QCD with 8 and 12 Flavors

Lattice Gauge Theory for LHC Physics
Boston University
November 6, 2009

Robert Mawhinney
Columbia University

Results of simulations with naive staggered fermions and the DBW2 gauge action at $T = 0$ and $T \neq 0$. A variety of hadron observables have been measured, f_π, m_π, m_ρ, σ, $<\overline{\psi}\psi>$

Done in collaboration with Xiao-Yong Jin

Xiao-Yong Jin and R.M., arXiv:0812.0413, 8 flavors, Lattice 2008
Xiao-Yong Jin and R.M., arXiv:0910.3216, 8 and 12 flavors, Lattice 2009

Motivation

T. Banks, A. Zaks / Phase structure of vector-like gauge theories

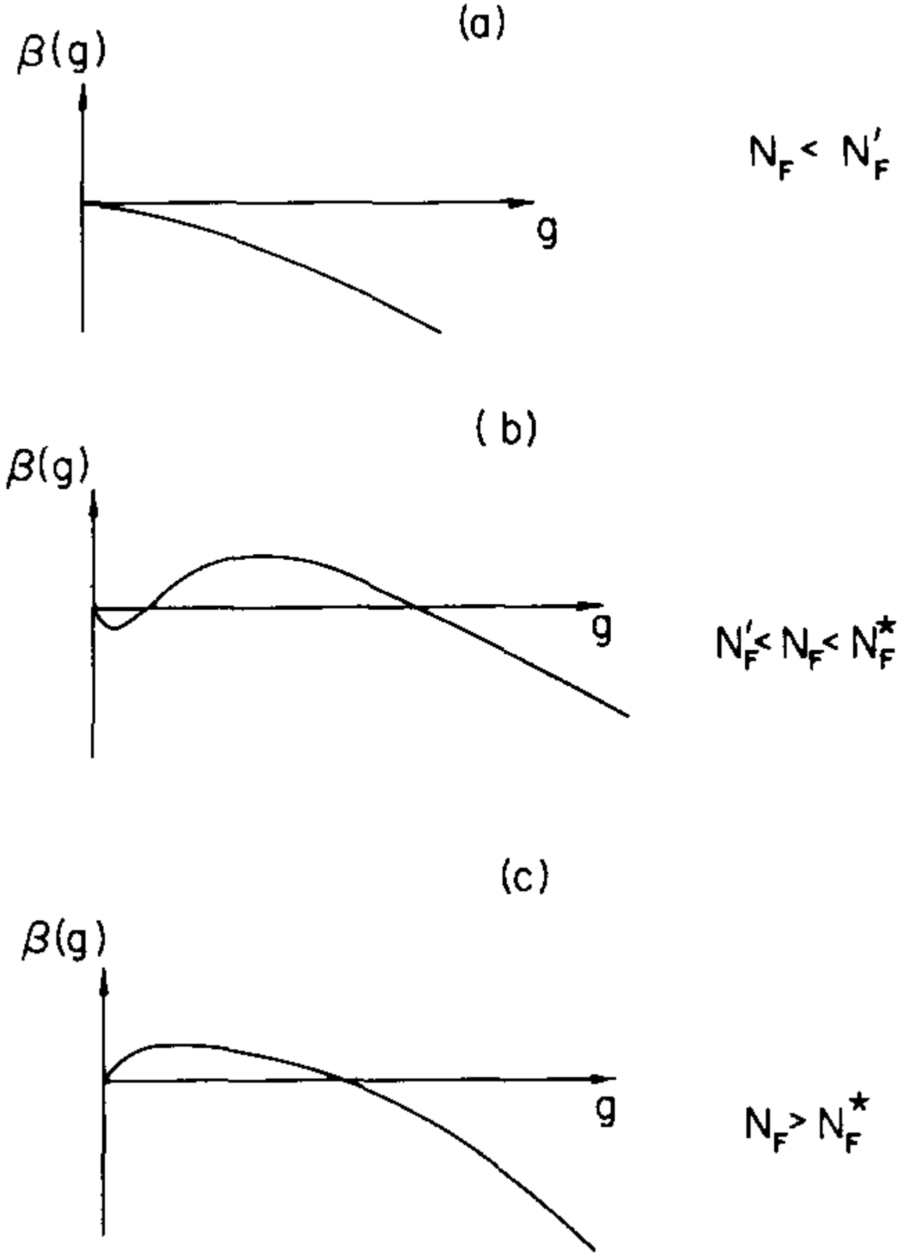
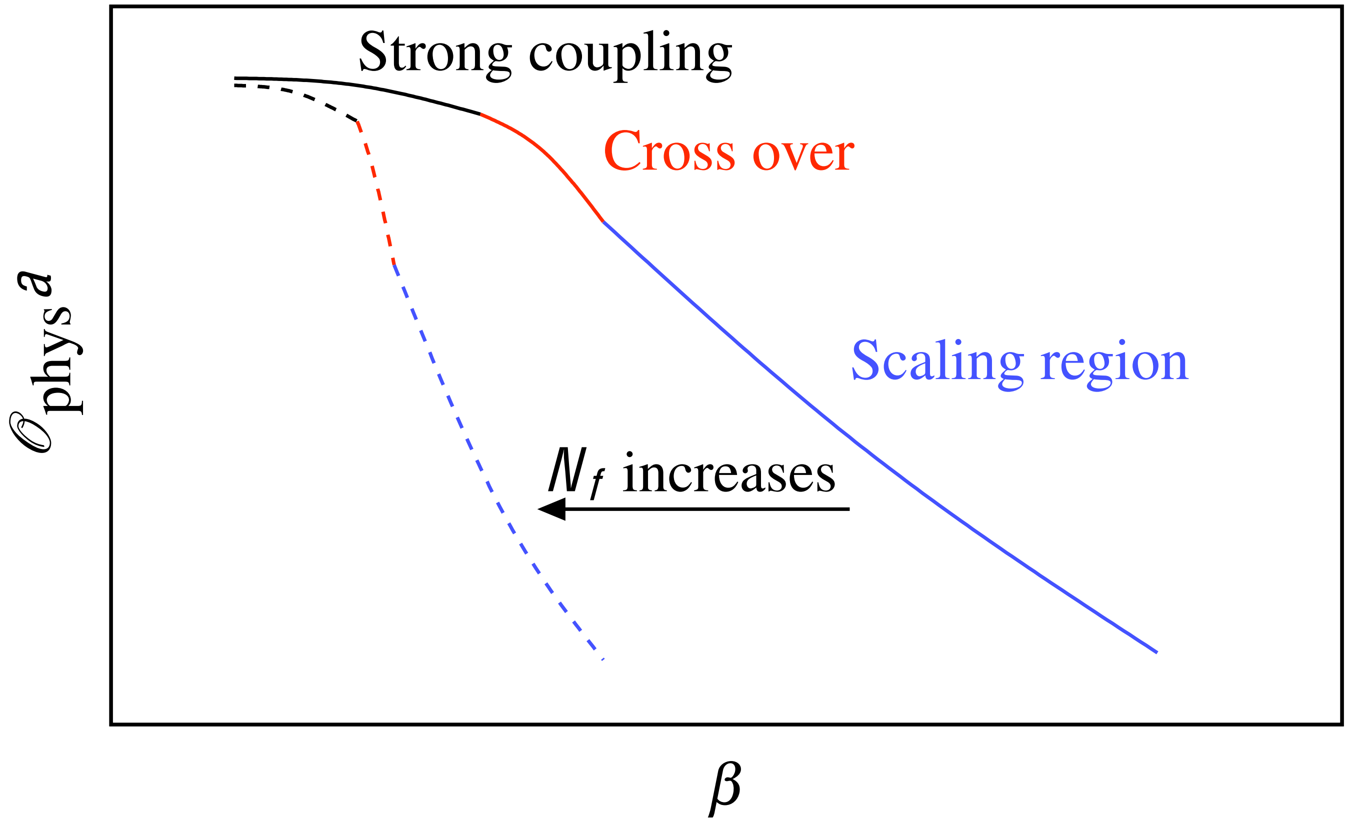


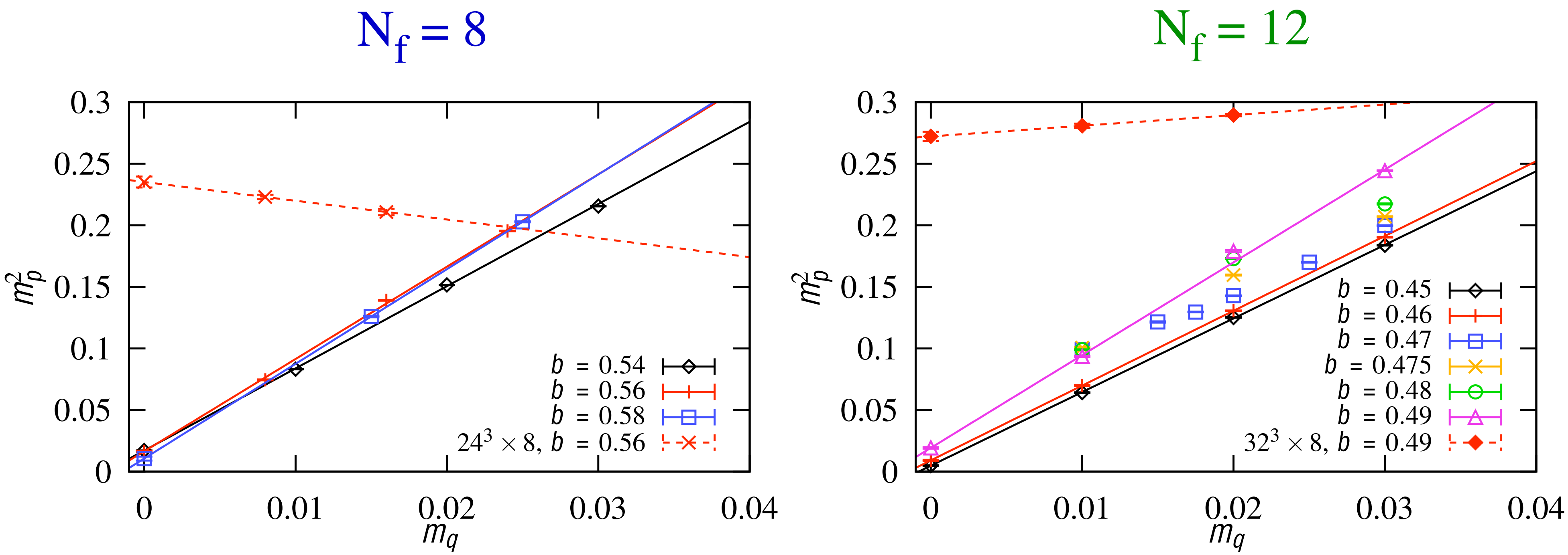
Fig. 1. 2 zero β function.

