Running coupling via Wilson loops and the lattice

Kieran Holland
University of the Pacific
November 7 2009

Lattice Higgs Collaboration

Zoltan Fodor, Julius Kuti, Daniel Nogradi, Chris Schroeder

arXiv: 0907.4562, PLB 681 (2009) 353

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even with 27 World Series titles

those damn Yankees STILL suck

outline

- running coupling role
- method
- test: pure gauge theory
- first results fundamental fermions
- conclusions

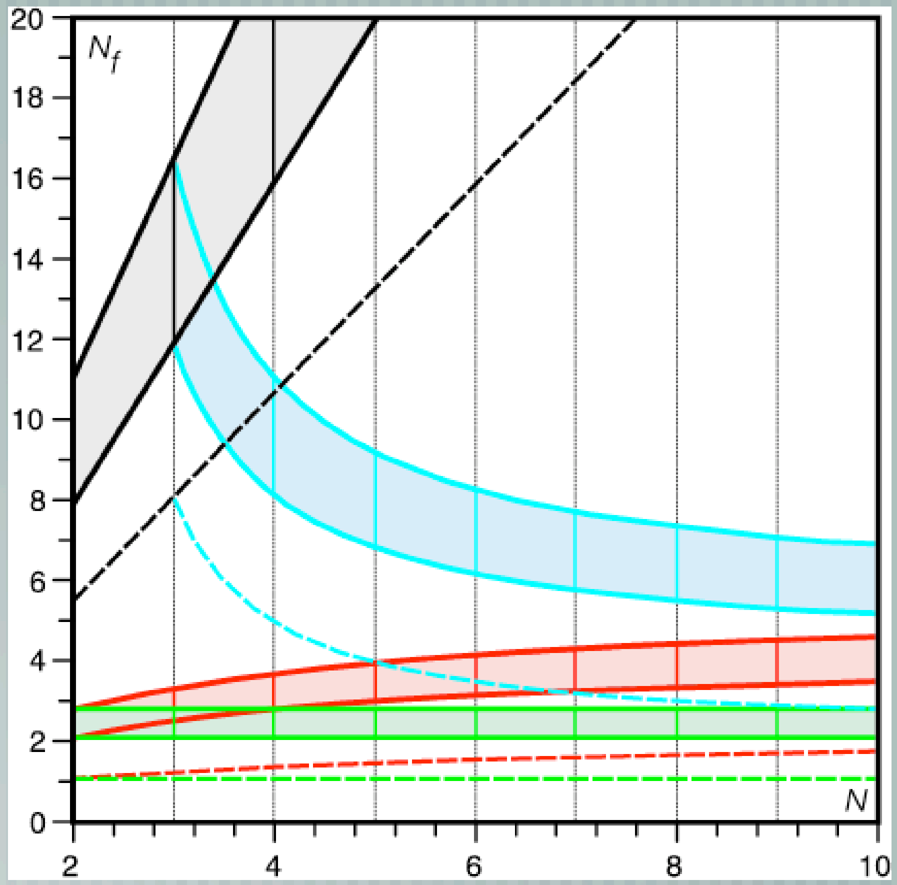
lattice stimulus package

yes/no question: is a theory conformal?

conformal windows

N_f

gray: fundamental
blue: 2-index antisymmetric
red: 2-index symmetric
green: adjoint



Dietrich & Sannino

N_c

hard question

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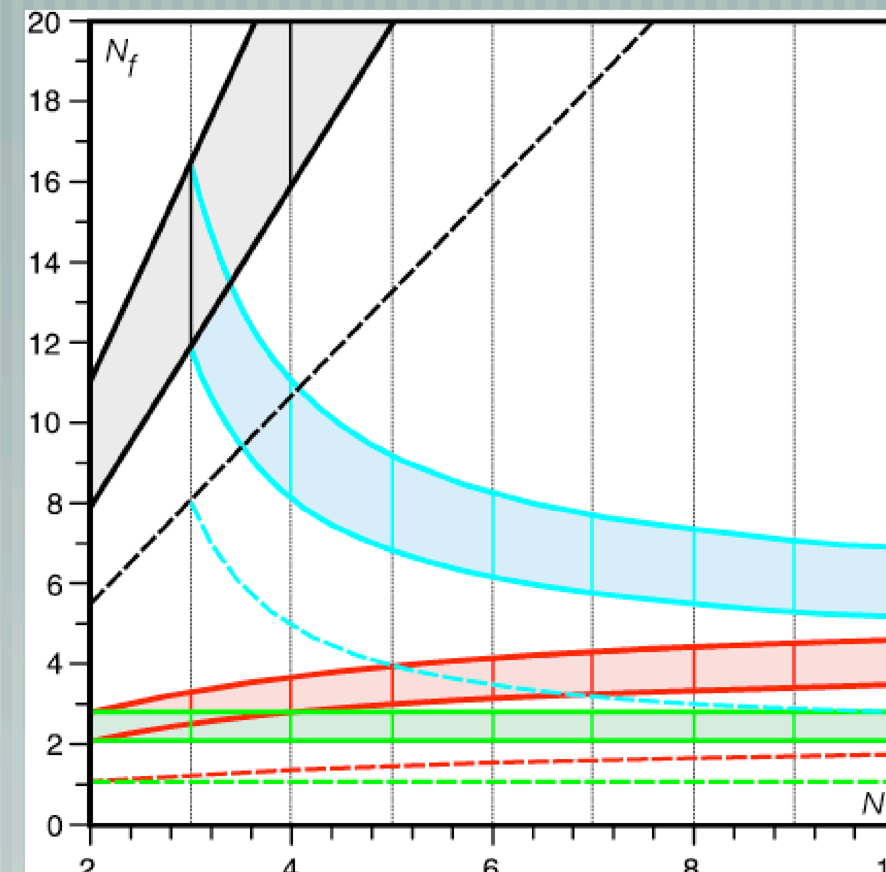


Dietrich & Sannino

N_c

hard question, but answerable - just give us more money, computers, ...

