Conformality Lost

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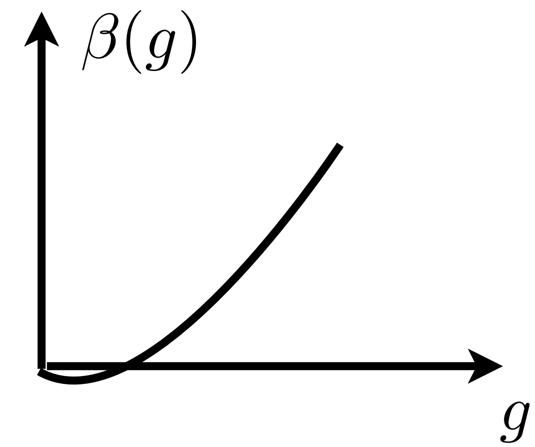
Motivation

Consider large N_c , large N_f QCD

N_f/N_c can be changed continuously

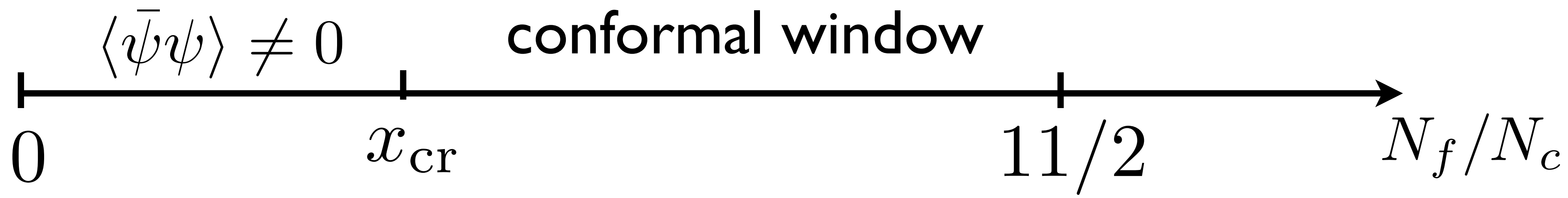


$\frac{N_f}{N_c} = \frac{11}{2} - \epsilon$ Banks-Zaks fixed point, CFT

$$\beta(g) = -\epsilon g^3 + \# g^5$$

$\frac{N_f}{N_c}$ small Confinement, chiral symmetry breaking

There exists a critical N_f/N_c where transition happens

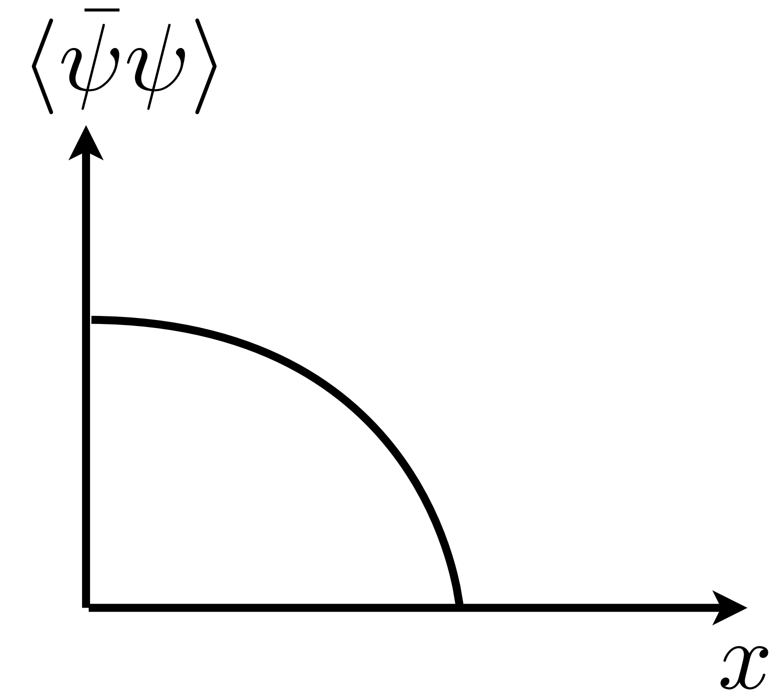


Possibilities:

- Power-law?

$$\langle\bar{\psi}\psi\rangle \sim (x_{\rm cr}-x)^{\beta}$$

$$x = \frac{N_f}{N_c}$$



typical of a 2nd order phase transition

But: in a 2nd order pt, conformal symmetry only at the phase transition

Here the system should be conformal for any $x > x_{\rm cr}$

- Another possibility:

  a Berezinskii-Kosterlitz-Thouless phase transition

BKT scaling:

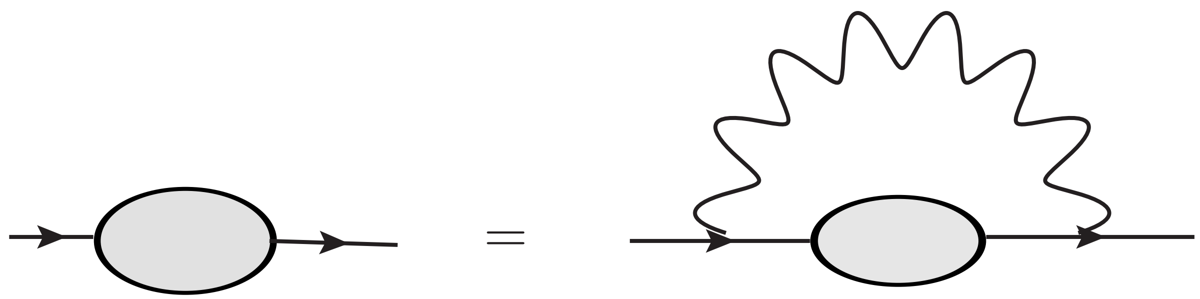
$$\xi^{-1}(T) = \begin{cases} \exp\left(-\frac{\#}{\sqrt{T-T_{\rm cr}}}\right) & T > T_{\rm cr} \\ 0 & T < T_{\rm cr} \end{cases}$$

If this is the case: chiral condensate goes to zero exponentially, with all derivatives vanishing as $x \to x_{\rm cr}$

physics: vortex condensation

But the BKT phase transition is very specific for 2D, while QCD is a 4D theory

Nevertheless: Schwinger-Dyson approach gives BKT scaling



Miransky 1985

Appelquist, Terning, Wijerwardhana 1996

Uncontrolled: Critical N_f/N_c is unreliable

May the scaling be right?

