Lattice studies of nearly conformal or really conformal four-dimensional gauge theories

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work done in collaboration with:
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Outline Part I: SU(2) with 4 adjoint Majorana flavors

- We describe our lattice data and try to characterize it with phenomenological formulas.
- We have tuned the bare mass, with significant computational efforts to study the small mass regime.
- Some open questions are highlighted.
- Because of lattice studies of Hietanan et al., we work under the assumption that MWTC has an IRFP, and we are driven away by

$$m_q \neq 0$$

- In addition we have to account for O(a) χ SB of lattice fermion and finite volume effects

$$V_3 = L^3, \quad L \neq \infty$$

Outline Part II: N=4 Super-Yang-Mills Using Ginsparg-Wilson Fermions

- The regulator breaks symmetry, and there exist lattice artifacts that do not automatically vanish as $a \to 0$.
- Minimize fine-tuning by preserving $SU(4)_R$
- Purely bosonic fine-tuning
- Fine-tuning by offline reweighting
- Multicanonical simulations to beat overlap problem
- Initial studies of Overlap (Neuberger) Fermion perturbative counterterms
- Goal: GPGPU code in next year or three.

An aside…

- My thesis supervisor used to warn against getting lost in two dimensions.
- I have discovered that on the lattice it is easy to get lost in four dimensions
  - Numerically demanding
  - Need for improvement (small L thus large a)
  - Need for efficient parallel code
  - Gradually drift into all-consuming computer science exercise

Ingredients of Part I studies

Quite standard…we want to look at chiral symmetry breaking, which is not spontaneous if theory has IRFP

$$G^{ab}_{PP}(t) = \int d^3x \, \langle P^a(t,\mathbf{x}) P^b(0,\mathbf{0}) \rangle, \quad G^{ab}_{AP}(t) = \int d^3x \, \langle A^a_0(t,\mathbf{x}) P^b(0,\mathbf{0}) \rangle, \tag{1}$$

where $P^a = \bar{\psi}\gamma_5 t^a \psi$ and $A^a_0 = \bar{\psi}\gamma_0\gamma_5 t^a \psi$, with $t^a \in \{\sigma^+, \sigma^-, \sigma^3\}$. For brevity we suppress the isospin indices a, b and leave it as implied that nonvanishing components G^{+-} (i.e. the ones without disconnected diagrams) of the Green's functions are used in the measurements.

$$G_{PCAC}(t) = \frac{\partial_t G_{AP}(t)}{G_{PP}(t)} \approx \frac{2 Z_m Z_P m_q}{Z_A} \equiv 2m_{\mathrm{PCAC}}, \quad 0 \ll t \ll T.$$

Measuring f_π , m_π

$$G_{PP}(t) = C_{PP}\cosh[m_\pi(\tfrac{T}{2} - t)]$$

$$f_\pi = (\tfrac{C_{PP}}{m_\pi^3})^{1/2} 2 Z_m Z_P m_q e^{m_\pi T/4}$$

So combined with the PCAC mass on the previous slide, we obtain f_π / Z_A.

4 Majorana Fermions χ

- Take a look at the mass term, with generic matrix M:

$$\mathrm{Tr}\chi\chi^T M = \chi_j^\alpha \chi_{\alpha i} M_{ij}$$

- SU(4) flavor symmetry $\chi \rightarrow \mathrm{V}\ \chi$ implies:

$$\Phi = \chi\chi^T \rightarrow V\Phi V^T$$

- Invariant if mass also transforms:

$$M \rightarrow V^* M V^\dagger$$

- This dictates mass term: $\mathrm{Tr}\Phi M + \mathrm{h.c.}$

Minimal Walking Technicolor

- 2 colors, 4 Majorana flavors (adjoint)
- Theory either nearly conformal or conformal
- Recent lattice results (Hietanen et al, 09) suggest theory is conformal (nontrivial IRFP).
- Good test-bed for studying conformal theories on lattice
- We can refine techniques, see what works, learn lessons, before tackling the much harder problem of N=4 SYM.

