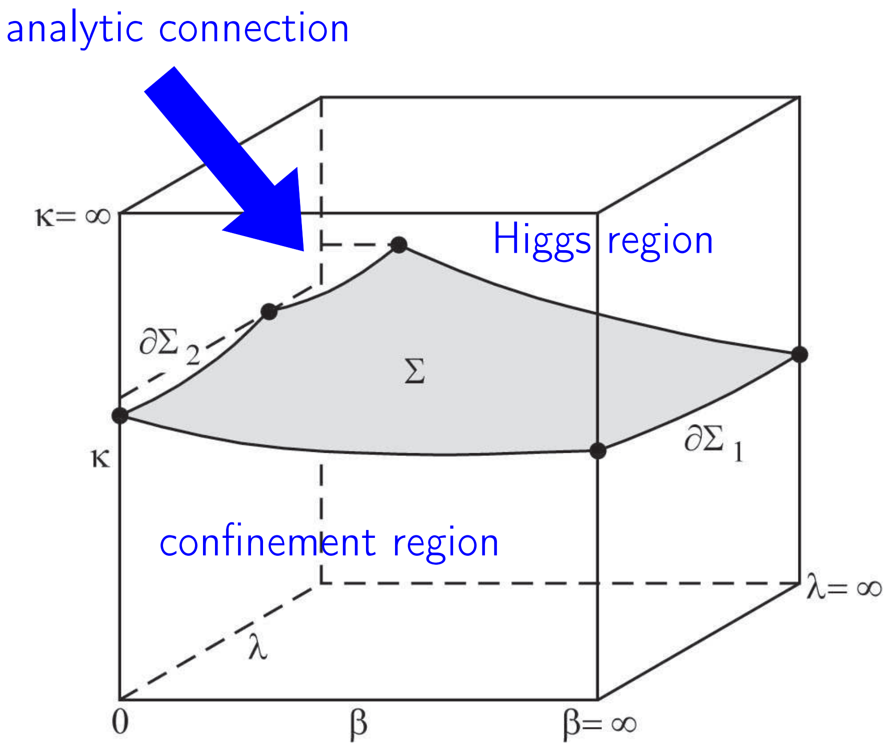
two-doublet lattice Higgs model

1. motivation
2. lattice action & symmetries
3. phases
4. symmetry breaking

(This is a preliminary report of work in progress with Richard Woloshyn.)

Recall that the standard SU(2)-Higgs model, with one scalar doublet, has two regions of parameter space *that are analytically connected*. (No gauge-invariant vev, no broken symmetry.)



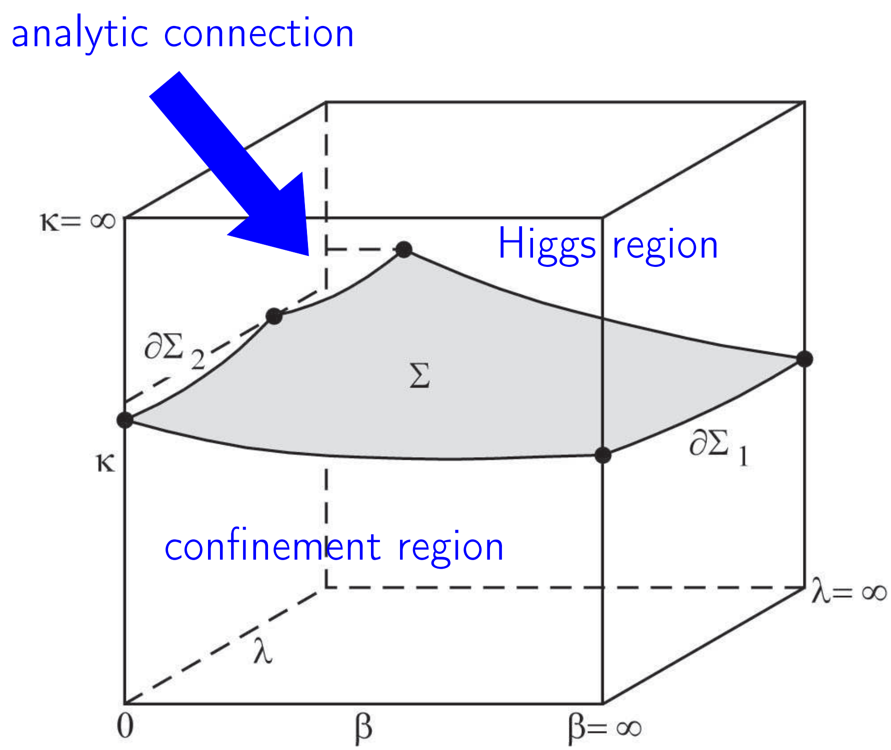
adapted from Straumann, astro-ph/0409042

as adapted from textbook by Montvay & Münster

Recall that the standard SU(2)-Higgs model, with one scalar doublet, has two regions of parameter space *that are analytically connected*. (No gauge-invariant vev, no broken symmetry.)

