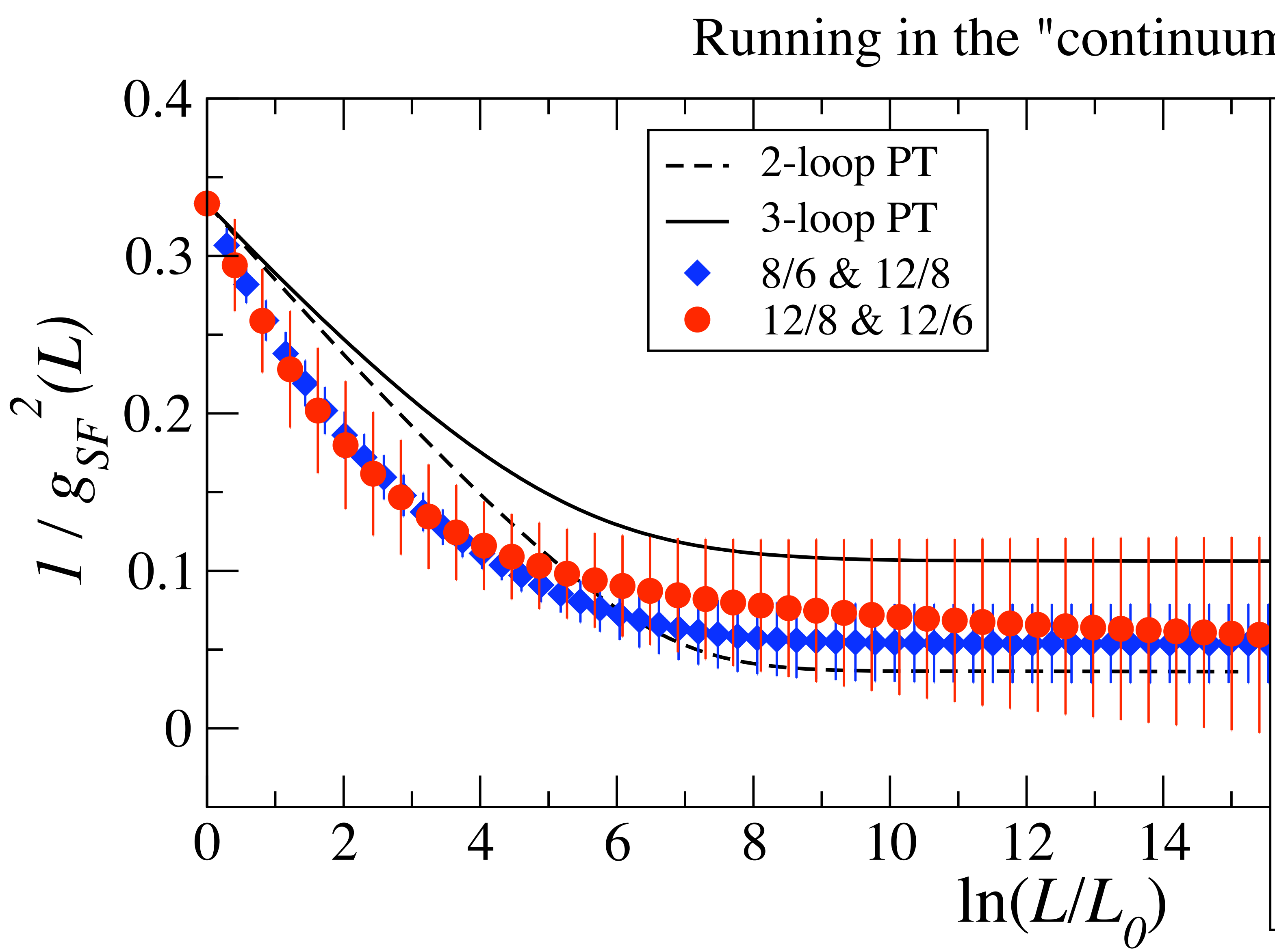
$$
\begin{aligned}
B^{l_1 l_2}(u_0,r) &= B^{\mathrm{SF}}(u_0,r) + e(u_0)\left(\frac{1}{l_1} - \frac{1}{r l_1}\right) \\
\frac{\ln(r)}{\ln(r')} B^{l'_1 l'_2}(u_0,r') &= B^{\mathrm{SF}}(u_0,r) + e(u_0)\frac{\ln(r)}{\ln(r')}\left(\frac{1}{l'_1} - \frac{1}{r' l'_1}\right)
\end{aligned}
$$

$$
B^{\mathrm{SF}}(u_0,r) \;\Rightarrow\; \frac{1}{u_1^{\mathrm{SF}}|_{u_0}} = \frac{1 + u_0\, B^{\mathrm{SF}}(u_0,r)}{u_0}
$$