consistency

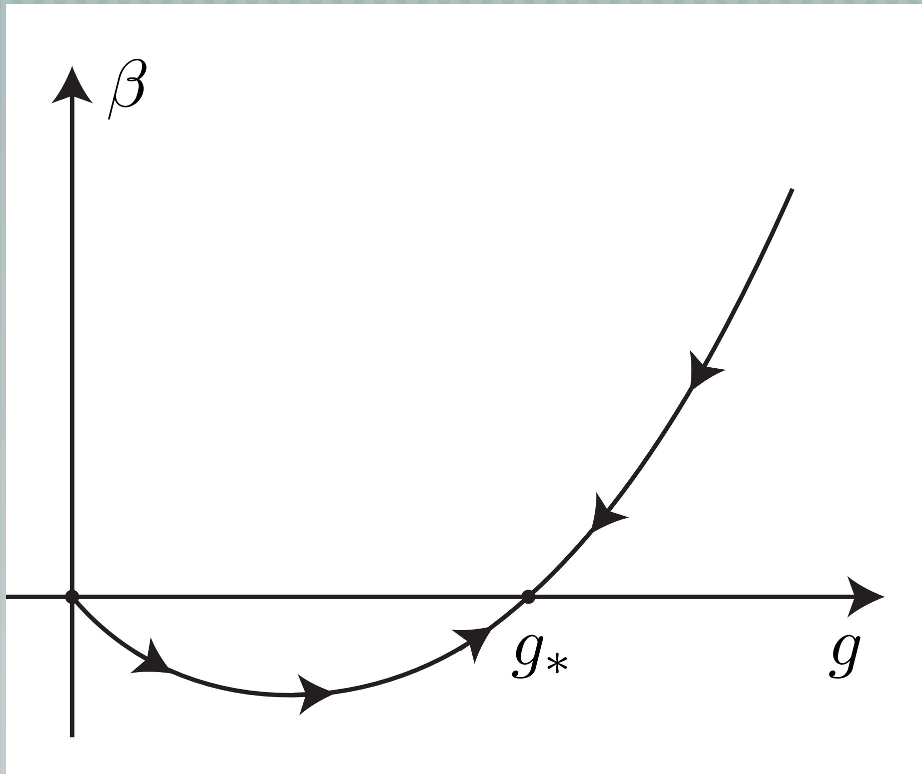
- more than 1 signal for conformal/non-conformal behavior
  - p-regime and techni-hadron spectrum
  - epsilon-regime and lowest Dirac eigenvalues
  - finite temperature transitions
  - running coupling (Schrodinger functional, Wilson loops)
  - ...

Julius Kuti
Friday

RG flow

if theory conformal, non-trivial zero of

$$\beta(g) = -\left(\beta_1 \frac{g^3}{16\pi^2} + \beta_2 \frac{g^5}{(16\pi^2)^2}\right)$$



infrared fixed point

Caswell-Banks-Zaks

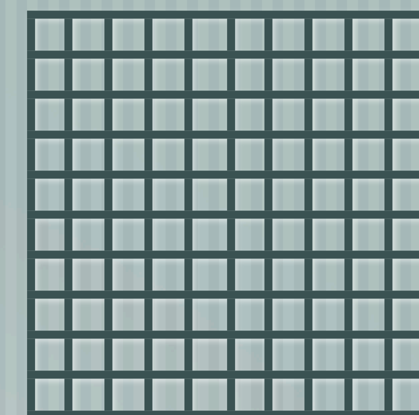
$$\frac{g_*^2}{16\pi^2} = -\frac{\beta_1}{\beta_2}$$

pert.theory trustworthy if g_* not too large

use lattice: non-perturbative

lattice in 2 minutes

discretize space-time: lattice spacing



gauge links $U_\mu = \exp(iaA_\mu)$

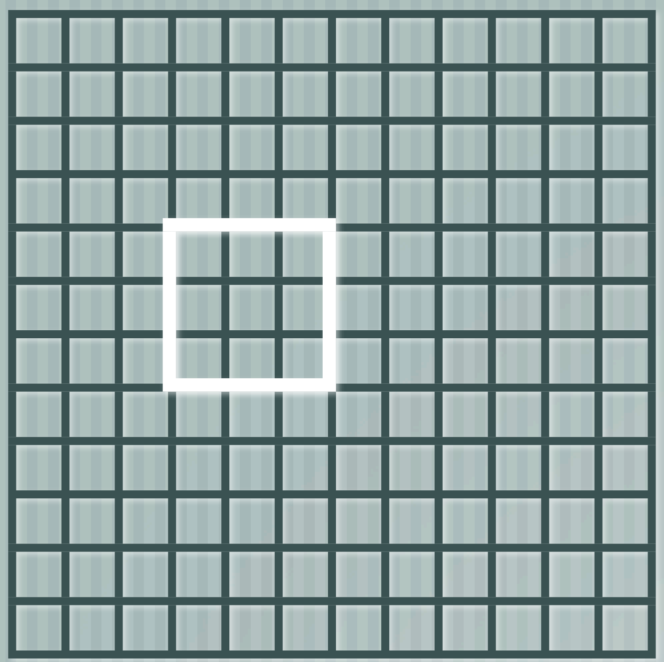
partition function

$$\mathcal{Z} = \int DU \; \{\mathrm{Det}(D[U])\}^{N_f} \exp(-S_{\mathrm{gauge}}[U])$$

Monte Carlo sampling $\displaystyle\int DU \rightarrow \sum_{\text{typical } U}$

non-local determinant: slow computation

running coupling



Wilson loop

continuum

$$g^2(R/L, L) = -\frac{R^2}{k(R/L)} \frac{\partial^2}{\partial R \partial T} \ln\langle W(R,T,L)\rangle \,|_{T=R}$$

lattice

$$g^2((R+1/2)/L, L) = \frac{1}{k(R/L)} (R+1/2)^2 \chi(R+1/2, L)\,,$$

$$\chi(R+1/2, L) = -\ln\left[\frac{W(R+1, T+1, L) W(R,T,L)}{W(R+1,T,L) W(R, T+1, L)}\right] |_{T=R}\,,$$