This phase diagram is easily mapped out by monitoring a quantity such as the

$$\text{gauge-invariant link} \equiv \left\langle \Phi^\dagger(x)U_\mu(x)\Phi(x+\mu) + \text{h.c.} \right\rangle.$$



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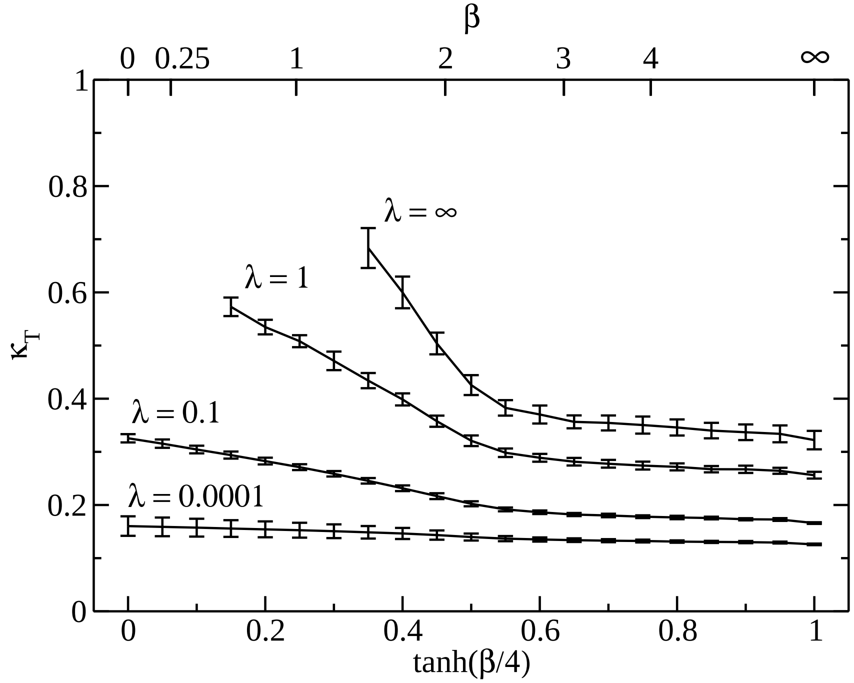
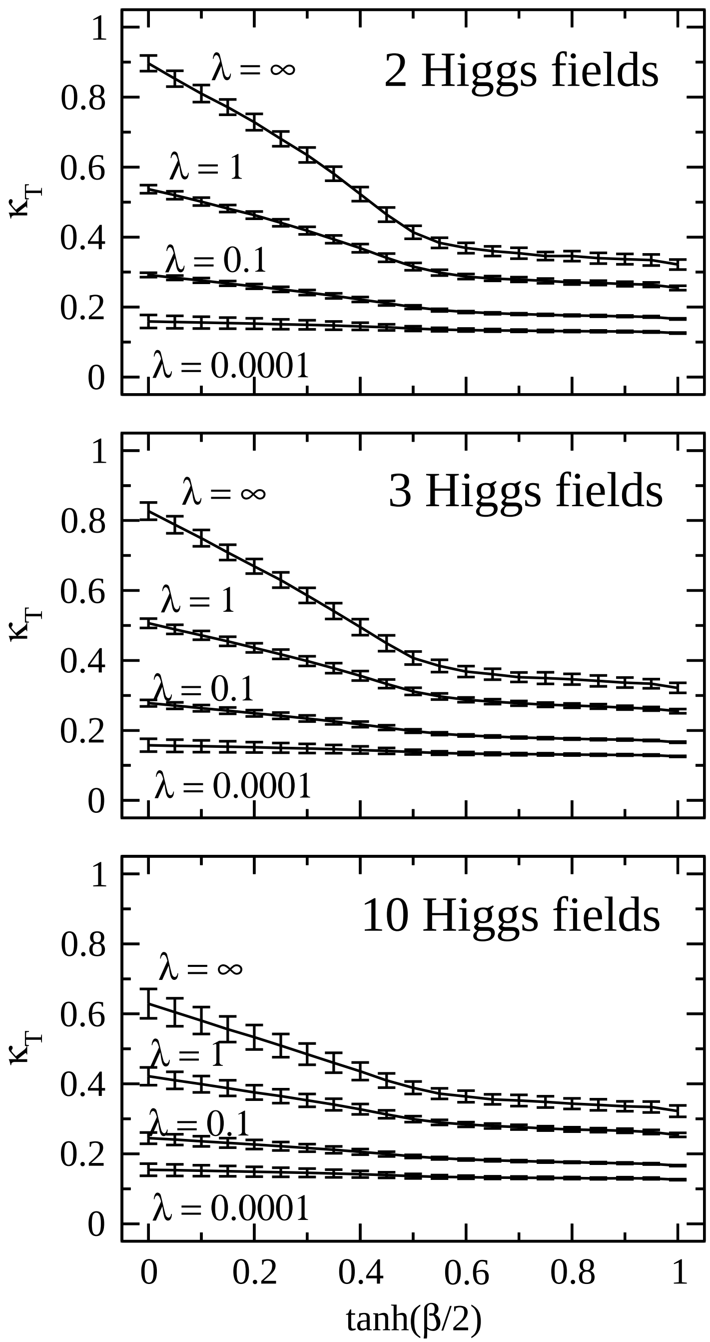
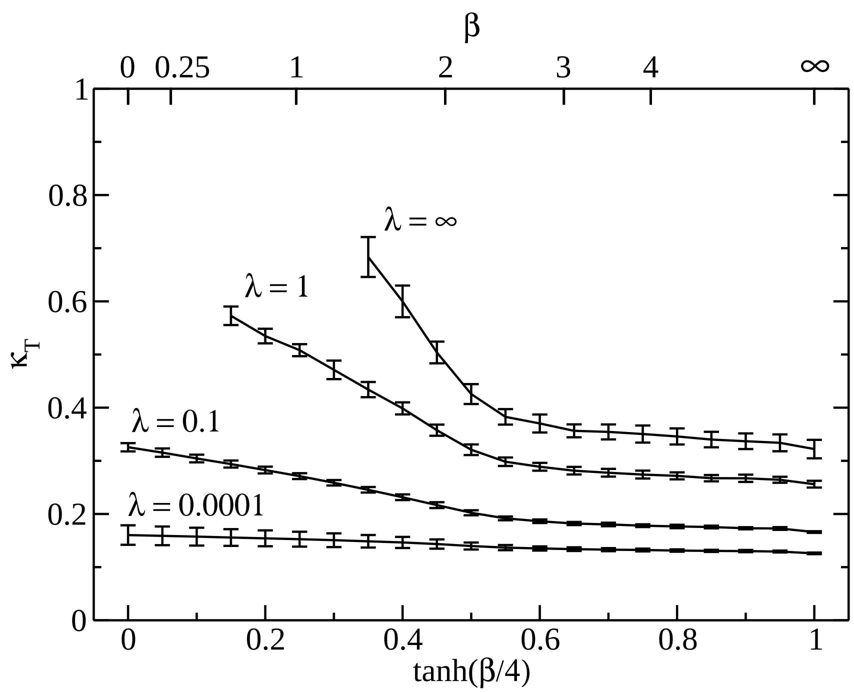


