The running coupling of many-fermion systems from Monte Carlo Renormalization Group studies

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Fermion - gauge models

- Can be QCD-like: confining, chirally broken

  or

- Can have an infrared fixed point: conformal

The most exciting cases are near the emergence of the IRFP

- Just below the theory could be “walking”
- Just above there is a strongly coupled conformal FP

Questions for lattice studies:

- Minimal N_f where the conformal regime develops
- Running of the coupling just below the conformal window
- Properties of the IRFP (anomalous dimension of the mass)

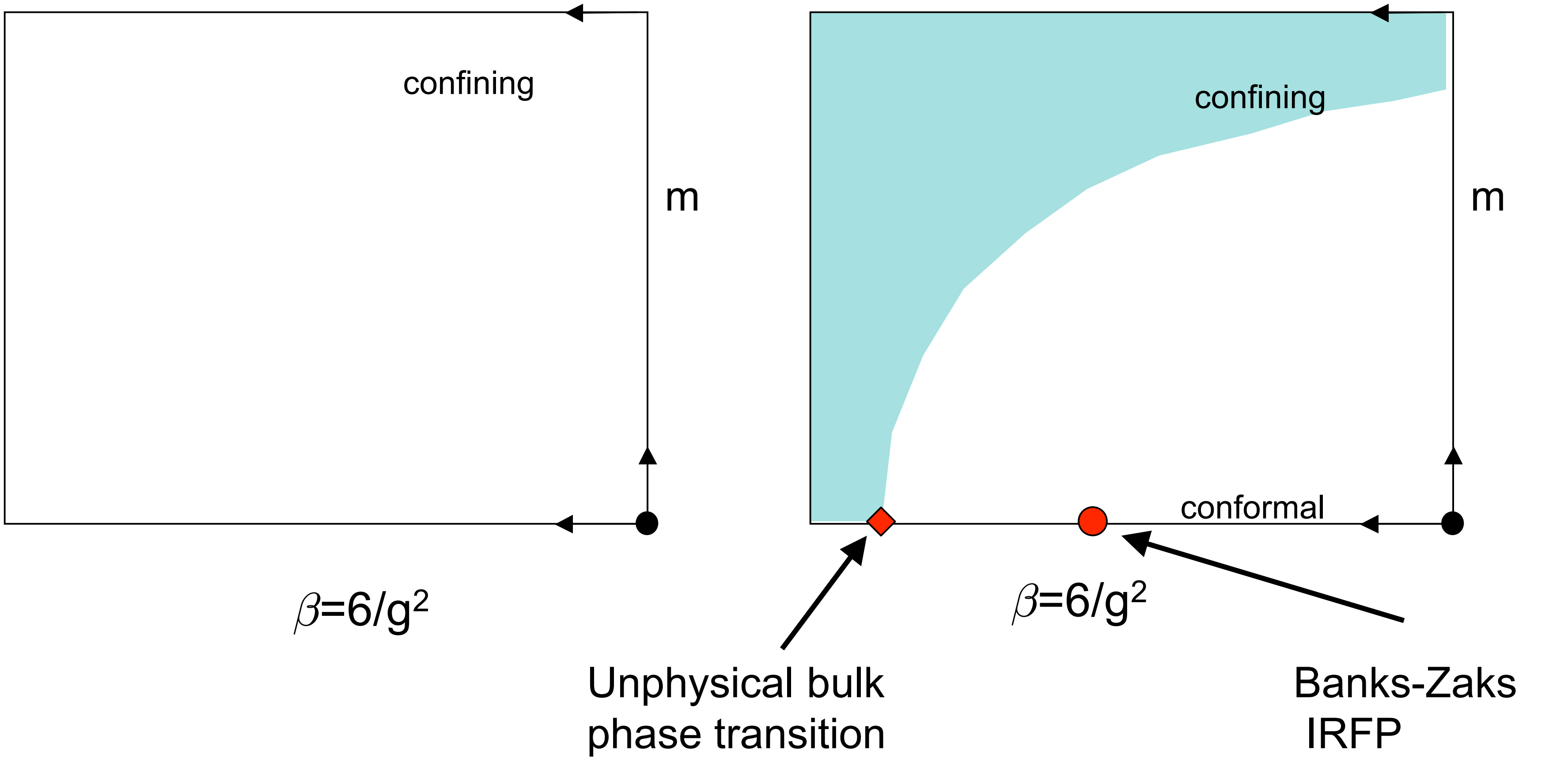
The most interesting questions are frequently the hardest ones

The lattice phase diagram

(arrows: UV to IR)

QCD like

Conformal



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(arrows: UV to IR)

