QCD with 2 colour-sextet quarks: Does it walk?

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Introduction

QCD with $1\frac{28}{125} \leq N_f < 3\frac{3}{10}$ flavours of massless colour-sextet quarks is expected to be either a Walking or a Conformal field theory.

For $N_f = 3$ conformal behaviour is expected. $N_f = 2$ could, a priori, exhibit either behaviour.

Lattice QCD can be used to try to determine which option the $N_f = 2$ theory chooses.

Simulations using Wilson fermions by DeGrand, Shamir and Svetitsky suggest $N_f = 2$ is conformal.

We are studying the $N_f = 2$ case using staggered fermions. We are currently performing simulations at finite temperature (T).

Our simulations suggest that this theory walks.

The scales of confinement and chiral symmetry breaking appear to be very different. (This also contrasts with what was reported by DeGrand, Shamir and Svetitsky for simulations using Wilson quarks.)

Hence the phenomenology is expected to be different from that of QCD with fundamental quarks and N_f in the walking window, where these two scales appear to be the same.

QCD with $N_f = 2$ staggered colour-sextet quarks at finite T

Wilson plaquette (triplet) action for gauge fields.

Standard staggered action for sextet quark fields, with sextet gauge fields on the links.

Staggered quarks have the advantage over Wilson quarks of having a simple chiral order parameter.

Exact RHMC algorithm used for simulations. Allows us to tune to $N_f = 2$.

Finite T allows us to study scales associated with confinement and χSB.

Changing N_t can distinguish between finite T and bulk transitions.

We are currently simulating on $8^3 \times 4$, $12^3 \times 4$ and $12^3 \times 6$ lattices. Typical runs away from transitions are 10,000 trajectories per (β, m). Close to transitions we use 50,000-100,000 trajectories per (β, m).

Simulations and Results

$N_t = 4$

For both $8^3 \times 4$ and $12^3 \times 4$ lattices we perform simulations with $m = 0.02$, $m = 0.01$ and $m = 0.005$ to attempt to access the chiral limit. We cover the range $5.0 \leq \beta = 6/g^2 \leq 7.0$.

The results on these 2 lattice sizes are consistent, so we will present results primarily from our $12^3 \times 4$ simulations.

In contrast to what was found by DeGrand, Shamir and Svetitsky, we find well separated deconfinement and chiral-symmetry restoration transitions.

Figure 1 shows the colour-triplet Wilson Line(Polyakov Loop) and the chiral condensate($\langle\bar{\psi}\psi\rangle$) as functions of $\beta = 6/g^2$, for each of the 3 quark masses on a $12^3 \times 4$ lattice.

Figure 2 shows a histogram of the triplet Wilson Line close to the deconfinement transition for $m = 0.02$.