Wurtz, Lewis, Steele, Phys. Rev. D79, 074501 (2009).



Notice the effect of additional scalar doublets.

(These are identical doublets interacting only through gauge couplings.)



Wurtz, Lewis, Steele, Phys. Rev. D79, 074501 (2009).

Two-doublet models have been studied extensively by many authors
using perturbative methods (continuum).

We want to understand the symmetries and symmetry breakings
from a nonperturbative method (lattice).

We consider an action that maintains a discrete symmetry for each scalar doublet:

$$
\begin{aligned}
S = & \frac{\beta}{2}\sum_x\sum_{\mu=1}^{4}\sum_{\nu=1}^{4}\left[1-\frac{1}{2}\mathrm{ReTr}U_\mu(x)U_\nu(x+\mu)U_\mu^\dagger(x+\nu)U_\nu^\dagger(x)\right] \\
& -\sum_x\sum_{n=1}^{2}\sum_{\mu=1}^{4}\kappa_n\left[\Phi_n^\dagger(x)U_\mu(x)\Phi_n(x+\mu)+\mathrm{h.c.}\right] \\
& +\sum_x\sum_{n=1}^{2}\left[\Phi_n^\dagger(x)\Phi_n(x)+\lambda_n\left(\Phi_n^\dagger(x)\Phi_n(x)-1\right)^2\right] \\
& +\sum_x\left[\xi\Phi_1^\dagger(x)\Phi_1(x)\Phi_2^\dagger(x)\Phi_2(x)+\zeta\left|\Phi_1^\dagger(x)\Phi_2(x)\right|^2\right]
\end{aligned}
$$

It is sometimes useful to change from doublet notation, $\Phi_n(x)$, to matrix notation,

$$
\varphi_n(x) = \begin{pmatrix} i\tau_2\Phi_n^*(x) & \Phi_n(x) \end{pmatrix}
$$

Heatbath algorithms, with additional accept-reject steps where necessary, are used for gauge updates and for scalar updates.

SU(2) gauge invariance:

$$\Phi_1(x) \to R(x)\Phi_1(x), \quad \Phi_2(x) \to R(x)\Phi_2(x), \qquad \text{and} \quad U_\mu(x) \to R(x)U_\mu(x)R^\dagger(x+\mu)$$

SU(2) global symmetry for scalar doublet #1:

$$\varphi_1(x) \to \varphi_1(x)R_{(1)}$$

SU(2) global symmetry for scalar doublet #2:

