Four dimensional large N gauge theories with adjoint fermions on a single site lattice - II

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Single site model with adjoint fermions

The theory is defined with action

$$S = S^g + S^f,$$

where

$$S^g = -bN \sum_{\mu \neq \nu = 1}^{4} \mathrm{Tr}\left[U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger\right]$$

and

$$S^f = -f \log\det D^f = -f\,\mathrm{Tr}\log D^f,$$

where f is related to number of Dirac fermion flavors and D^f the fermion operator in some lattice regularization.

HMC-update algorithm

- Define a molecular dynamics Hamiltonian

$$\mathcal{H} = \frac{1}{2}\sum_{\mu=1}^{4} \mathrm{Tr} H_\mu^2 + S$$

- Evolve fields in fictitious time τ using equations of motions (MD equations)
- The equations of motions for U is

$$\frac{dU_\mu}{d\tau} = iH_\mu U$$

Equation of motion for H_μ

- The equation of motion for H_μ can be obtained from

$$\frac{d\mathcal{H}}{d\tau} = 0 \Leftrightarrow \sum_{\mu=1}^{4} \mathrm{Tr}\left[H_\mu \frac{dH_\mu}{d\tau}\right] + \frac{dS^g}{d\tau} + \frac{dS^f}{d\tau} = 0,$$

- Contribution from gauge part

$$\frac{dS^g}{d\tau} = -ibN \sum_{\mu,\nu=1}^{4} \mathrm{Tr} H_\mu \left[U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger - U_\nu^\dagger U_\mu U_\nu U_\mu^\dagger - h.c.\right]$$

Naïve fermions

- The fermion action with naive fermions reads

$$S^f = -f \log\det \begin{pmatrix} m & C \\ -C^\dagger & m \end{pmatrix} = \mathrm{Tr}\log\left[m^2 + CC^\dagger\right],$$

where

$$C = \sum_\mu \sigma_\mu \left(V_\mu - V_\mu^T\right)$$

- The adjoint representation link matrices V are obtained from U's with

$$V_\mu^{ab} = \frac{1}{2}\mathrm{Tr}\left[T^a U_\mu T^b U_\mu^\dagger\right],$$

Naive fermion contribution H_μ

- The fermion contribution

$$\frac{dS^f_{\text{naive}}}{d\tau} = -f\,\mathrm{Tr}\frac{1}{m^2 + CC^\dagger}\left(\frac{dC}{d\tau}C^\dagger + C\frac{dC^\dagger}{d\tau}\right)$$

- After some algebra

$$\frac{dS^f_{\text{naive}}}{d\tau} = i\frac{f}{2}\sum_{\mu=1}^{4} TrH_\mu \sum_{ab} \bar{A}^a_{\mu b}[T^a, T^b],$$

where

$$\bar{A}_\mu = \left[V_\mu \sum_{\nu=1}^{4}(V_\nu - V^t_\nu)M_{\mu,\nu} + \sum_{\nu=1}^{4}(V_\nu - V^t_\nu)M_{\mu,\nu}V^{\mathrm{T}}_\mu\right] - \text{transpose}$$

$$M_{\mu,\nu} = \mathrm{Tr}_{\text{spin}}\left[\frac{1}{m^2 + CC^\dagger}\sigma_\mu\sigma^\dagger_\nu\right]$$

Naive fermion simulations

- The simulations times scale as $\mathcal{O}(N^6)$
- Simulations were performed with $b = 5$, $N = 11$ and $\mu = 0.01$.
- 100 iterations takes about 6h