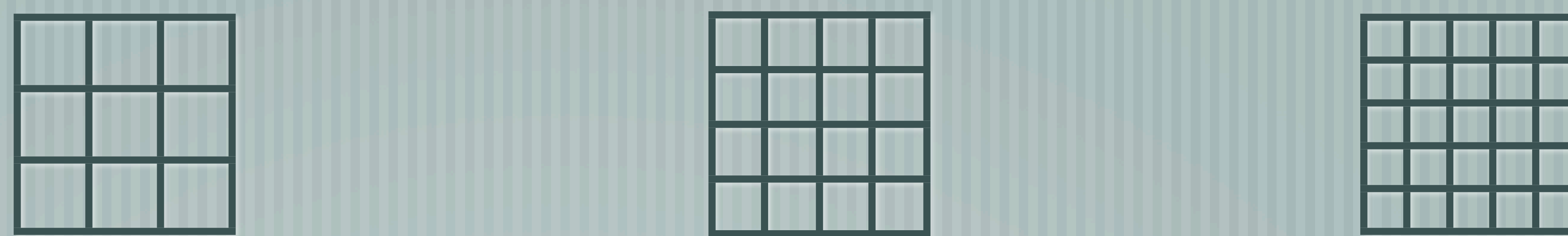
keep $(R+1/2)/L$ fixed: coupling flows with L

old idea: Campostrini et al. PLB349, 499 (1995) also Bilgici et al 0902.3768

step-scaling to continuum

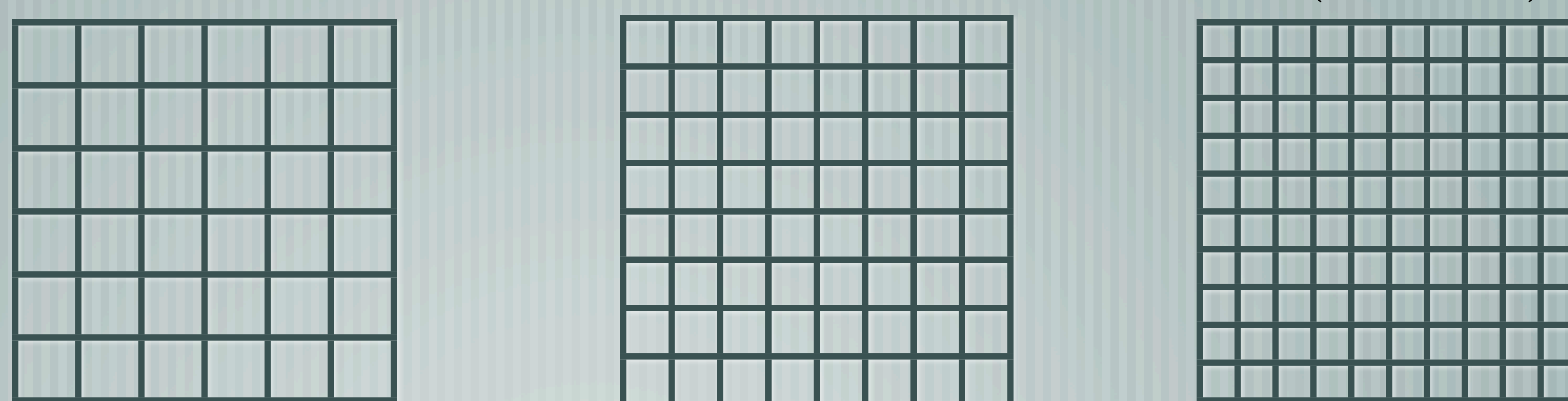
$$g^2(L_1, \beta_1) = g^2(L_2, \beta_2) = g^2(L_3, \beta_3)$$

tune:



$g^2(2L_1, \beta_1)$ $g^2(2L_2, \beta_2)$ $g^2(2L_3, \beta_3)$

double:



extrapolate RG step to continuum $g^2(L) \to g^2(2L)$

test: pure gauge theory

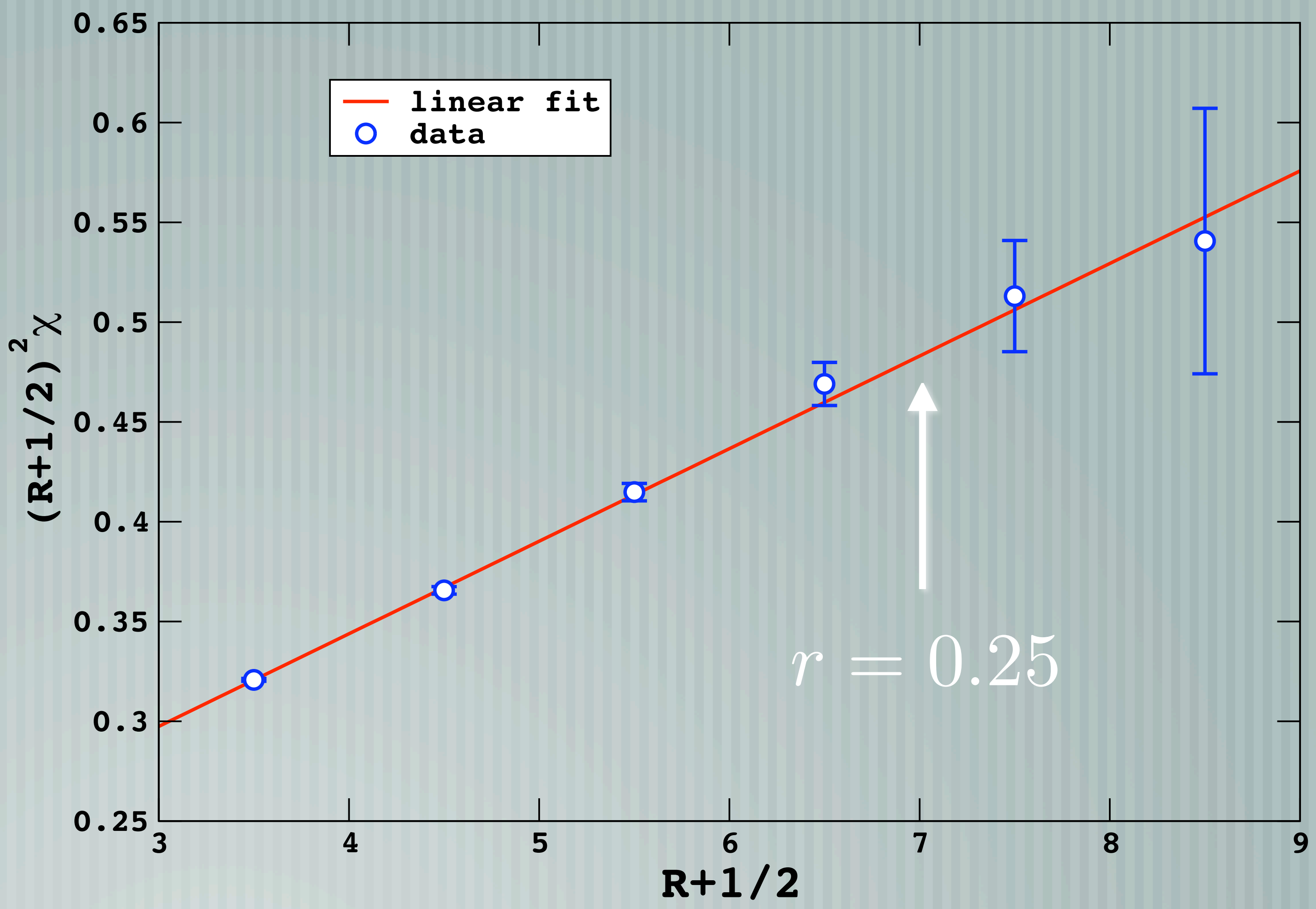
4-dimensional $SU(3)$

example: 28^4 lattice

scheme:

$r = (R+1/2)/L = 0.25$

accurate measurement



test: pure gauge theory

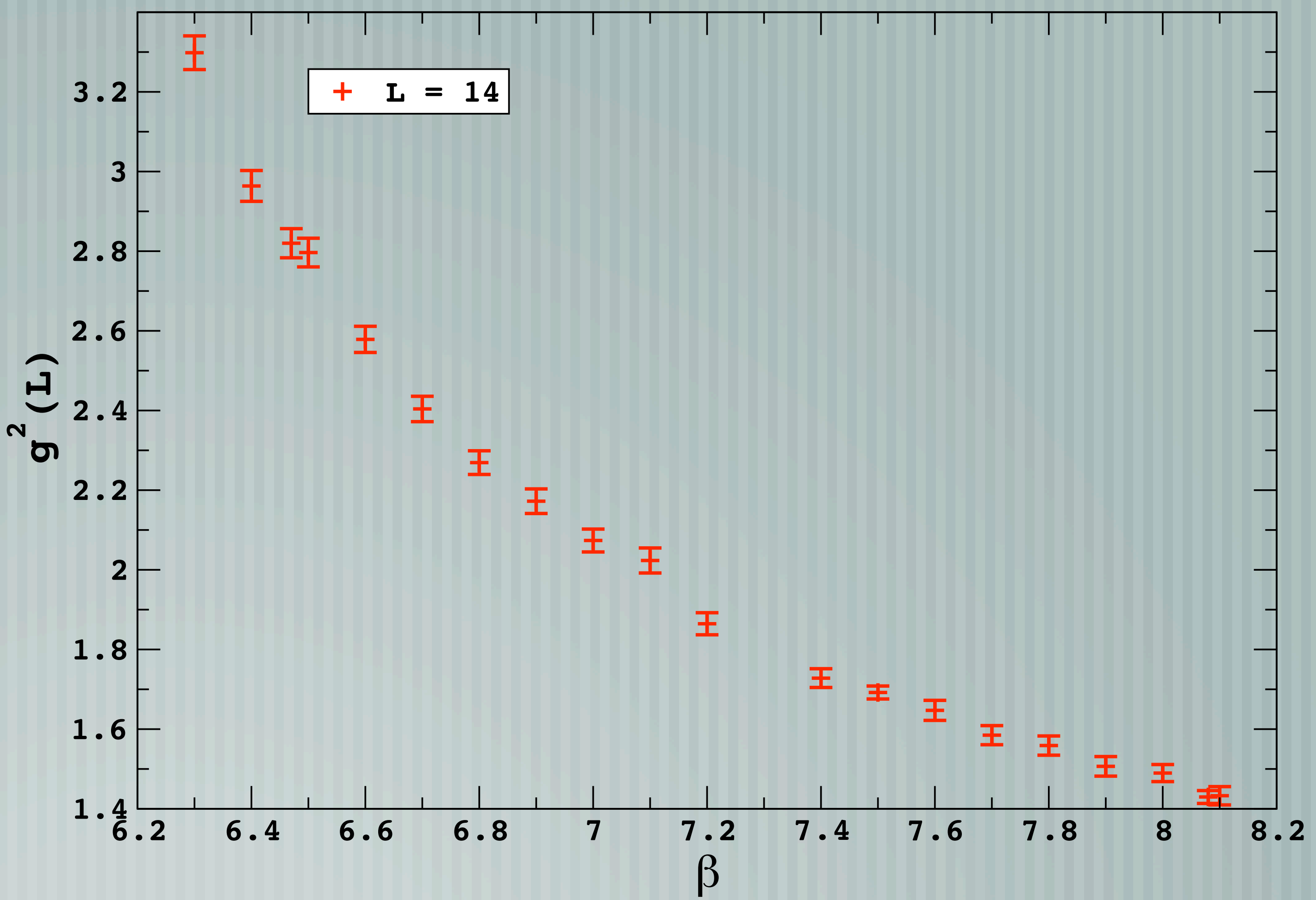
many runs on small lattices to tune coupling