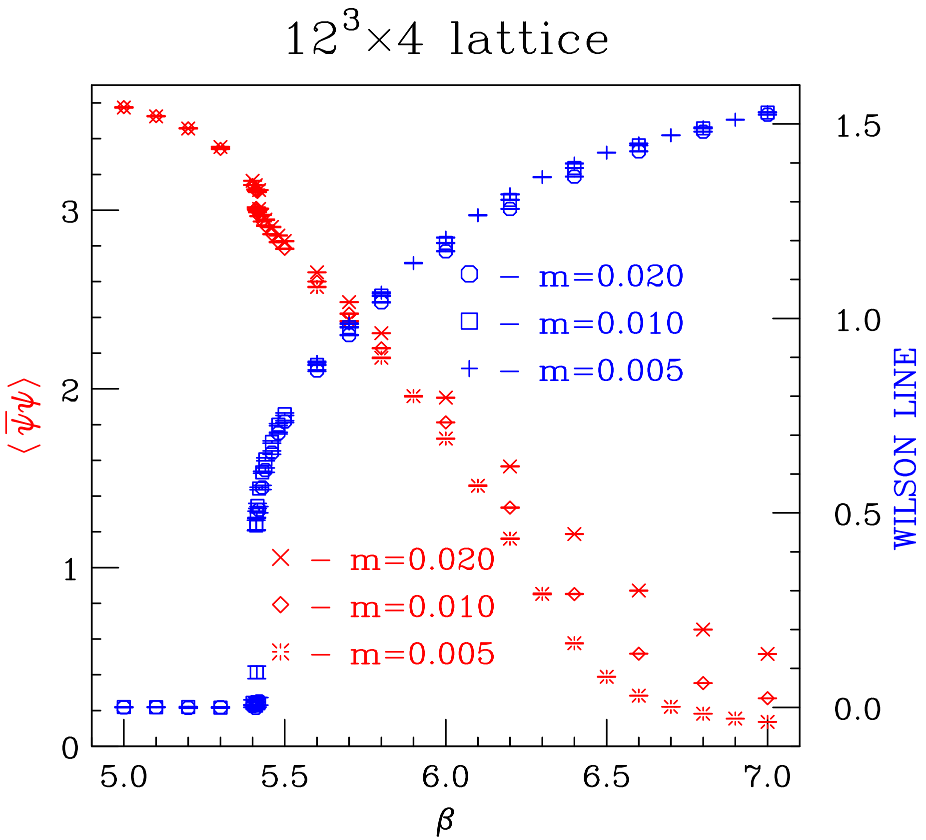

Figure 1: Wilson line and $\langle\bar{\psi}\psi\rangle$ as functions of β on a $12^3 \times 4$ lattice.

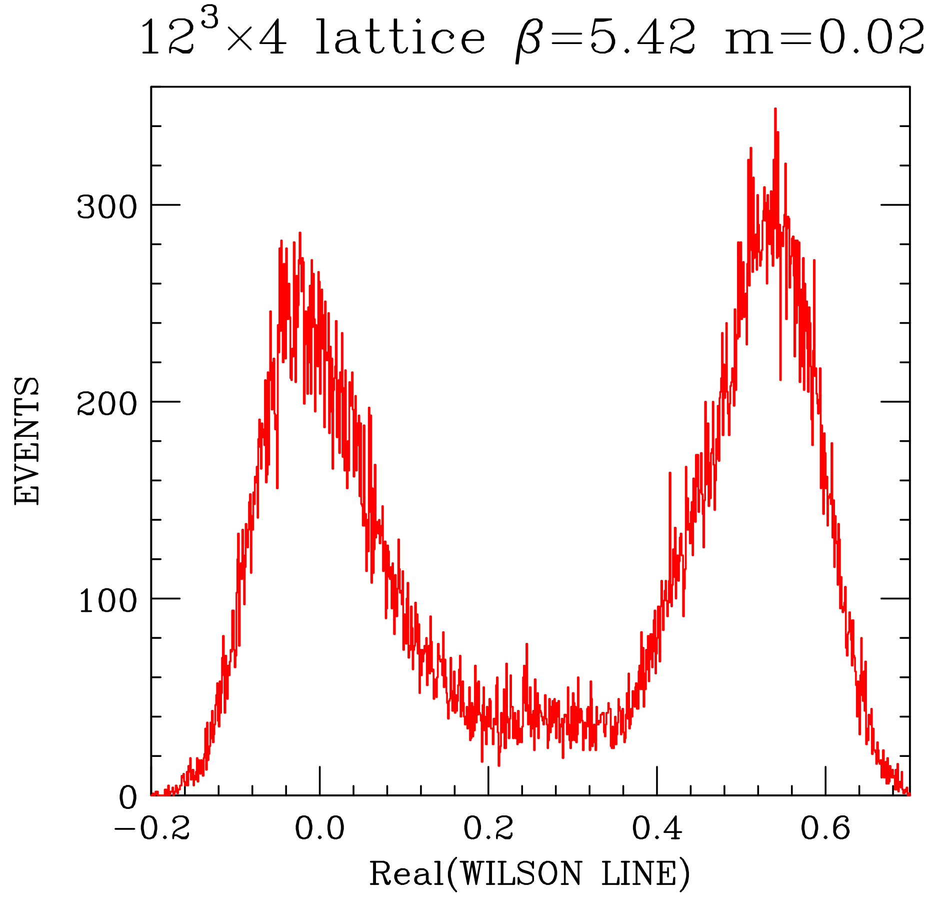

Figure 2: Histogram of the Wilson Line at $\beta = 5.42$, $m = 0.02$ on a $12^3 \times 4$ lattice.

