Chiral Symmetry Breaking in Nearly Conformal Gauge Theories

Julius Kuti

Boston University Workshop
November 6-7, 2009

With Lattice Higgs Collaboration members:
Z. Fodor, K. Holland, D. Nogradi, C. Schroeder

Outline

1. Overview of three coordinated projects of our LHC program
   - SU(3) color, fundamental rep, staggered Nf=4-20
   - 2-index symmetric representation with SU(3) color (not discussed)
   - Running coupling (talk by Kieran Holland)
2. Chiral symmetry breaking
   - Finite volume p-regime, delta-regime, epsilon-regime
   - Goldstone spectra and staggered CHPT
   - New results at Nf=4,8,9,12 will be presented
3. Inside and above the conformal window
   - Zero momentum dynamics at Nf=16,20
4. Conclusions and Outlook
   - Prospects for model building ?
   - Can lattice studies be transformational ?
   - Is peta-scale to exa scale power needed for ?

Talk is base on the published results:

1. **Topology and higher dimensional representations.**
   Published in **JHEP 0908:084,2009**.
   e-Print: **arXiv:0905.3586** [hep-lat]

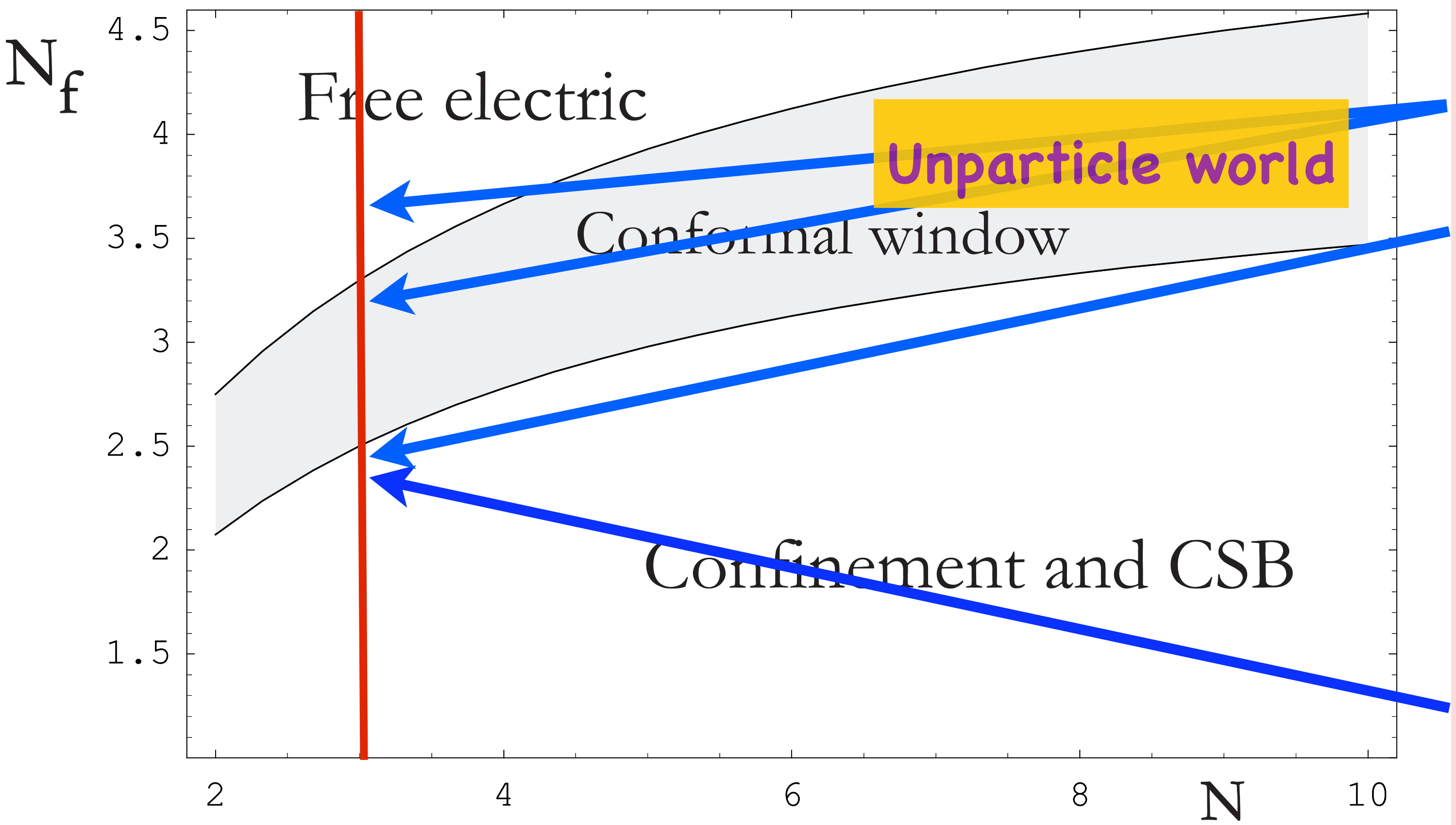
2. **Nearly conformal gauge theories in finite volume.**
   Phys.Lett.B681:353-361,2009.
   e-Print: **arXiv:0907.4562** [hep-lat]

3. **Chiral properties of SU(3) sextet fermions**
   e-Print: **arXiv:0908.2466** [hep-lat]

4. **Chiral symmetry breaking in nearly conformal gauge theories**
   e-Print: **arXiv:0911.xxx** [hep-lat] **to be posted next week**

and some unpublished analysis

Phase diagram of TWO projects as nearly conformal gauge theories in flavor-color space ?



Project 1:

Fundamental rep Nf=4,8-12,14,16,20 flavors and three colors with staggered fermions.

Project 2:

2-index symmetric rep with Nf=2 flavors and three colors with overlap chiral fermions → staggered

(will not be discussed here, but quenched results are published)

Our unified GPU/MPI code ready

near the conformal window (walking):

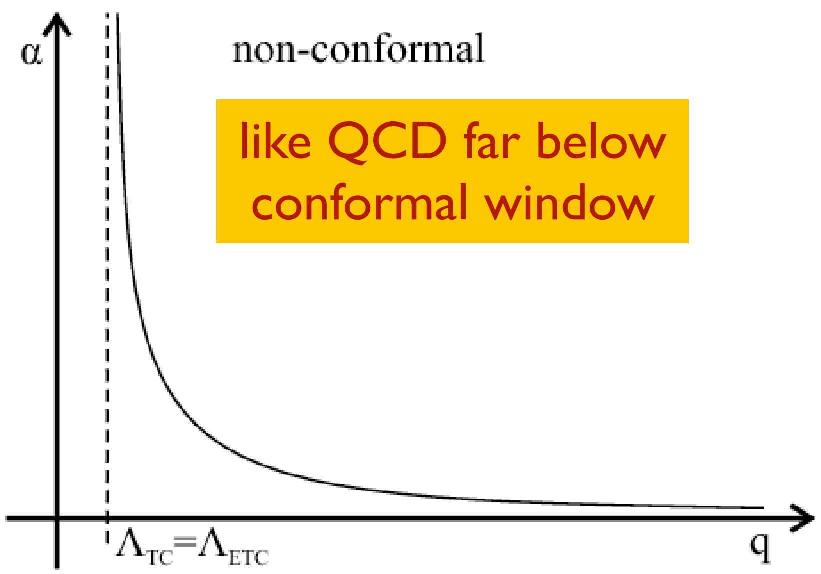
Higgs phenomenology with nearly vanishing beta function

Phenomenology goal: nearly conformal gauge theory with minimal realization of the composite Higgs mechanism consistent with ElectroWeak Precision Data ?

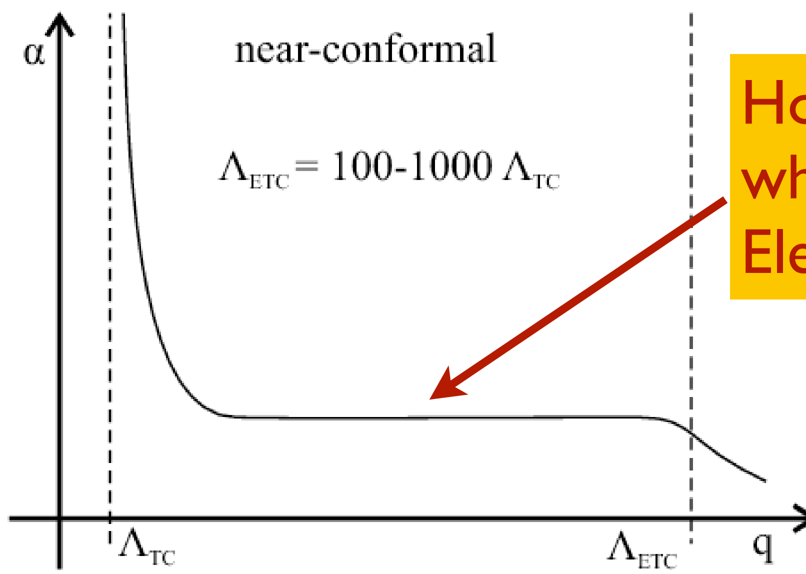
They are fun lattice field theories anyway!