Quantum mechanics with $1/r^2$ potential

Schrödinger equation $$-\psi'' - \frac{2}{r}\psi'(r) + \frac{\alpha}{r^2}\psi(r) = E\psi(r)$$

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Short-distance behavior:

$$\psi(r) \sim \frac{1}{r^\nu} \qquad \nu = \frac{1}{2}(1 \pm \sqrt{1+4\alpha})$$

$\alpha > -1/4$ conformal QM

$\alpha < -1/4$ nonconformal: cutoff needed

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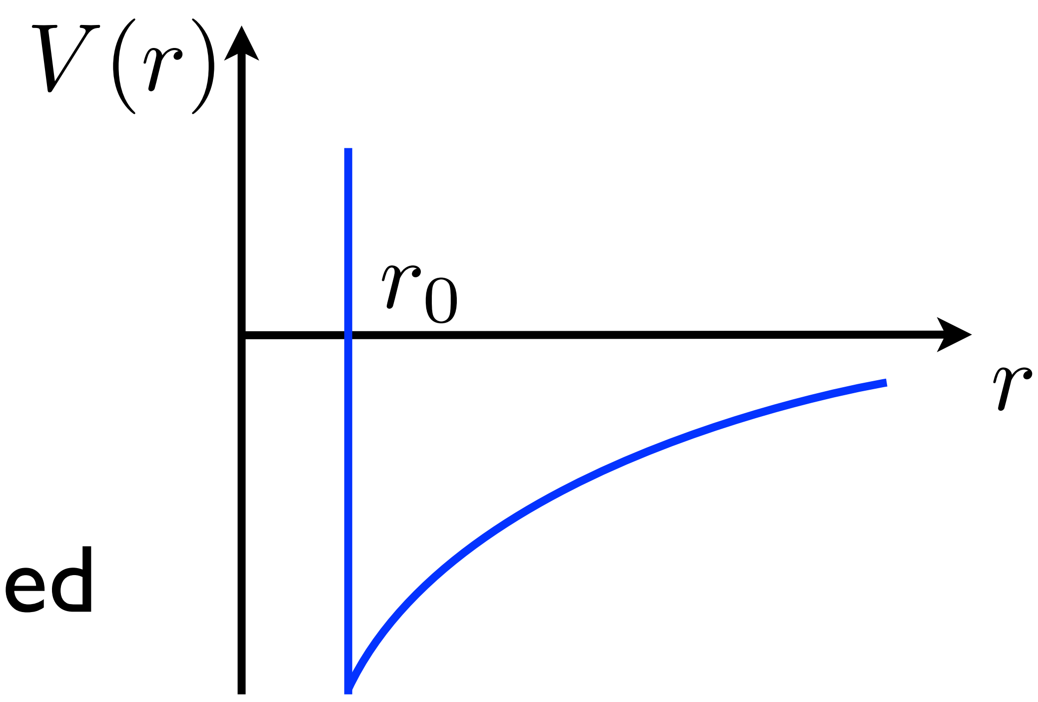

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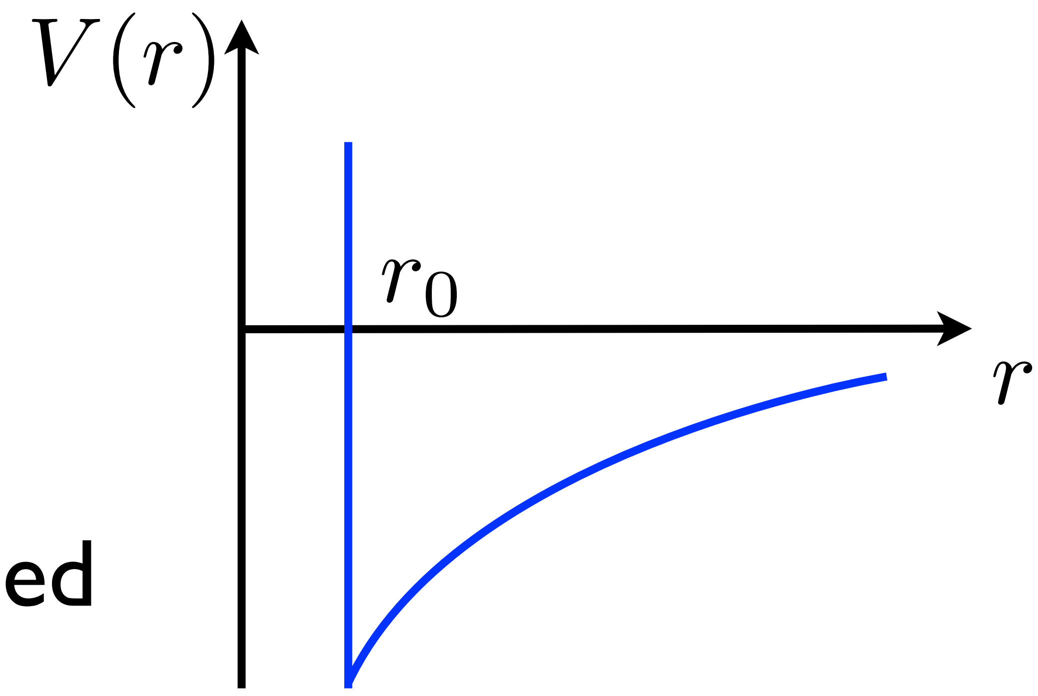

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Let us put an infinite repulsive core for $r < r_0$