$$\varphi_2(x) \to \varphi_2(x)R_{(2)}$$

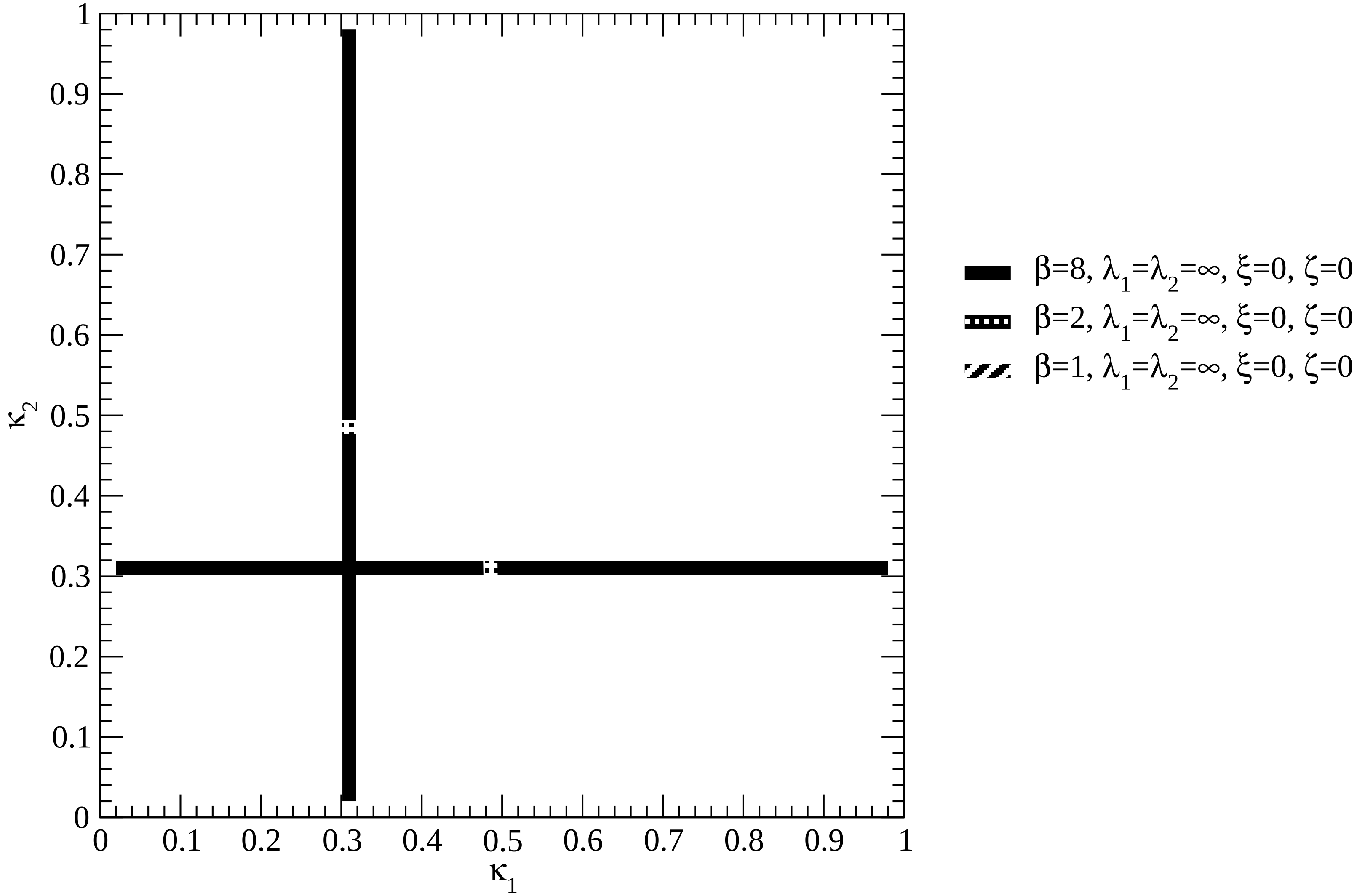
In the special case where *the action and all constraints* are functions of $\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2$ rather than being functions of $\Phi_1^\dagger\Phi_1$ and $\Phi_2^\dagger\Phi_2$ separately, we also have a global U(2) symmetry,

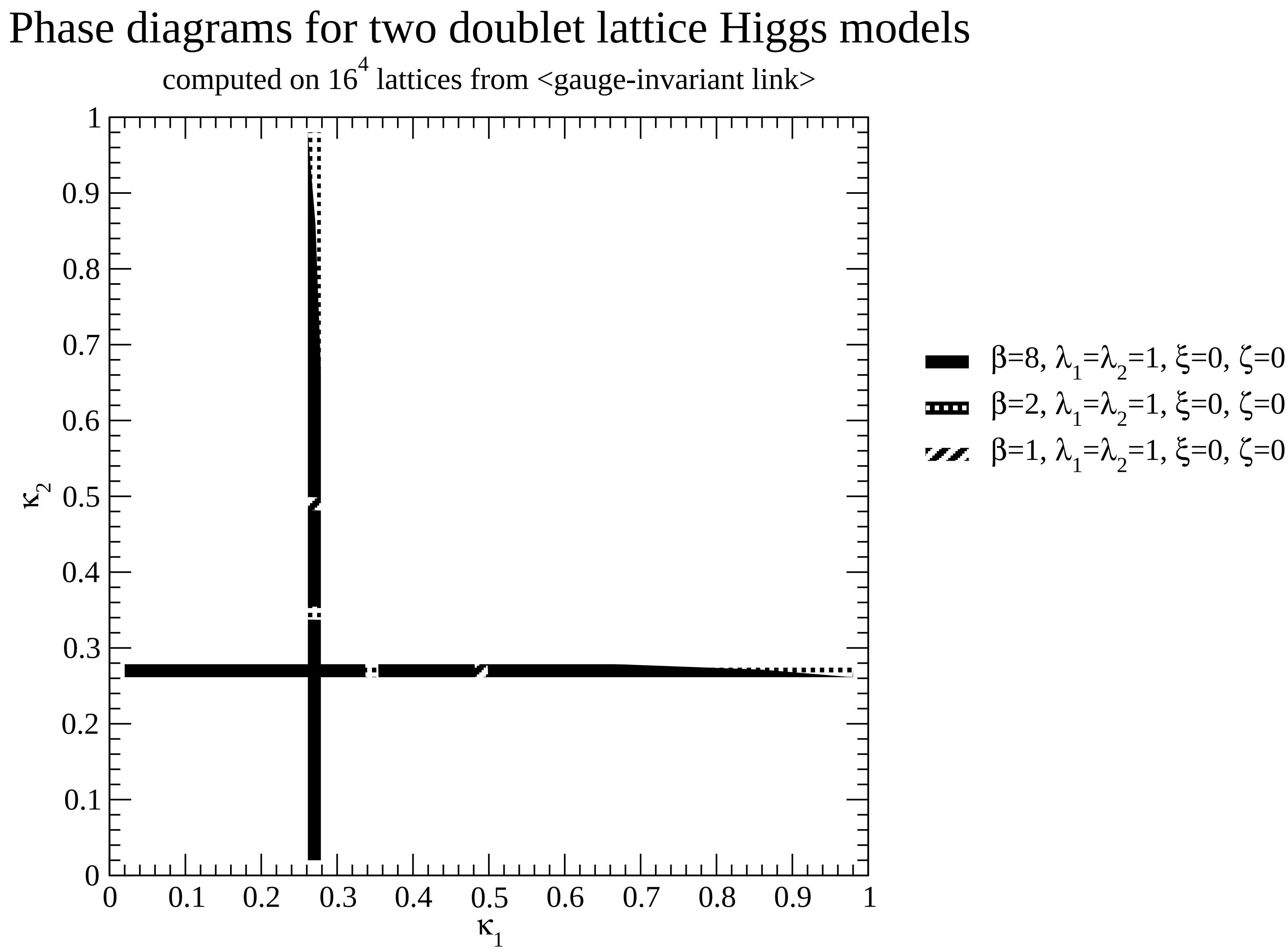
$$\begin{pmatrix} \Phi_1(x) \\ \Phi_2(x) \end{pmatrix} \to R_{\text{flavor}} \begin{pmatrix} \Phi_1(x) \\ \Phi_2(x) \end{pmatrix}$$

