Four dimensional large N gauge theories with adjoint fermions on a single site lattice – I

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Large N QCD in the 't Hooft limit

Large N QCD in the 't Hooft limit (finite number of fermion flavors in the fundamental representation) on a l^4 torus is in the confined phase if $l > T_c^{-1}$ and there is no dependence on l – Continuum reduction.

- Fermions are quenched in the 't Hooft limit.
- Fermion boundary conditions do not matter in the confined phase – Quark momentum can be force-fed.
- Chiral condensate exists in finite volume in the large N limit and has been computed using chiral random matrix theory as a tool.
- Meson masses can be calculated by computing the propagator in momentum space. Momenta are in units of $\frac{2\pi}{LN}$ and become continuous on a finite lattice in the large N limit.
- Chiral symmetry is restored for $T > T_c$ and fermion boundary conditions matter in the T direction and anti-periodic boundary condition is favored.

Large N QCD with adjoint fermions

There are arguments that the continuum limit can be obtained by working on a single site lattice:

- R.L. Mkrtchyan and S.B. Khokhlachëv, *Reduction of the $U(\infty)$ supersymmetry theory to a random matrix model*, Pis'ma Zh. Eksp. Teor. Fiz. **37** No 3, 160-161 (1983): Continuum argument for Wess-Zumino model.
- P. Kovtun, M. Ünsal and L.G. Yaffe, *Volume independence in large N_c QCD-like gauge theories* hep-th/0702021:
  - Large N Yang-Mills theory with fermions in the adjoint representation (not necessarily super-symmetric) can be reduced to a single site lattice with periodic boundary conditions for fermions.
  - This is not the case for fermions in the anti-symmetric representation.
  - They considered the theory on $\mathbf{R}^3 \times S^1$ and studied the one-loop effective potential for the Wilson line in the compact direction and showed that the potential favors a uniform distribution of the eigenvalues on the unit circle.

More on the one-loop effective potential for the Wilson line

More analysis on $\mathbf{R}^3 \times S^1$.
Does the Z_N symmetry of the Wilson line remain unbroken?

- P.F. Bedaque, M.I. Buchoff, A. Cherman and R.P. Springer, *Can fermions save large N dimensional reduction?,* 0904.0277.
- B. Bringoltz:
  - *Large-N volume reduction of lattice QCD with adjoint Wilson fermions at weak-coupling,* 0905.2406.
  - Partial breakdown of center symmetry in large-N QCD with adjoint Wilson fermions, 0911.0352.
- E. Poppitz and M. Ünsal, *Comments on large N volume independence*, 0911.0358.

Numerical analysis

Do the Z_N^4 symmetries on a single site lattice remain unbroken? B. Bringoltz and S.R. Sharpe, *Non-perturbative volume reduction of large N QCD with adjoint fermions* 0906.3538.

- $8 \leq N \leq 15$.
- Wilson fermions with periodic boundary conditions.
- SU(2) updates using standard Metropolis algorithm. Scales like N^8.
    - $N(N-1)/2$ SU(2) updates.
    - Fermion determinant computation for each update is a $(N^2-1)^3$ order computation.
- Reasonable range for the gauge coupling: $b = \frac{1}{g^2 N} \leq 1$.
- Finds a wide range of quark masses (including light and heavy) for which Z_N^4 symmetry is not broken.

Fermion boundary conditions

Unlike the case of the ’t Hooft limit of QCD, boundary conditions of fermions in the adjoint representation matter.

- P. Kovtun, M. Ünsal and L.G. Yaffe, *Volume independence in large N_c QCD-like gauge theories* hep-th/0702021: The one-loop effective potential does not favor the Z_N symmetric phase if the fermion has anti-periodic boundary condition in the compact direction.

It is therefore possible for this theory to have a deconfining transition at some temperature.

Are there doublers on a single site lattice?

Do we need to use Wilson fermions or Overlap fermions on a single site lattice?

With periodic boundary conditons, there is only the zero momentum mode for free fermions on a single site lattice – $U - U^\dagger$ is zero when $U = 1$.