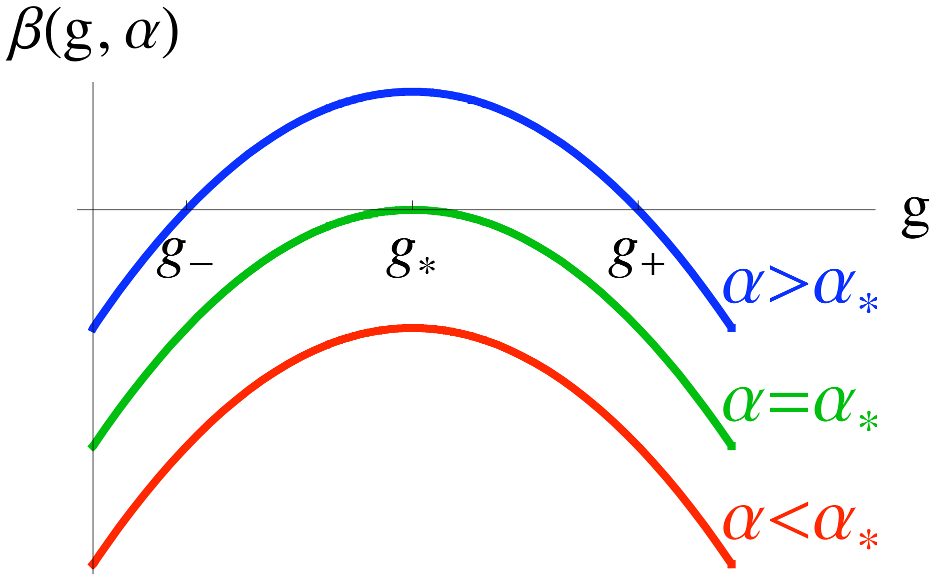
The potential always has bound state, and the energy is

$$\frac{1}{mr_0^2} \exp\left(-\frac{2\pi}{\sqrt{-1/4-\alpha}}\right)$$ BKT scaling again!

- Three transitions have the same scaling:
  - The BKT phase transition
  - The transition in QM with $1/r^2$ potential
  - Chiral phase transition in SD approach
- Pure coincidence, or there is a deeper reason?

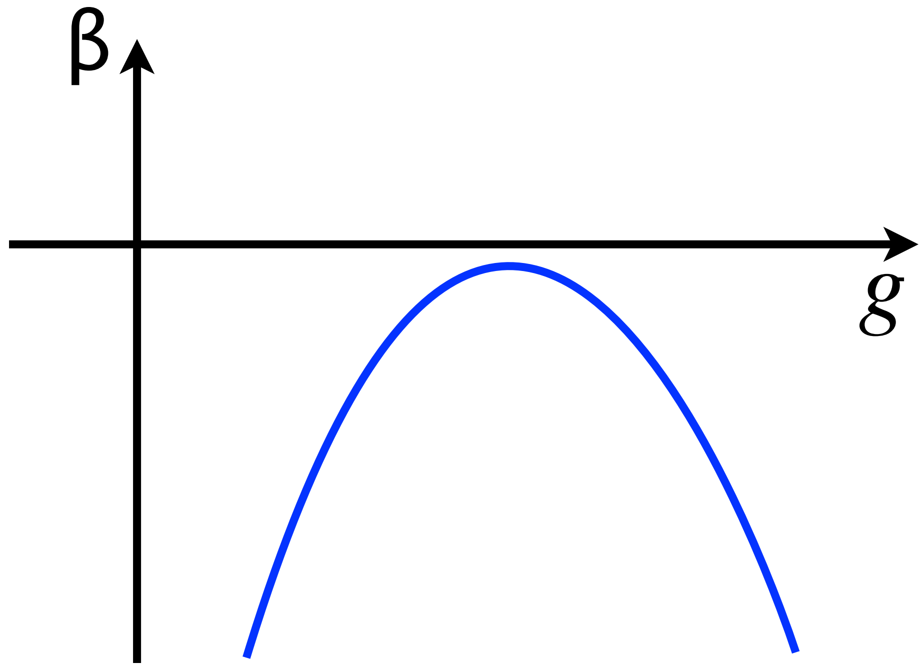
In the language of renormalization group, conformality may be lost due to

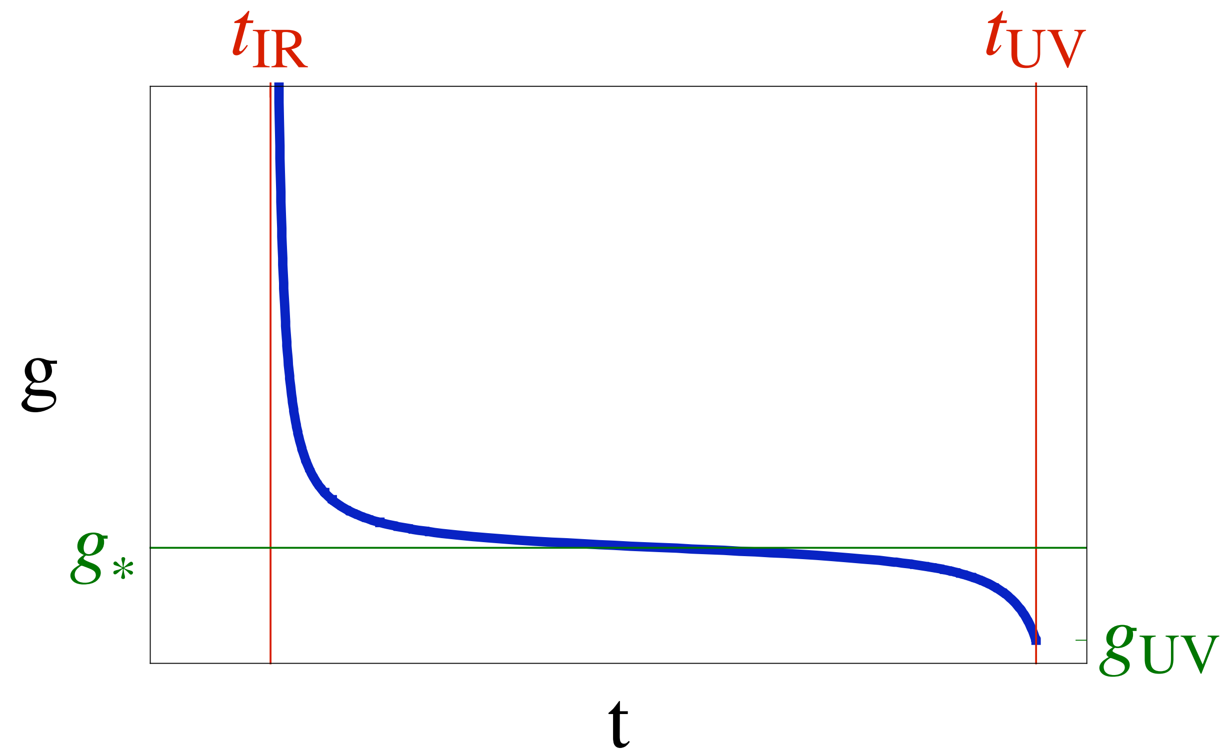
- Fixed point moving to zero   SQCD $N_f/N_c = 3$
- Fixed point moving to infinity   SQCD $N_f/N_c = 3/2$?
- Fixed point merger and annihilation



$$\beta(g;\alpha) = \frac{\partial g}{\partial t} = (\alpha - \alpha_*) - (g - g_*)^2$$