Project 3: Important to complement the search for chirally broken phase with running coupling and beta function

Talk by Kieran Holland



like QCD far below conformal window



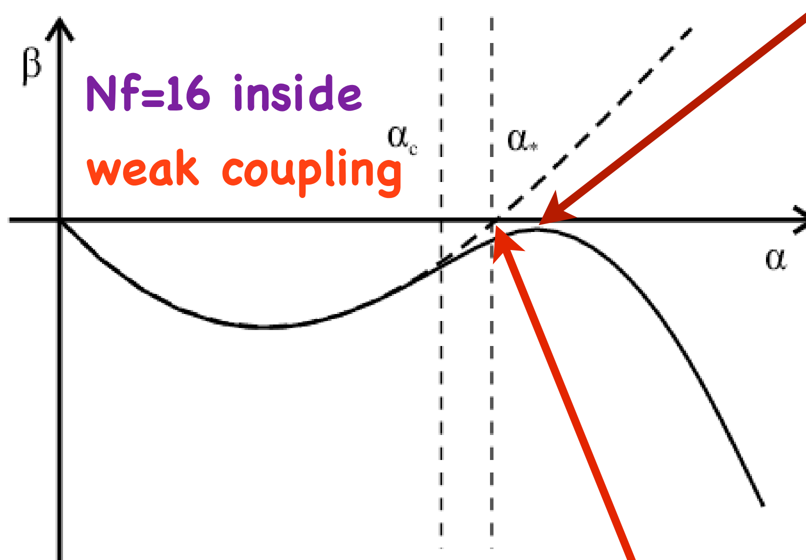
How to reach walking scale which is required by ElectroWeak Precision Tests?

Is 2-index symmetric rep nearly conformal?

Fundamental rep with Nf=4,8,9 should be similar

Nf=10,11,12 under continued study

Nf=12 controversial



Nf=16 inside weak coupling

would be Banks-Zaks FP

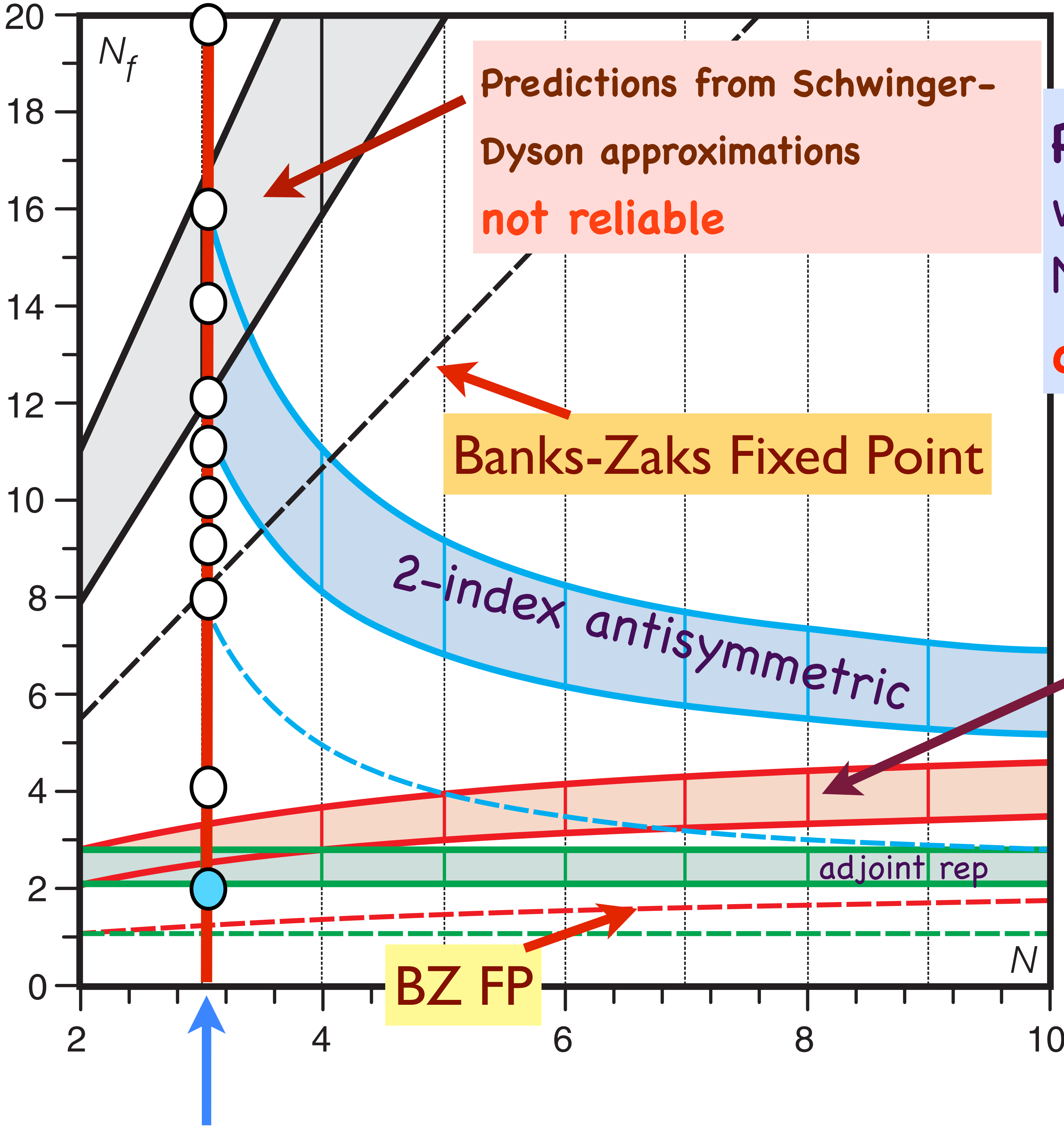
DeGrand et al. (conformal?)

our staggered code is running

testing our tool set

important in model building

Theory space and conformal windows



We only run with N=3 colors

Important early work by Bardeen, Leung, Love on Schwinger-Dyson

Project 1: in fundamental rep with N=3 colors with Nf=4,8,9,10,11,12,14,16,20 flavors

dynamical staggered

Project 2: 2-index symmetric rep (sextet)

N=3 colors and Nf=2 flavors

dynamical overlap

Running on CPU clusters and GPU clusters

Very demanding

Unified code

We are supported by the Wuppertal hardware/software infrastructure

Zoltan Fodor
Kalman Szabo
Sandor Katz

GPU Hardware

CUDA code:
Kalman Szabo
Sandor Katz

GTX 280
Flops: single 1 Tflop, double 80 Gflops
Memory 1GB, Bandwidth 141 GBs^{-1}
230 Watts, \$350

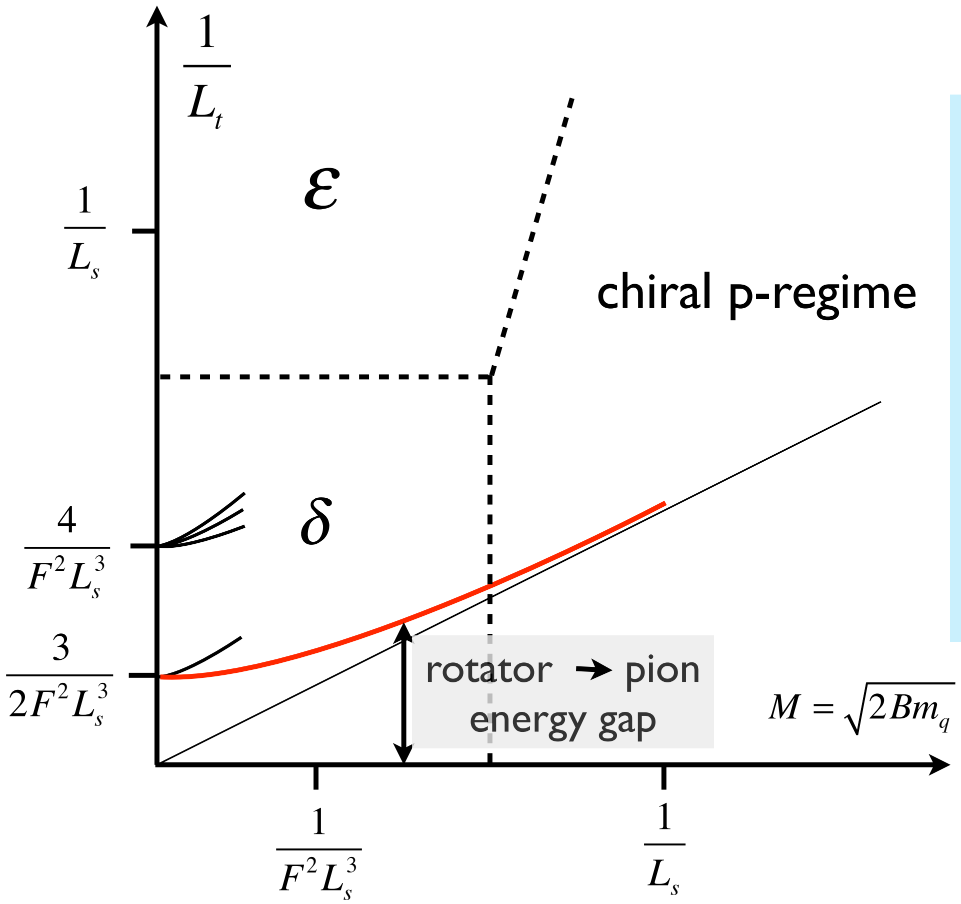
also USQCD CPU
cluster support

UCSD Tesla cluster
ARRA funded by DOE
waiting for Fermi cards

Tesla 1060
Flops: single 1 Tflop, double 80 Gflops
Memory 4GB, Bandwidth 102 GBs^{-1}
230 Watts, \$1200