But $U \neq 1$ in the weak-coupling limit unless the Z_N^4 symmetries are fully broken.

What are the chances an eigenvalue of U equal to $e^{i\pi}$ in one direction matches with an eigenvalue of 1 or $e^{i\pi}$ in another direction when the eigenvalues of U are uniformly spread out on the unit circle?

Chance for a doubler mode is unlikely if the Z_N^4 symmetries are unbroken.

One-loop effective potential on a single site lattice

Follow G. Bhanot, U.M. Heller and H. Neuberger, *The Quenched Eguchi-Kawai Model*, Phys. Lett. **B113** 47 (1982). Set

$$U_\mu = e^{ia_\mu} D_\mu e^{-ia_\mu}; \quad D_\mu^{ij} = e^{i\theta_\mu^i}\delta_{ij}.$$

- First integrate out a_μ for fixed θ_μ^i.
- Study the effective potential as a function of θ_μ^i.

The lowest contribution from the gauge sector is from the quadratic term in a_μ and the result after integration of a_μ to this order is

$$S_g = \sum_{i \neq j} \ln \left[\sum_\mu \sin^2 \frac{1}{2} \left(\theta_\mu^i - \theta_\mu^j \right) \right]$$

Fermionic contribution to the effective potential

The lowest order contribution comes from setting $a_\mu = 0$.
The gauge field effectively has $(N-1)$ zero momentum modes and $N(N-1)$ non-zero momentum modes of the form $e^{i(\theta^i_\mu - \theta^j_\mu)}$ with $1 \le i \neq j \le N$.
The fermionic contrbution has the form

$$S_f = -2f \sum_{i \neq j} \ln d(\theta^i - \theta^j + \phi)$$

- $e^{i\phi_\mu}$ is the phase associated with the boundary condition in the μ direction.
- f is the number of Dirac flavors.

Naïve fermions and Overlap fermions

Naïve fermions

$$d(p) = \mu^2 + \sum_{\mu} \sin^2 p_\mu$$

Overlap fermions

$$d(p) = \frac{1+\mu^2}{2} + \frac{1-\mu^2}{2} \frac{2\sum_\mu \sin^2 \frac{p_\mu}{2} - m}{\sqrt{\left(2\sum_\mu \sin^2 \frac{p_\mu}{2} - m\right)^2 + \sum_\mu \sin^2 p_\mu}}$$

Using HMC to find the mininum of the one-loop effetive action

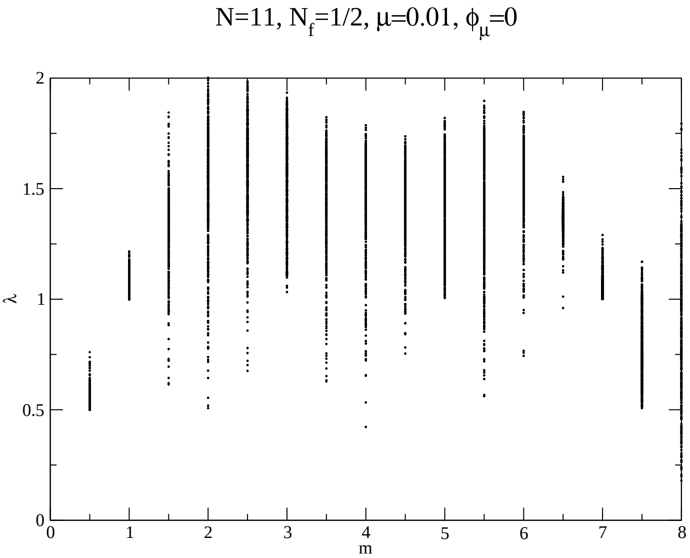
Consider

$$H = \frac{1}{2}\sum_{\mu,i}\left(\pi^i_\mu\right)^2 + \beta S$$

$$S = \sum_{i\neq j}\ln\left[\sum_{\mu}\sin^2\frac{1}{2}\left(\theta^i_\mu - \theta^j_\mu\right)\right] - 2f\sum_{i\neq j}\ln d(\theta^i - \theta^j + \phi)$$

- View π^i_μ as conjugate of θ^i_μ.
- $\sum_i \theta^i_\mu$ and $\sum_i \pi^i_\mu$ are constants of motion and we can set them to zero to remain in $SU(N)$.