If we were to carry over our QCD intuition…

- Explicit chiral symmetry breaking
- Should be order parameter of that
- Use fermion bilinear discussed above
- Write general effective theory for that order parameter
- Take limit of very small explicit chiral symmetry breaking

Linear σ model description – order parameter of symmetry breaking (LG)

$$p^2 \sim m \sim a$$

Introduce the explicit O(a) breaking: $A = Wa$

Standard spurion analysis

$$\mathcal{L} = \mathrm{Tr}[\Phi(M+A) + \mathrm{h.c.}]$$
$$+ \sum_k g_k \mathrm{Tr}(\Phi\Phi^\dagger)^k + \mathrm{Tr}|F(\Phi\Phi^\dagger)\partial_\mu \Phi|^2$$
$$+ \sum_{k,\ell} y_{k,\ell} \mathrm{Tr}(\Phi\Phi^\dagger)^k \mathrm{Tr}(\Phi\Phi^\dagger)^\ell + \cdots$$

Double-trace $1/N_C$ supressed, for fundamental flavors,etc.
Coleman-Witten 1980, eq. (3) assumption.

NGB (if they exist) or "conformal pions"

The pions enter through polar decomposition: $\Phi = HU$

In case that <H> = v 1 (unit matrix), and <U>=1 (unit matrix), U parameterizes SU(4)/SO(4).

Since $\mathrm{Tr}\Phi M + \mathrm{h.c.}$ is our mass term, we (naively) get

$$M_\pi^2 \propto M$$

Where we get into trouble: v=v(M) if only explicit breaking of chiral symmetry (IRFP).

As a result, we have no idea how M_π depends on M.

I.e., chiral perturbation theory is useless.

Expectations

- If IRFP, no χ SB, so no reason to expect light pions.
- We don't know if $\bar{\psi}\gamma_5 t_a \psi$ is an interpolating operator for a primary state in the CFT.
- Let's turn to the numerical approach and just look at states
- Also measure current mass "m_q" and decay constant "f_π" even though meaning in CFT is totally unclear, since χ PT is rubbish.