Tesla 1070
Flops: single 4 Tflops, double 320 Gflops
Memory 16GB, Bandwidth 408 GBs^{-1}
900 Watts, \$8000

Chiral regimes to identify in theory space:



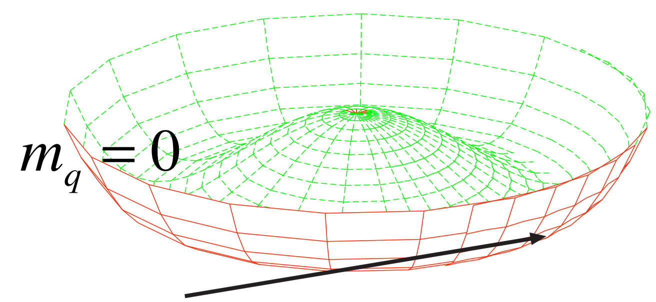
Goldstone dynamics is different in each regime

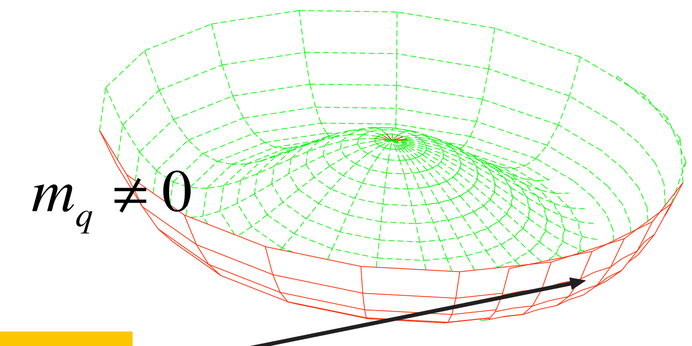
We study δ and ε -regimes (RMT)

and p-regime (probing chiral loops)

complement each other

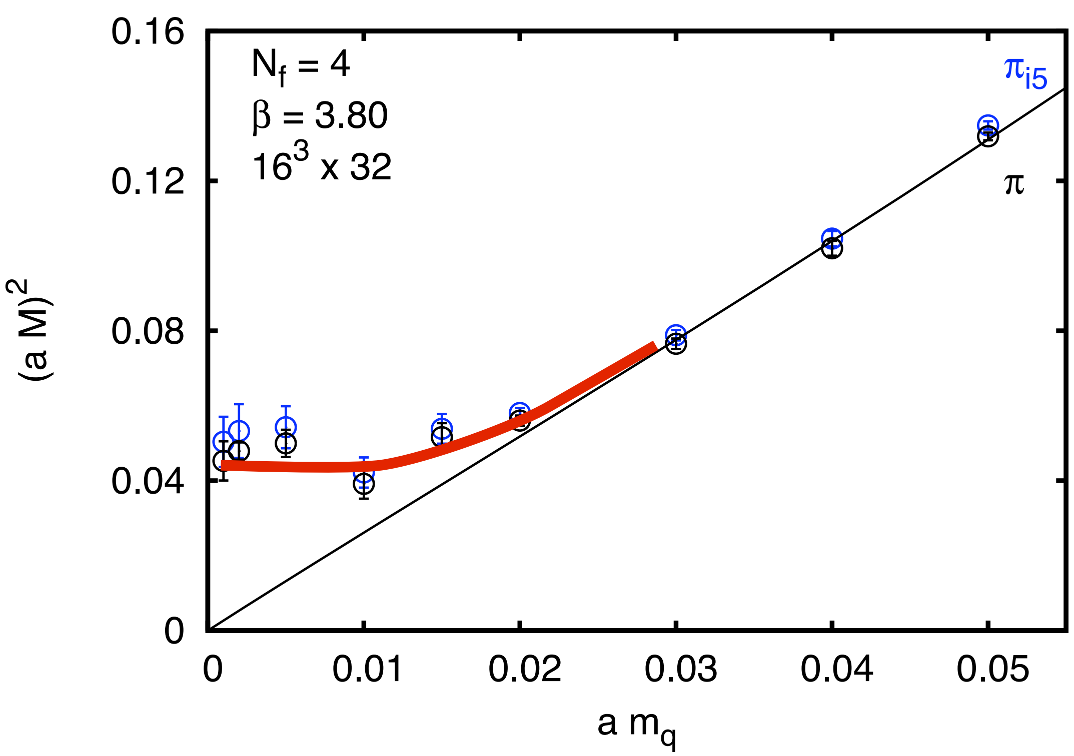
interpretation of rotator levels in $m_q \to 0$ limit:

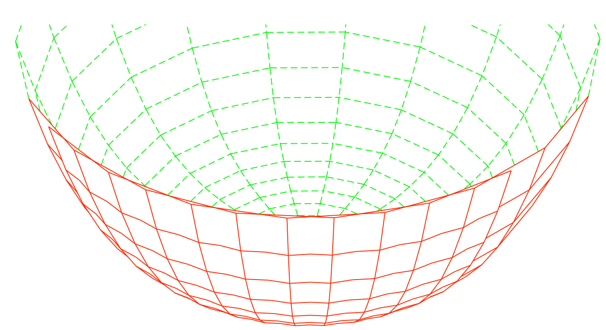




Veff: chiral condensate in flavor space
arbitrary orientation of condensate

tilted condensate



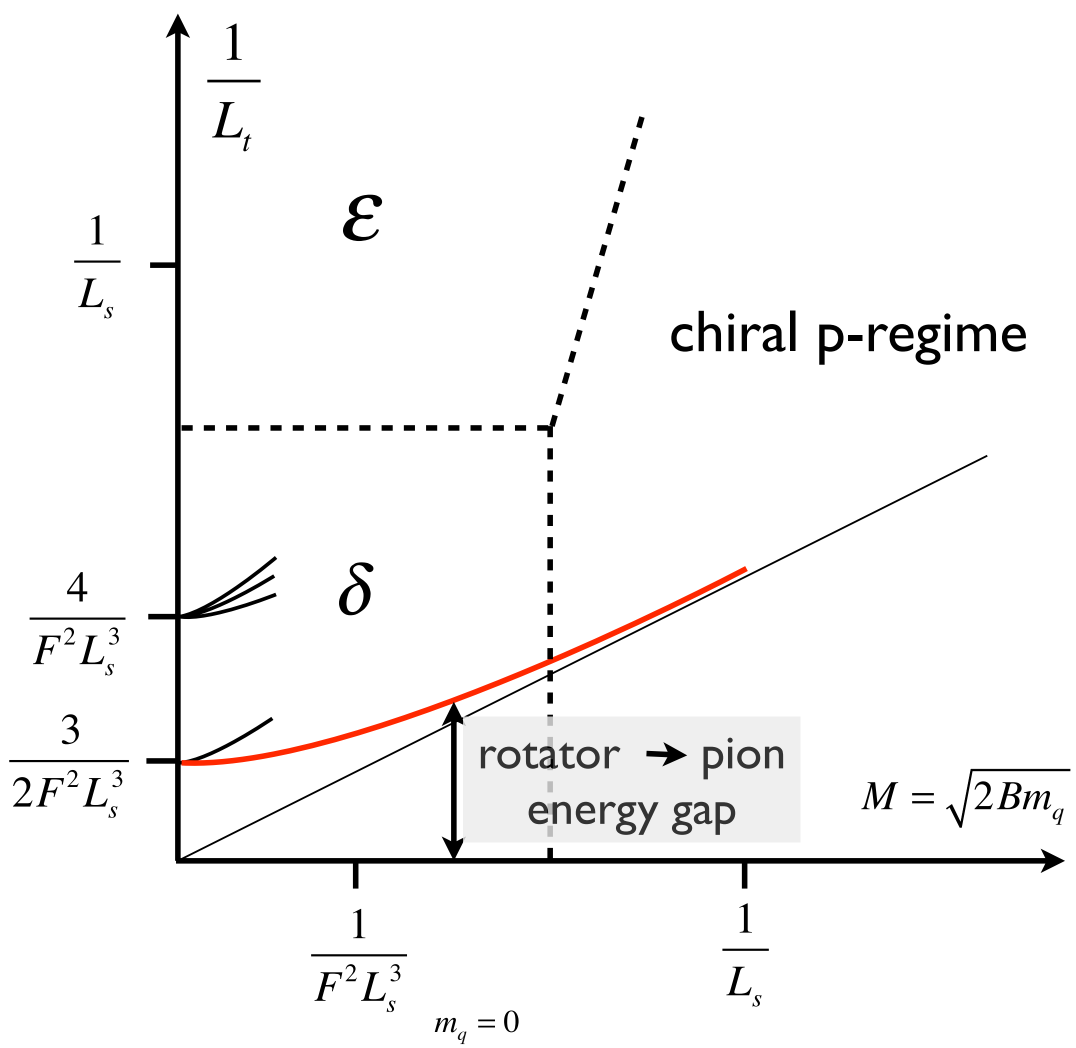


Not to misidentify rotator gaps
as evidence of chirally symmetric
phase !!