No doubters

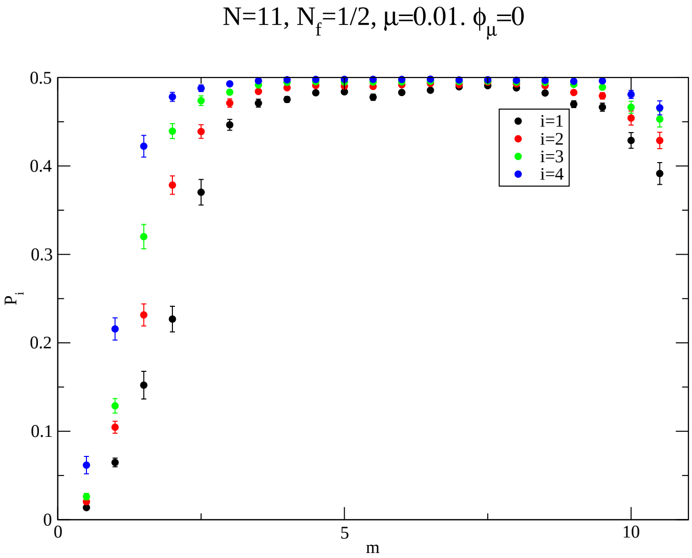


Order parameter for Z_N symmetry

$$P_\mu = \frac{1}{2}\left(1 - |\mathrm{Tr}\, U_\mu|^2\right) = \frac{1}{N^2}\sum_{i,j} \sin^2 \frac{1}{2}\left(\theta_\mu^i - \theta_\mu^j\right)$$

- $P_\mu = 0$ implies Z_N symmetry is broken in that direction.
- $P_\mu = \frac{1}{2}$ implies Z_N symmetry is not broken in that direction.

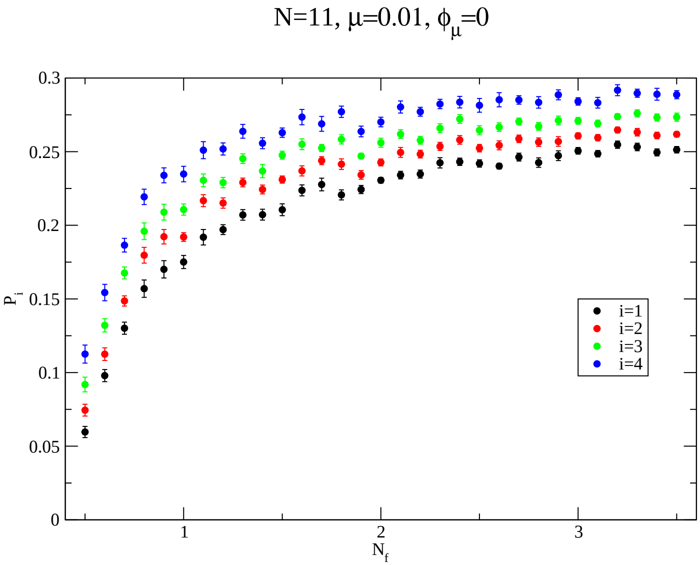
Order parameter for overlap fermions as a function of the Wilson mass parameter



The Wilson mass parameter

- One cannot take the Wilson mass parameter to zero in the weak coupling limit. This is intimately tied to the fact that the Z_N symmetries are not broken in this limit.
- We cannot take the Wilson mass parameter to be arbitrarily large. Overlap fermions approach Naïve fermions in the limit.