example: 14^4

each point requires separate simulation

AND

additional simulations on doubled lattices at tuned couplings



continuum extrapolation

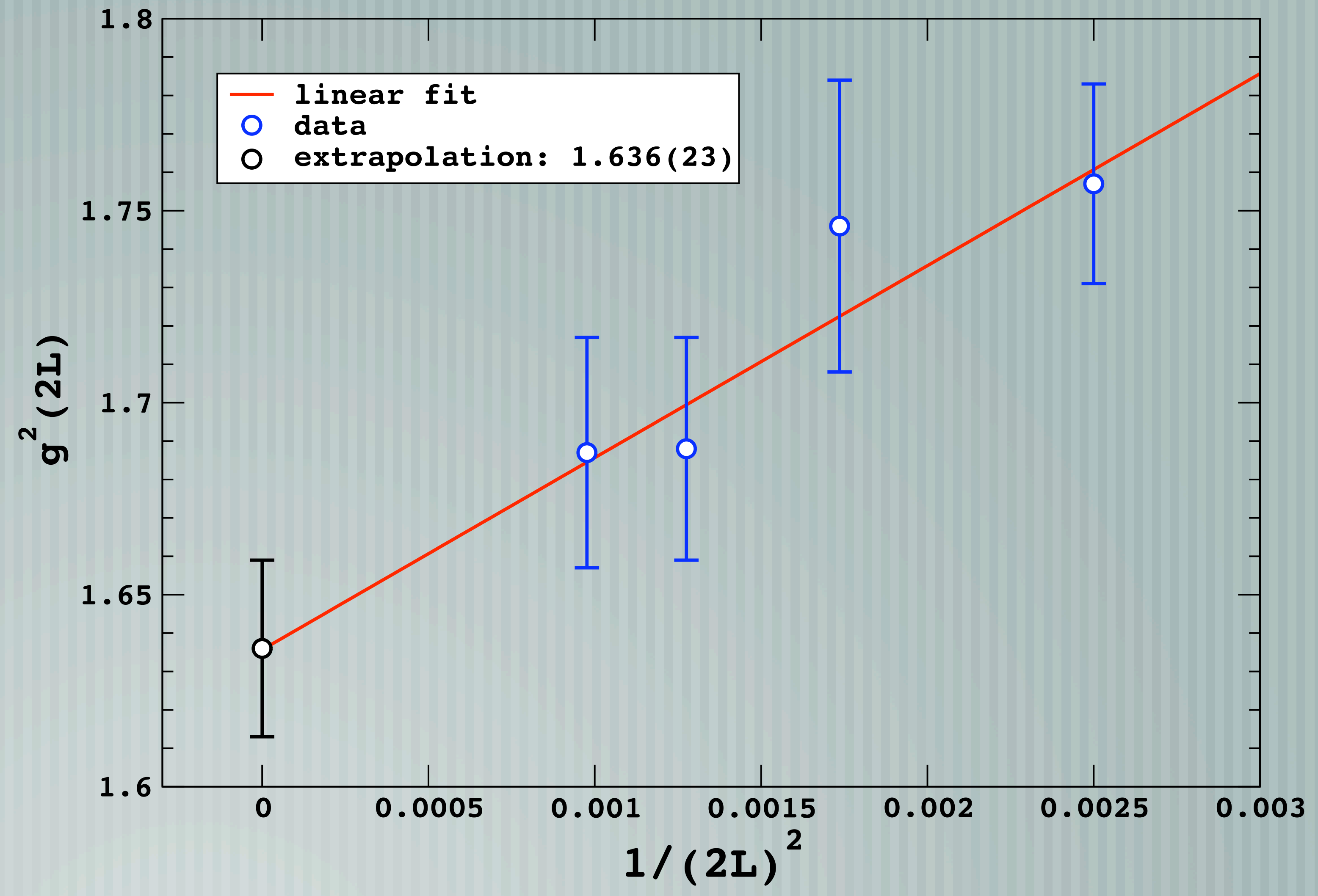
tune: $g^2(L_i, \beta_i) = 1.44$

4 doubled lattices

$$2L = 20, 24, 28, 32$$

cut-off effects $\mathcal{O}(a^2)$

i.e. $\mathcal{O}(1/L^2)$



connect RG steps