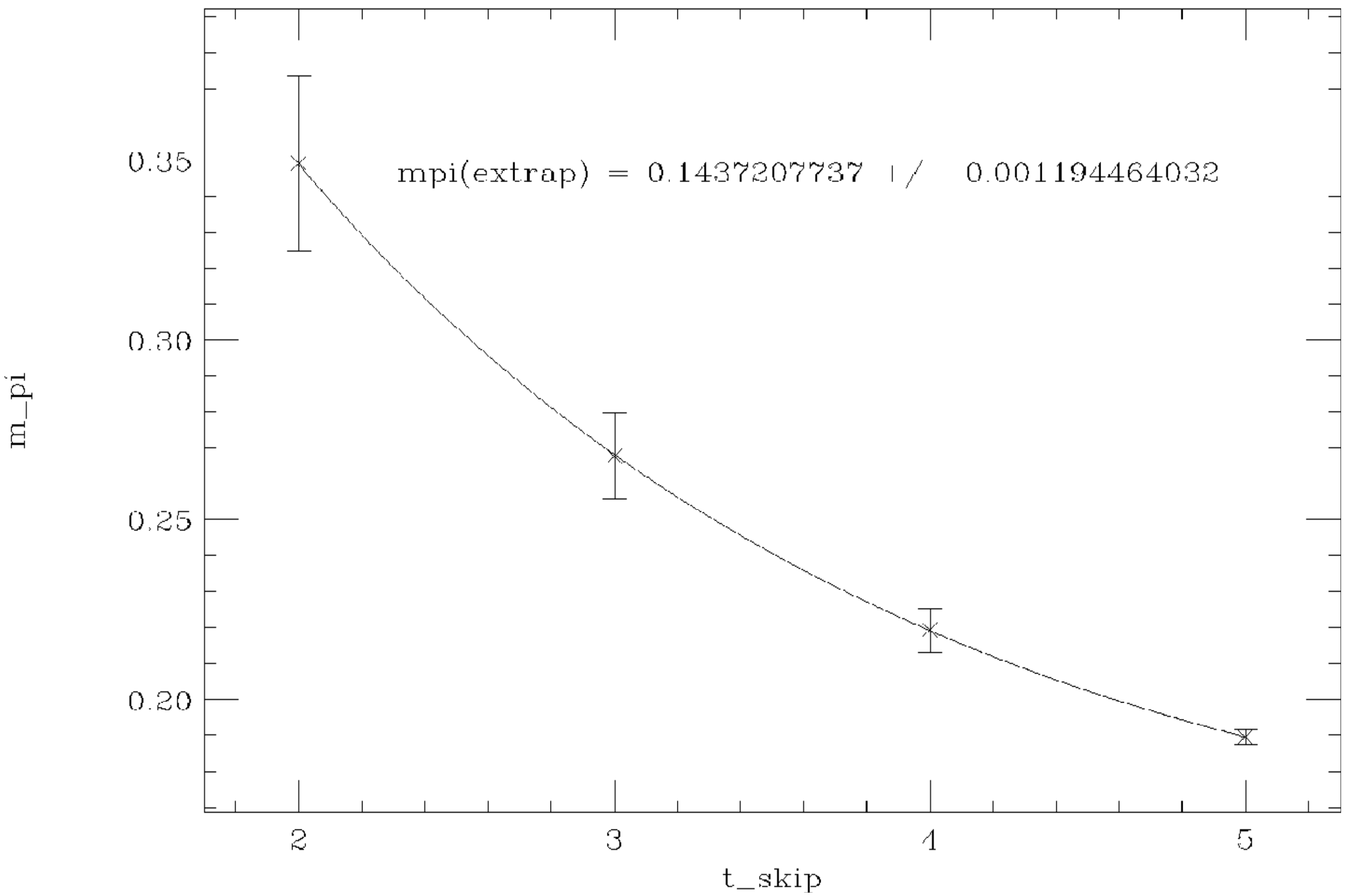
Measurements

- We looked in great detail at the dependence on how the fit was performed.
- We used unimproved Wilson valence quarks.
- Future work: nHYP smeared clover and mass reweighting, as was done by [Hasenfratz, Hoffmann, Schaefer 08].
- We also explored mixed action: unimproved sea quarks, clover valence, and found that very light masses could be reached.
- However, unknown systematic errors --- perhaps could be nonperturbatively corrected for by det reweighting scheme.

t_{first} and t_{skip}

- We are measuring exponential decay in correlation fuctions $G(t) \sim \exp(-m\ t)$
- However, excited states m' also contribute, more so at small t.
- To identify the t regime where the excited state contribution is sufficiently suppressed, we only do a **cosh**[m((T/2)-t)] fit for times in the range $t_{first} \leq t \leq T-t_{first}$.
- A related parameter is $t_{skip} = t_{first}-1$.
- Also note the lattice inverse coupling parameter $\beta=4/g^2$