The colour-triplet Wilson Line shows a (first order?) deconfinement transition at $\beta = 5.420(5)$ ($m = 0.02$) and $\beta = 5.412(1)$ ($m = 0.01$). It is close to zero below this transition and real and positive above.

The chiral symmetry restoration transition occurs at $\beta \approx 6.5$.

For $\beta < 5.9$ but significantly above the deconfinement transition we find vestiges of the broken Z_3 symmetry with a 3-state signal in the Wilson Line. However, those states with the Wilson Line oriented in the direction of one of the non-trivial cube roots of unity, while long lived, are only metastable, decaying into the state with a real positive Wilson Line. (See figure 3.)

At $\beta \approx 5.9$ these metastable states undergo a transition to a state with a real negative Polyakov Loop.

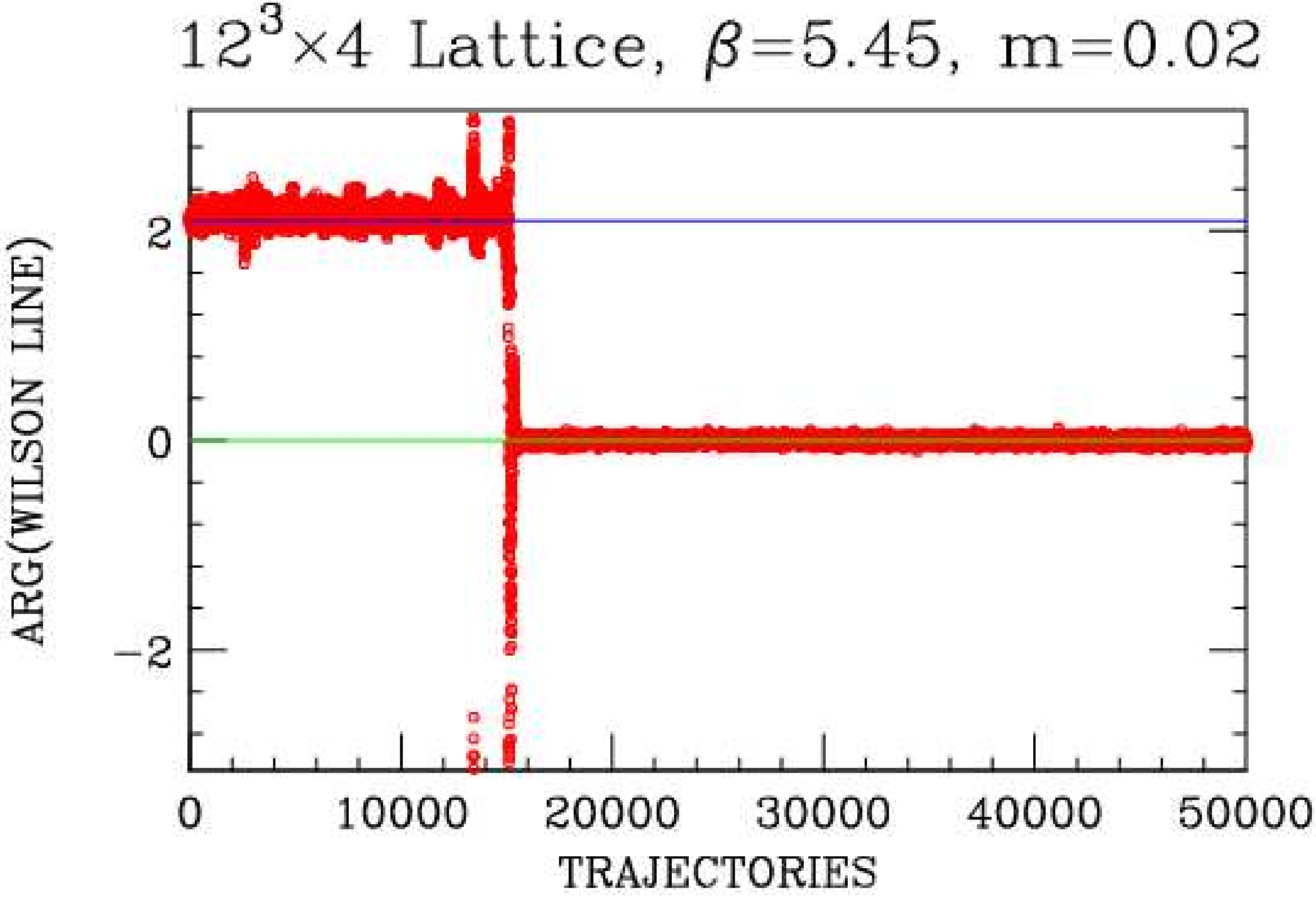

Figure 3: Time evolution of the argument of the triplet Wilson Line at $\beta = 5.45$ on a $12^3 \times 4$ lattice.