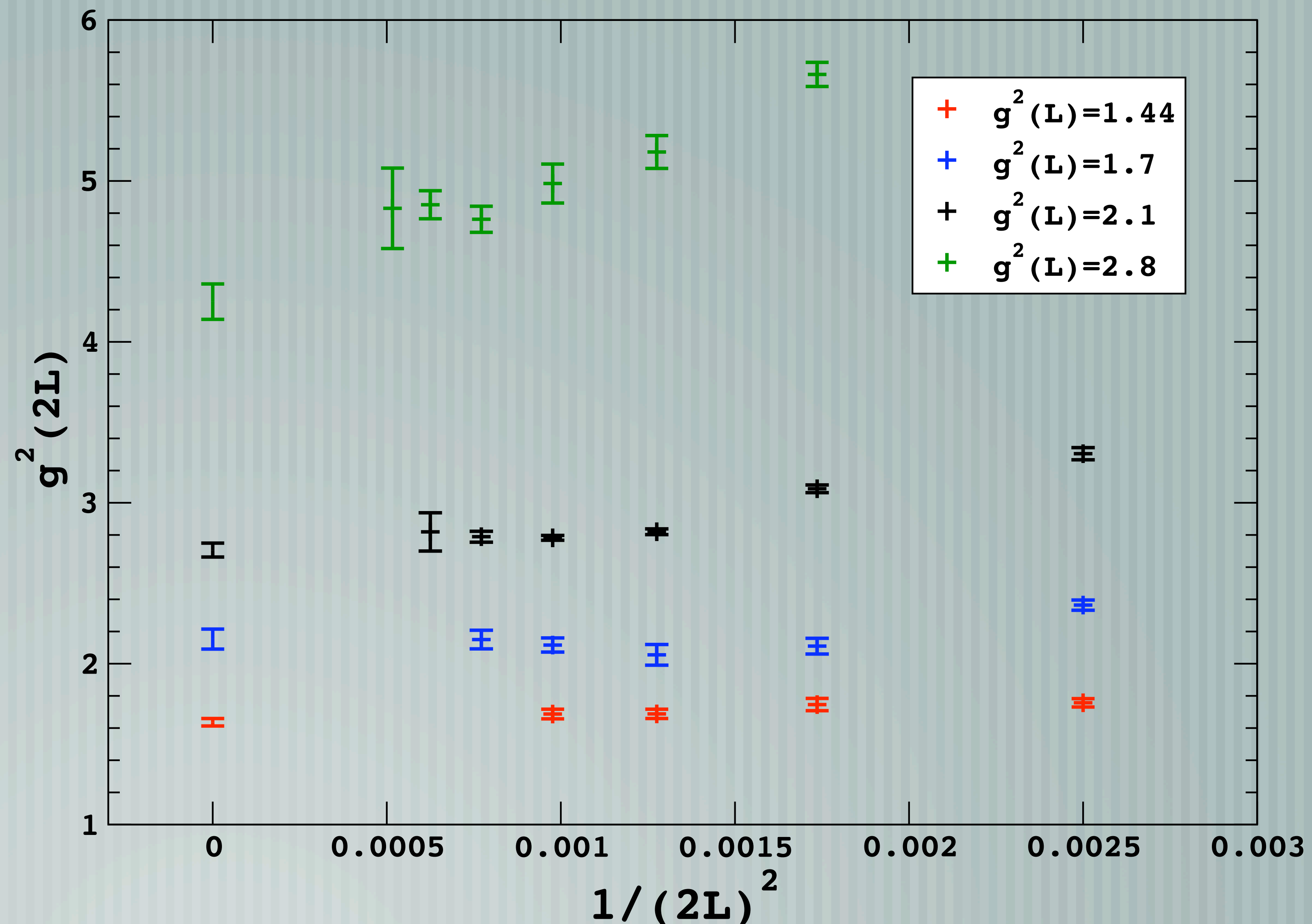
iterate procedure

tune $g^2(L_i, \beta_i)$ to previous extrapolation

range of doubled lattices

$$2L = 20, ..., 44$$

actually, not that cheap

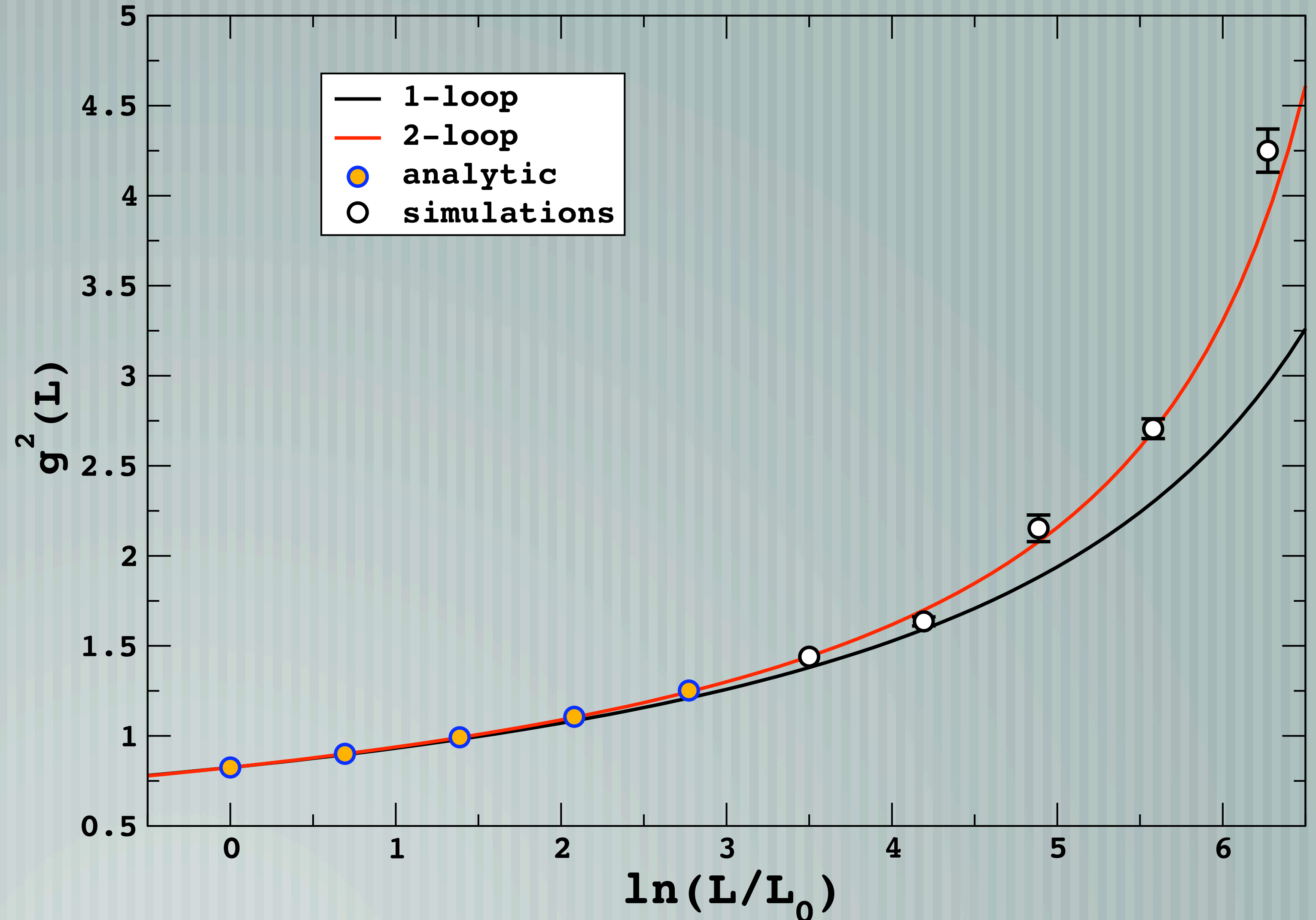


RG flow

simulation: 4 RG steps

weak coupling: calculate Wilson loop in pert.theory (with improvement - Julius)