T = 0 QCD versus β



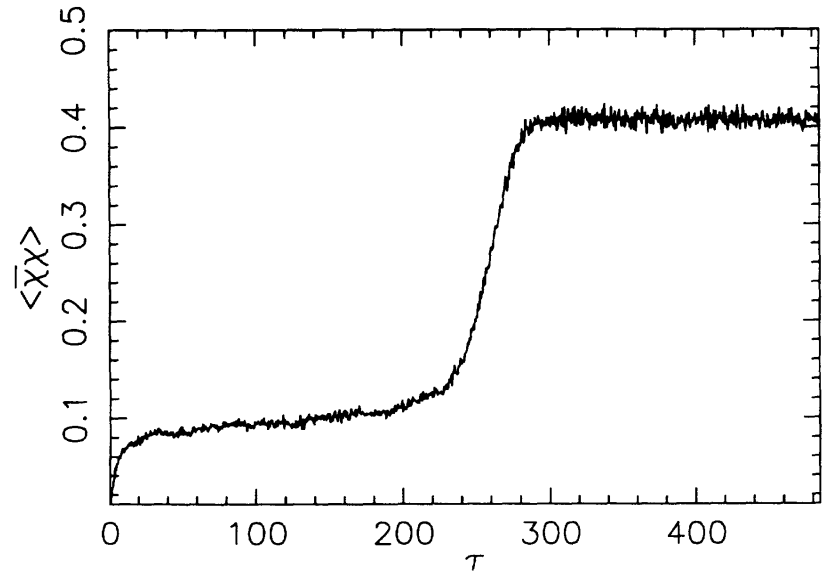
- Need $\beta > \beta_{\text{crossover}}$ to be continuously connected to continuum physics
- Irrelevant (in continuum) operators can play a large role $\beta < \beta_{\text{crossover}}$
- Strong to weak coupling may be discontinuous
- Steepness of crossover region may be hard to distinguish from walking.

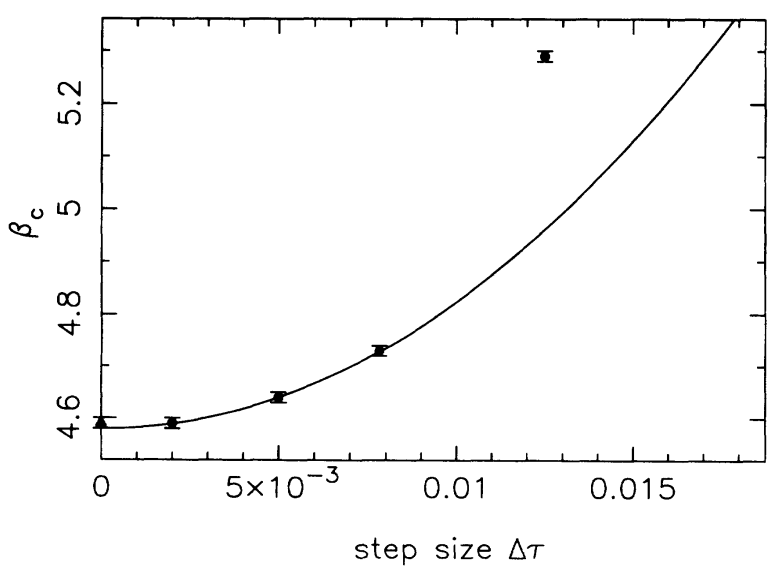
m_π^2 vs. m_f for 8 and 12 flavors, $T = 0$ and $T \neq 0$



- $T = 0$ simulations at a few lattice spacings show $m_\pi^2 = 2Bm_f$
- At weaker coupling, change from $N_t = 32$ to $N_t = 8$. No Goldsone boson
- Clear evidence that the system has a chirally broken phase at $T = 0$ and a phase with chiral symmetry restored for large T for $N_f = 8$ and 12.
- Will now look at Polyakov loop, lattice artifact transitions, algorithm issues, massless extrapolation, many other observables, finite volume effects and taste breaking

Bulk Transition Signal for $N_f = 8$ (1992)

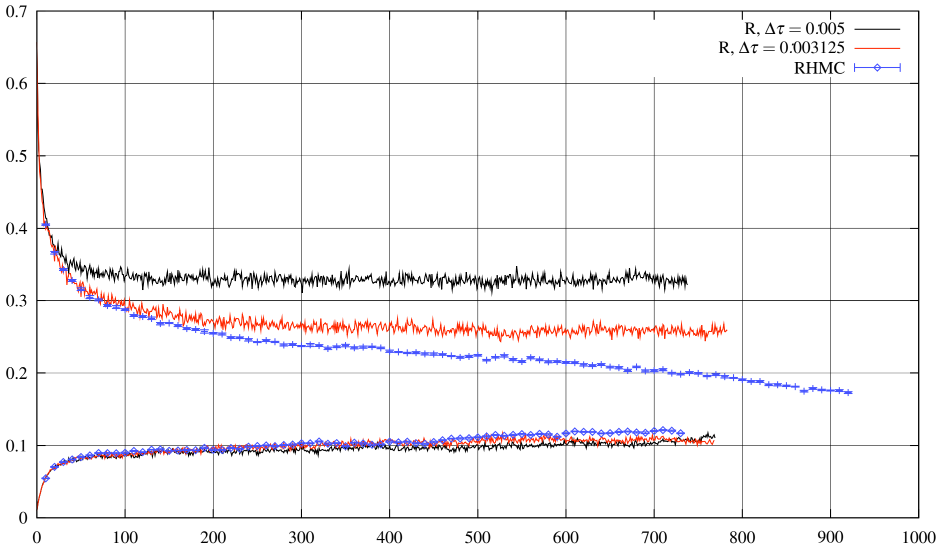




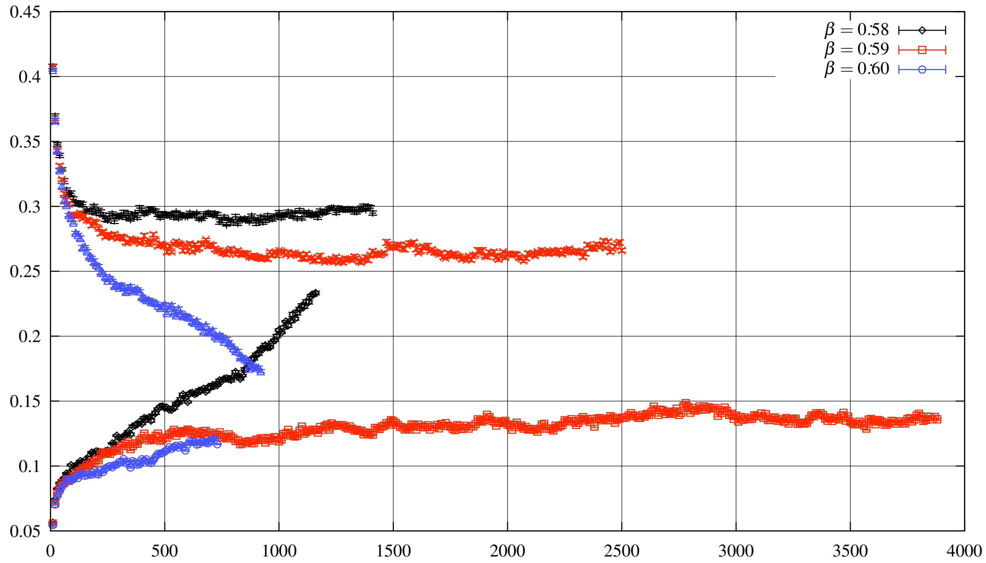
- Brown, et. al. (PRD 46 (1992) 5655)
- Naive staggered fermions with Wilson gauge action, $m_q = 0.015$
- Inexact R algorithm, $16^3 \times 32$ volume
- Bulk phase transition seen - argue that system has χSB on both sides
- Extrapolating in step size says transtion at $\beta = 4.58(1)$ with $\Delta\tau = 0.0$
- Look for this with exact RHMC

Bulk Transition Signal for $N_f = 8$ (2008)

$<\overline{\psi}\psi>$ for R and RHMC, $\beta = 4.60$



$<\overline{\psi}\psi>$ for RHMC, different β's



- Naive staggered fermions with Wilson gauge action, $m_q = 0.015$
- Inexact R algorithm and exact RHMC, $16^3 \times 32$ volume
- R algorithm transition at $\beta = 4.60$ disappears with RHMC
- RHMC sees bulk, first order transition at $\beta = 4.59$. Smaller discontinuity
- Unexpected conclusion: R algorithm, with conventional step size, makes it easier to see bulk transition, as it exists for a wider range of β.
- No bulk transition seen with DBW2, so far.