but this is partially redundant. Only a U(1)×U(1) global symmetry is new.

Phase diagrams for two doublet lattice Higgs models

computed on 16^4 lattices from <gauge-invariant link>

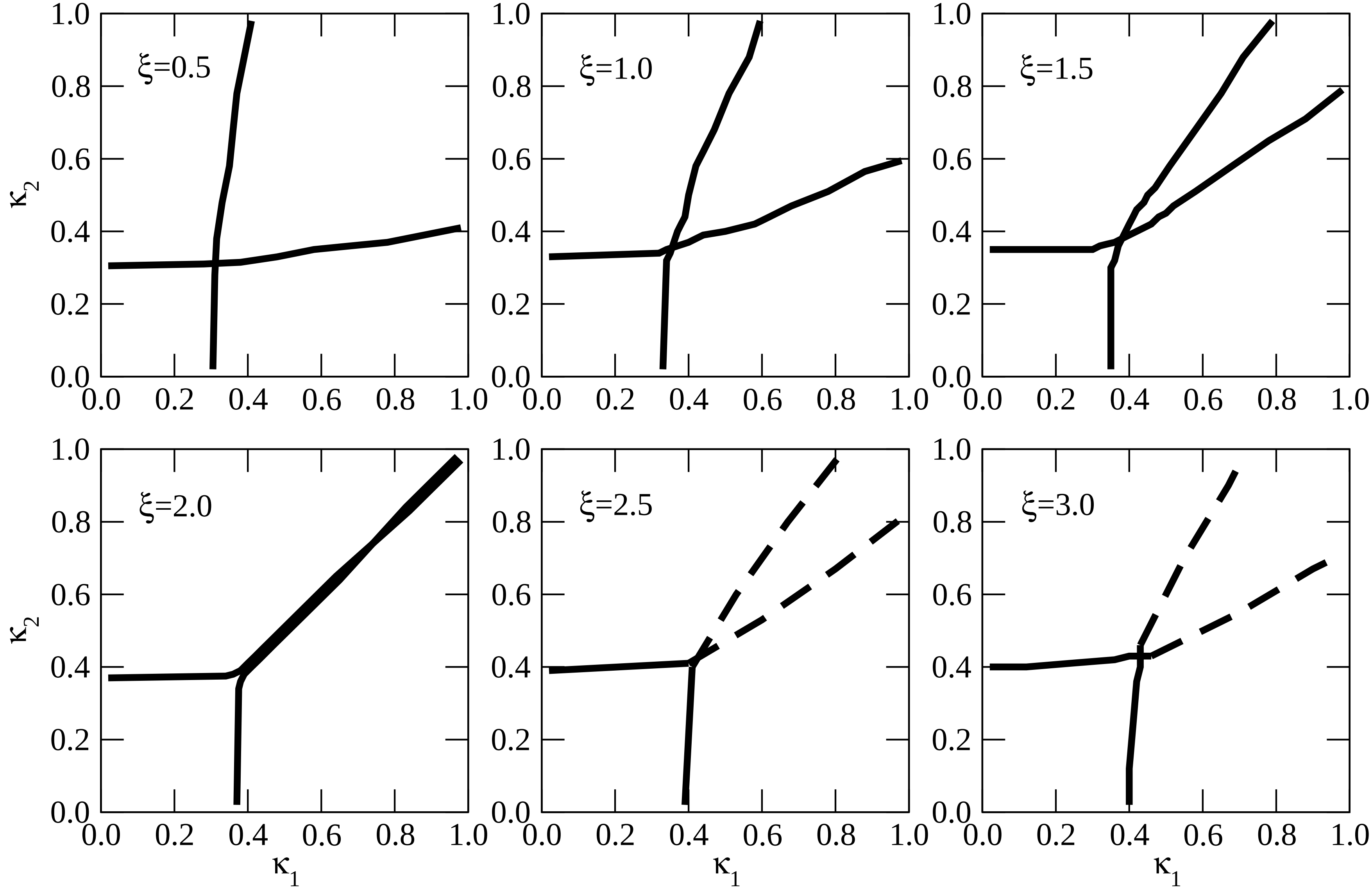




Phase diagrams for two doublet lattice Higgs models
computed on 16^4 lattices from <gauge-invariant link>
κ_2
κ_1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
$\beta=8,\ \lambda_1=\lambda_2=1,\ \xi=0,\ \zeta=0$
$\beta=2,\ \lambda_1=\lambda_2=1,\ \xi=0,\ \zeta=0$
$\beta=1,\ \lambda_1=\lambda_2=1,\ \xi=0,\ \zeta=0$