Running of coupling for $\alpha = \alpha_* - \epsilon$





$$\frac{\Lambda_{\mathrm{IR}}}{\Lambda_{\mathrm{UV}}} = \exp\left[t_{\mathrm{IR}} - t_{\mathrm{UV}}\right] = \exp\left[\int_{g_{\mathrm{UV}}}^{g_{\mathrm{IR}}} \frac{dg}{\beta(g;\alpha)}\right] \simeq e^{-\pi/\sqrt{(\alpha_* - \alpha)}}$$

This may be the explanation!

If that picture is correct: two fixed points for $\alpha > \alpha_*$

AdS/CFT correspondence:

Operator $\Leftrightarrow$ Field

$$\Delta(\Delta - d) = m^2 R^2$$

Breitenlohner-Freedman bound $m^2 > -d^2/4$

$-d^2/4 < m^2 < -d^2/4 + 1$: two different boundary theories

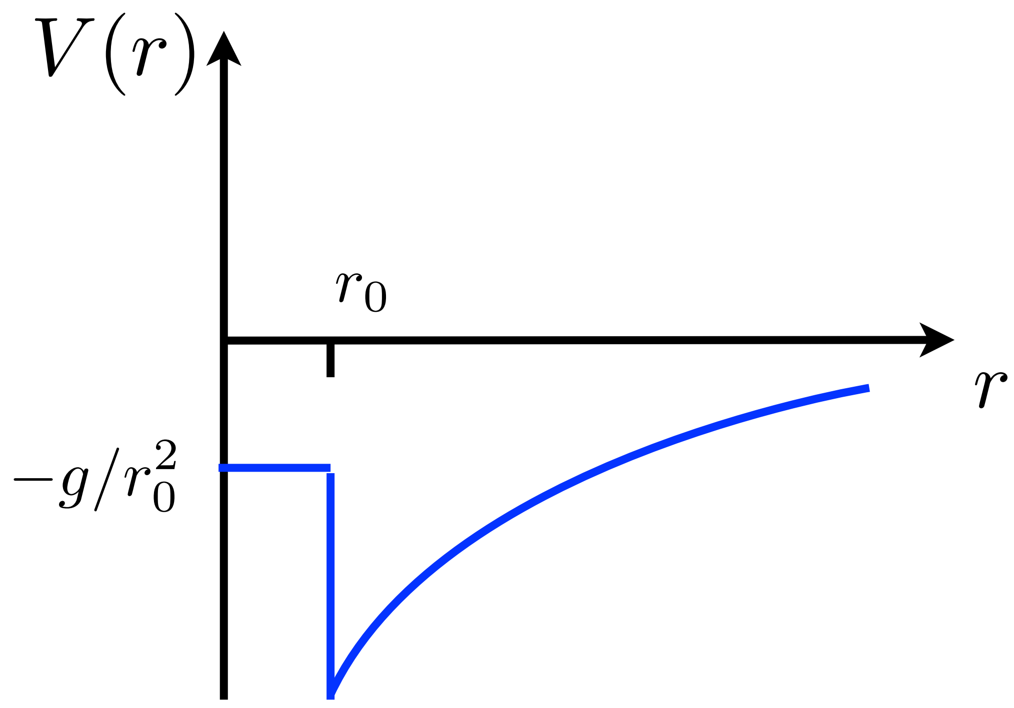
Klebanov, Witten

$$\boxed{\Delta_+ + \Delta_- = d}$$

$$\Delta_+ - \Delta_- < 1$$

Lost of conformality: m^2 drops below the BF bound

RG for quantum mechanics with $1/r^2$ potential



Regularize potential by a square-well core

Change g and r_0, preserving low-energy physics

Get beta function

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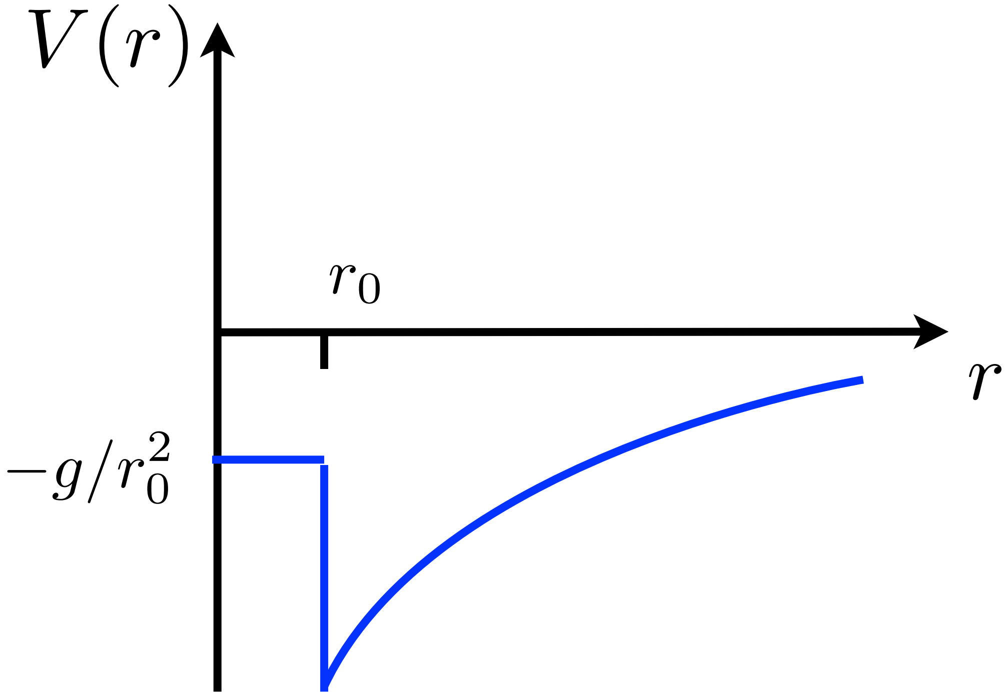