$N_t = 6$

At $N_t = 6$, the remnant Z_3 symmetry is again manifest above the deconfinement transition and there is a clear 3-state signal. Tunnelings occur between these 3 states. There are tunnelings in all directions, indicating that all 3 states are stable, in contrast to $N_t = 4$.

We separate the contribution of the state with a real Wilson Line from that from states where the Wilson line is oriented in the direction of either complex cube root of unity.

Figures 4,5 show the Wilson Lines (Polyakov Loops) and the chiral condensates. The first graph is for the states with real positive Wilson Lines. The second is for those with complex (or negative) Wilson Lines.

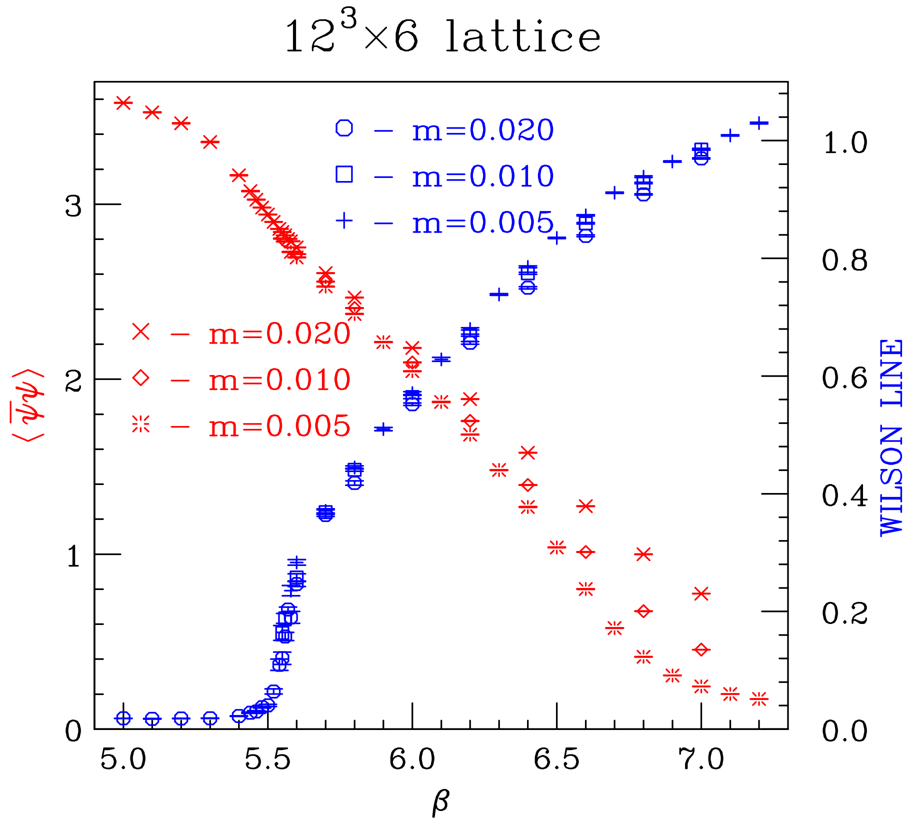

Figure 4: Triplet Wilson Line and $\langle\bar{\psi}\psi\rangle$ as functions of β for the state with a real positive Wilson Line.