Naïve fermions break all four Z_N symmetries



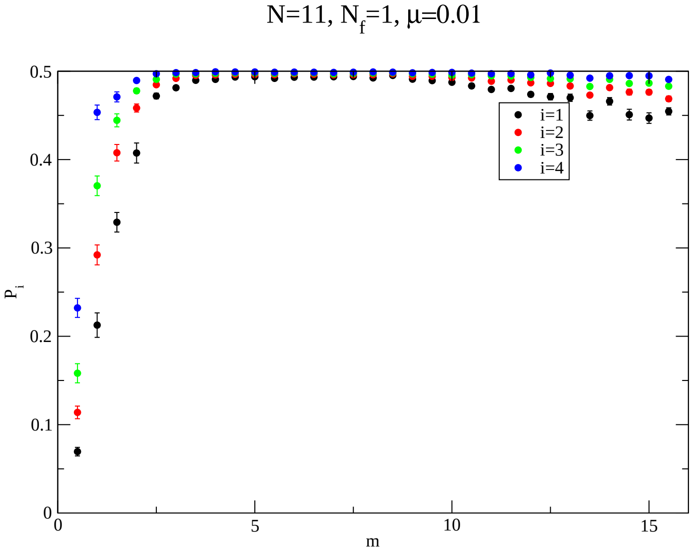
Failure of Naïve fermions

$$S = \sum_{i \neq j} \ln \left[\sum_{\mu} \sin^2 \frac{1}{2} \left(\theta^i_\mu - \theta^j_\mu \right) \right] - 2f \sum_{i \neq j} \ln \left[\mu^2 + \sum_{\mu} \sin^2 (\theta^i_\mu - \theta^j_\mu) \right]$$

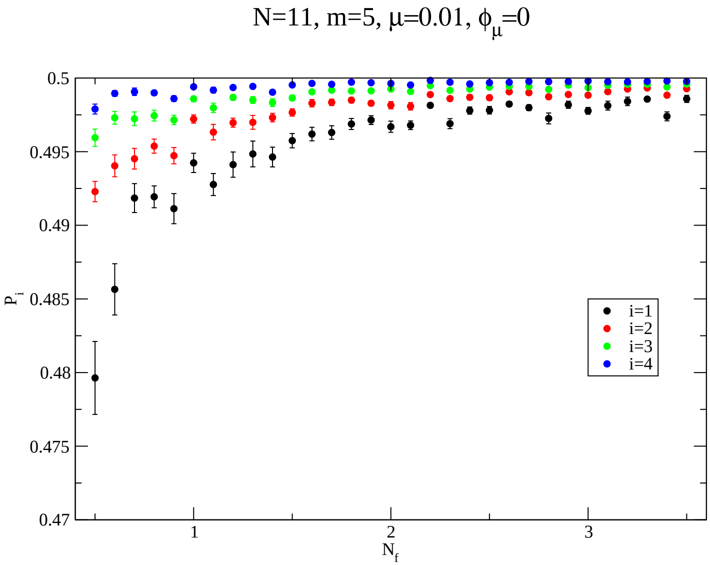
The fermionic contribution cannot separate $\theta^i_\mu = \theta^j_\mu$ from $\theta^i_\mu = \theta^j_\mu + \pi$.

Overlap fermions with one Dirac flavor

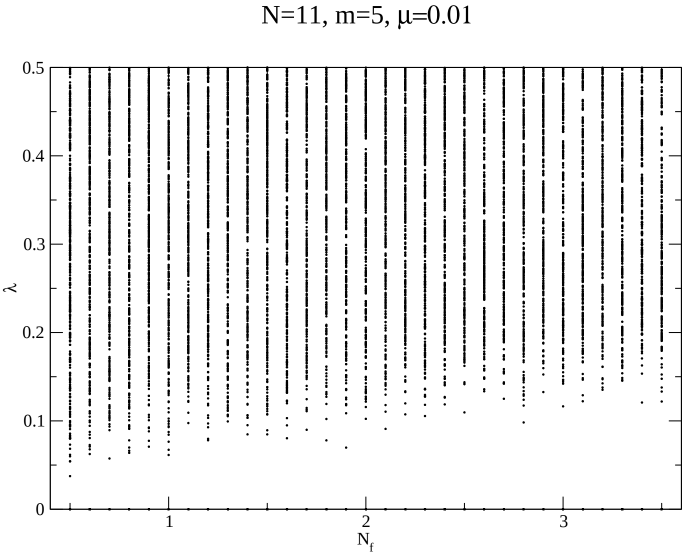
The upper range of the validity of the Wilson mass parameter goes up.