Pion mass near critical m_0, T=16



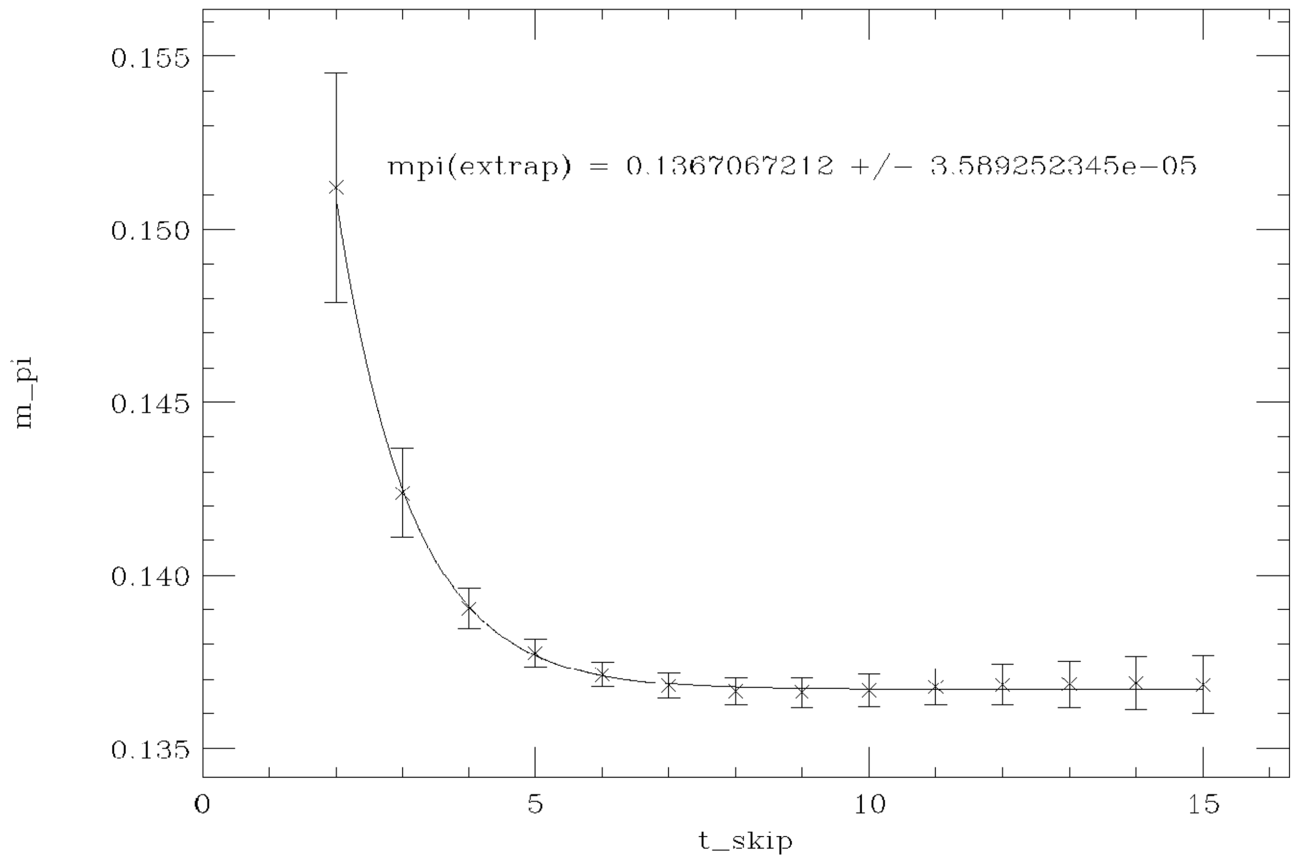
Apologies for obscene error format -- these are automated plots that need to be polished

β = 2.5, 8^3 x 16, m= -1.1

No plateau

- In the previous slide, no plateau is seen
- So, the mass extraction is not reliable
- Must extend time direction to find plateau
- That is shown in next slide

Pion mass near critical m_0, T=64



β = 2.5, 8^3 x 64, m= -1.1

Pion mass near critical m0

- Thus we see that some care is required
- 0.35 to 0.20 if just use data on T=16 lattice
- 0.144(1) for extrapolation on T=16, of form:

$$Q(t_{first}) = Q_\infty + c_0 \exp(-c_1 t_{first})$$

- Cf., 0.13671(4) on T=64 lattice is reliable
- $m_\pi L=1.1$
- $m_\pi T=2.2, 8.8$
- $m_q L < 1$ (I.e. $L < \lambda_q$! We're always squeezing it – don't see how to avoid such violence if IRFP. One approach: hold $m_q L$ = fixed, take $L \to \infty$.)

Other quantities

- m_ρ
- f_π
- R = $(f_\pi m_\pi)^2/m_q^2$ = Σ / m_q
- Each requires care as we approach the critical mass. Generally get good fit from

$$Q(t_{first}) = Q_\infty + c_0 \exp(-c_1 t_{first})$$

- We've not found any massless $\pi^\pm$ as would occur in Aoki phase [Aoki 84, 86; Sharpe, Singleton 98]