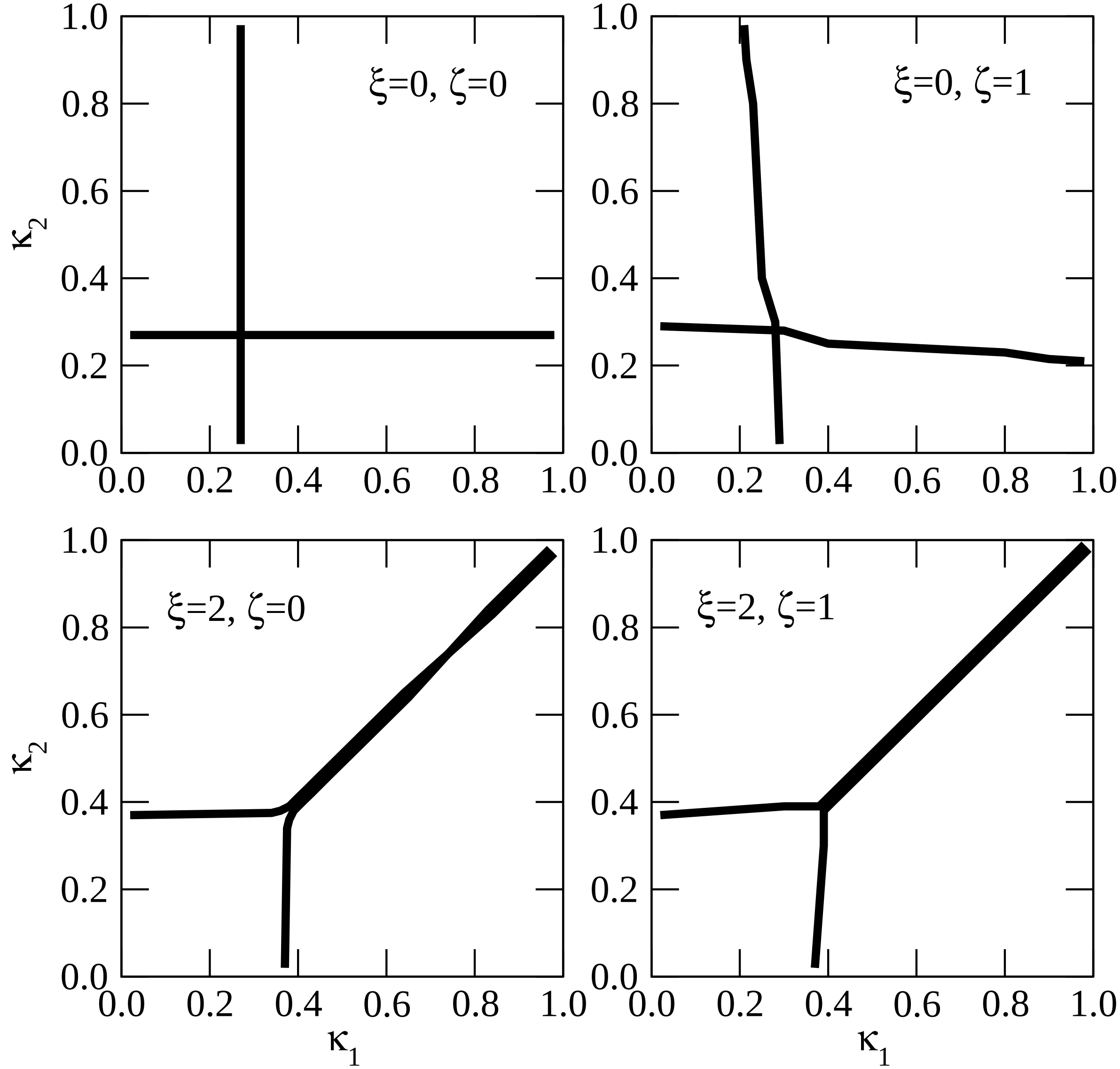
Phase diagrams for two doublet lattice Higgs models

computed on 16^4 lattices with β=8 and λ_1=λ_2=1 and ζ=0, using <gauge-invariant link>