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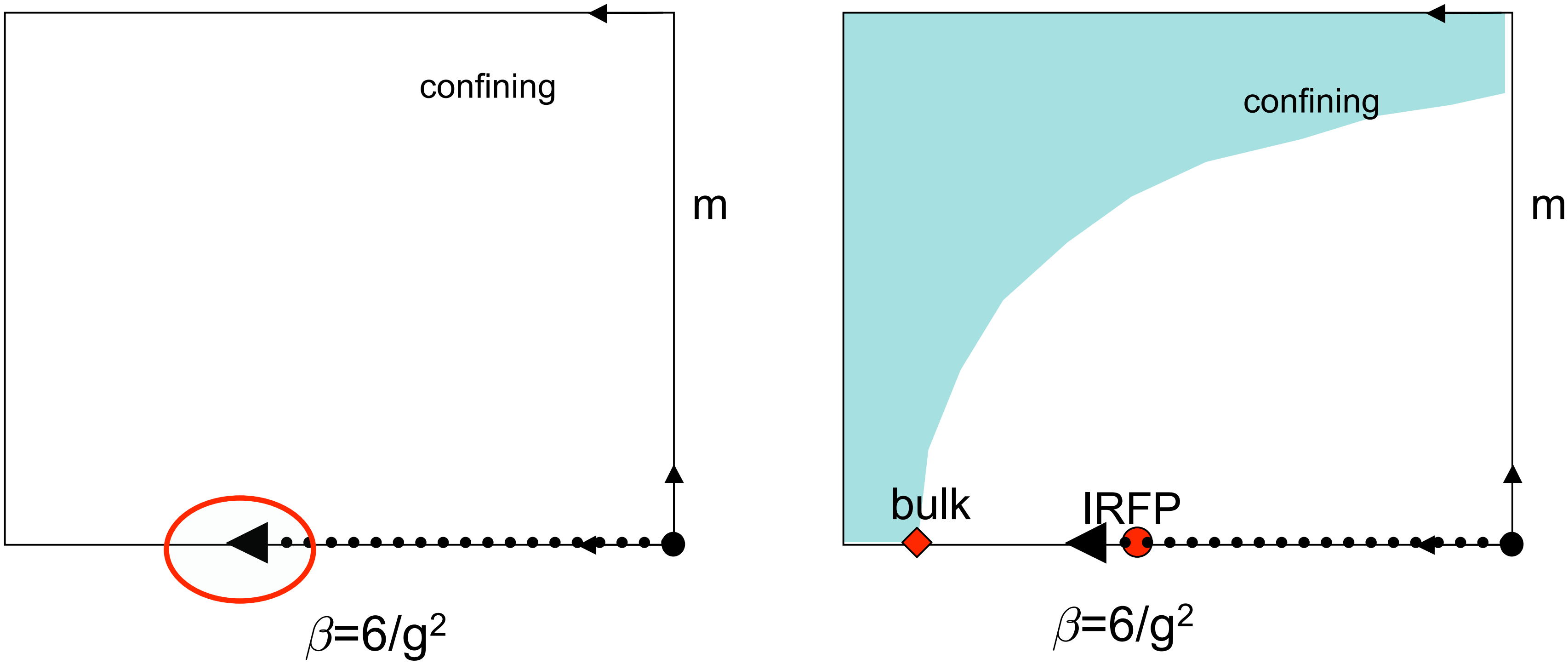
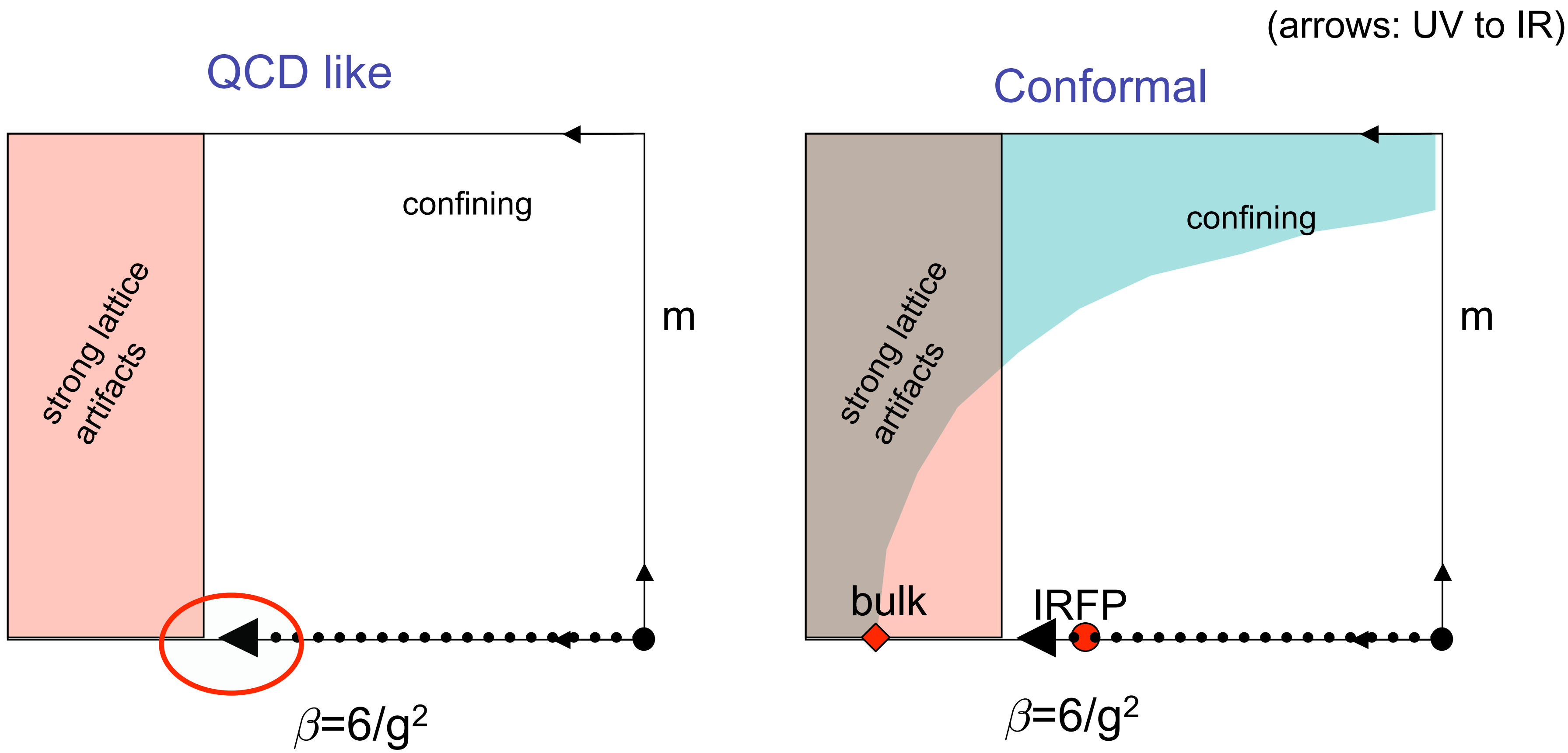