Taking Massless Limit

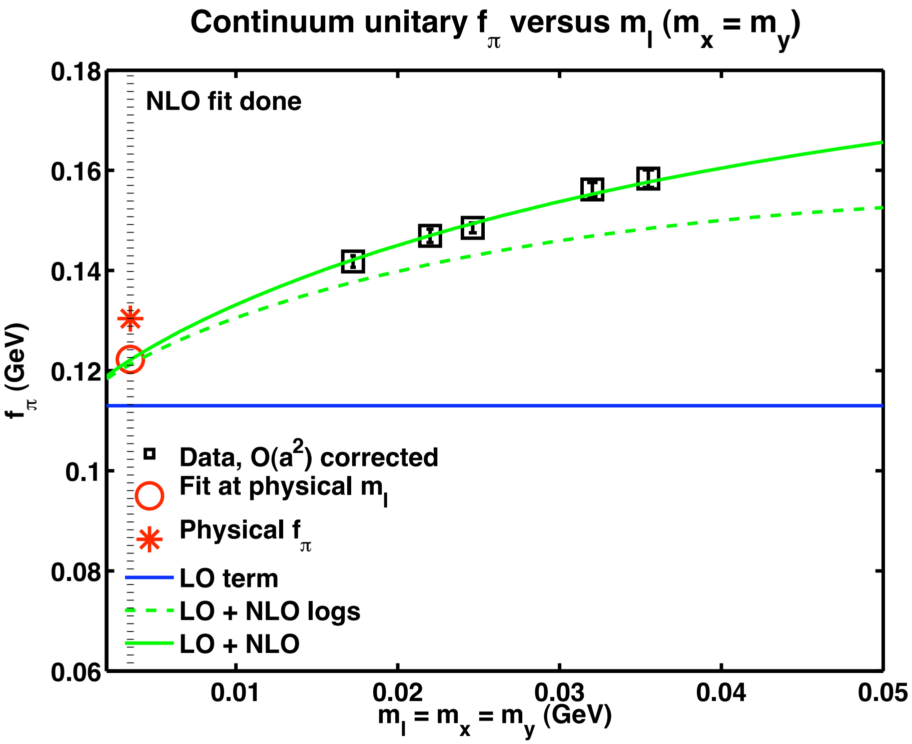
- Want statements about theories with $m_f = 0$, but simulate at finite m_f.
- Extrapolate to $m_f = 0$.
- What about chiral perturbation theory? There are logs in continuum result

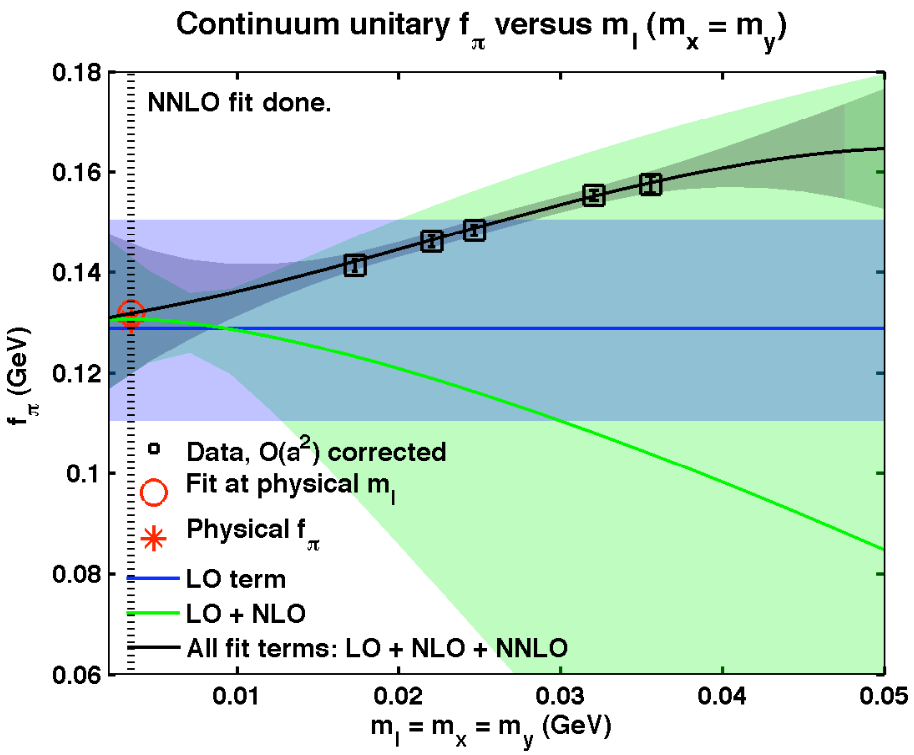
$$m_\pi^2 = 2Bm_f \left(1 + \frac{2}{N_f}\frac{2Bm_f}{(4\pi f)^2}\ln\left[\frac{2Bm_f}{\Lambda_{\chi\mathrm{PT}}^2}\right]\right)$$

$$f_\pi = f\left(1 - N_f\frac{2Bm_f}{(4\pi f)^2}\ln\left[\frac{2Bm_f}{\Lambda_{\chi\mathrm{PT}}^2}\right]\right)$$

- Coefficients have explicit N_f dependence, as well as implicit dependence, via f and B. Convergence may be poor
- Flavor breaking of staggered fermions will also modify logs.
- Will use simple linear extrapolations in m_f here. For high precision 2+1 flavor QCD work, logs are ~ 10 percent correction for physical m_q

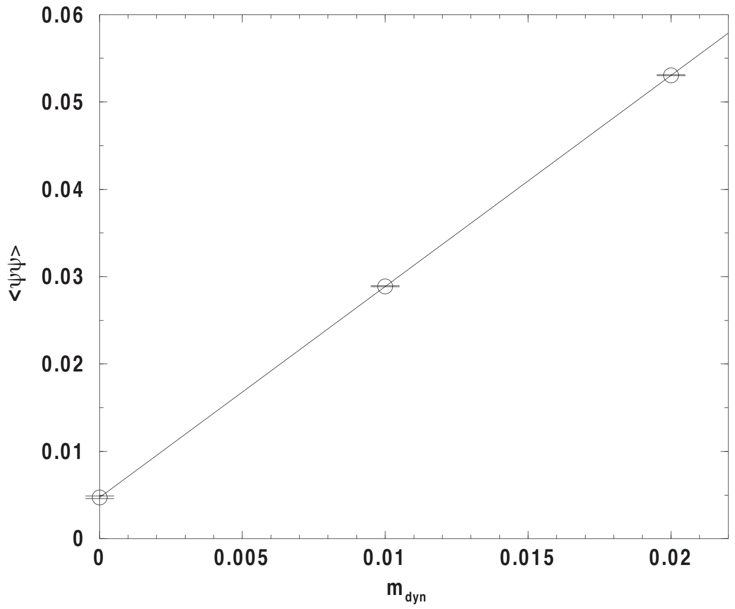
Complete NNLO ChPT May Not Help





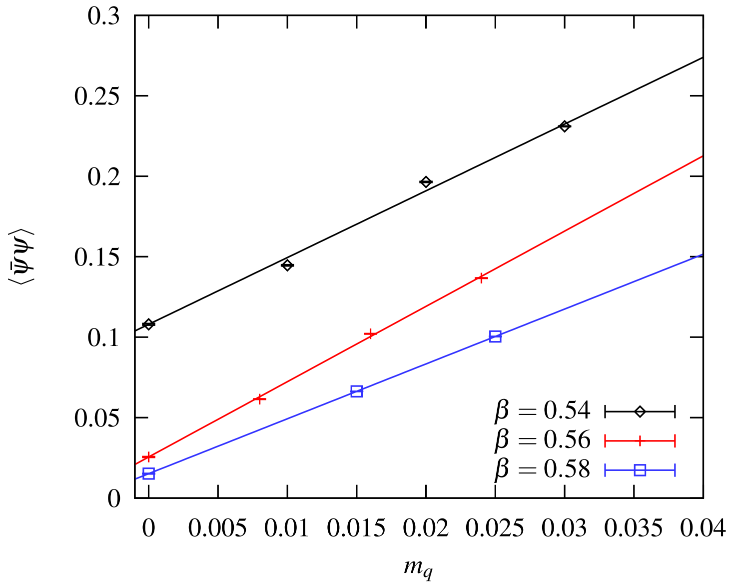
- 2+1 flavor DWF QCD (RBC and UKQCD) arXiv:0910.3194, Lat 09
- 2 a's, 5 dynamical m_l, 84 partially quenched values for m_π and f_π
- NLO gives $f_\pi = 122.2$ +/- 3.4_{stat} +/- 7.3_{ChPT}
- NNLO adds 13 additional parameters and gives $f_\pi = 133$ +/- 13_{stat}
- NNLO looks linear to physical m_l. For $m_f \to 0$, logs may be a small correction. Analytic expansion may be reasonable guide, but no proof

Extrapolation of Chiral Condensate

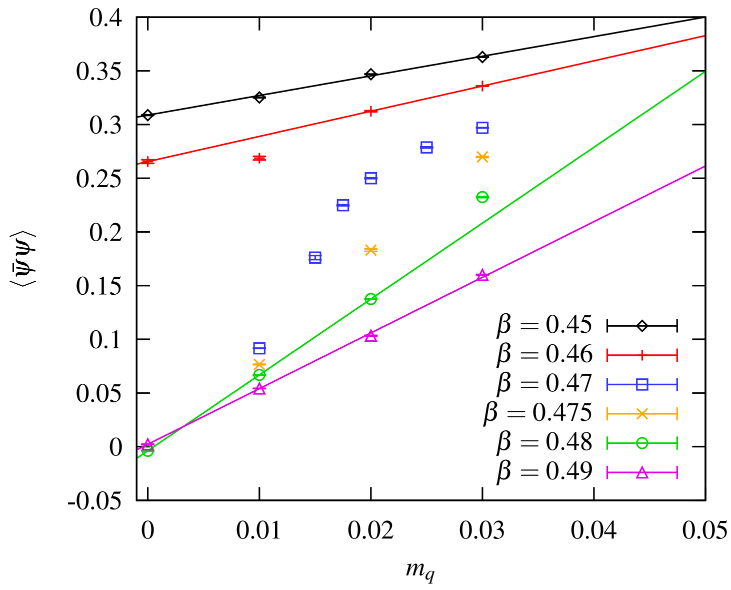


$N_f = 4$

- $N_f = 4$ and 8 extrapolate linearly to non-zero values.
- $N_f = 12$ at strong coupling shows χSB in massless limit
- $N_f = 12$ at weak coupling shows a rapid change in the system.