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$$\beta(g) = \frac{2\sqrt{g}\left(\alpha + \sqrt{g}\cot\sqrt{g} - g\cot^2\sqrt{g}\right)}{-\cot\sqrt{g} + \sqrt{g}\csc^2\sqrt{g}}$$

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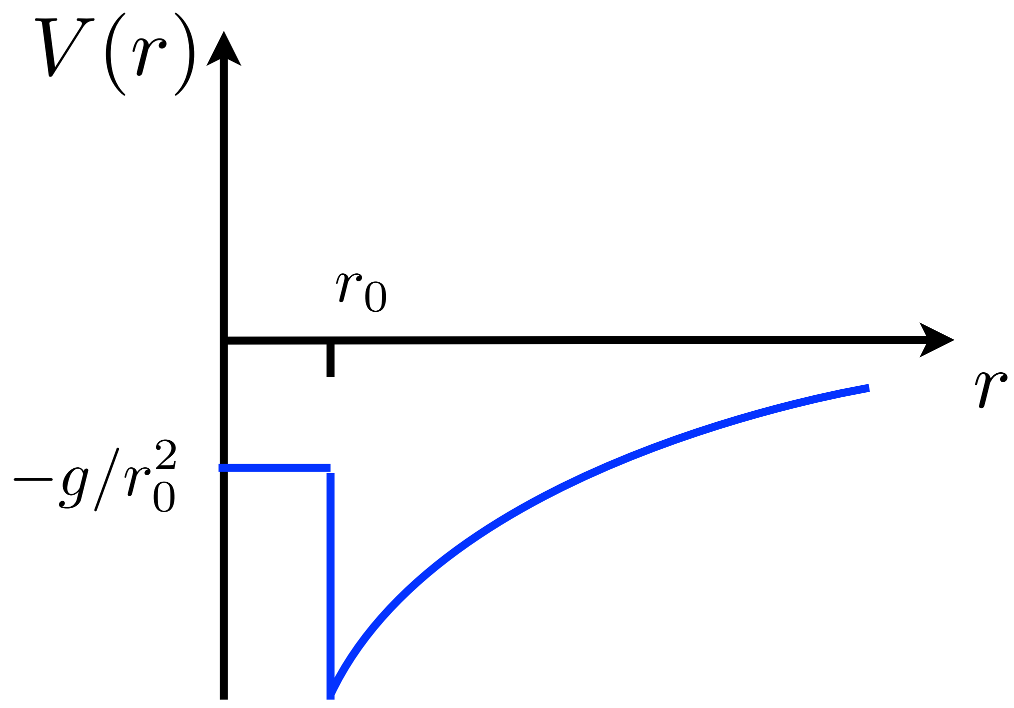

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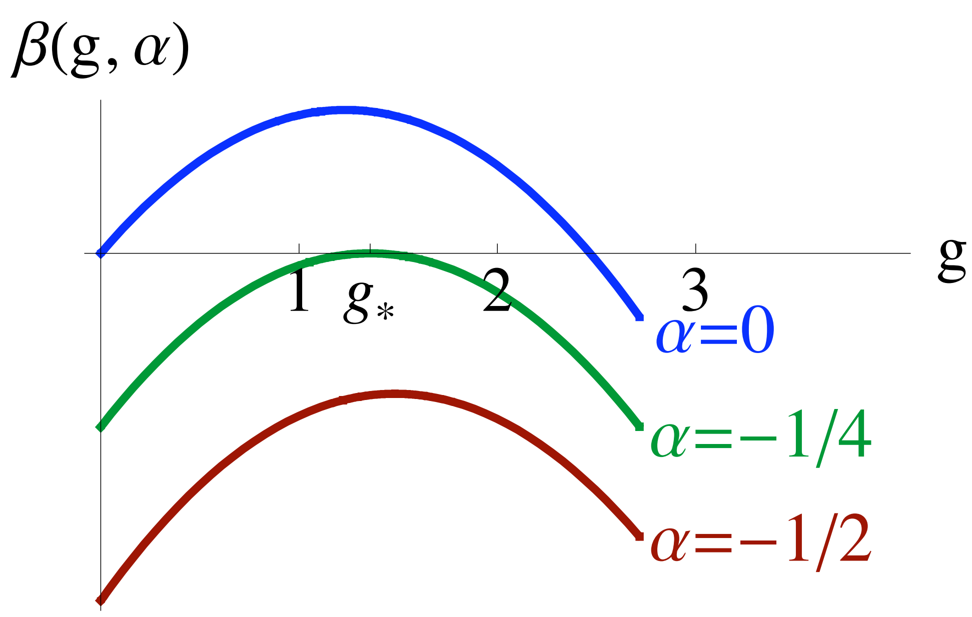

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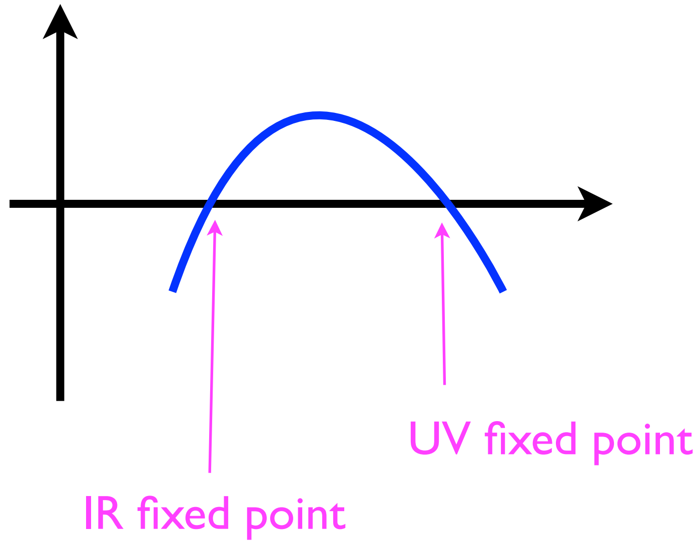
What are the two fixed points?