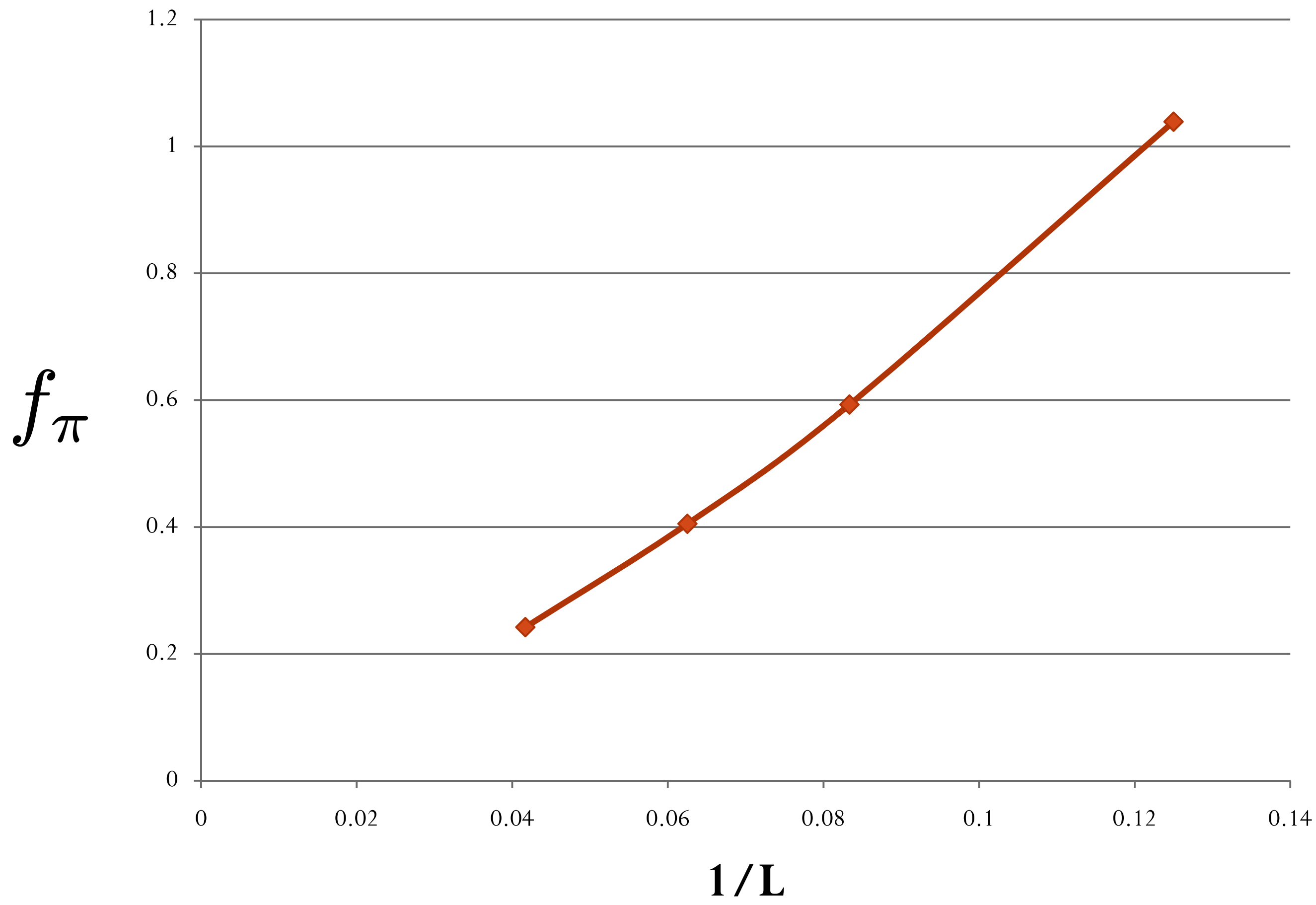
f_π and R= $(f_\pi m_\pi)^2/m_q^2$ show decrease with L

L	ma	$m_\pi a$	$m_\rho a$	$f_\pi a$	$m_q a$	Ra^2
8	-1.1	0.13625(7)	0.14531(5)	1.039(12)	0.03834(3)	13.621(15)
12	-1.1	0.12260(7)	0.13537(15)	0.593(11)	0.02900(11)	6.091(6)
16	-1.1	0.1204(2)	0.1251(7)	0.405(5)	0.0284(4)	2.957(13)
24	-1.1	0.1344(12)	0.1497(6)	0.242(2)	0.0266(3)	1.49(3)