nice agreement with 2-loop RG flow



fundamental fermions

$SU(3), N_f = 16$ fundamental $g_*^2 \approx 0.5$ 2-loop

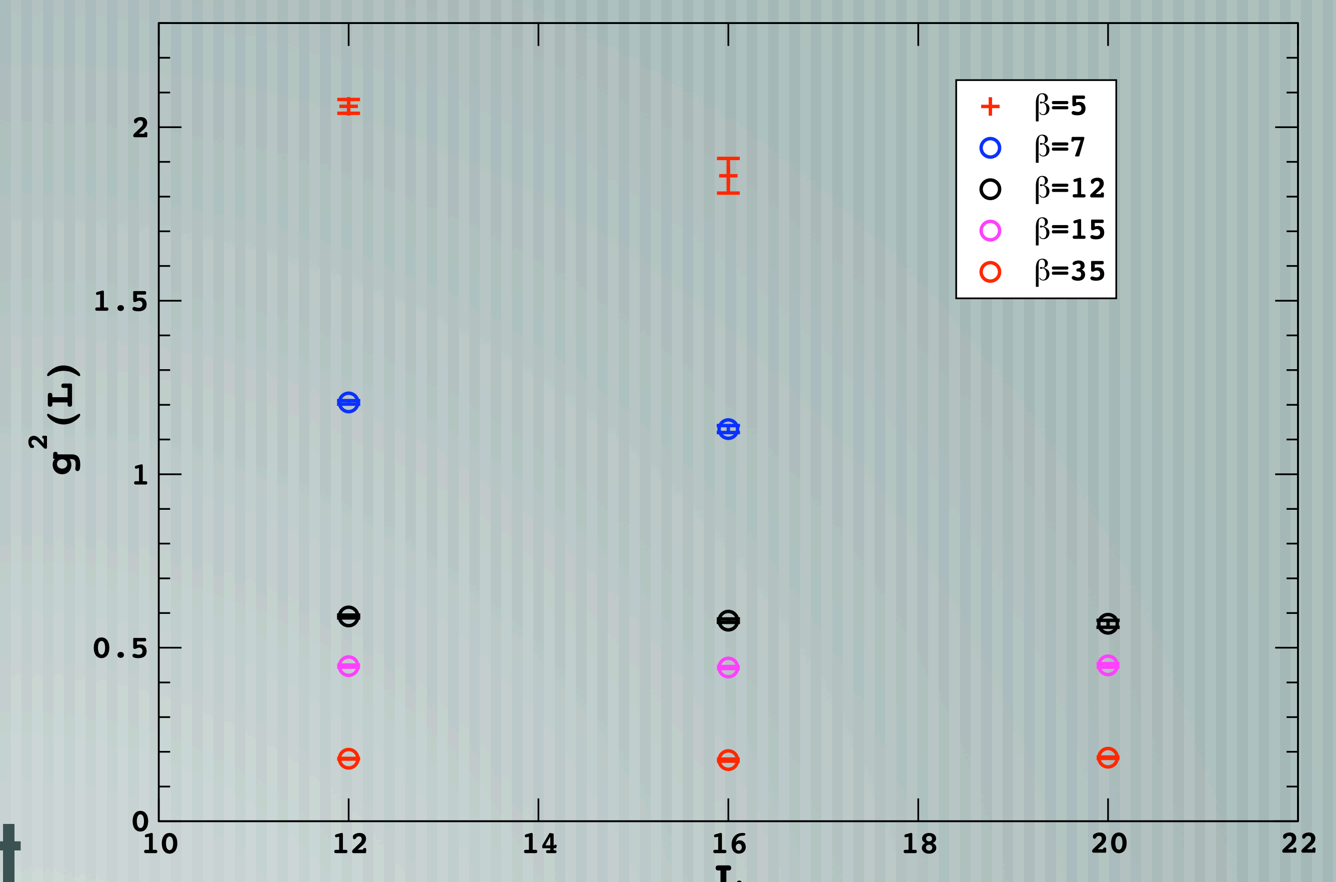
guaranteed(?) conformal

lattice: staggered fermions

no step-scaling

$$g^2(\beta, \Lambda_{\mathrm{scheme}} L, a^2/L^2)$$

consistent with flow to fixed point



similar to Hietanen at al 0904.0864 $SU(2), N_f = 2$ adjoint, Del Debbio et al

fundamental fermions

$SU(3), N_f = 12$ fundamental

$g_*^2 \approx 9$ 2-loop

$g_*^2 \approx 4.3$ 3-loop Wilson-loop scheme

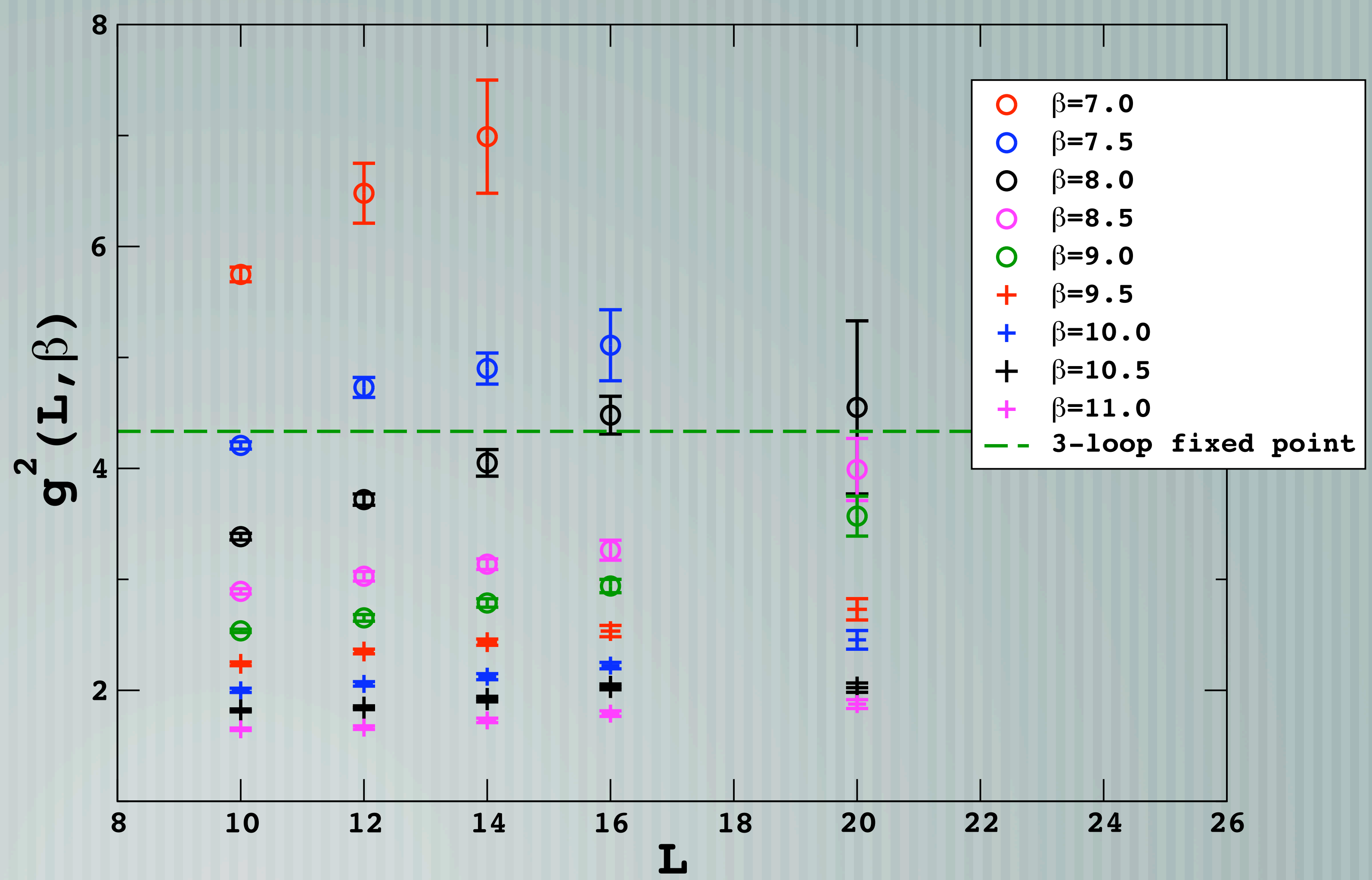
interesting & controversial

lattice: staggered

no step-scaling

$$g^2(\beta, \Lambda_{\text{scheme}} L, a^2/L^2)$$