One-loop expansion in our analysis of p-regime:

Leutwyer, Gasser, P. Hasenfratz,

Niedermayer, Hansen, Neuberger, ...



$$M_\pi^2 = M^2\left[1 - \frac{M^2}{8\pi^2 N_f F^2} ln\left(\frac{\Lambda_3}{M}\right)\right],$$

$$F_\pi = F\left[1 + \frac{N_f M^2}{16\pi^2 F^2} ln\left(\frac{\Lambda_4}{M}\right)\right],$$

Note Nf scaling!

$$\lambda = ML_s$$

$$M_\pi(L_s,\eta) = M_\pi\left[1 + \frac{1}{2N_f}\frac{M^2}{16\pi^2F^2}\cdot\widetilde{g}_1(\lambda,\eta)\right],$$

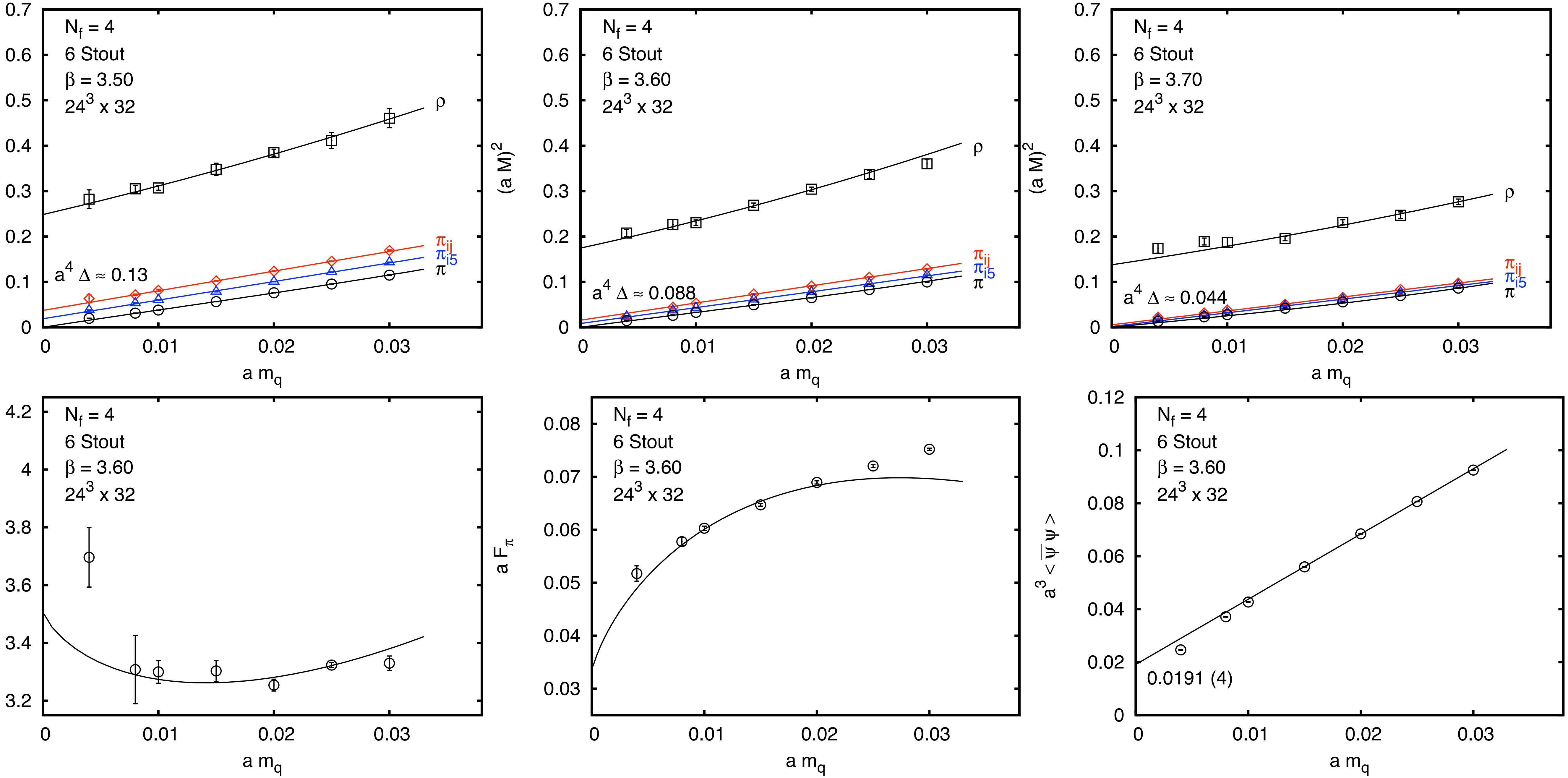
$$F_\pi(L_s,\eta) = F_\pi\left[1 - \frac{N_f}{2}\frac{M^2}{16\pi^2F^2}\cdot\widetilde{g}_1(\lambda,\eta)\right],$$

We use staggered action with stout smearing

Taste breaking included in staggered perturbation theory!

structure changing as Nf grows

Nf=4 NLO chiral analysis in p-regime:

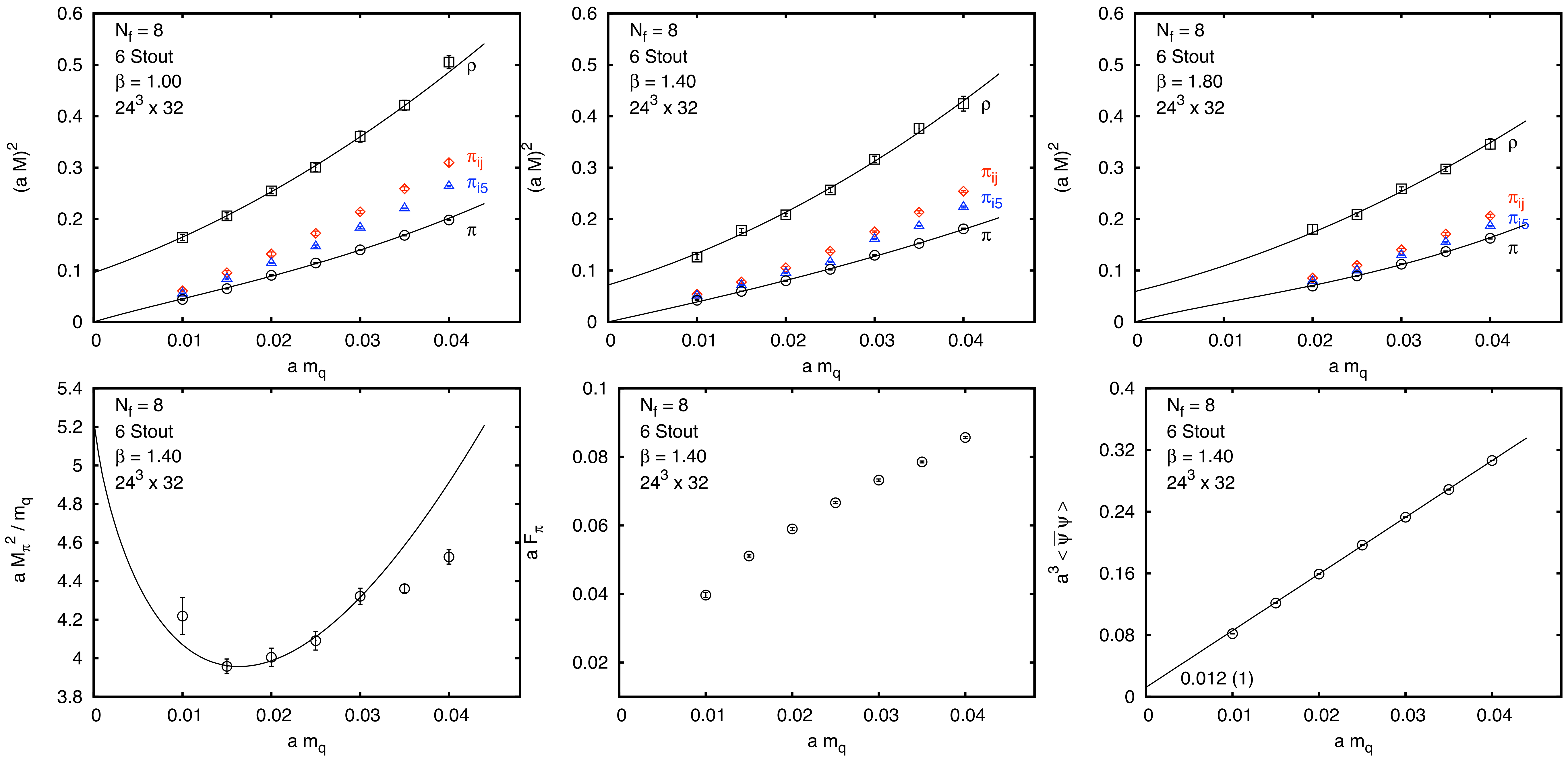


$$\mathcal{L}_\chi^{(4)} = \frac{F^2}{4}\mathrm{Tr}(\partial_\mu\Sigma\partial_\mu\Sigma^\dagger) - \frac{1}{2}B\,m_q\,F^2\mathrm{Tr}(\Sigma + \Sigma^\dagger) \;+ V_\chi^{(6)}$$

$$M_\pi(T_a)^2 = 2Bm_q + a^2\Delta(T_a) + O(a^2 m_q) + O(a^4)$$

$$\begin{aligned}
\Delta(\xi_5) &= 0\,,\\
\Delta(\xi_\mu) &= \frac{8}{F^2}(C_1 + C_2 + C_3 + 3C_4 + C_5 + 3C_6)\,,\\
\Delta(\xi_{\mu 5}) &= \frac{8}{F^2}(C_1 + C_2 + 3C_3 + C_4 - C_5 + 3C_6)\,,\\
\Delta(\xi_{\mu\nu}) &= \frac{8}{F^2}(2C_3 + 2C_4 + 4C_6)\,.
\end{aligned}$$