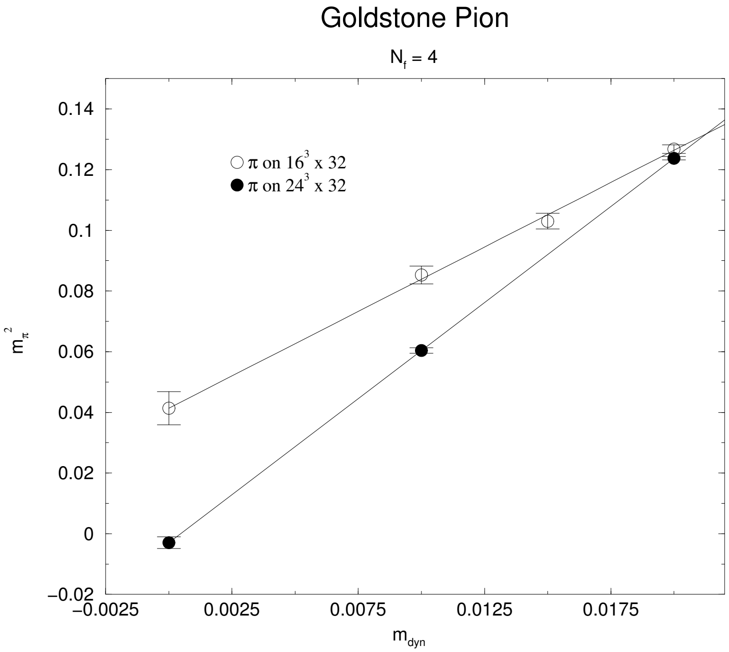


$N_f = 8$

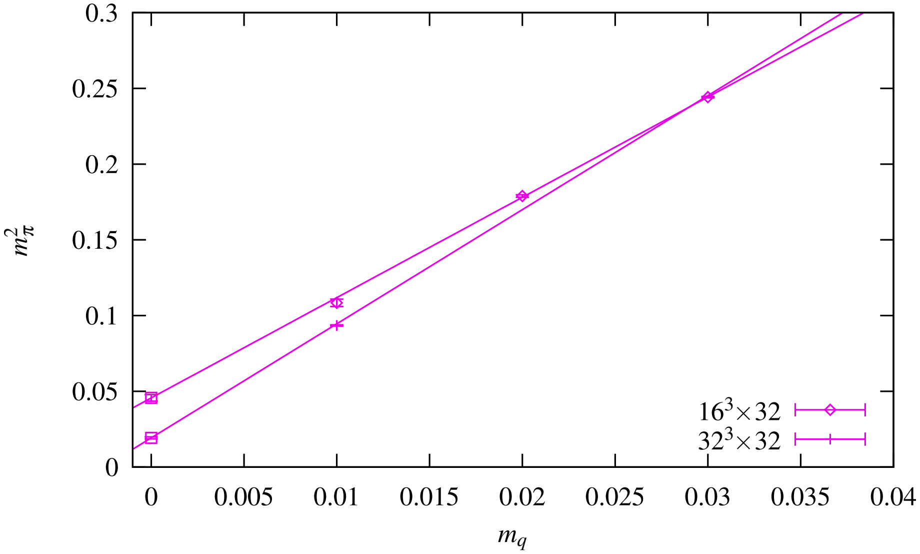


$N_f = 12$

Extrapolation of m_π^2 for $N_f = 4$ and 12



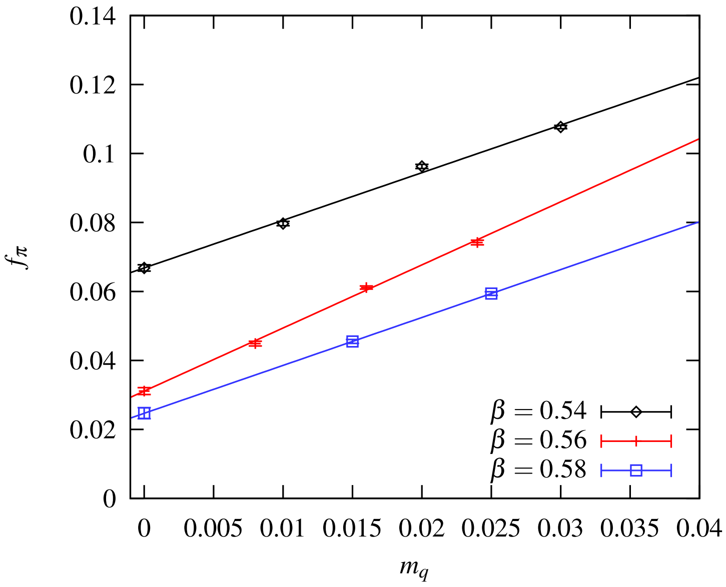
$N_f = 4$



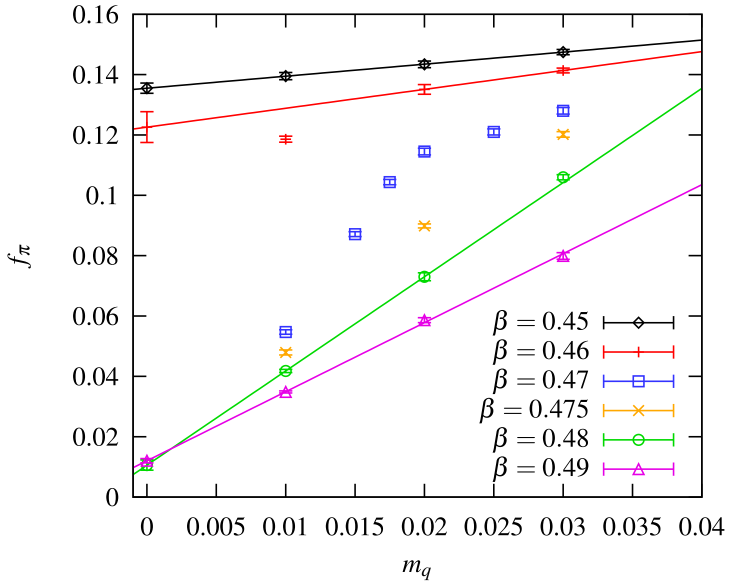
$N_f = 12$

They look very similar

Extrapolation of f_π



$N_f = 8$

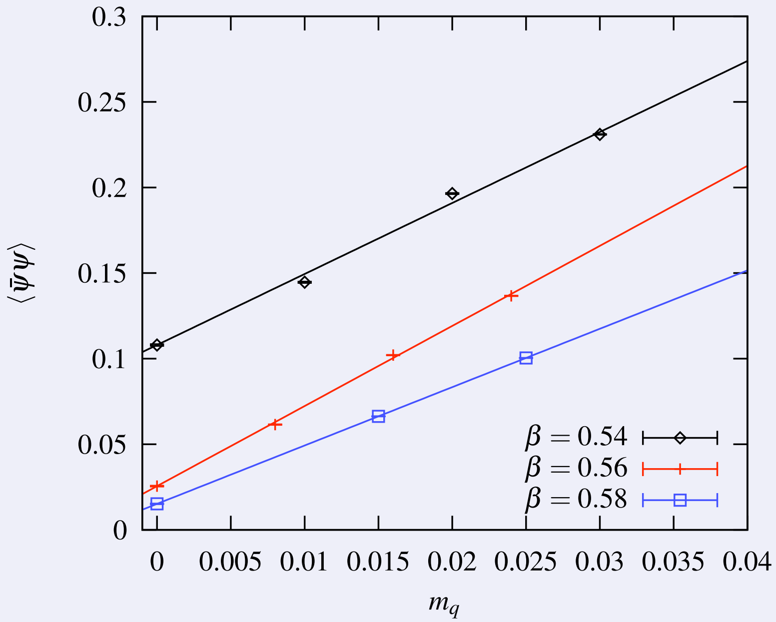


$N_f = 12$

- Extrapolated f_π for $N_f = 8$ shows 2× change across the region of rapid evolution.
- Extrapolated f_π for $N_f = 12$ shows ~10× change across the region of rapid evolution.
- Extrapolated, weak coupling f_π for $N_f = 12$ is non-zero.

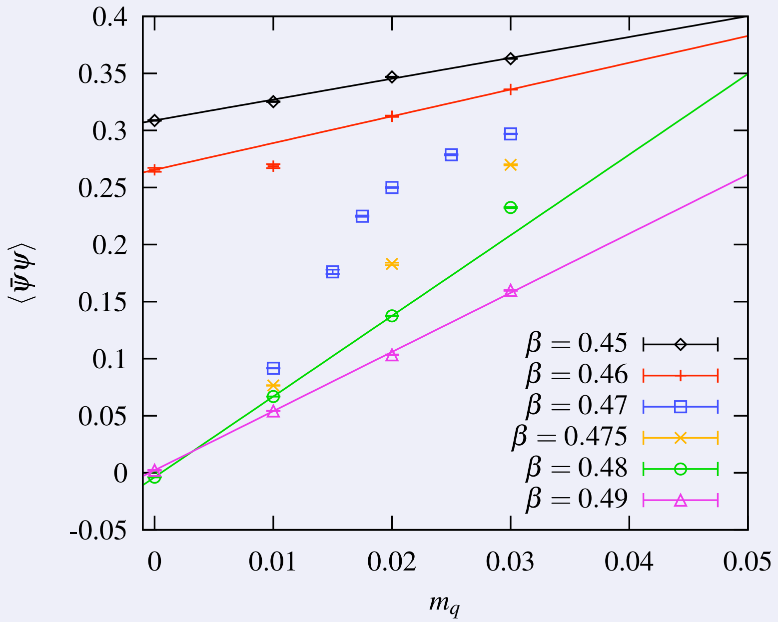
Extrapolation of Chiral Condensate, again

8 flavors



- $\langle\bar{\psi}\psi\rangle(\beta = 0.56, m_q = 0) = 0.02555(23)$.
- $\langle\bar{\psi}\psi\rangle(\beta = 0.58, m_q = 0) = 0.01519(16)$.

12 flavors



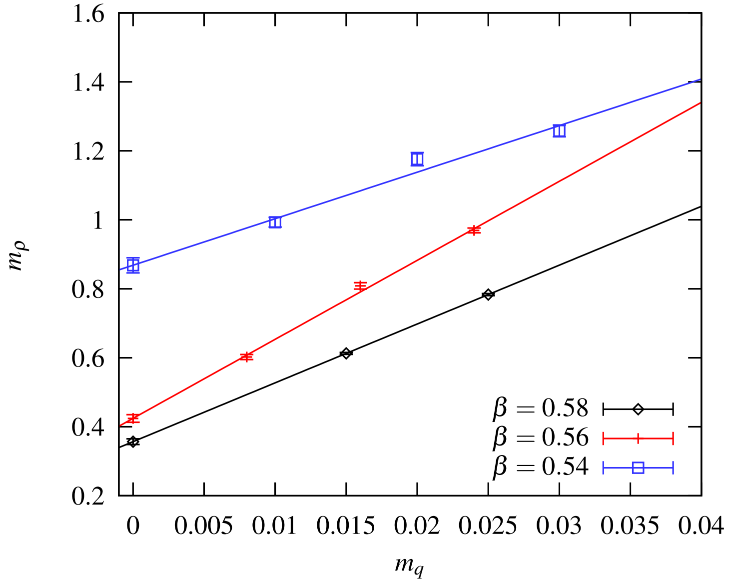
- $\langle\bar{\psi}\psi\rangle(\beta = 0.48, m_q = 0) = -0.00382(25)$.
- $\langle\bar{\psi}\psi\rangle(\beta = 0.49, m_q = 0) = 0.00220(27)$.