The Schrödinger equation

$$-\psi'' - \frac{2}{r}\psi' + \frac{\alpha}{r^2}\psi = E\psi$$

has two solutions at small r: $\psi = \frac{c_+}{r^{\nu_+}} + \frac{c_-}{r^{\nu_-}}$ $\nu = \frac{1}{2}(1 \pm \sqrt{1+4\alpha})$

c_+/c_- depends on the short-range part of the potential

Choosing $c_+ = 0$ or $c_- = 0$ conformal theory

Generic short range potential “flows” to $c_+ = 0$

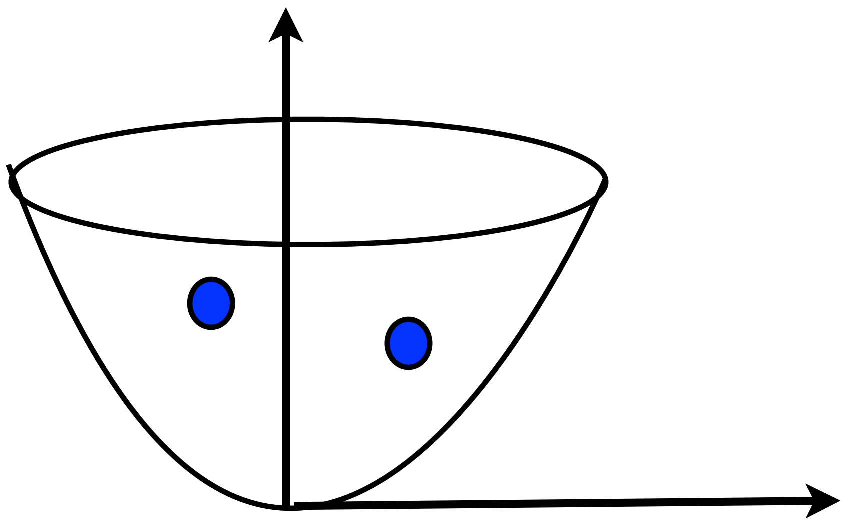
Fine-tuning required to achieve $c_- = 0$

Operator dimensions at fixed points

If AdS/CFT intuition is correct: there is an operator that has different dimensions at two fixed points

Nonrelativistic conformal symmetry: Nishida, DTS

dimensions of primary operators = Eigenstates in harmonic potentials, in unit of oscillator frequency ω



In our case: the operator $\psi\psi$

corresponds to ground state of 2 particles in harmonic potential

$$\psi(\mathbf{x},\mathbf{y}) = \frac{e^{-\omega(x^2+y^2)/2}}{|\mathbf{x}-\mathbf{y}|^{\nu_\pm}}$$

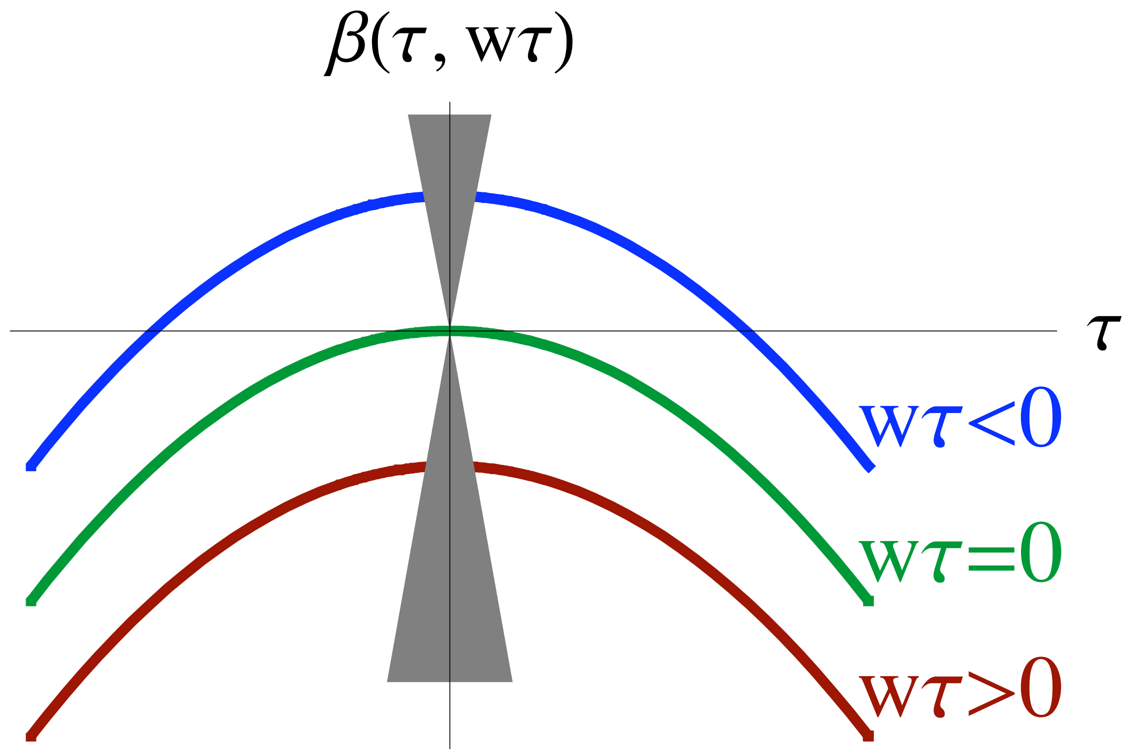
$$\Delta_\pm = \frac{E_\pm}{\omega} = \frac{d+2}{2} \pm \sqrt{\alpha - \alpha_*}$$

$$\Delta_+ + \Delta_- = d + 2$$

space (→ d), NR time (→ 2)