Lattice simulations can connect the perturbative FP and strong coupling

- Found IRFP ? Done ✔
- No IRFP? Show that it is confining before a bulk transition is reached
- Strong lattice artifacts can interfere

The lattice phase diagram



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Monte Carlo Renormalization Group method

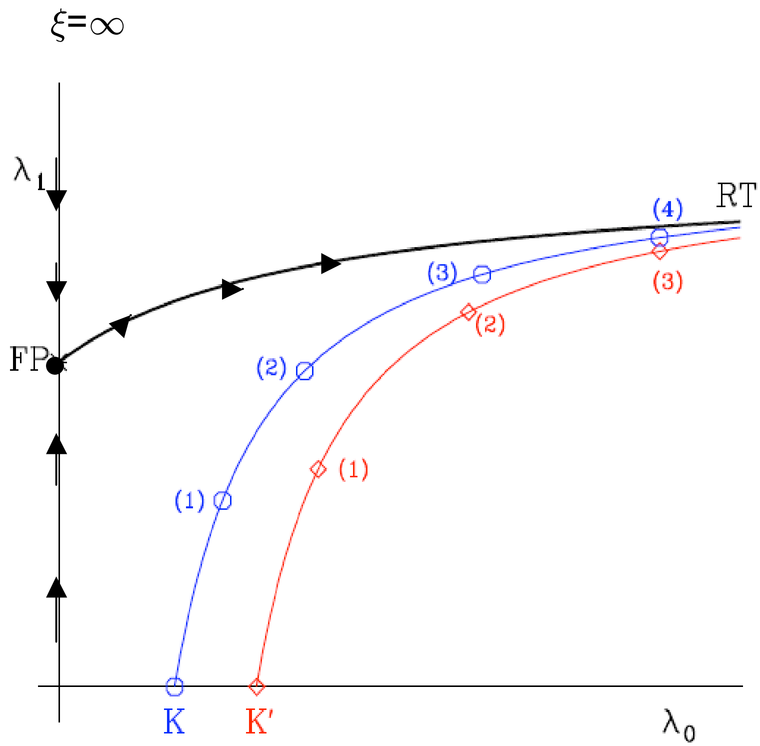
- MCRG was designed to study phase the diagram and critical exponents
- It is well suited to connect the lattice weak and strong coupling phases, identify an IRFP and measure critical exponents
- MCRG works in bare coupling space

2-lattice matching method

The method identifies pairs of bare couplings (β,β') where $a(\beta) = a(\beta') / 2$
and predicts the (differential) bare step scaling function $s_b(\beta;2) = \beta-\beta'$