4× change

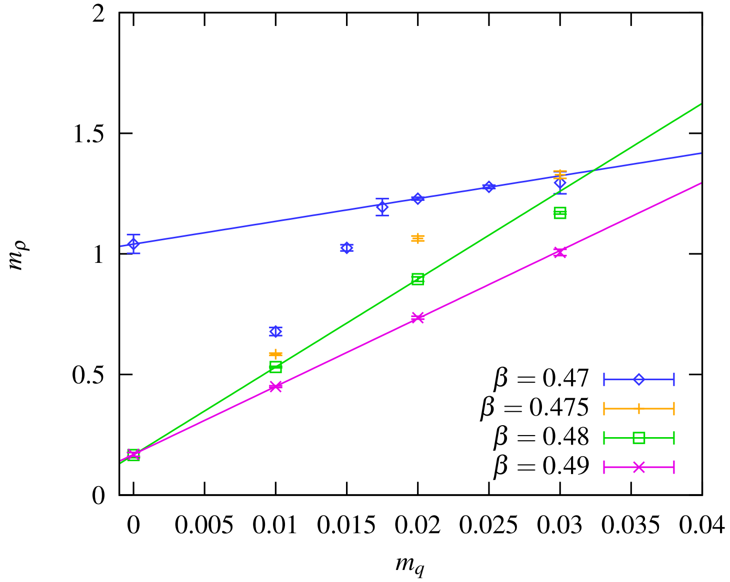
100× change

$$\langle\bar{\psi}\psi\rangle \sim \frac{m_\pi^2 f_\pi^2}{m_q}$$

Extrapolation of m_ϱ



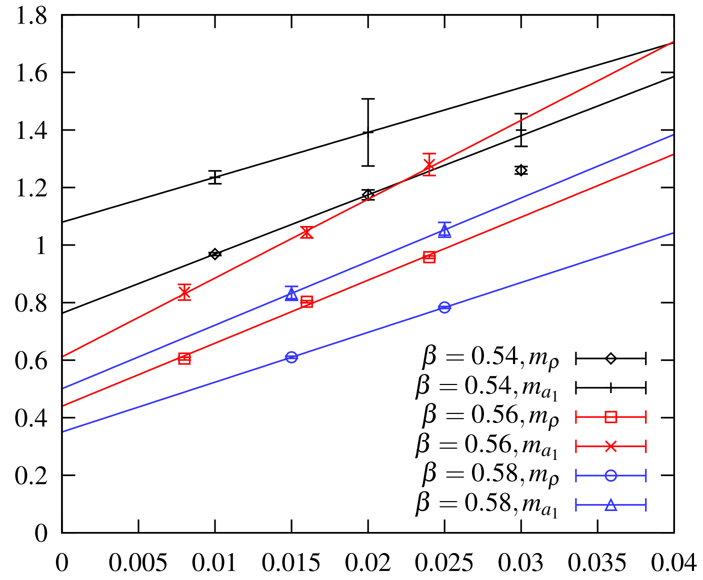
$N_f = 8$



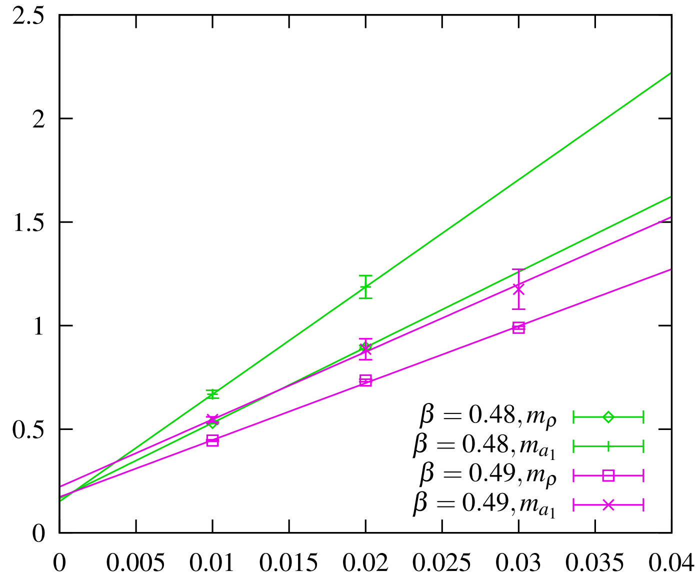
$N_f = 12$

- Extrapolated m_ϱ for Nf = 8 shows 2× change across the region of rapid evolution.
- Extrapolated m_ϱ for $N_f = 12$ shows ~6× change across the region of rapid evolution, less than the 10x seen in f_π
- Look at other particles to see what is happening with m_ϱ

Parity Doubling



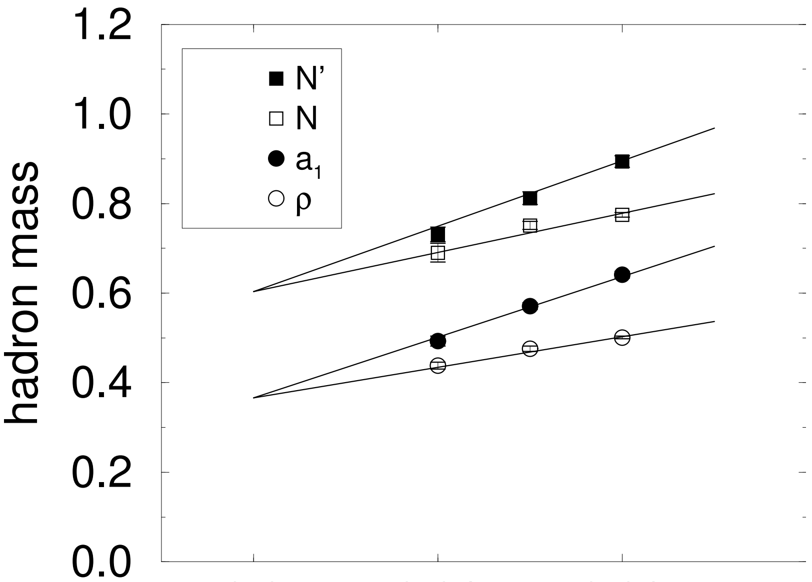
$N_f = 8$



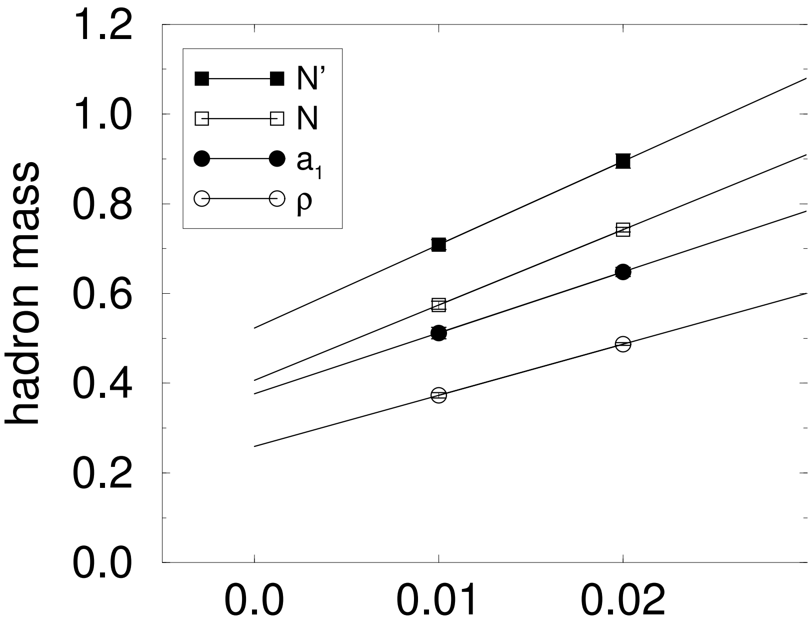
$N_f = 12$

- If chiral symmetry is not broken, particle spectrum is parity doubled
- Spectrum for Nf = 8 shows mass of ϱ and a^1 not degenerate when $m_f = 0$.
- Spectrum for Nf = 12 shows parity doubling.
- What happened to χSB?

Parity Doubling for 4 flavors



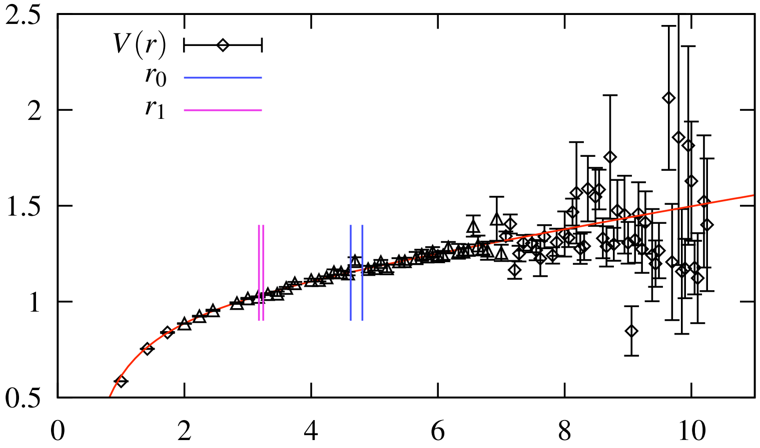
$16^3 \times 32$ volume



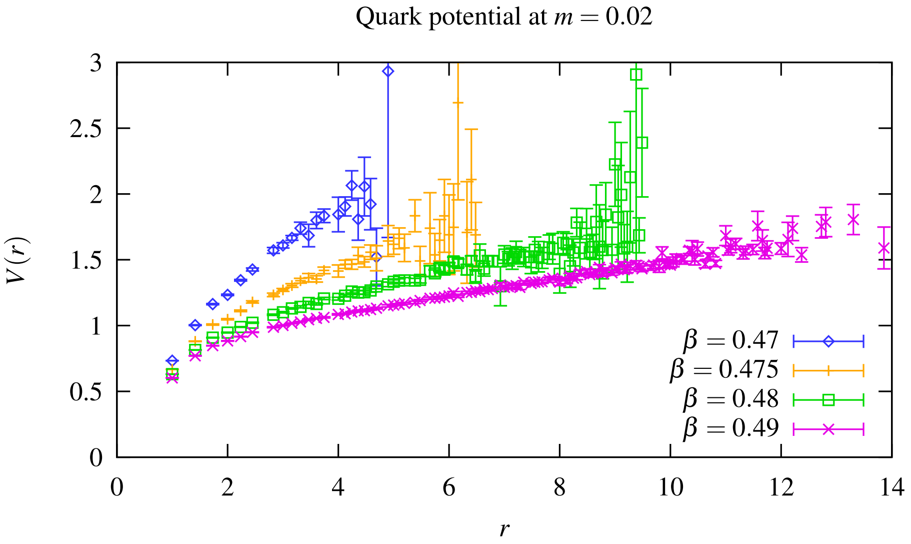
$24^3 \times 32$ volume

- Parity doubling disappears for larger volume system
- System still has χSB, but parity doubling is a sensitive indicator of finite volume effects