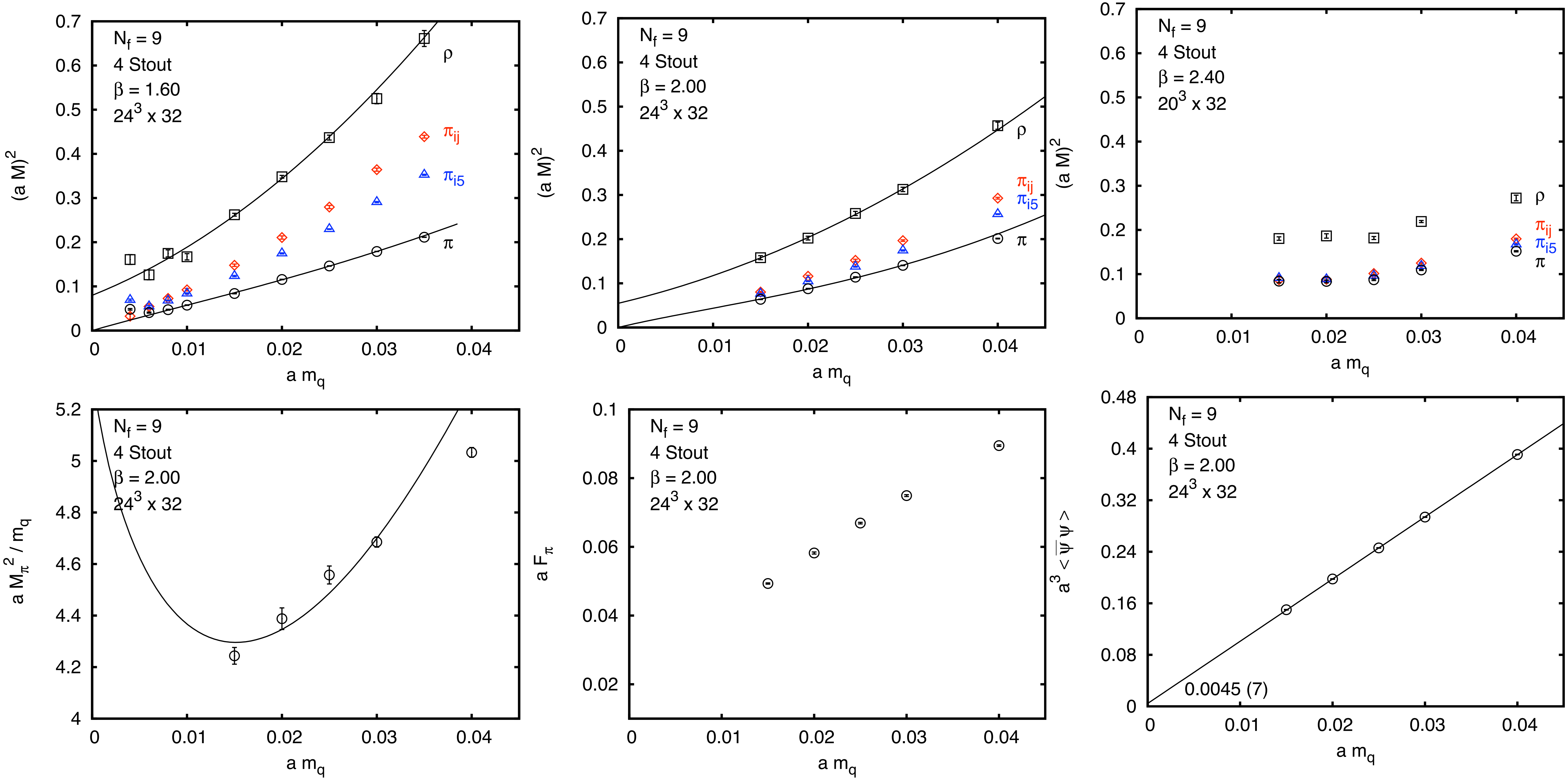
Nf=8 NLO chiral analysis in p-regime:



$$
\frac{(m_{\pi_5^+}^{1-\mathrm{loop}})^2}{2m} = \mu\Bigg\{1 + \frac{1}{16\pi^2 f^2}\Bigg(\frac{2}{N_F}\Big[\ell(m^2_{\eta'_V}) - \ell(m^2_{\pi_V})\Big] + \frac{2}{N_F}\Big[\ell(m^2_{\eta'_A}) - \ell(m^2_{\pi_A})\Big] + \frac{1}{2N_F}\ell(m^2_{\pi_I})\Bigg) + \frac{16\mu}{f^2}(2L_8 - L_5)(2m) + \frac{32\mu}{f^2}(2L_6 - L_4)(4N_F m) + a^2 C\Bigg\}
$$

Nf=8 staggered NLO

Nf=9 NLO chiral analysis in p-regime:

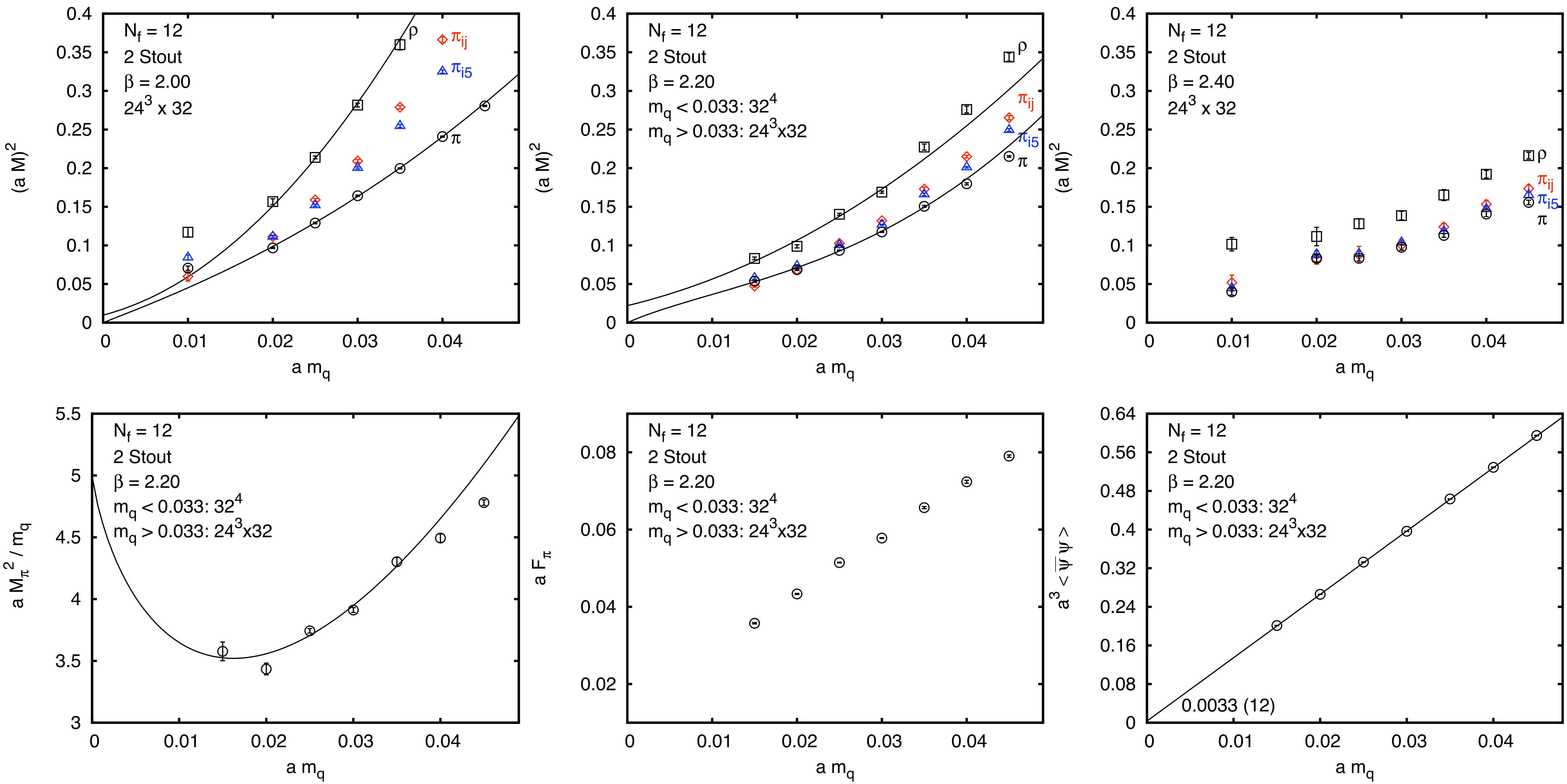


Testing rooting (nothing unusual happens)

(useful for rooted sextet code, complete and running with Nf=2)

Provides additional independent info on chiral condensate trend

Nf=12 NLO chiral analysis in p-regime:



Similar pattern to Nf=8 case!

more work is needed

Some features of Nf=4,8,9,12 runs:

Nearly degenerate Goldstone spectra

stout action performs very well

Chiral condensate measured in F unit

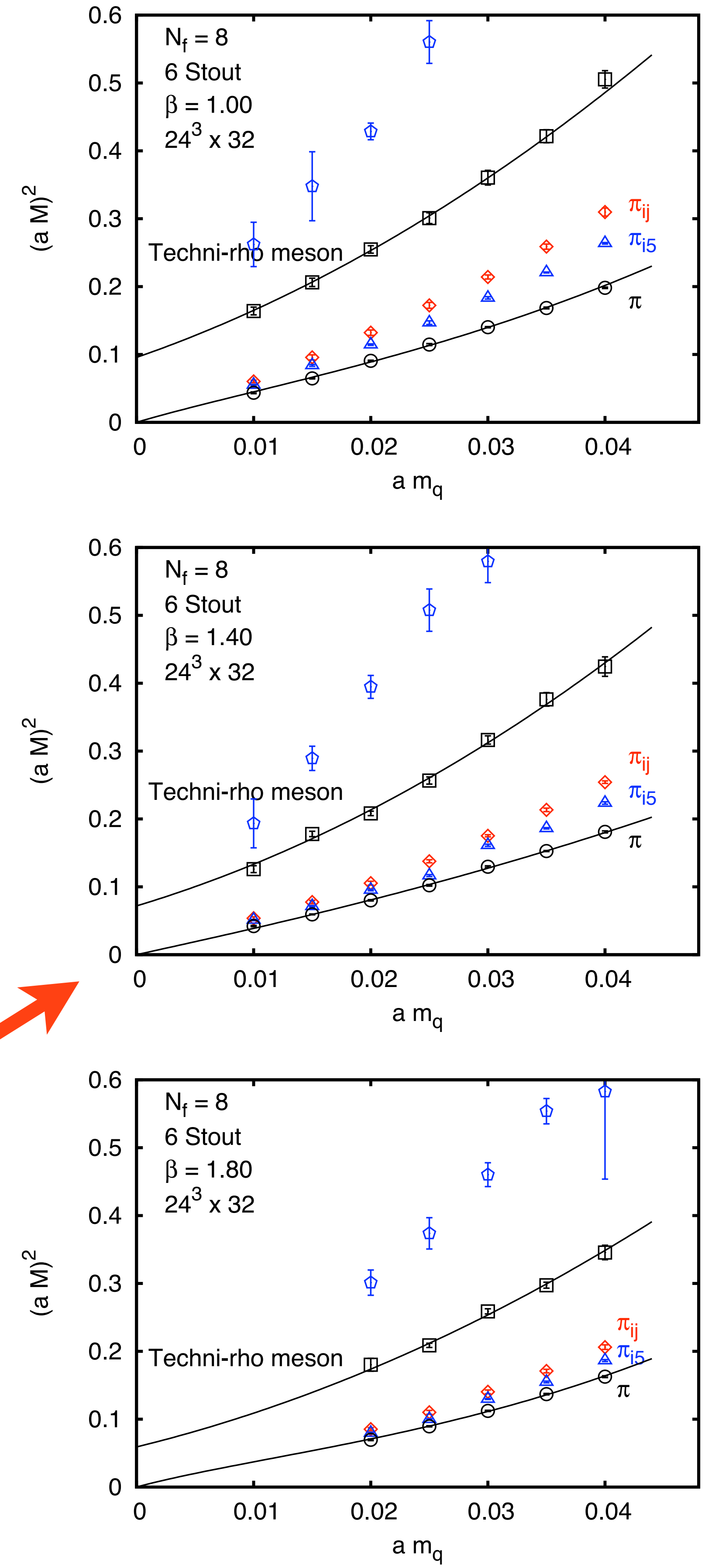
is enhanced as Nf increases $\sim \frac{<\bar{\psi}\psi>^{1/3}}{F}$

renormalization scale is not set

Nf=4	B/F = 53(6)
Nf=8	B/F = 157(17)
Nf=9	B/F = 125(19)
Nf=12	B/F = 209(64)

large errors, preliminary,limited to 32^4!

rho - A1 splitting



Better separtaion of rho and

Goldstones at Nf=12 would require

bigger runs at smaller fermion masses

Problem when F*Ls is not large enough?
This can be quantified
(delta and p regimes connected)

$$E_l = \frac{1}{2\theta} l(l+2) \text{ with } l = 0,1,2,\ldots \quad \text{rotator spectrum for SU(2)}$$

$$\text{with } \theta = F^2 L_s^3 (1 + \frac{C(N_f = 2)}{F^2 L_s^2} + O(1/F^4 L_s^4)) \quad \text{(P. Hasenfratz and F. Niedermayer)}$$

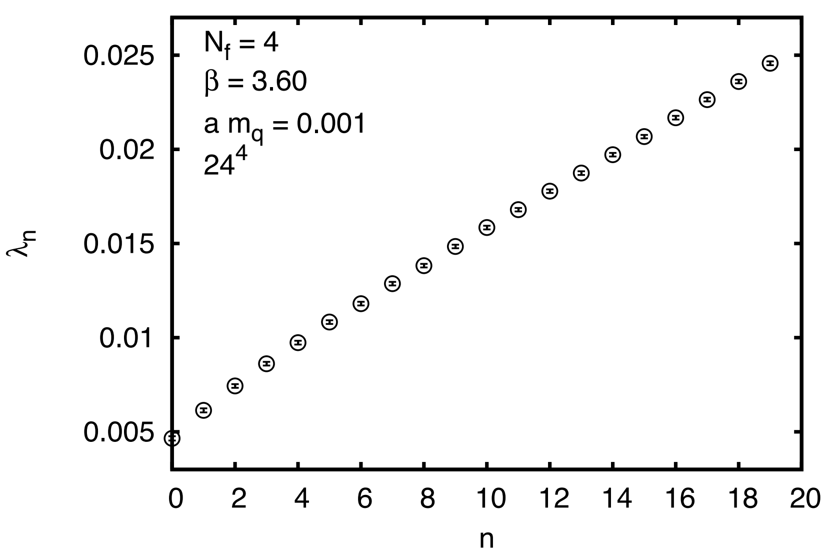
there is overall factor $\frac{N_f^2 - 1}{N_f}$ for arbitrary N_f

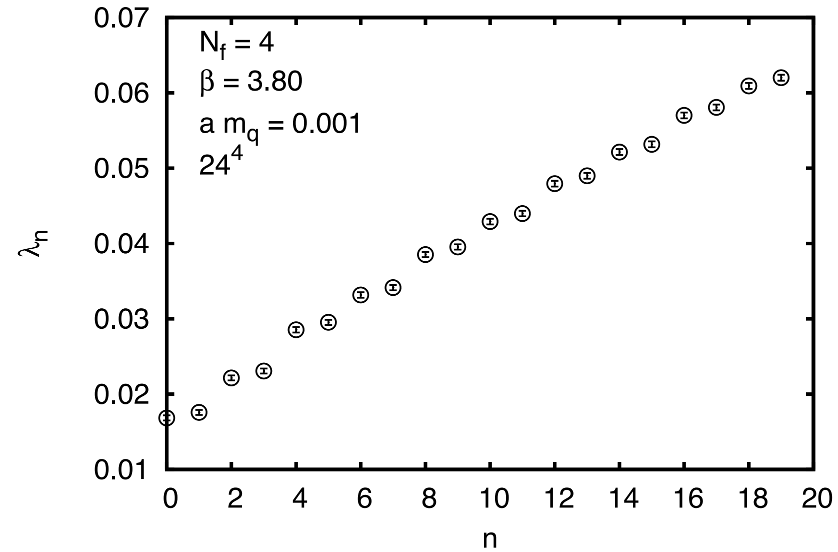
$C(N_f = 2) = 0.45$ expected to grow with N_f

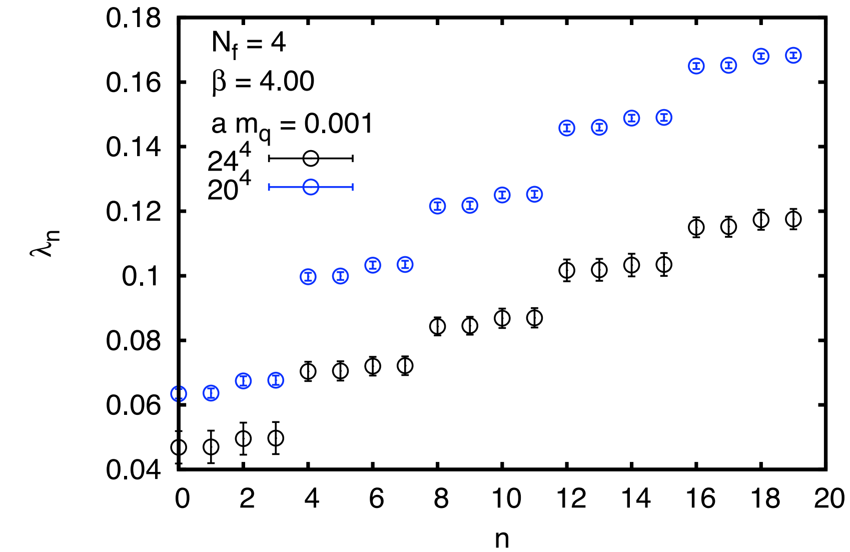
At $FL_s = 0.8$ the correction is 70% and grows with N_f