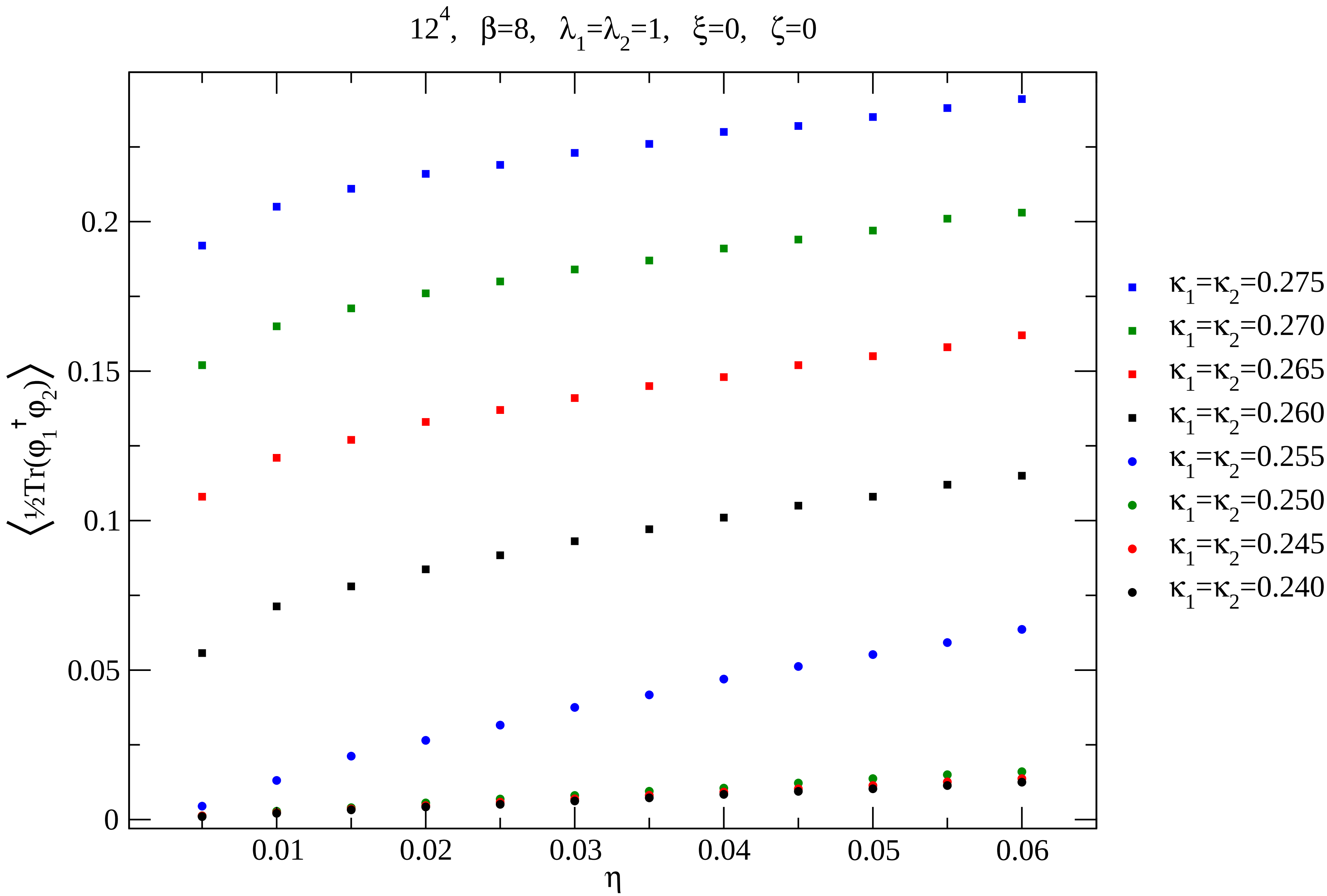
Phase diagrams for two doublet lattice Higgs models