String Tension



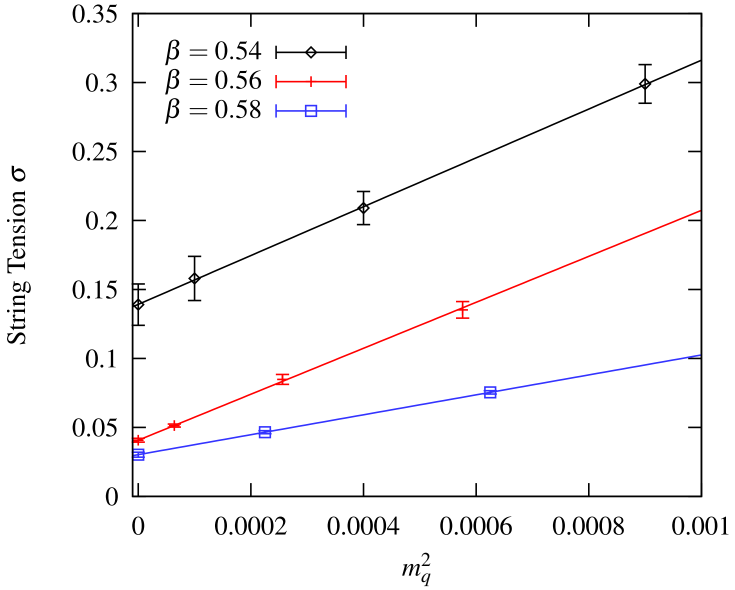
$N_f = 8, \beta = 0.56, m_f = 0.008,$
$24^3 \times 32$



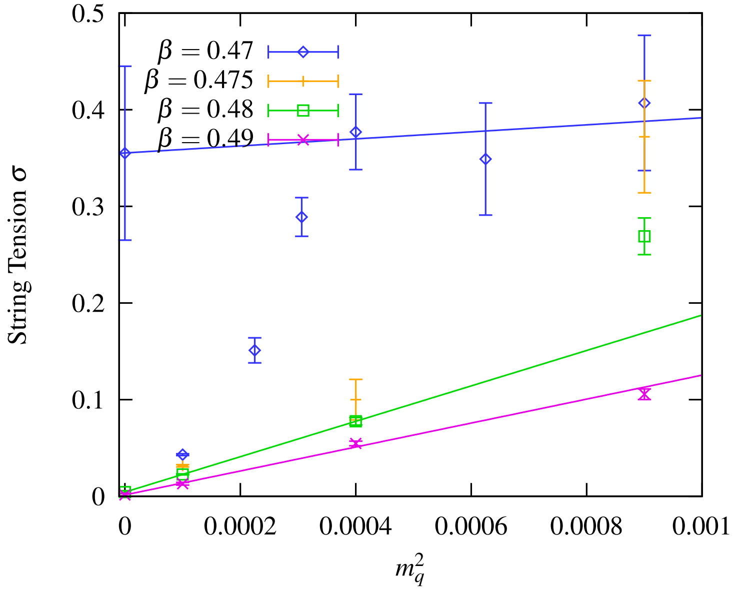
$N_f = 12$

- String tension is small for $N_f = 12$, making it easy to measure
- Our Coulomb gauge fixing method has larger finite volume errors than other methods
- For non-zero quark mass, system exhibits confinement

String Tension versus m_f



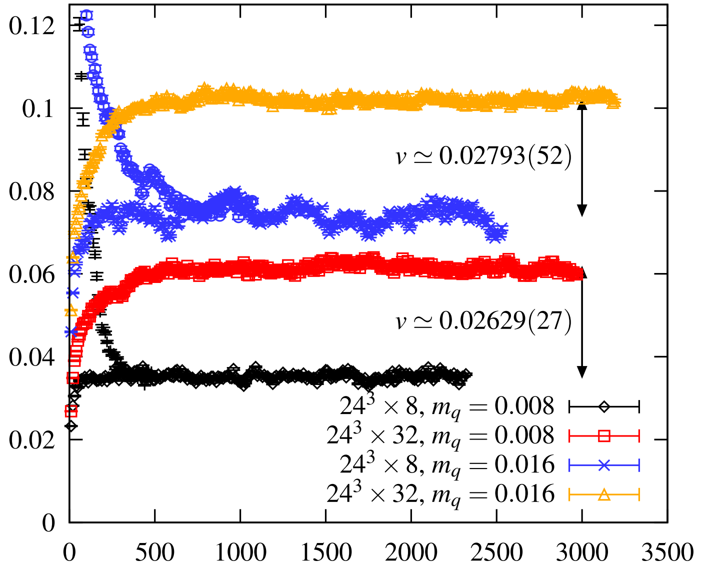
$N_f = 8$



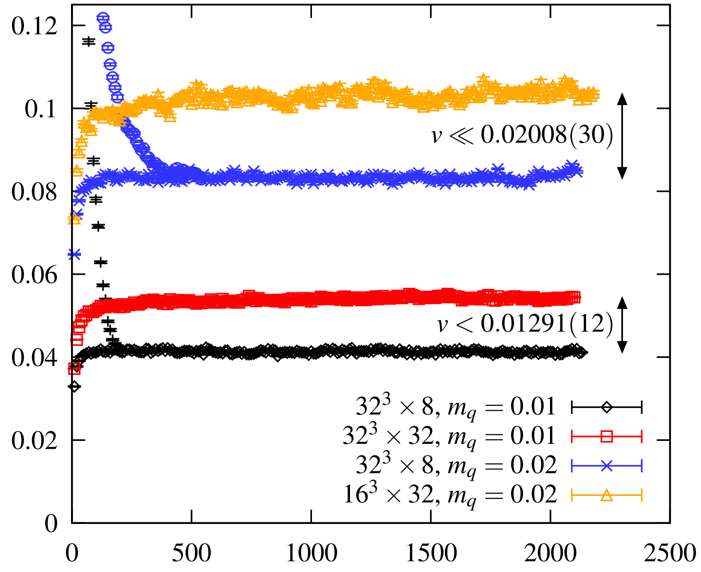
$N_f = 12$

- String tension for $N_f = 8$ drops by 4× across rapid evolution region
- For $N_f = 12$, $\sigma(\beta = 0.48, m_f = 0) = 0.0043(15)$
- For $N_f = 12$, $\sigma(m_f = 0)$ is consistent with 100× change across region of rapid evolution of system.

Evolution of Chiral Condensate at $N_t = 8$



$N_f = 8$



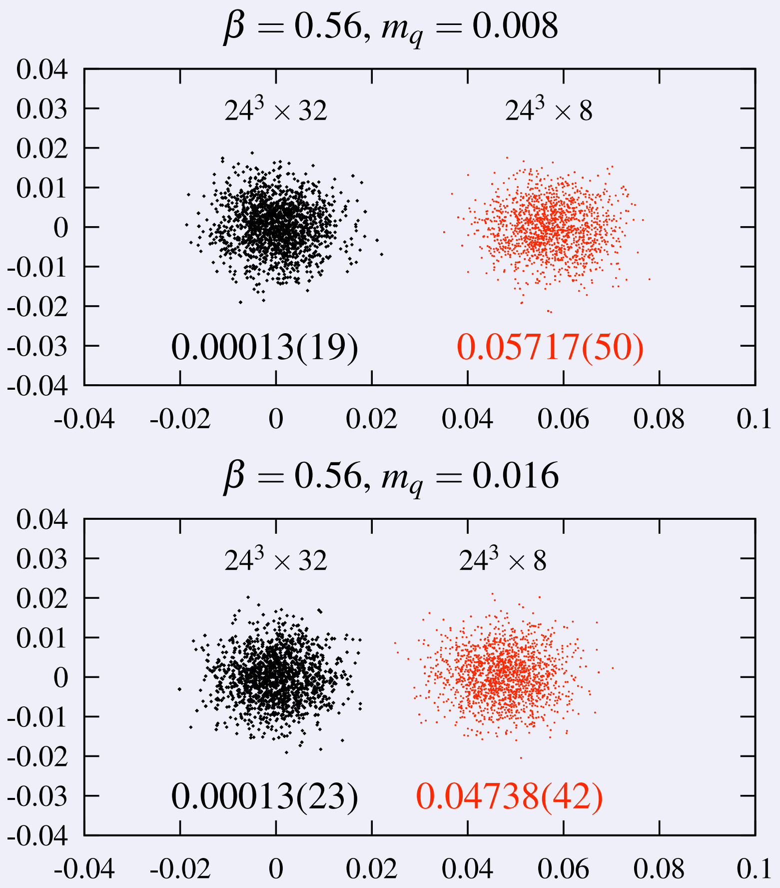
$N_f = 12$

- For $N_f = 8$, at T = 0 we find v = 0.02555(23) by linear extrapolation, very close to the observed shift in v (at non-zero quark mass) when going from $N_t = 32$ to $N_t = 8$.
- For $N_f = 12$, $\beta = 0.49$, we find the shift in v (at non-zero quark mass) from $N_t = 32$ to $N_t = 8$ to be larger than the v when $m_f \to 0$ for T=0. However, we are not sure we are on the weak coupling side of the rapid evolution region at this β, so our T = 0 v is not very reliable.