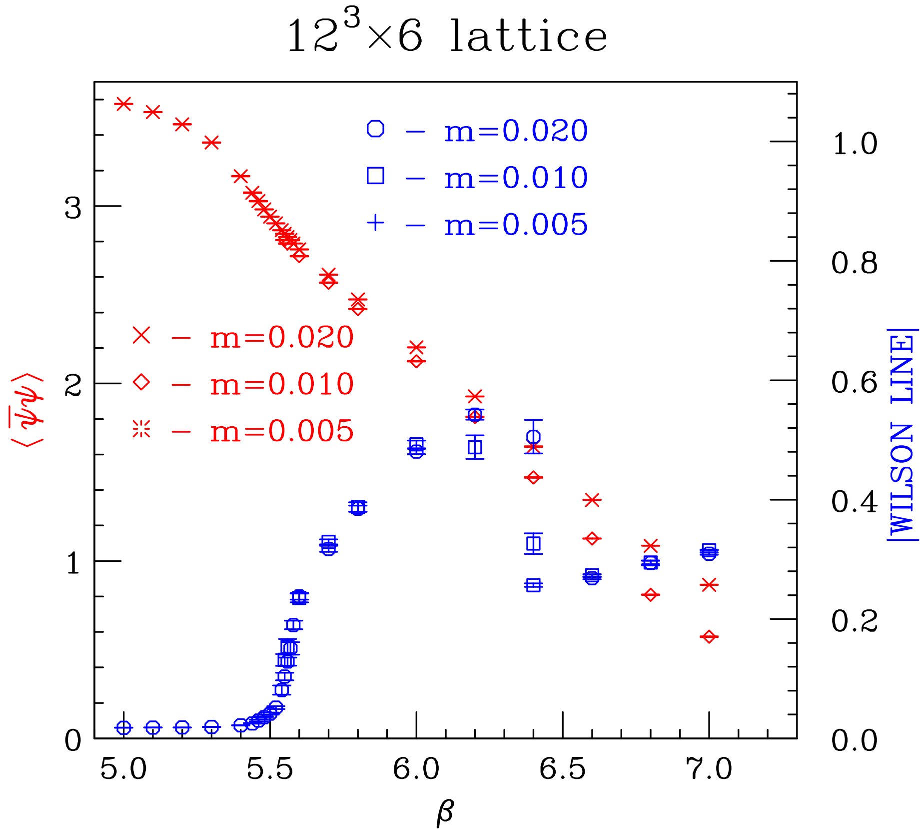

Figure 5: Triplet Wilson Line and $\langle\bar{\psi}\psi\rangle$ as functions of β for states with complex or negative Wilson Line.

The deconfinement transition is at $\beta = 5.56(1)$ for $m = 0.02$ and $\beta = 5.55(1)$ for $m = 0.01$. The increase in β from $N_t = 4$ is what would be expected for a finite temperature transition in an asymptotically free field theory.

The chiral-symmetry restoration transition is at $\beta \approx 6.8$. Increase over $N_t = 4$ suggests finite T transition with asymptotic freedom.

The states with complex Wilson Lines show a transition to states with real negative Wilson Lines at $\beta \approx 6.5$. Large increase in β over that for $N_t = 4$ suggests that this transition could be a lattice artifact, or else could merge with the chiral transition for larger N_t.

Figure 6 shows the 3-state signal above the deconfinement transition on a $12^3 \times 6$ lattice. This graph represents 100,000 trajectories. Figure 7 shows histograms of the magnitude of the triplet Wilson line, close to the deconfinement transition.