no sign yet of IRFP



controversy

Julius Kuti: p- and epsilon-regime

Wilson-loop g(L) consistent with this

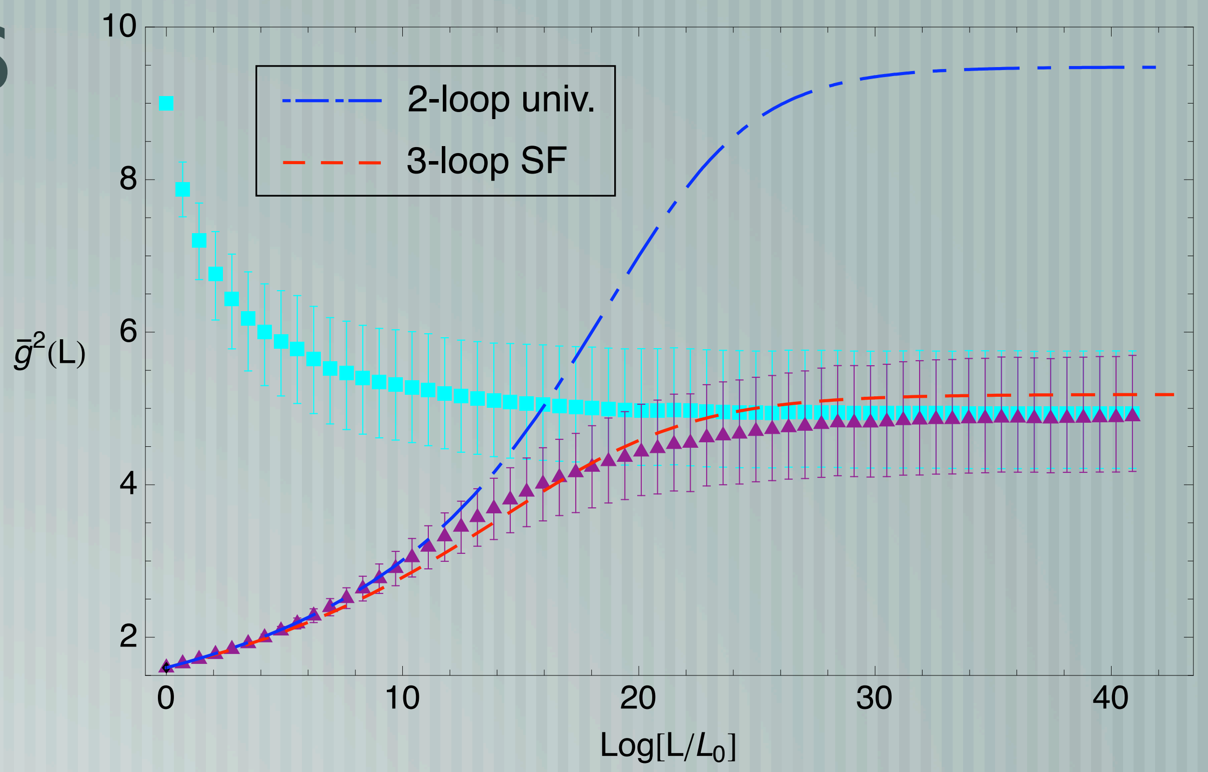
Appelquist et al: different definition running coupling - flows to IRFP

also Deuzeman et al: conformal

new work: Hasenfratz; Jin & Mawhinney

12 flavor theory unresolved

12 flavor is chirally broken



Appelquist, Fleming, Neil
arXiv: 0712.0609, 0901.3766

conclusions

- method works in pure gauge theory
- fundamental $SU(3), N_f = 16$ looks conformal
- no sign yet of IRFP for $SU(3), N_f = 12$ fundamental
- long-term: consistency with other signals
- speculative theories + expensive computers = fun