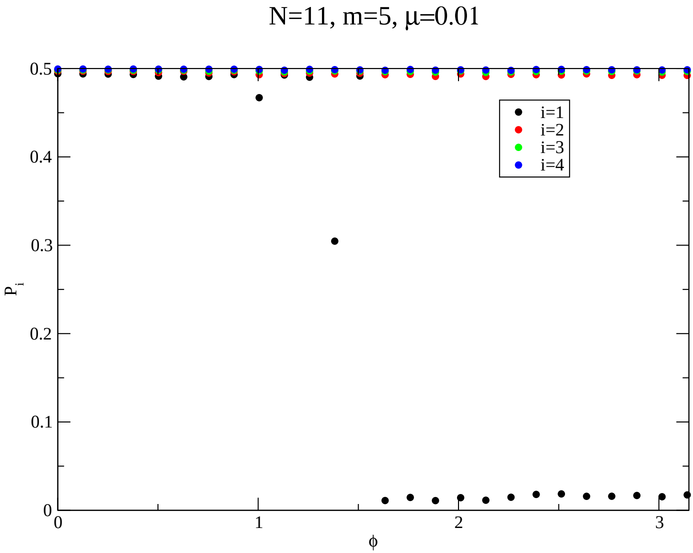
Order parameter as a function of fermion flavors



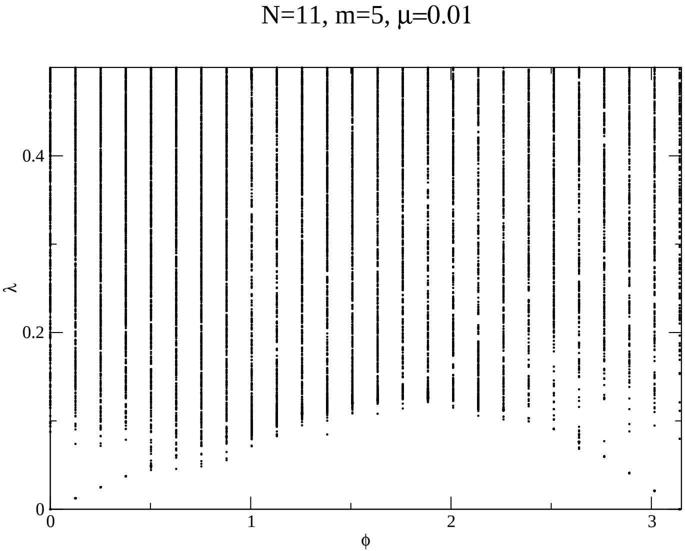
Overlap fermion spectrum as a function of fermion flavors



Effect of boundary condition



Overlap fermion spectrum as a function of boundary conditions



What next?

- Set up a numerical algorithm to study large N gauge theories with adjoint overlap fermions on a single site lattice.
- Check that there are no doublers and that one can work with a wide range of the Wilson mass parameter.
- Compute physical quantities like the string tension and chiral condensate.
  - Use $\mathrm{Tr}\left[U_\mu^m U_\nu^n \left(U_\nu^n U_\mu^m\right)^\dagger\right]$ to compute the string tension.
  - Set $z_i = \lambda_i N \Sigma$ where λ_i is the ith eigenvalue of the overlap Dirac operator. Use chRMT to obtain $< z_i >$ and hence compute the chiral condensate, Σ.
- See if physical quantities scale as one takes the continuum limit.

Ari will now show some progress we have made in this direction.

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