- At a fixed point $s_b(\beta^*)=0$
  - $s_b(\beta)$ identifies a FP just like the renormalized one!
- The value of s_b is related to the scaling dimension of the coupling
  - for AF models $s_b = 3\ln(2)/(4\pi^2)\, b_0 + O(g^2)$
  - $s_b > 0$ where the RG β-function is $\beta(g) < 0$ (sorry)
  - s_b in the mass predicts the anomalous dimension of the mass
    $m = m' \, 2^{1/y}$

RG flow lines & 2-lattice matching



at a FP with 1 relevant direction:

– do simulations at K and K'

– do RG blocking and compare the blocked actions

– if $K^{(n)} = K'^{(n-1)}$ --> a(K)=a(K')/2

– the step scaling function is

$$s_b(K)=\lim_{n_b \to \infty} (K-K')$$

Every RG transformation should predict the same $s_b(K)$, but

- the location of the FP depends on the RG transformation
- tuning the free parameters in the RG transformation can pull the FP and its RT close, reducing systematical errors

2- lattice matching MCRG - in practice:

Identifies matched couplings (β,β') by comparing expectations values after n_b $(n_b -1)$ RG blocking steps

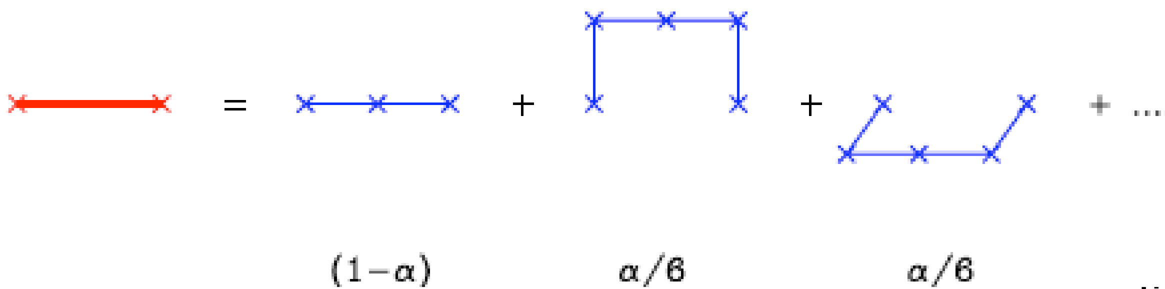
- Can be optimized by tuning the free parameter(s) of the RG transformation
- Finite volume effects are largely controlled
- Requires relatively small statistics
- Has a lot of built-in consistency checks
  - compare several blocking levels
  - compare several operators
  - compare different RG transformations

Renormalization Group transformation

A real space block transformation averages out the short distance modes

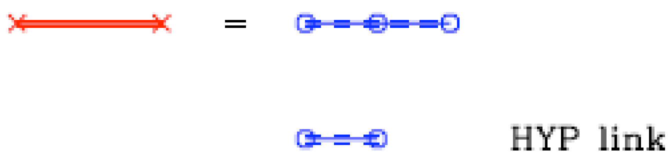
Many possibilities - I tried 2 types:

Original



optimize with α

HYP



optimize with α_1

(play with α_2, α_3)

Simulations & results

- I use nHYP smeared staggered fermions, N_f flavors in fundamental representation
- Start with well understood models:
  - N_f = 0,4,8 : QCD like
  - N_f = 16 : conformal with IRFP
  - N_f = 12 : borderline; needs a new approach

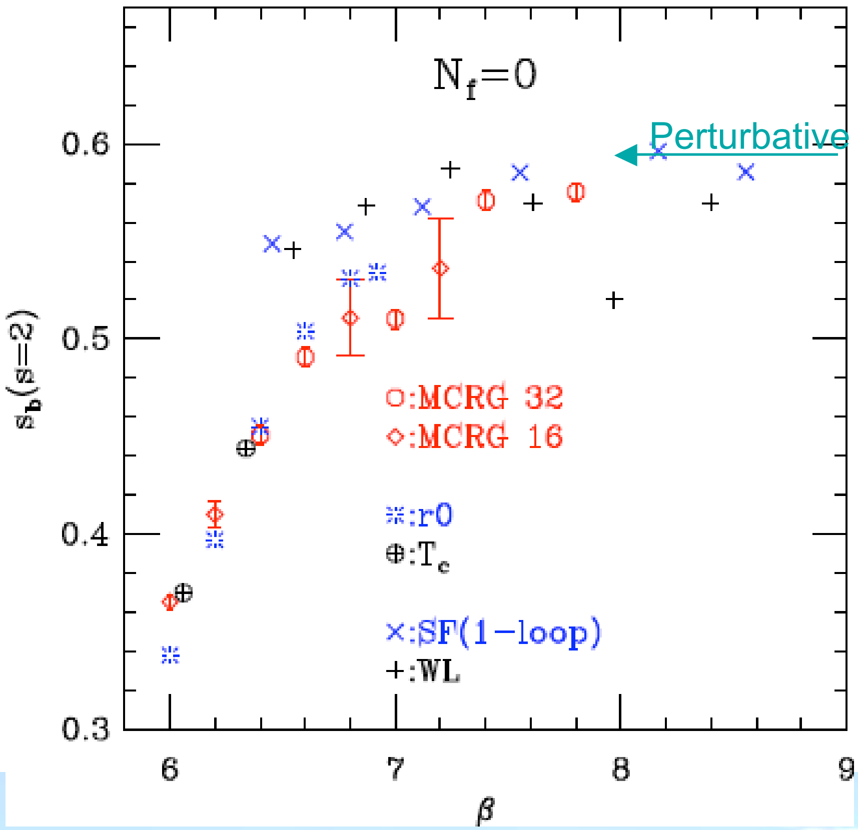
$\rightarrow$ Consider different block transformations:

  - Cover deeper range of couplings
  - Might distinguish QCD-like, walikng and conformal behavior

SU(3) pure gauge - test case

The bare step scaling function can be calculated in many ways

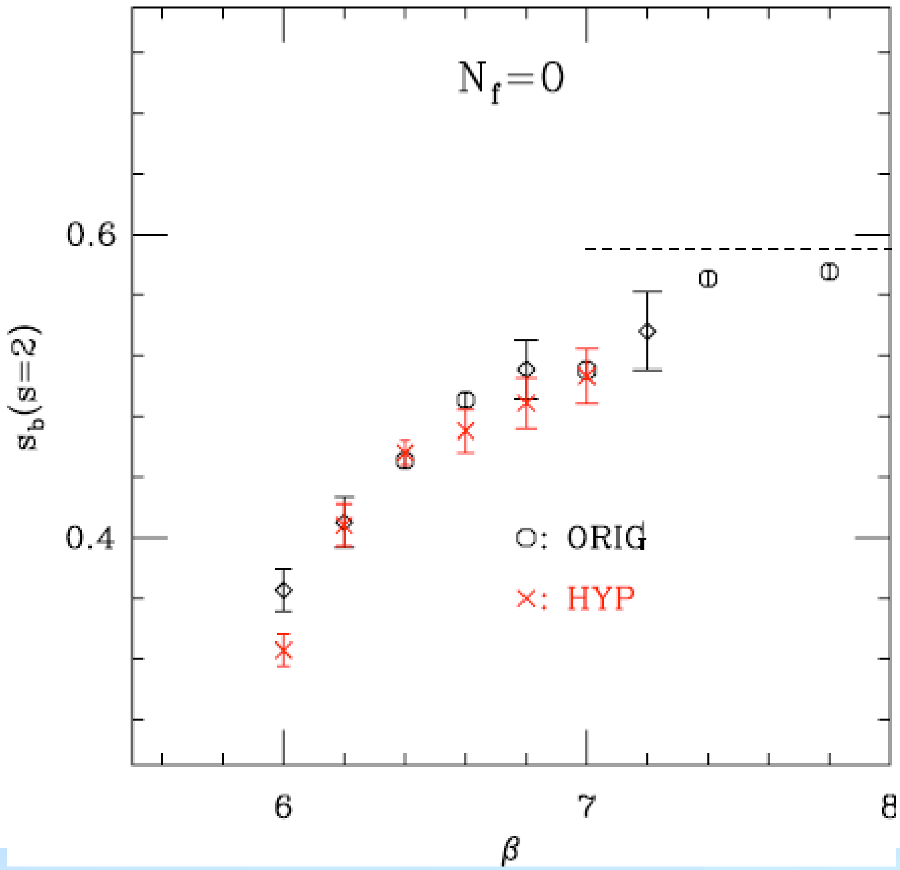
- Schrodinger fn; Wilson loop ratios,
- physical observables r_0, T_c
- RG matching: $32^4 \rightarrow 16^4$ and $16^4 \rightarrow 8^4$



- Excellent agreement between r_0, T_c and MCRG
- Both SF and MCRG approach the perturbative value
- Since at β=6 we can test confinement, we know there is no physical IRFP

SU(3) pure gauge - test case

- Compare different RG transformations:

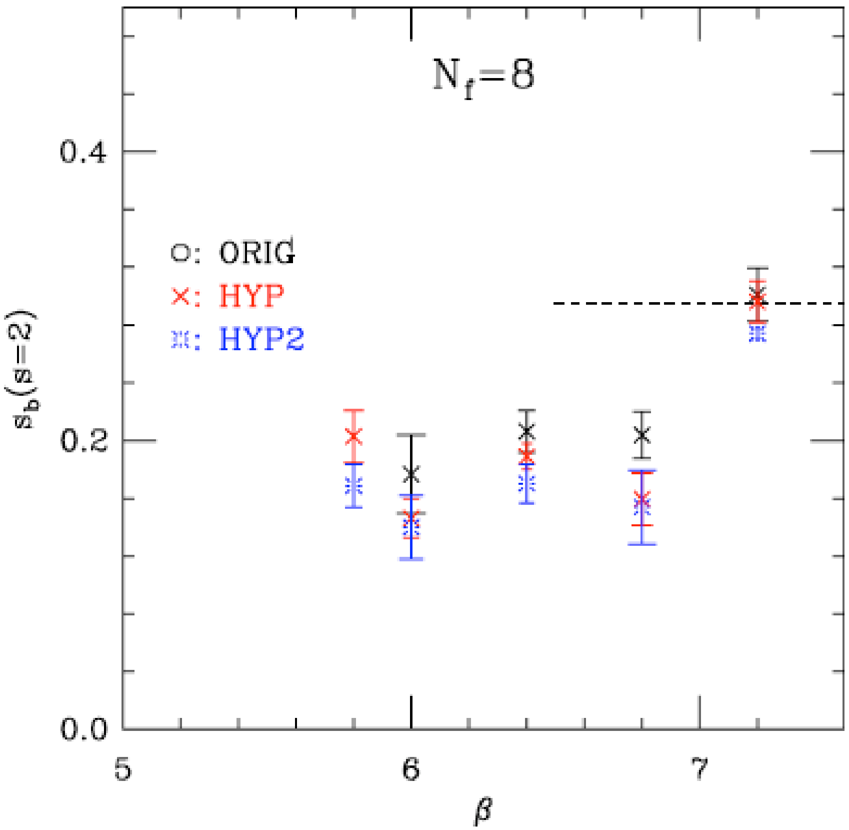


Excellent agreement between the 2 RG blockings

When the RG flow is governed by an UVFP, $s_b(\beta)$ is the property of the action, independent of the RG.

N_f=8 flavors

- Compare different RG transformations:



N_f=8 is QCD-like, RG flow is governed by the UVFP at g=0

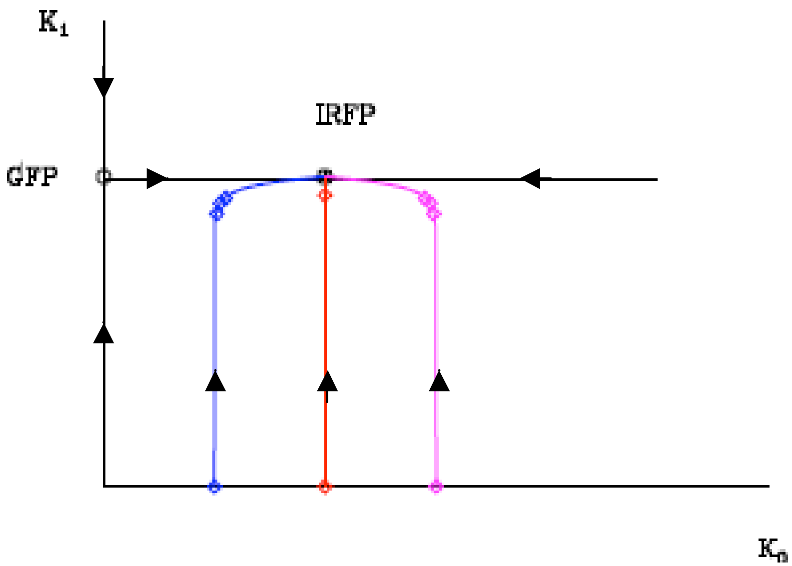
Good agreement between the 3 RG blockings

N_f=16 flavor SU(3) model

Do we see a difference?

On the critical surface around an IRFP the flows converge to the FP when $n_b \to \infty$