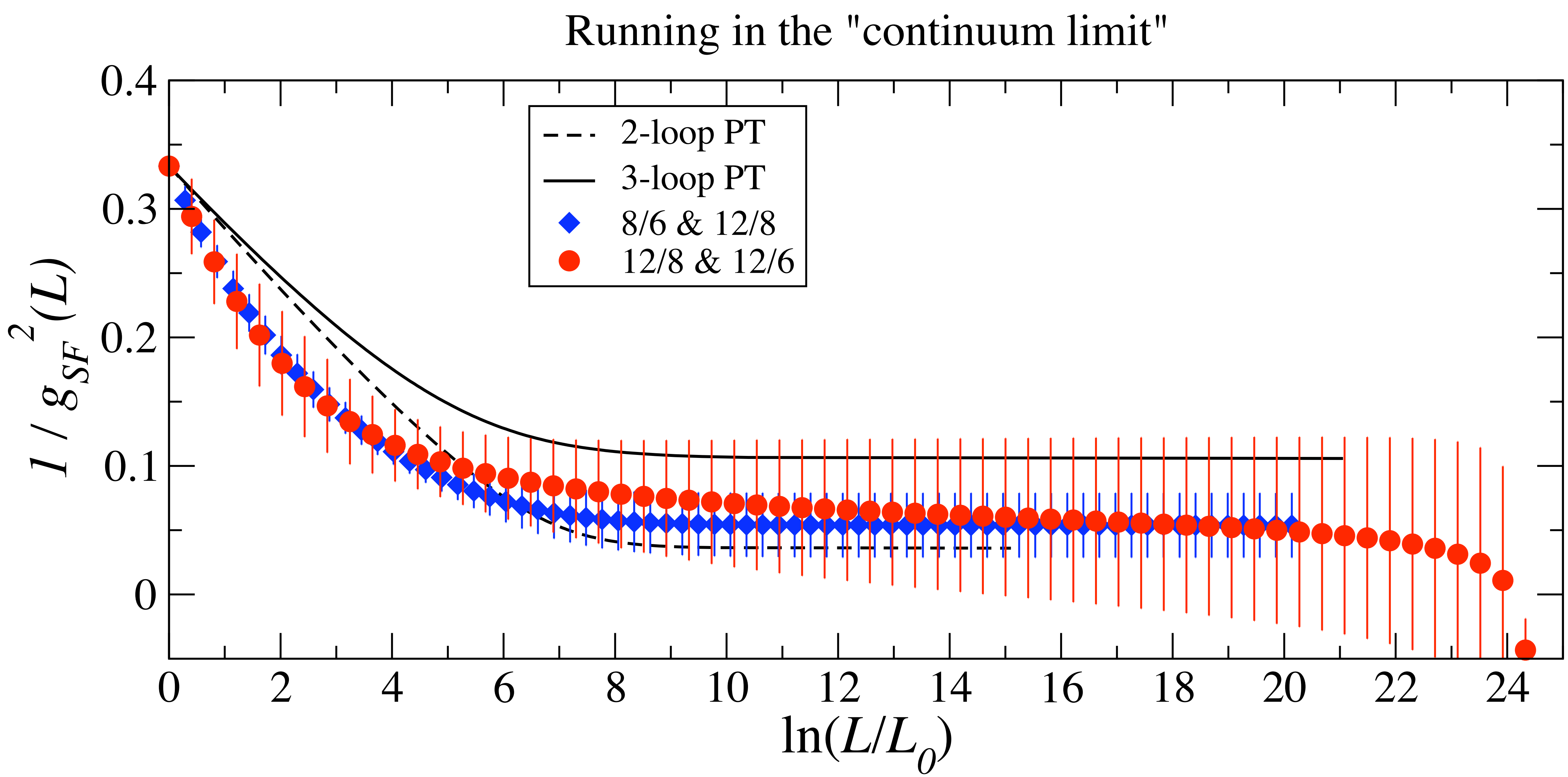
Repeat with $u_i \rightarrow u_{i+1}$ to determine u_{i+2}

Running in “continuum limit”



- $(l_1, l_2, l_1', l_2')=(8, 6, 12, 8)$
- $(l_1, l_2, l_1', l_2')=(12, 8, 12, 6)$
- Two results agree and show an inflection point.
- $L/a=16$ data will be used to check the validity of the two assumptions and to confirm this behavior.

Running in “continuum limit”



TQ gets constituent quark mass and decouples? This is what is expected for ideal WTC!

although this is statistically unstable and the “divergent region” is not covered by raw data.

Will Matsui be in the pinstripes next year?

Summary and Outlook

✓Running coupling of 10-flavor QCD is studied.

✓Putting two reasonable assumptions, we took the continuum limit.

✓Preliminary analysis suggests that this theory is really close to the lower end of the conformal window.

✓L/a=16 data will help to confirm this statement.

✓We will proceed to γ.