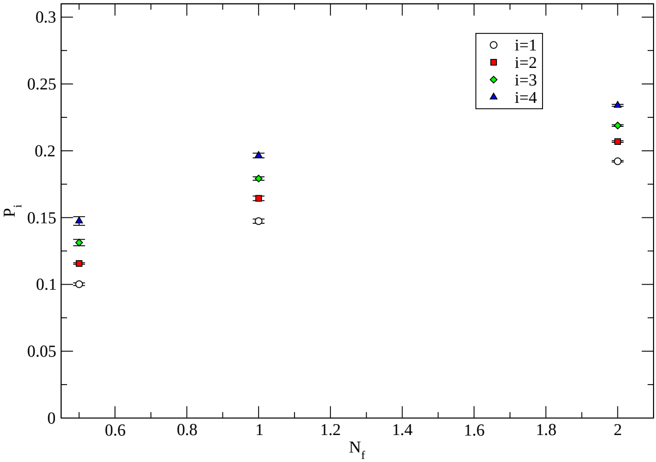

Figure: Polyakov loop with $b = 5$

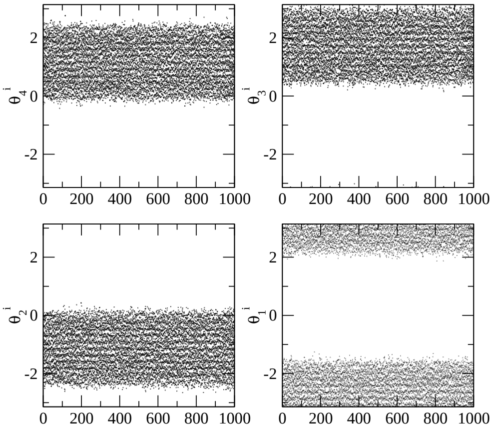

Figure: Histogram of the angles of the eigenvalues of the polyakov loop

Overlap fermions

$$S^f = -f \,\mathrm{Tr} \log \left[\frac{1}{2} \left[(1+\mu)\,\gamma_5 + (1-\mu)\,\epsilon(H) \right] \right] \equiv -f \,\mathrm{Tr} \log \left[H_o \right],$$

where H is Hermitean Wilson Dirac operator in adjoint representation

$$H = \begin{pmatrix} c - \frac{1}{2}\sum_\mu \left(V_\mu + V_\mu^t\right) & \frac{1}{2}\sum_\mu \sigma_\mu \left(V_\mu - V_\mu^t\right) \\ -\frac{1}{2}\sum_\mu \sigma_\mu^\dagger \left(V_\mu - V_\mu^t\right) & -c + \frac{1}{2}\sum_\mu \left(V_\mu + V_\mu^t\right) \end{pmatrix},$$

where $m \in] \sim 2, \sim 10[$ (normally $m \in]0,2[$) and $c = 4 - m$

HMC equation of motion

- The fermion contribution

$$\frac{dS^f}{d\tau} = -f\,\mathrm{Tr}\frac{1}{H_o}\frac{1-\mu}{2}\frac{d\epsilon(H)}{d\tau}.$$

- $\epsilon(H)$ can be written as

$$\epsilon(H) = \sum_i |\lambda_i\rangle\langle\lambda_i| - \sum_i |\omega_i\rangle\langle\omega_i|,$$

where $|\lambda_i\rangle$ and $|\omega_i\rangle$ are eigenvectors of H with positive and negative eigenvalues. Thus,

$$\frac{d\epsilon(H)}{d\tau} = \sum_i \frac{d|\lambda_i\rangle}{d\tau}\langle\lambda_i| + \sum_i |\lambda_i\rangle\frac{d\langle\lambda_i|}{d\tau} - (\lambda \to \omega)$$

- Single sign changes corresponds to non-physical fractional topological charges.

- After some manipulation

$$\frac{dS^f}{d\tau} = f(1-\mu)\mathrm{Tr} H_\mu i \frac{f}{2} \sum_{ab} [T^a, T^b] \left[\bar{A}^{ab}_\mu - (\bar{A}^{ab}_\mu)^T \right],$$

where

$$\bar{A}_\mu = V_\mu \mathrm{Tr}_{\mathrm{spin}} \left[\sum_{j,k} \frac{|\lambda_j\rangle\langle\lambda_j| \frac{1}{H_o} |\omega_k\rangle\langle\omega_k|}{\omega_k - \lambda_j} + \mathrm{h.c.} \right]$$

- The algorithm efficiency is $\mathcal{O}(N^6)$.
- Can be parallelized.

Simulations with adjoint fermions

- All the results are with $\mu = 0.01$ (physical quark mass) and $N = 11$
- 100 configurations with $b = 5$ takes around 30h ($\tau = 0.01$ and 100 step in trajectory)
- Thermalization might be challenging and require smaller τ with less steps