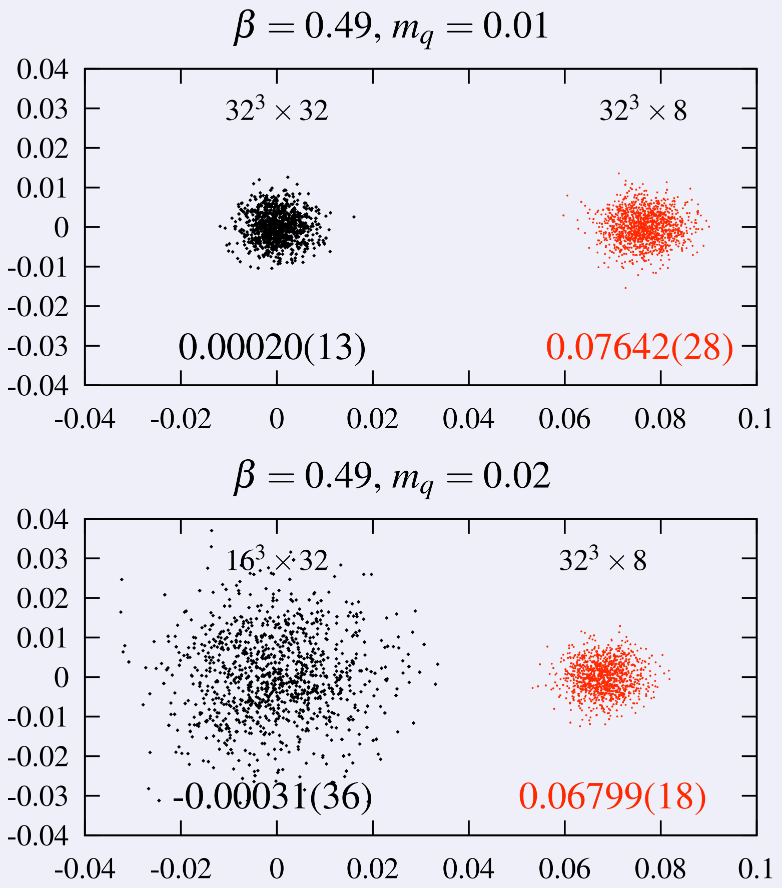
Unrenormalized Polyakov Loop

Clearly $\langle P \rangle \neq 0$ at $N_\tau = 8$

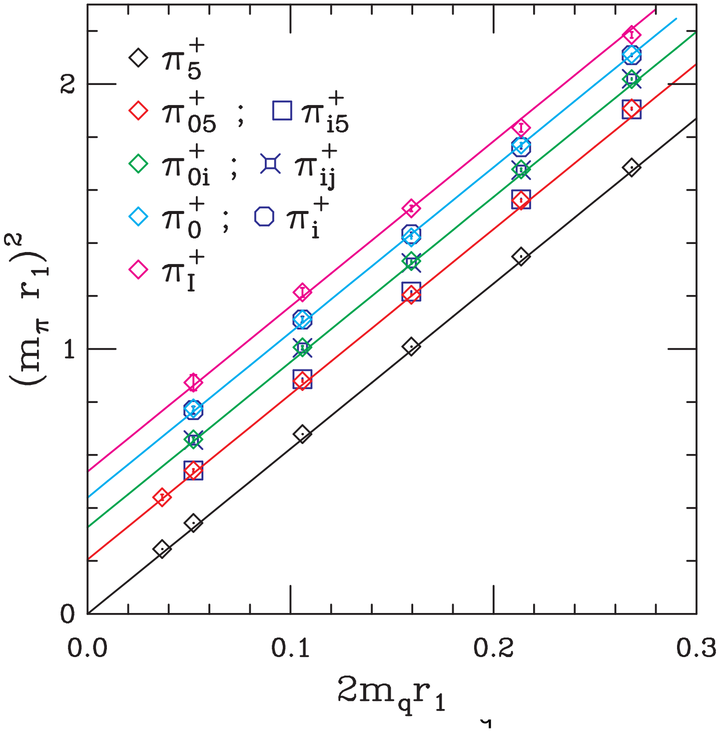
8 flavors



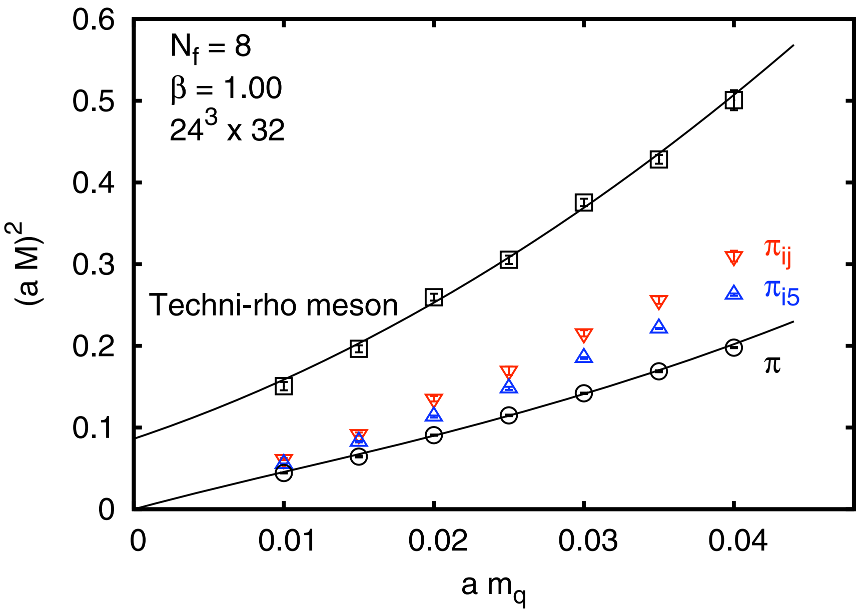
12 flavors



Staggered Flavor (taste) Breaking

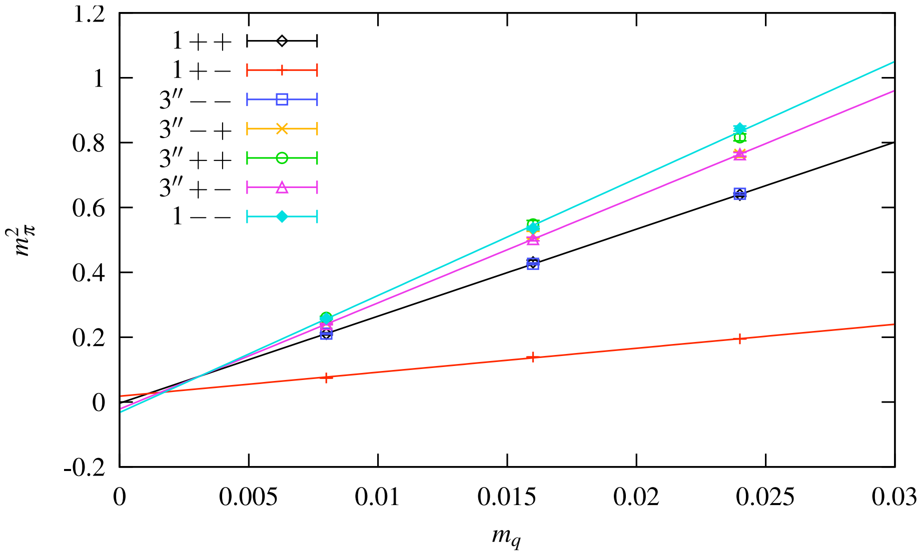


- MILC Phys.Rev.D70:114501,2004
- 2+1 flavor ASQTAD, a = 0.12 fm
- Visible splitting when $m_q = 0$
- Same slope for all tastes
- $m_\rho \sim 0.5$ in lattice units

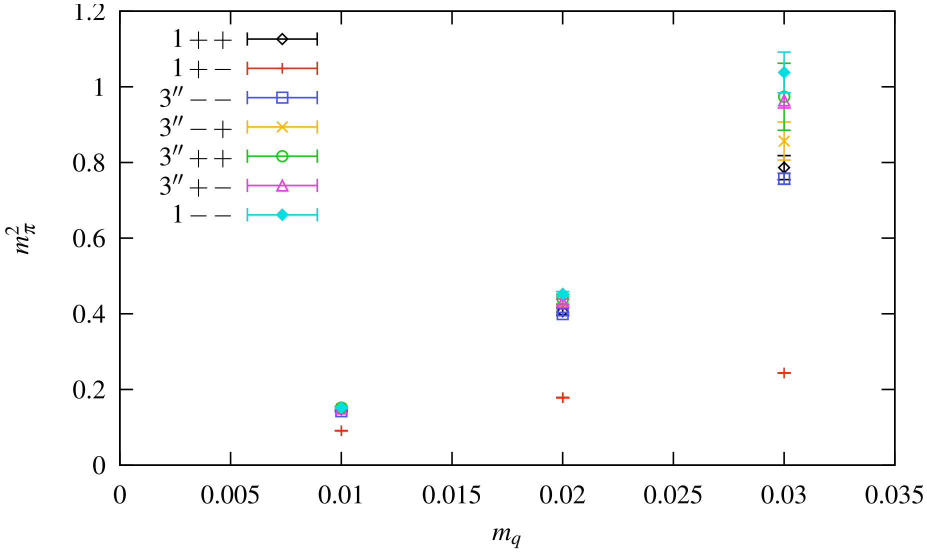


- Fodor, et.al., PLB 681, 353 (2009)
- 6 stout smearing steps
- $m_\rho \sim 0.3$ in lattice units
- Slope varies with taste
- Small splittings when $m_q = 0$

Staggered Flavor (taste) Breaking: DBW2 action



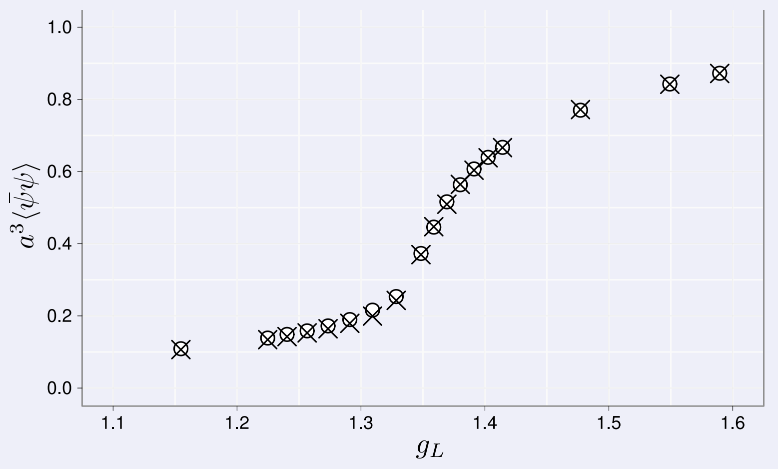
- Measurements for β = 0.56, $24^3 \times 32$ volumes
- Different slopes for all tastes
- Small splittings when $m_q = 0$
- $m_\pi^2 \sim m_q$ for all tastes
- $m_\rho \sim 0.4$ in lattice units