BKT phase transition

Can be interpreted as the merging of fixed points



Back to QCD

- Imagine that the four-fermi coupling c runs

$$\beta(c) = (2-\gamma)c - c^2 - \alpha_s N_c$$

dimension

Iterating 4-fermi interaction

One-gluon exchange generates four-fermi coupling

at some value of gauge coupling there is no fixed point

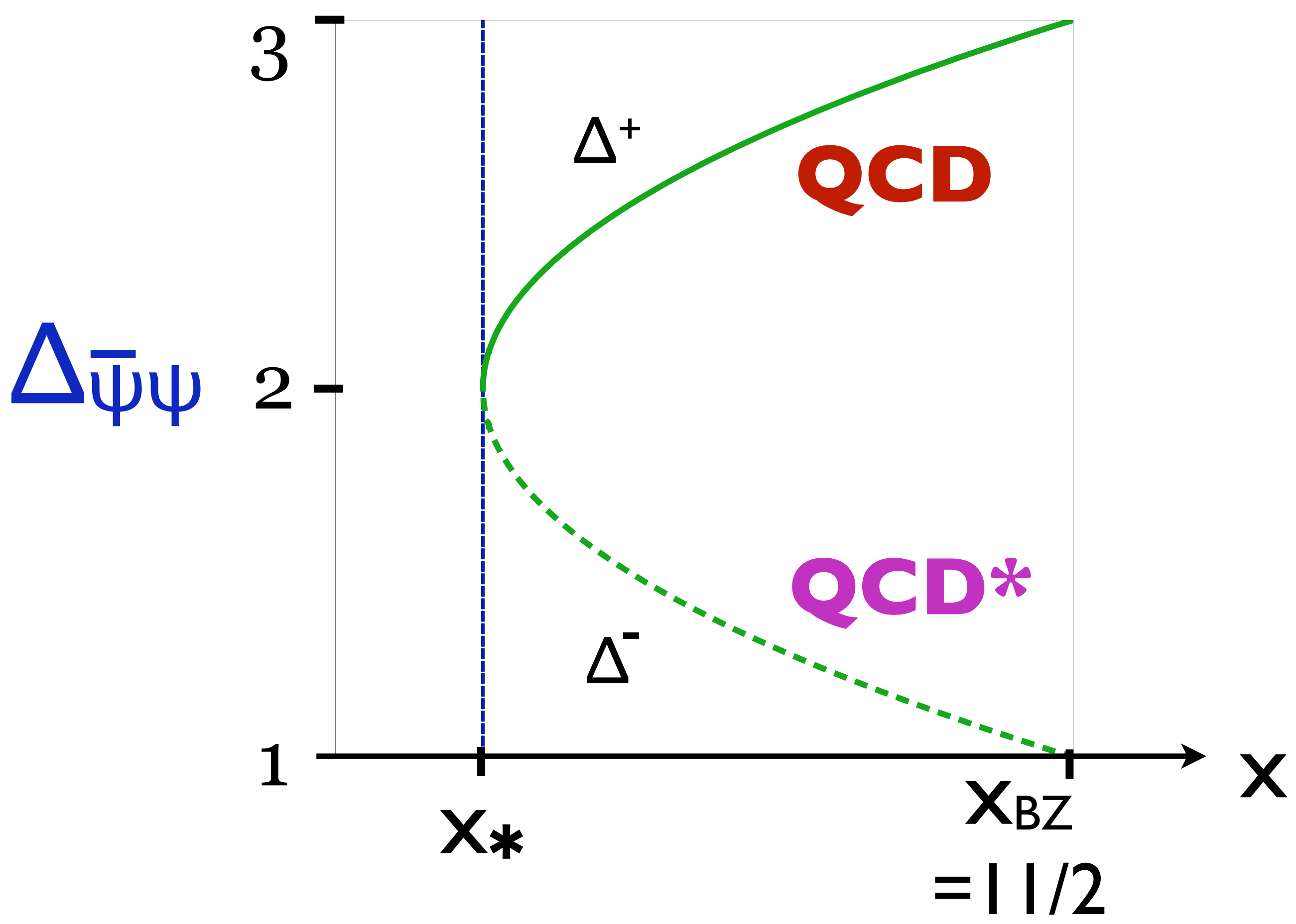
- This would give rise to BKT scaling

- Consequence: near the lower end of the conformal window, there is a UV fixed point in addition to the usual IR fixed point

$$\Delta[\bar{\psi}\psi]|_{\mathrm{UVfp}} \neq \Delta[\bar{\psi}\psi]|_{\mathrm{IRfp}}$$

- It would be nice (though not guaranteed) if this fixed point can be found in theweak coupling regime $N_f/N_c = 11/2 - \epsilon$

- We begin gently: can one find a perturbative fixed point where one fermion bilinear changes dimension?



$\Delta_{\bar{\psi}\psi}$
3
2
1
Δ^+
QCD
QCD*
Δ^-
x_*
x_{BZ}
=11/2
x

Model A

Start with perturbative Banks-Zaks fixed point,

$$\Delta(\bar{\psi}\psi) = 3 - \#\lambda$$

In the UV fixed point: we should have a scalar with Δ=1+#λ, almost a free scalar

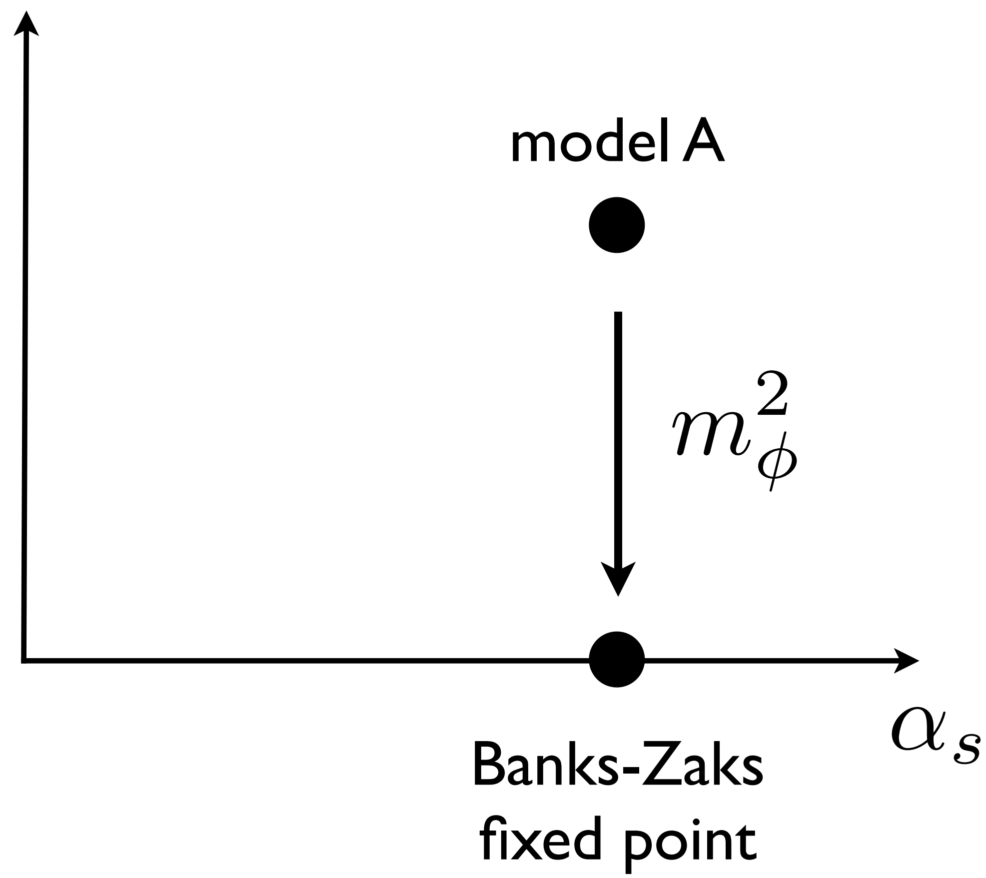
This suggests how to construct such a theory