Eigenvalues of overlap and Wilson operator

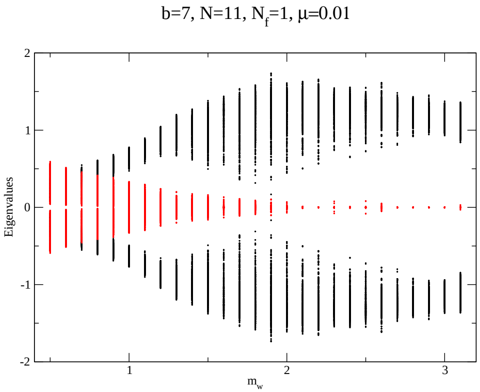

Figure: Simulations with $f = 1$, $m_w = 2$ and $b = 5$

Eigenvalues of overlap and Wilson operator

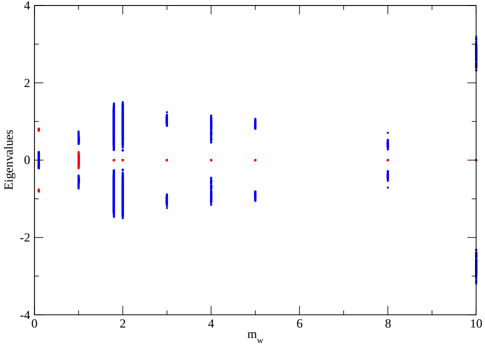

Figure: Simulations with $f = 1$ and $b = 5$

Zero modes of overlap operator

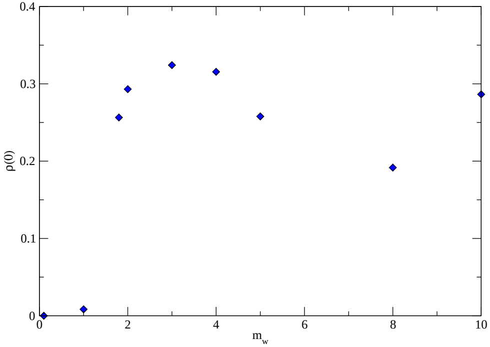

Figure: Simulations with $f = 1$ and $b = 5$

Center symmetry restoration

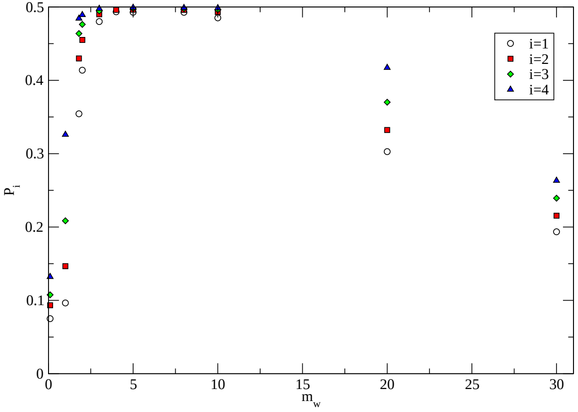

Figure: Simulations with $f = 1$ and $b = 5$

Ratio between two smallest eigenvalues

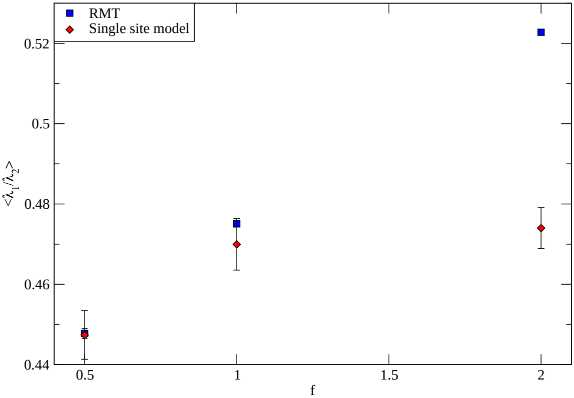

Figure: Ratio between two smallest eigenvalues of fermion matrix at $b = 5$ and $m_w = 5$ in random matrix theory and single site model

Cumulative probability density

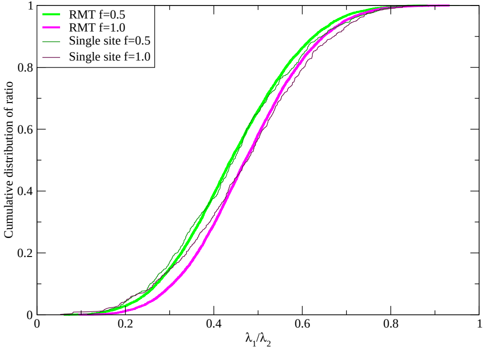

Figure: Cumulative probability density between two smallest eigenvalues of fermion matrix $b = 5$ and $m_w = 5$

Summary

- We presented the framework of simulating EK-model with dynamical fermions in adjoint representation.
- Even if no doublers naive fermsions do not work
- The center symmetry is restored for Overlap fermions for high enough Wilson mass.
- Still need to calculate physical observables