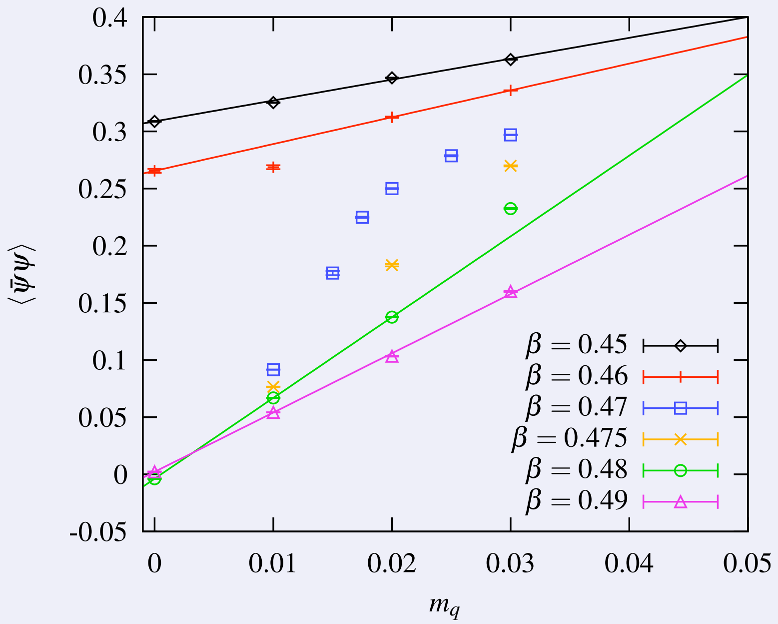
- Measurements for β = 0.49, $16^3 \times 32$, $32^3 \times 32$ volumes
- Small splittings when $m_q \leq 0.01$
- Only Goldstone has $m_\pi^2 \sim m_q$
- $m_\rho \sim 0.1$ in lattice units

12 flavors results comparison with Deuzeman *et al.*, 2009

arXiv:0904.4662v1



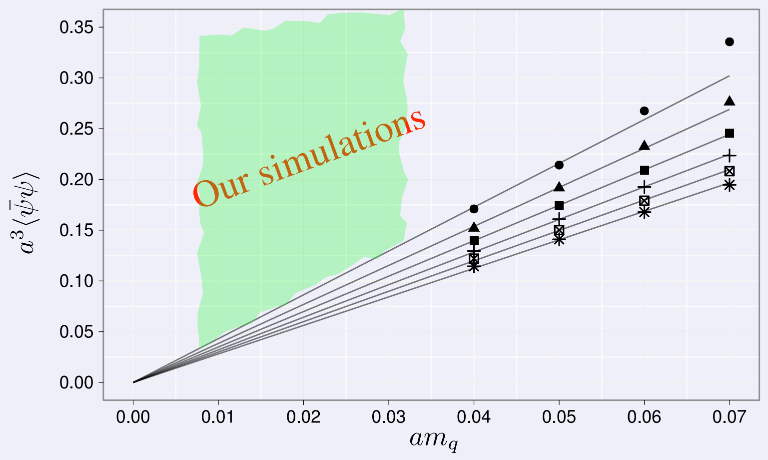
$m_q = 0.05$, spatial volume 16^3, circles for $N_\tau = 16$ and crosses for $N_\tau = 8$.



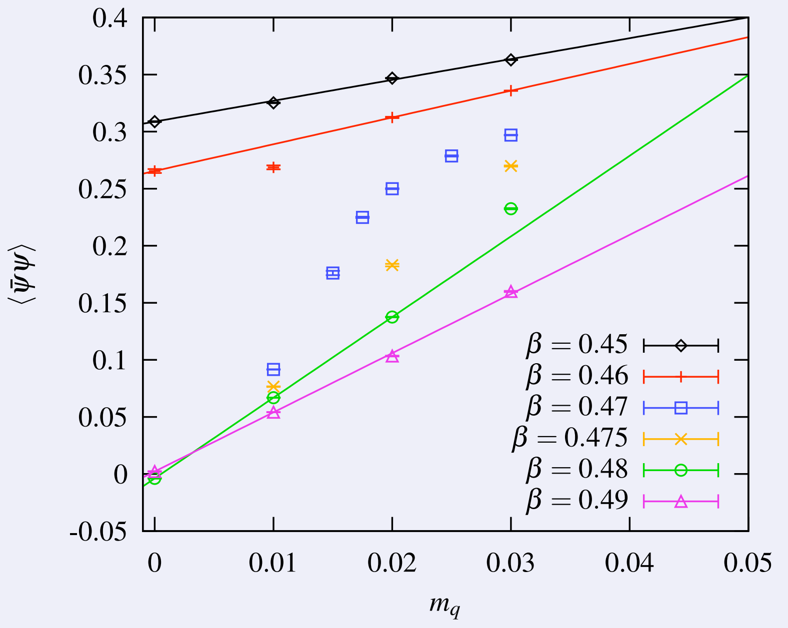
- Bulk transition observed.

12 flavors results comparison with Deuzeman *et al.*, 2009

arXiv:0904.4662v1



Weaker coupling side of the transition.

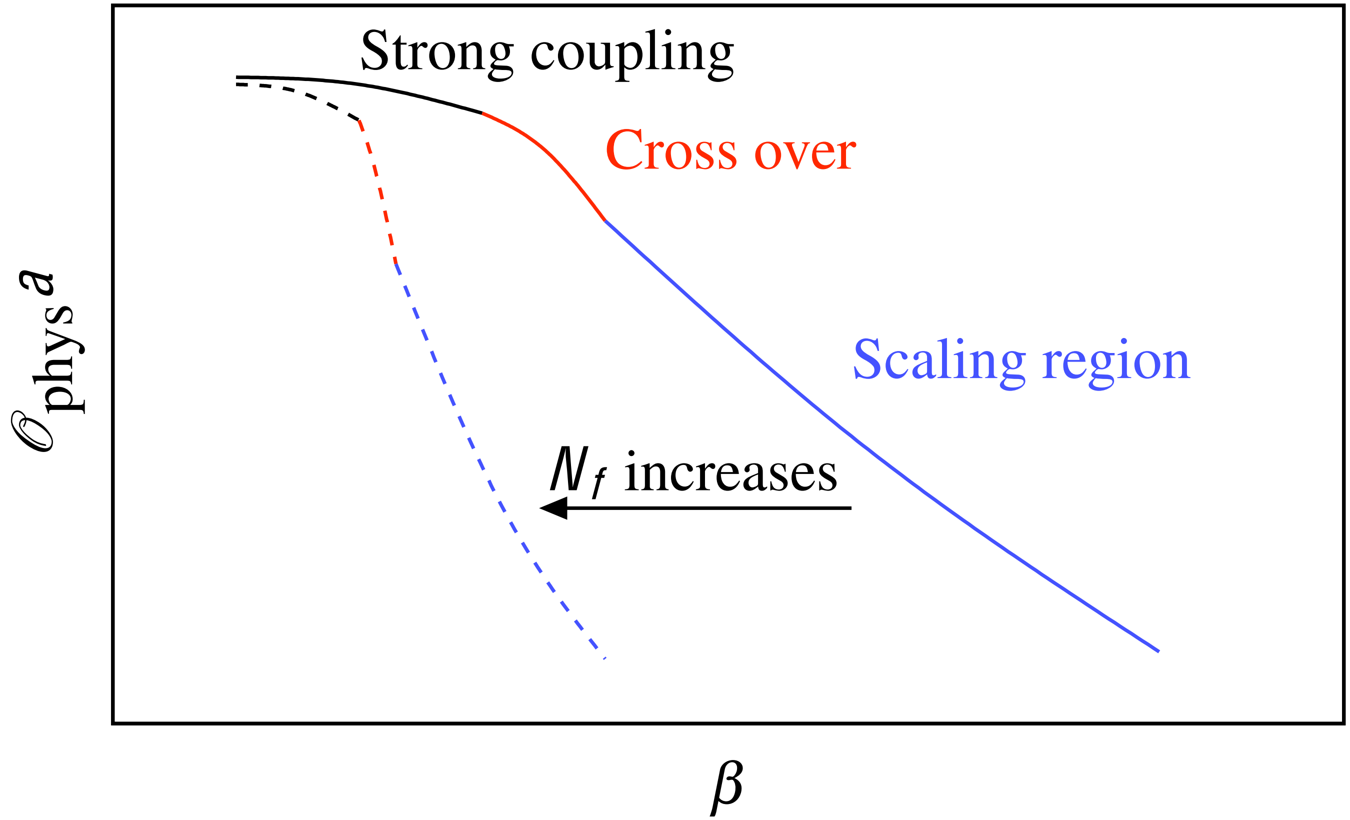


- Very small $\langle\bar\psi\psi\rangle$ at chiral limit.
- Our simulations run in smaller m_q's.
- We studied more observables.

Summary

- Direct calculation of hadronic observables shows that $N_f = 8$ and 12 QCD with the DBW2 gauge action and naive staggered fermions
  a. Is in a chirally-broken phase at zero temperature, with $m_\pi^2 \sim m_f$
  b. Has a chirally symmetric phase for high temperature, with no Goldstone boson
  c. Exhibits a region of rapid evolution of the system, with respect to the bare lattice coupling β, which is more pronounced for $N_f = 12$.
- The extrapolations to the massless limit that were used here are simple linear extrapolations. ChPT may not be helpful to improve this, since convergence may be poor due to explicit N_f factors, large B and small f.
- The weak coupling side of $N_f = 12$ region has very small QCD scales, in lattice units. This requires simulations in large volumes. Have some data on finite V effects and they are measureable, but perhaps not dominating.
- Our conclusions differ from Appelquist, Fleming and Neil (PRL 100 171607 (2008)), who measure β-functions and claim $N_f = 12$ is conformal

T = 0 QCD versus β



- Discontinuities can exist in strong to weak coupling region.
  - * Likely lattice artifacts
  - * Can be small or large, depending on irrelevant operators
- β for strong-to-weak transition strongly mass dependent for $N_f = 12$.
- What is $\beta_{\rm min}$ for Schrödinger functional and RG methods?

Outlook

- Have measured staggered fermion flavor breaking in meson spectrum for $N_f = 8$ and 12
  - Found $m_q = 0$ splittings surprisingly small.
  - Pronounced differences in slope of m_π^2 with m_q
  - What arguments can one use to decide whether flavor breaking is relevant to phase of system?
- Considering further large volume runs for $N_f = 12$ at weak coupling. Difficult to make sure we are completely through the crossover.
- Further increases in N_f would make hadronic signal even smaller and require large volumes. Vanishing signal/noise
- Likely need better observables if simulations with larger N_f are done.

Acknowledgements

- Simulations done on the Columbia and RBRC QCDOC computers, and NYBlue at BNL
- We have used the existing CPS software for our simulations
- Thanks to members of the RBC collaboration, and particularly Norman Christ, for helpful discussions.