$$L = L_{\rm QCD} + \frac{1}{2}(\partial_\mu \phi)^2 - y\bar{\psi}\psi\phi - \frac{\lambda}{24}\phi^4$$

$$\beta_y = \frac{y}{16\pi^2}(y^2 N_f N_c - 3g^2 N_c)$$

Fixed point for y and λ exists

Running of α unaffected



model A
m_ϕ^2
α_s
Banks-Zaks
fixed point

Model C

Previous model does not have chiral symmetry: cannot be a candidate for the UV fixed point

To restore chiral symmetry: introduce O(Nf^2) scalars

$$L = L_{\rm QCD} - y(\bar{\psi} t^A \psi \phi^A + i \bar{\psi} t^A \gamma^5 \psi \pi^A)$$ + action for scalar

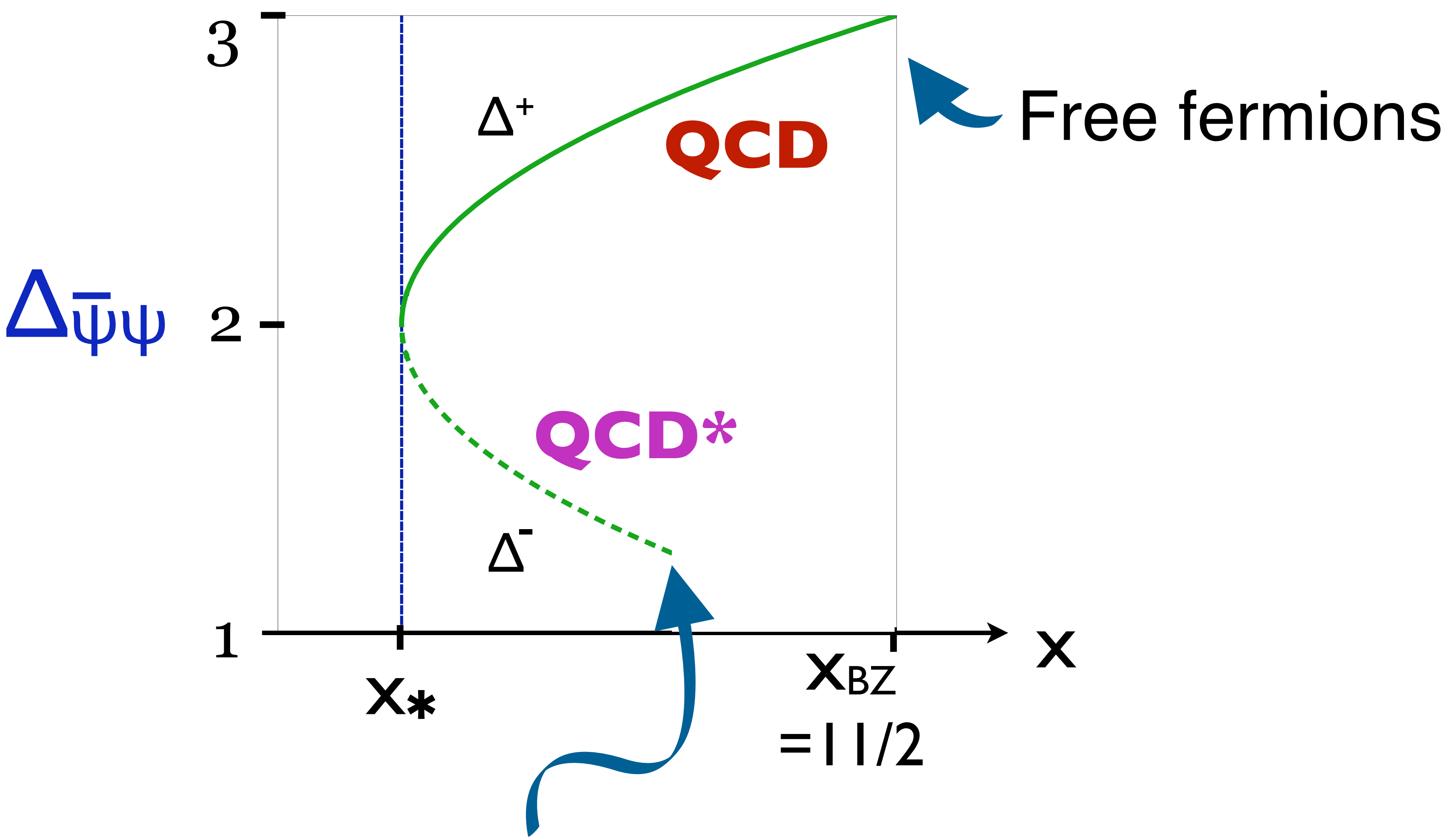
But now the running of α is affected: too many scalars

This model turns out not to have perturbative fixed point

Perhaps the UV fixed point exists only sufficiently close to the critical Nf/Nc?

This fixed point should be looked for on the lattice

QCD* ?



UV fixed point starts at strong-ish coupling?

Conclusions

- Merger and annihilation of fixed points explains the loss of conformality in a variety of systems
- The scaling of the IR scale near the phase transition coincides with that of a Berezinskii-Kosterlitz-Thouless phase transition
- It is conceivable that the chiral phase transition in Nf/Nc also has BKT scaling
  - a UV fixed point in QCD with fine-tuned four-fermi interaction
- Loss of conformality ~ violation of the Breitenlohner-Freedman bound in gravity dual
  - implies $\Delta[\bar{\psi}\psi] = 2$ exactly at the phase transition