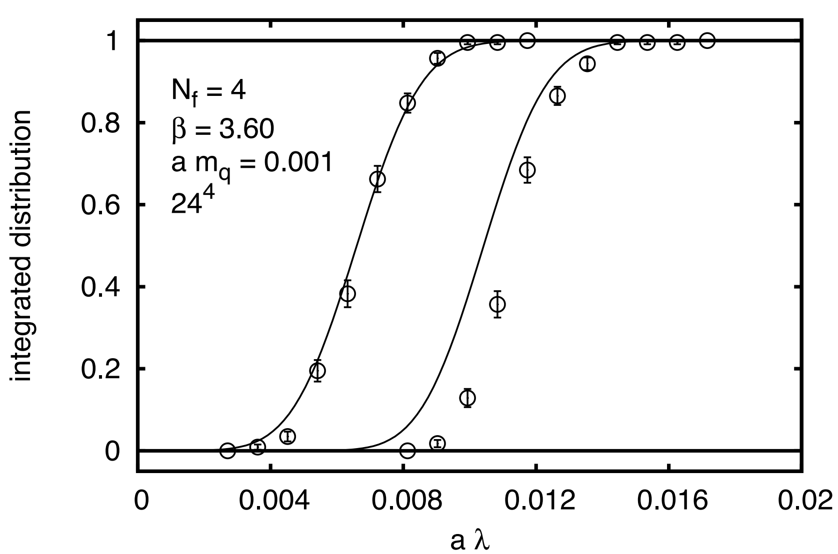
When expansion collapses in $\delta -$regime, the p-regime analysis also becomes suspect

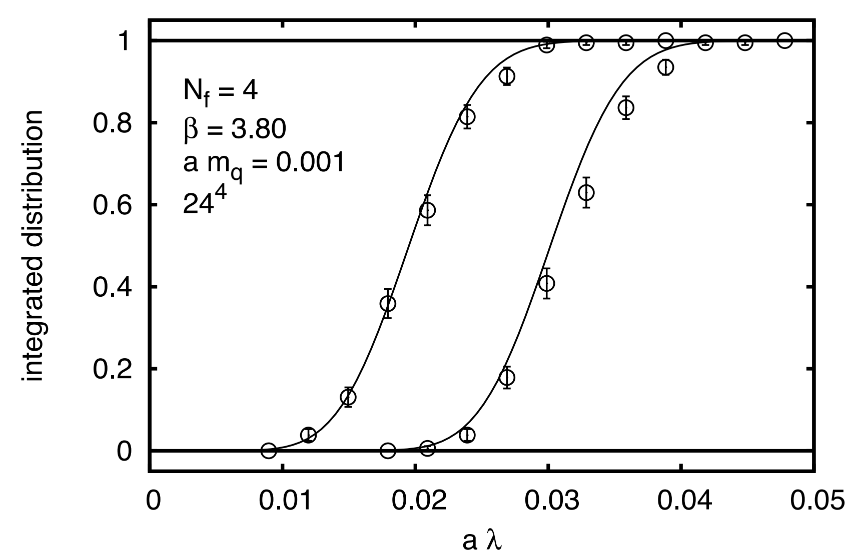
Random Matrix Theory tests in epsilon regime:

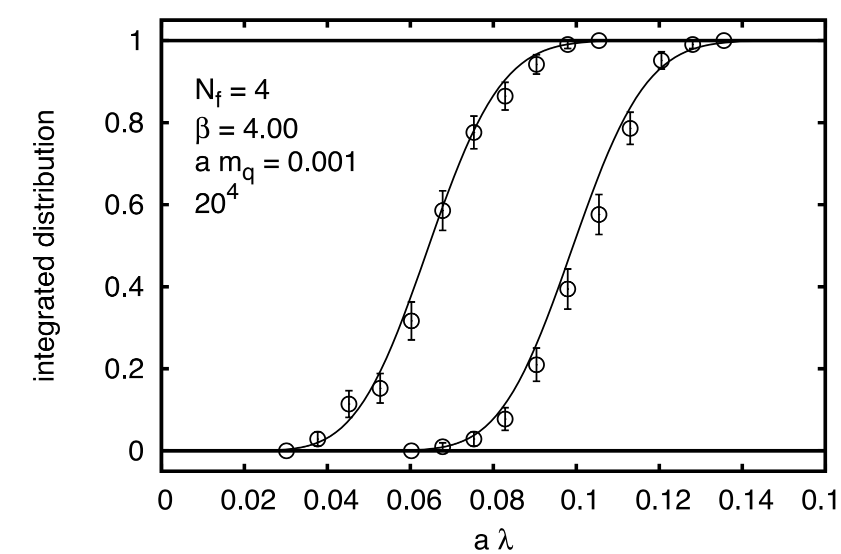


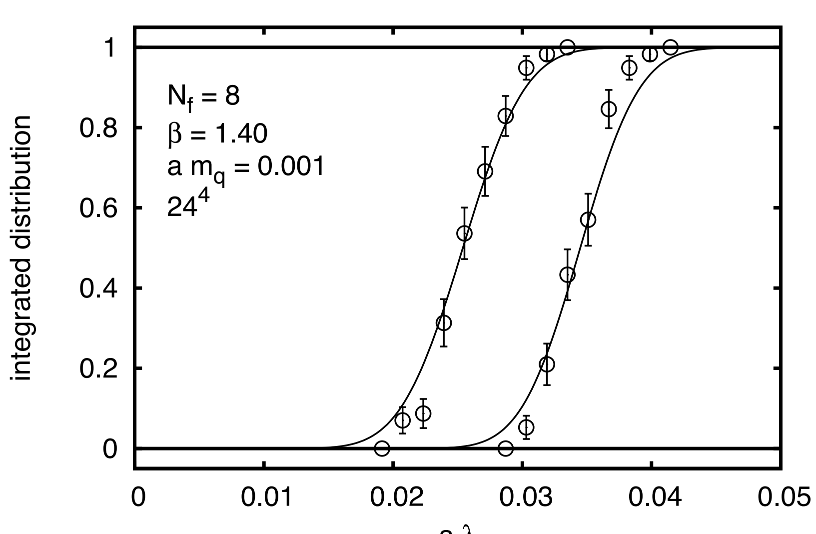












Dirac spectrum

Integrated eigenvalue distrubutions of RMT

--> quartet degeneracy

--> RMT

Inside the conformal window Nf=16 case study

Nf=16 is most accessible to analysis

What is the finite volume spectrum?

How does the running coupling $g^2(L)$ evolve with L?

From 2-loop beta function $g^{*2} \approx 0.5$

$$g^2(L) \to g^{*2}, \text{ as } L \to \infty$$

Nontrivial small volume dynamics in QCD turns into large volume dynamics around weak coupling fixed point of conformal window

At small $g^2(L)$ the zero momentum components of the gauge field dominate the dynamics: Born-Oppenheimer approximation

Originally it was applied to pure-gauge system Luscher, van Baal

SU(3) pure-gauge model: 27 inequivalent vacua

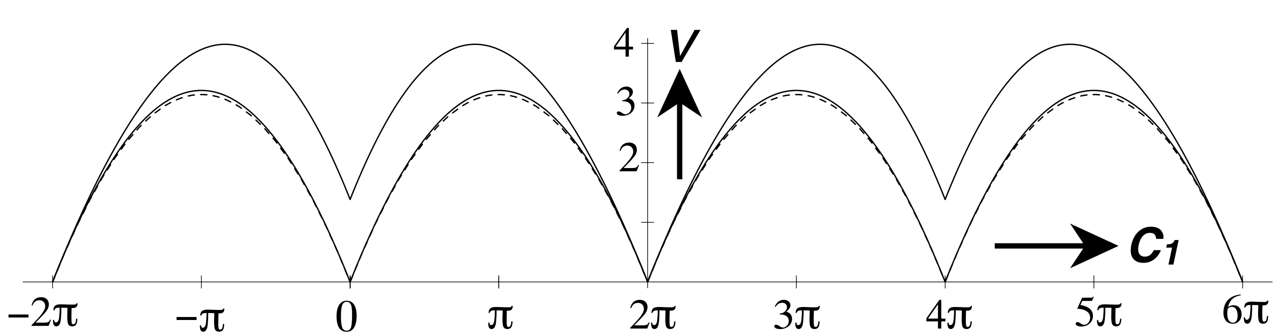
Low excitations of Hamiltonian (Transfer Matrix) scale with $\sim g^{2/3}(L)/L$
will evolve into glueball states for large L

Three scales of dynamics on smallest scale WF is localized on one vacuum
tunneling accross vacua on second scale
over the barrier: confinement scale (third)

$A_i(\mathbf{x}) = T^a C_i^a / L$ <-- zero momentum mode of gauge field

For $SU(3)$, $T_1 = \lambda_3/2$ and $T_2 = \lambda_8/2$

$$V_{\rm eff}^{\mathbf{k}}(\mathbf{C}^b) = \sum_{i>j} V(\mathbf{C}^b[\mu_b^{(i)} - \mu_b^{(j)}]) - N_f \sum_i V(\mathbf{C}^b \mu_b^{(i)} + \pi\mathbf{k}) \qquad \mu^{(1)} = (1,1,-2)/\sqrt{12} \text{ and } \mu^{(2)} = \tfrac{1}{2}(1,-1,0)$$