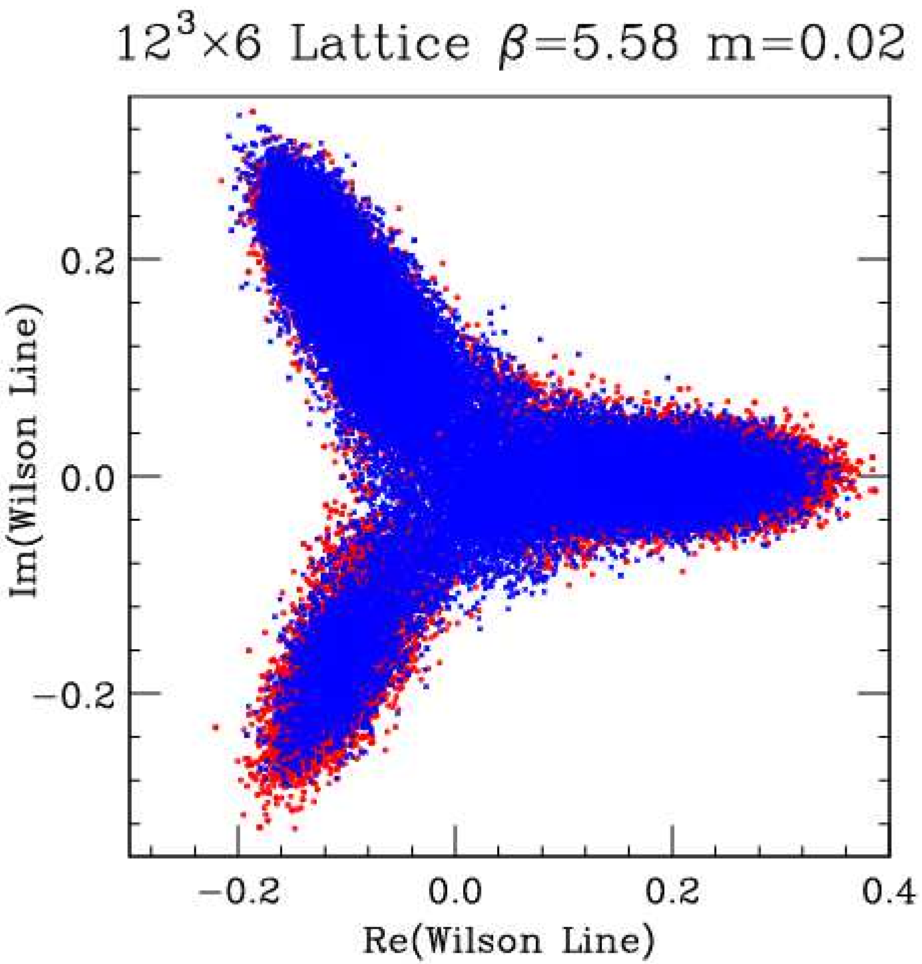

Figure 6: Scatterplot of triplet Wilson Lines at $\beta = 5.58$, $m = 0.02$ in the deconfined regime on a $12^3 \times 6$ lattice.