With finite n_b the flow picks up the slowest flowing operator



The location of the IRFP depends on
The RG transfomation

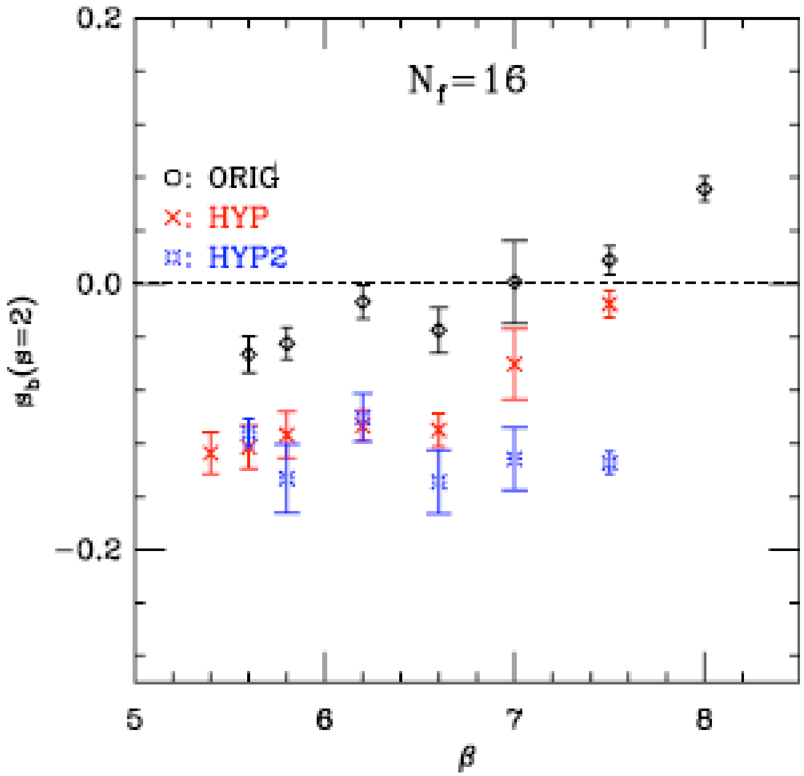
$s_b(\beta)$ can depend on the blocking
(scheme dependent)

This is a signal for non-QCD-like
behavior

N_f=16 flavor SU(3) model

Do we see a difference?

$16^4 \to 8^4$ MCRG



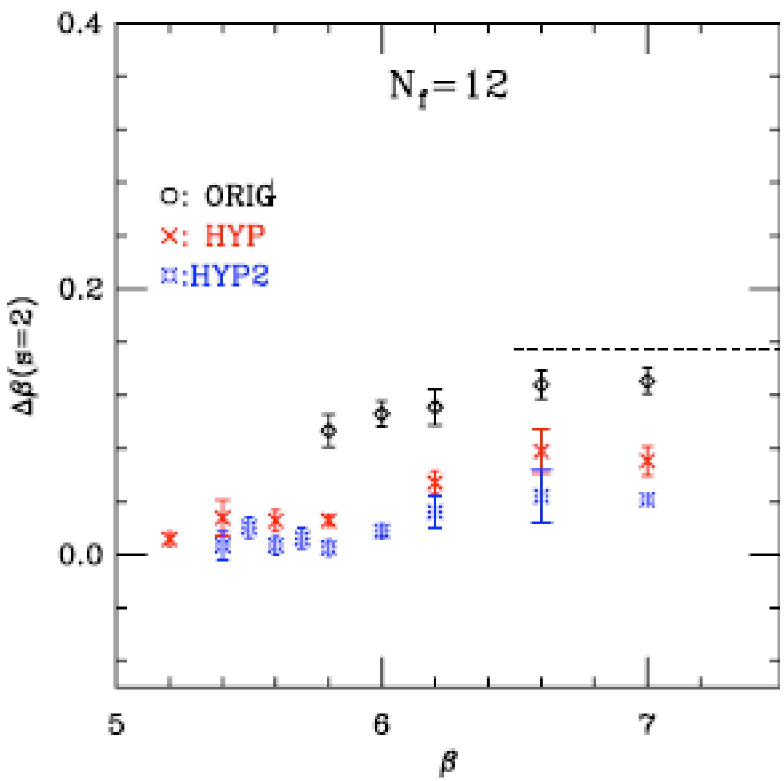
ORIG blocking shows $s_b(\beta)$=0 around β=7.0

HYP blocking has an IRFP around β=8.0

HYP2 blocking (α_2,α_3 different) is yet an other IRFP

N_f=12 flavors

Use the same techniques as before; $16^4 \rightarrow 8^4$



- Step scaling function connects to perturbative regime, remains positive

- HYP blocking predicts different $s_b(\beta)$~0
  - Could be scaling violation, but β=5.0-7.0 is very weak coupling
  - Could develop an IRFP later
  - Could be "walking", the flow is governed by the small value of the β function

N_f=12 flavors

Flow is not (fully) controlled by the perturbative UVFP

- Can one find the IRFP (back flow) or find a confining phase?
  - RG matching dies for β <5.2 (strong lattice artifacts)
  - In the range β = (2.5,5.0) there is no sign for a bulk phase transition
  - 16^4 lattices are not confined at β =4.5,
    $8^3\times$ 16 lattices are not confined at β =2.5
  - β =2.5 is deep in strong coupling

Conclusion

MCRG is an effective alternative method to study the phase structure and scaling properties of lattice QFT's

- o The method is very universal, straightforward to implement for any other system (sextet fermions are under study)
- o MCRG requires only limited statistics
- o MCRG can predict the anomalous dimension of the mass

N_f=0-8,16 as expected. N_f=12 is difficult:

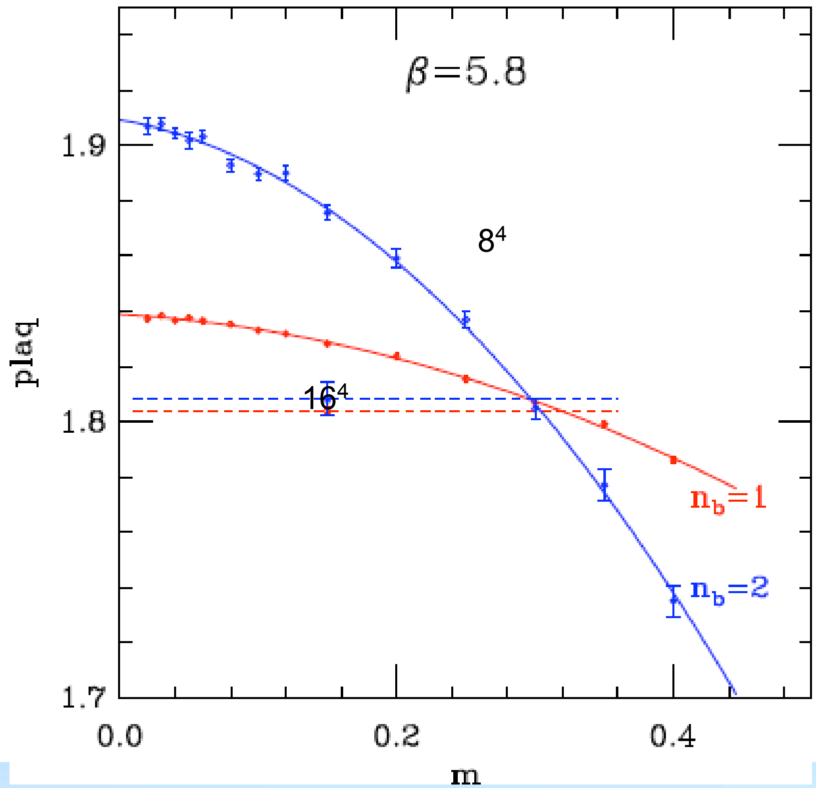
- – $s_b(\beta)$ hovers above zero, the flow is extremely slow
- – Different RG transformations predict different flow
    - • Flow is not controlled by the perturbative UVFP

MCRG to find the mass anomalous dimension

N_f=16 flavor SU(3) model

Matching in the mass at fixed $\beta = 5.8$

$m_2 = m_1\, 2^{1/\nu}$

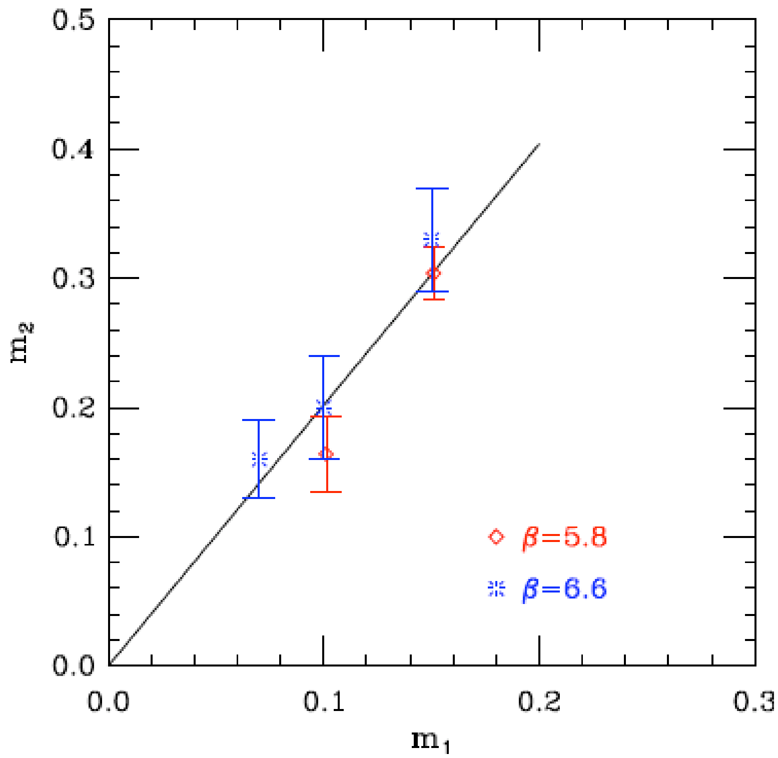


- use the same gauge observables (probably not the best choice)

-at α_{opt} both n_b=2(1) and 3(2) predicts the same matching pair

The critical exponent for the mass

At several couplings, mass values



$m_2 = m_1\, 2^{1/\nu}$

ν=1.0(1)

Free field exponent (close to GFP)