Effective potential shows the effects of massless fermions

Fermions develop a gap in the spectrum

~ 1/L k=(0,0,0) periodic

k=(1,1,1) antiperiodic

van Baal

Quantum vacuum is at minimum of Veff(C) when massless fermions are turned on

early work by van Baal, Kripfganz and Michael

Fermions develop a gap $\sim \pi/L$ in the spectrum

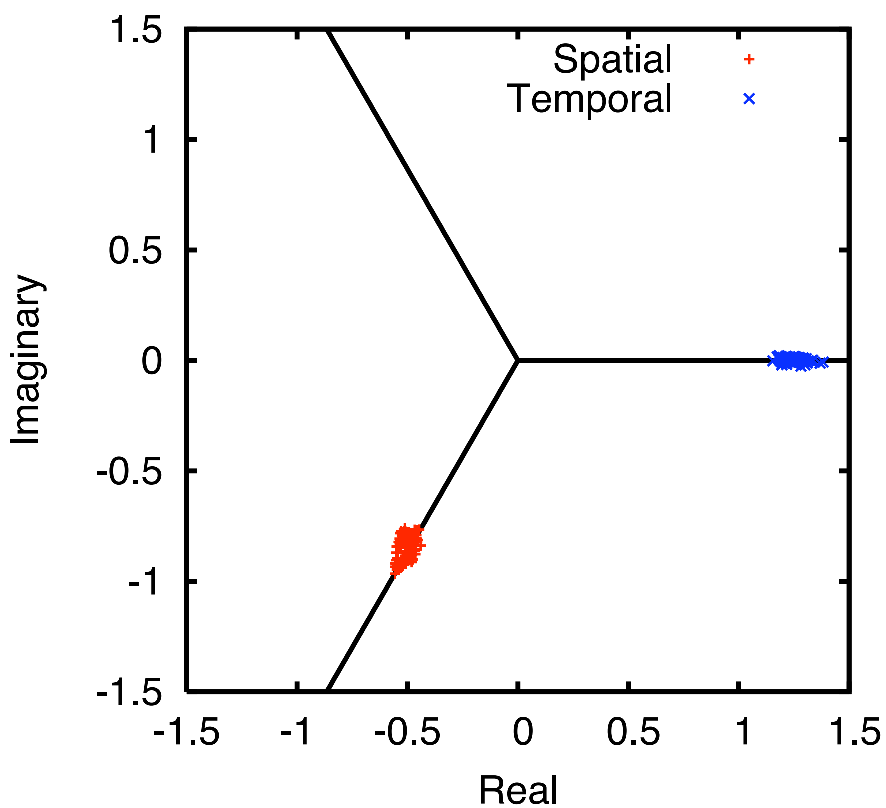
$$\mu_b^{(n)} \mathbf{C}^b = 2\pi \mathbf{l}/N \ (\mathrm{mod}\ 2\pi)$$

$$V_{\mathrm{eff}}^{\mathbf{k}} = -N_f N \check{V}(2\pi \mathbf{l}/N + \pi \mathbf{k})$$

k=(1,1,1) antiperiodic minimal when l=0 (mod 2π) A=0

k=(0,0,0) periodic minimal when $\vec{l} \neq 0$ nontrivial vacua

Polyakov loop distributions probe the vacua



$$P_j = \frac{1}{N} \mathrm{tr}\left(\exp(iC_j^b T_b)\right) = \frac{1}{N} \sum_n \exp(i\mu_b^{(n)} C_j^b) = \exp(2\pi i l_j/N)$$

k=(0,0,0) antiperiodic $A = 0 \ (P_j = 1)$

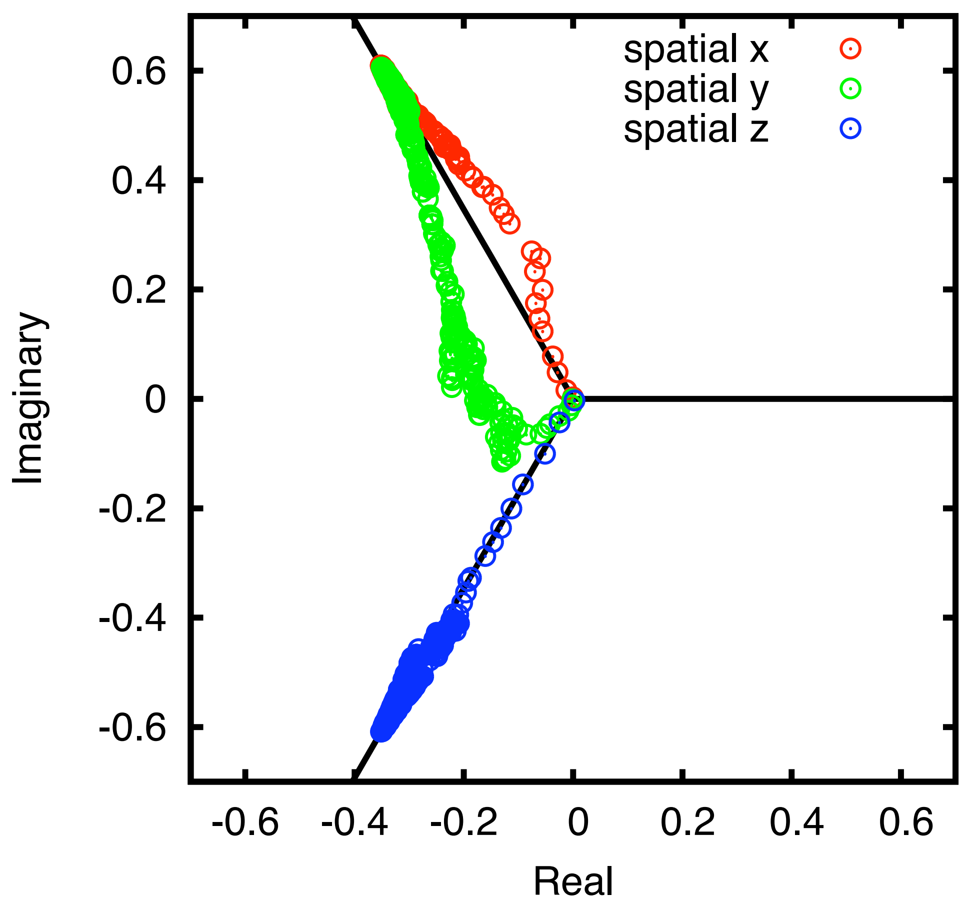
k=(0,0,0) periodic $P_j = \exp(\pm 2\pi i/3)$

16^4 lattice simulation at $\beta = 18$

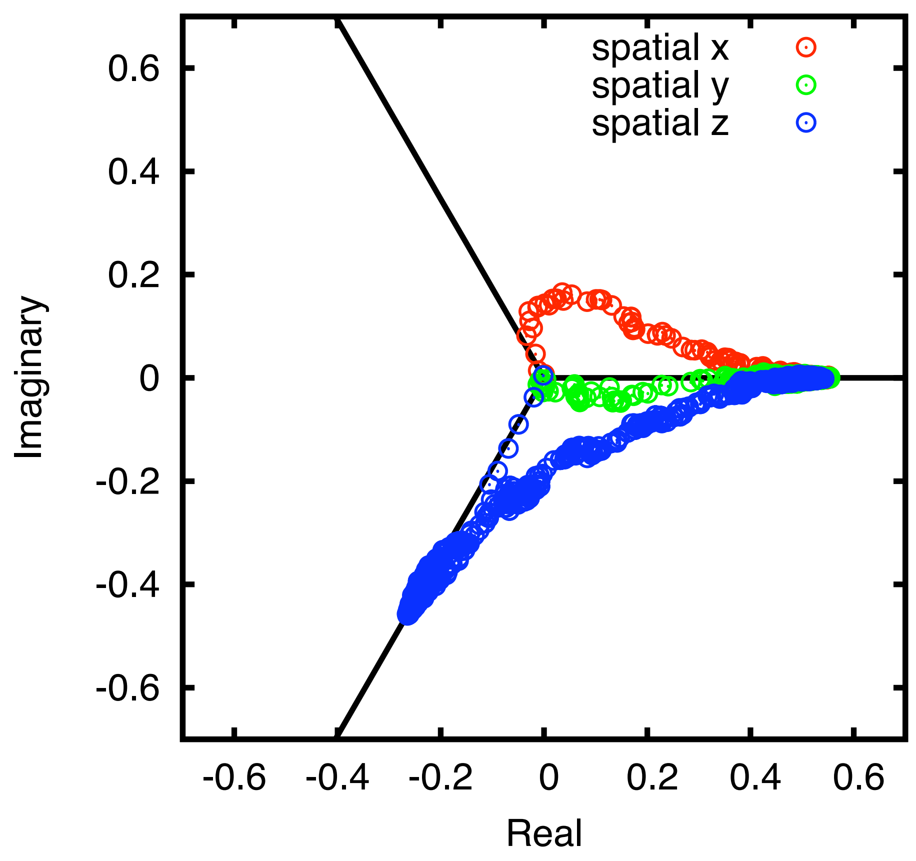
If there is strong coupling inside the conformal window, transition over the barrier into third regime (confinement in QCD) where this picture qualitatively changes

Nf=16 inside conformal window
femto volume and tunneling volume

3-stout, N_f=16, 12^3x36, beta=30.0, m=0.005, pbc



3-stout, N_f=16, 12^3x36, beta=18.0, m=0.001, apbc



Conclusions and Outlook

- Our focus is shifted to Nf=10-16 range (and beyond?)
  primary focus: Nf=12 chiral symmetry breaking (?)

- Zero-mode -->Low lying glueball spectrum relative to mesons!

- Nf=12 might be close enough to realize walking technicolor

- What is the fate of the Nf=2 sextet model?

- Reliable lattice studies will be very demanding on computing

- Reliable EW precision quantities (S/T/U) will be real hard