computed on 16^4 lattices with β=8 and λ_1=λ_2=1, using <gauge-invariant link>

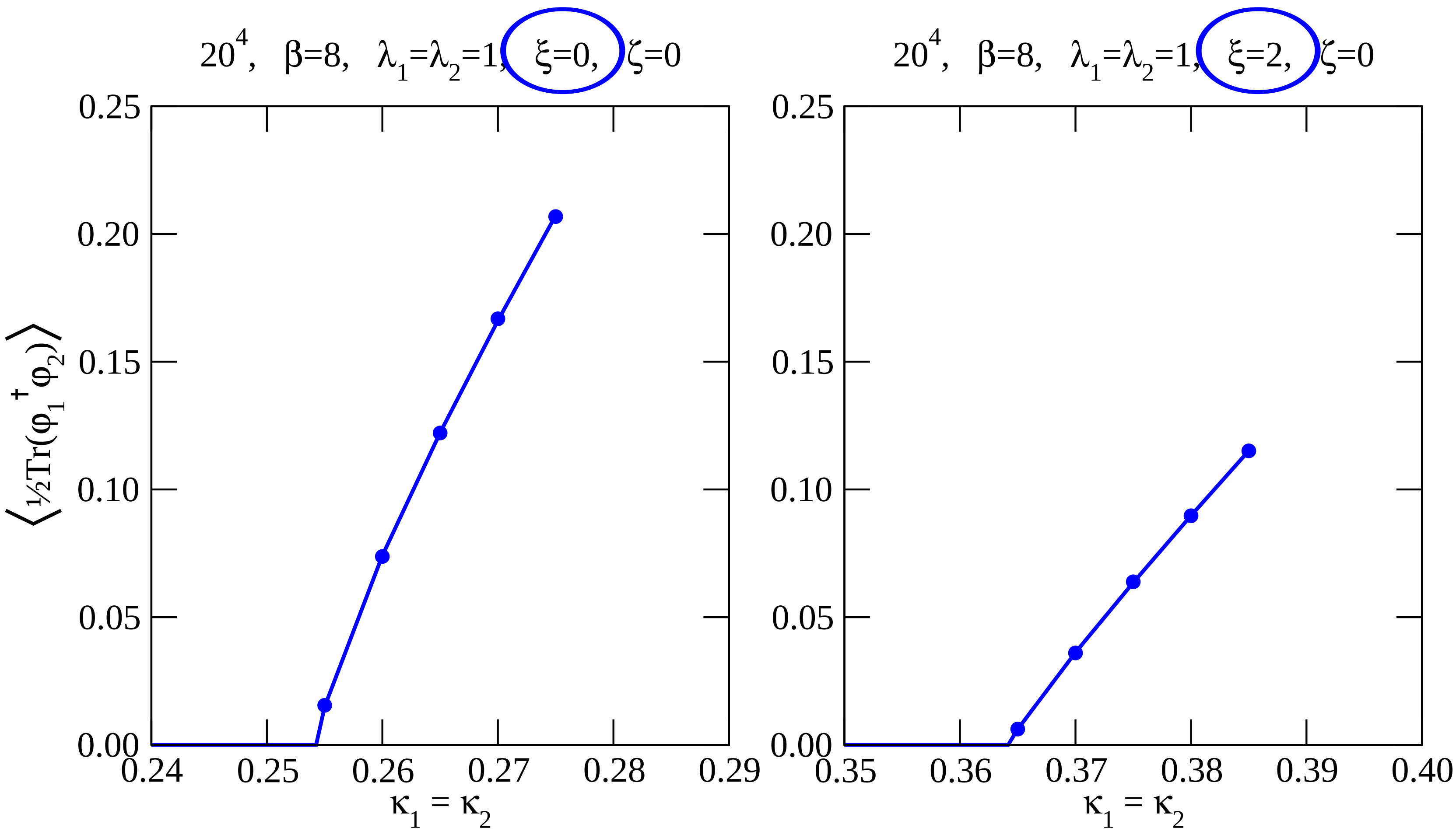


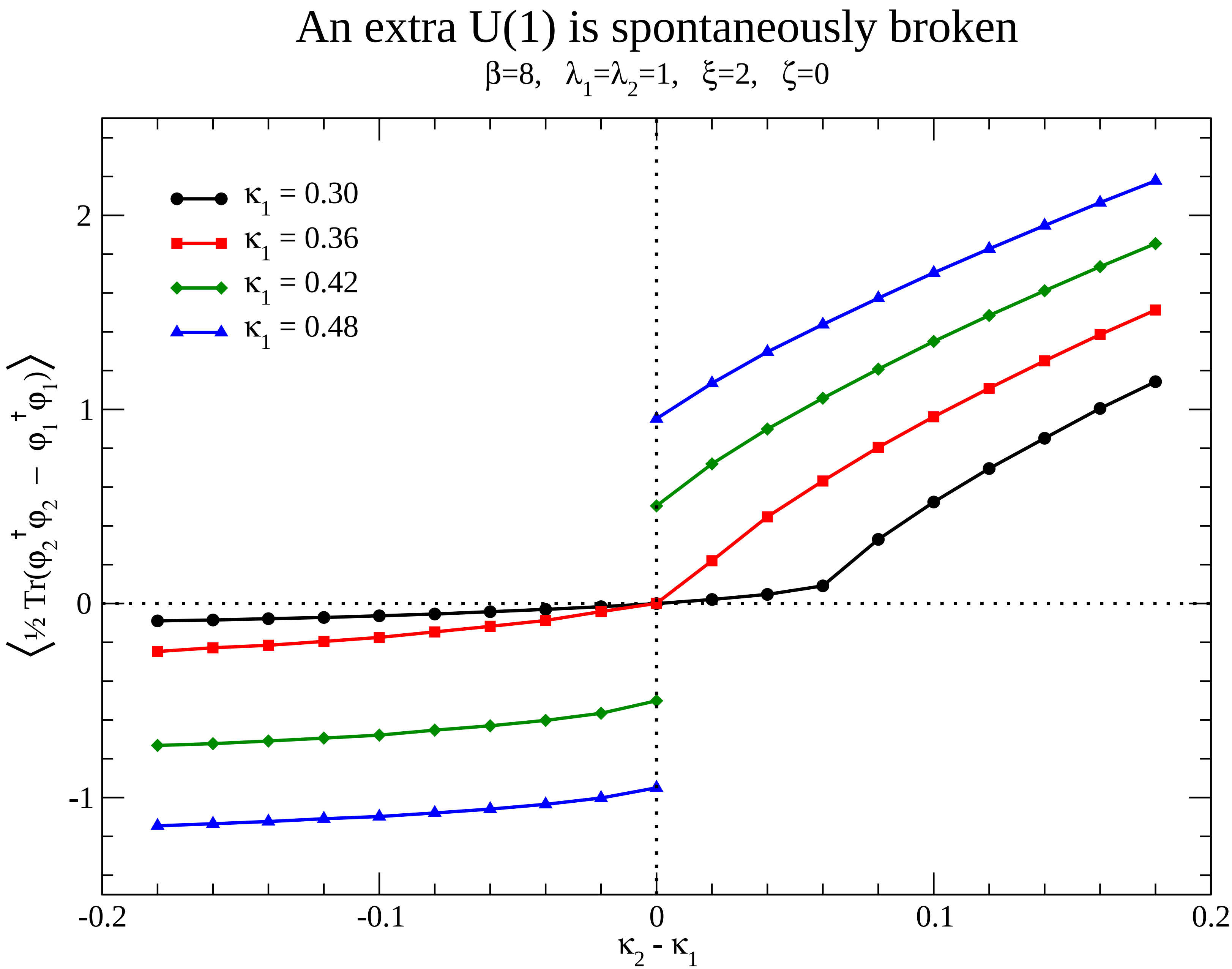
To test for spontaneous symmetry breaking of SU(2)×SU(2), add a small explicit breaking term:

$$\delta S = \frac{\eta}{2}\mathrm{Tr}\left[\varphi_1^\dagger(x)\varphi_2(x)\right]$$



An order parameter after extrapolation to $\eta = 0$.





An extra U(1) is spontaneously broken
β=8, λ1=λ2=1, ξ=2, ζ=0
κ1 = 0.30
κ1 = 0.36
κ1 = 0.42
κ1 = 0.48
⟨½ Tr(φ2†φ2 − φ1†φ1)⟩
κ2 - κ1
-0.2
-0.1
0
0.1
0.2
2
1
0
-1

summary

We have mapped out the phase diagram of a two-doublet Higgs model through lattice simulations.

The simulations provide evidence of spontaneously broken global symmetries (in contrast to the one-doublet Higgs model).

The lattice formulation is gauge invariant, and the gauge symmetry is not broken in this formulation (which is also true in the one-doublet lattice Higgs model).