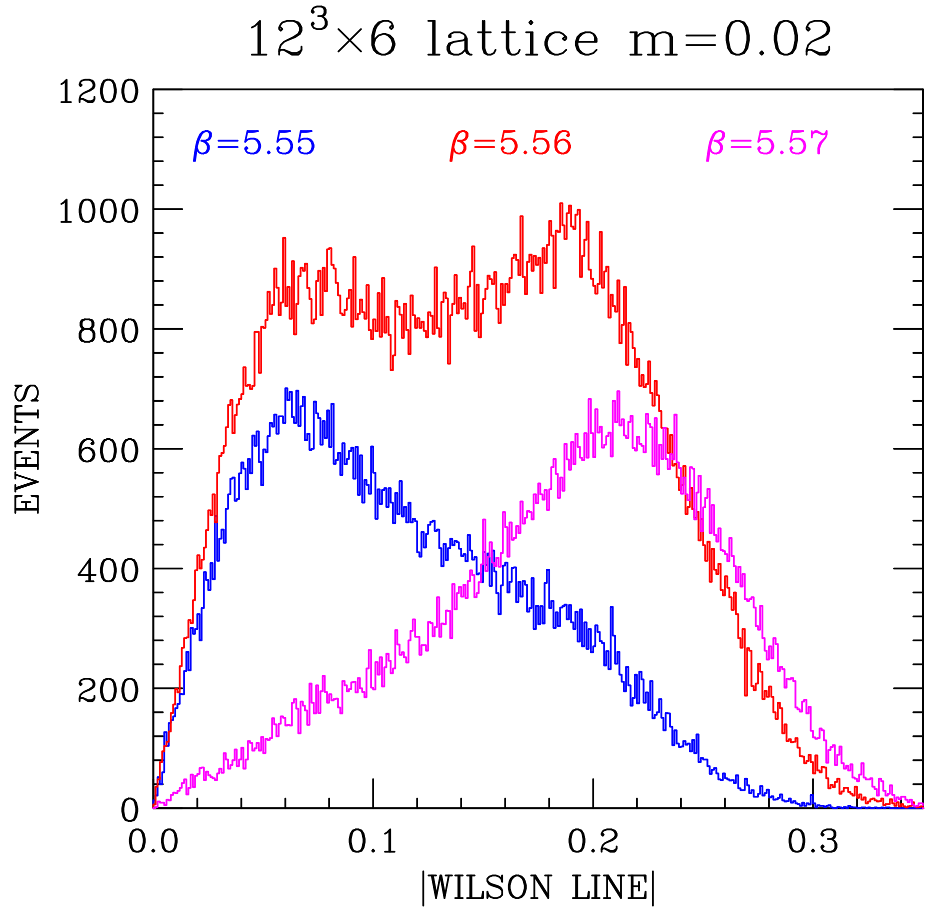

Figure 7: Histograms of Wilson Line distributions at $\beta = 5.55, 5.56, 5.57$, $m = 0.02$, spanning the deconfinement transition on a $12^3 \times 6$ lattice.

Discussion and Conclusions

- We are studying the thermodynamics of Lattice QCD with 2 flavours of staggered colour-sextet quarks. We find well separated deconfinement and chiral-symmetry restoration transitions. This contrasts with the case of fundamental quarks, where these 2 transitions are coincident, but is similar to the case of adjoint quarks where again these 2 transitions are separate.
- At $N_t = 4$, $\beta_d(m = 0.02) = 5.420(5)$, $\beta_d(m = 0.01) = 5.412(1)$; deconfinement appears first order. $\beta_\chi \approx 6.5$.
- At $N_t = 6$, $\beta_d(m = 0.02) = 5.56(1)$, $\beta_d(m = 0.01) = 5.55(1)$; $\beta_\chi \approx 6.8$.
- The increase in the βs for both transitions from $N_t = 4$ to $N_t = 6$ is consistent with their being finite temperature transitions for an asymptotically free theory (rather than bulk transitions).
- If there is an IR fixed point, we have yet to observe it. Our results suggest a Walking rather than a conformal behaviour – very preliminary.

- Why is this phase diagram so different from that for Wilson quarks (DeGrand, Shamir & Svetitsky)? Is it because there is an infrared fixed point, and we are on the strong-coupling side of it? Are our quark masses too large to see the chiral limit? Is it because the flavour breaking of staggered quarks does not allow a true chiral limit at fixed lattice spacing?

- For the deconfined phase there is a 3-state signal, the remnant of now-broken Z_3 symmetry. For $N_t = 4$ the states with complex Polyakov Loops appear metastable. For $N_t = 6$ all 3 states appear stable. Breaking of Z_3 symmetry is seen in the magnitudes of the Polyakov Loops for the real versus complex states.

- Between the deconfinement and chiral transitions, we find a third transition where the Wilson Lines in the directions of the 2 non-trivial roots of unity change to real negative Wilson Lines. This transition occurs for $\beta \approx 5.9$ ($N_t = 4$) and $\beta \approx 6.5$ ($N_t = 6$). This rapid increase suggests that the transition is a lattice artifact.

- If, however, this third transition is real, the fact that the magnitude of the negative Polyakov Loop is roughly one third of that for the positive Polyakov Loop, suggests that this transition is associated with colour symmetry breaking
$SU(3) \longrightarrow SU(2) \times U(1)$.
- Drawing conclusions from $N_t = 4$ and $N_t = 6$ is dangerous. We need $N_t = 8$ or greater. We should also use smaller quark masses. At $N_t = 6$ we need a second spatial lattice size.
- To understand this theory more fully, we need to study its zero temperature behaviour, measuring its spectrum, string tension, potential, f_π... Measurement of the running of the coupling constant for weak coupling is needed.
- We should also study $N_f = 3$, which is almost certainly conformal, to see if we observe qualitative differences.
- These simulations were performed on the Cray XT4 (Franklin) at NERSC, and on the Linux Cluster (Abe) at NCSA under an LRAC grant.