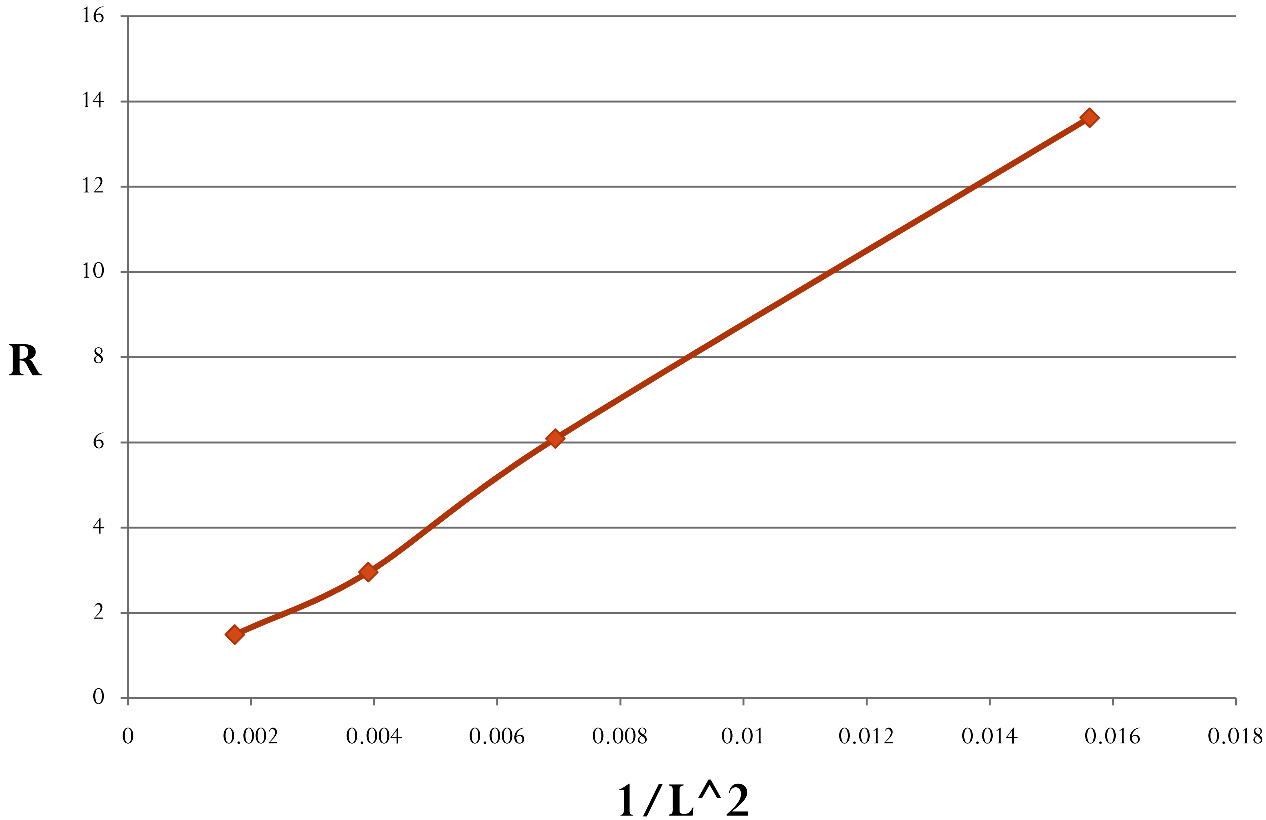
Table 1: Estimates for the $\beta = 2.5$, $L^3 \times 64$ PBC lattice, with unimproved Wilson quarks.

Pion basically flat, rho only slightly heavier, m_q, f_π , R, decreasing

f_π approaches zero



$R=\Sigma/m_q$ appears to vanish as $L \to \infty$



Evidence for...?

- Clearly, this is tantalizing b/c if IRFP exists then f_π and Σ should vanish in the limit

$$m_q \to 0, \quad L \to \infty, \quad a \to 0$$

- However, we now have to carefully study the effects of these three quantities being away from the limit
- Also, one should keep in mind that L is introduced to set the scale if we are really dealing w/ a CFT.
- In a QCD-like theory we would have the additional scale Λ_{QCD}

A first guess at phenomenological fit

- We can roughly fit the $\beta = 2.5$ data with

$$f_\pi = c m_q^\delta L^{\delta-1} + O(a\Lambda_U^2), \quad m_\pi = c' m_q^{1-\delta} L^{-\delta} + O(a\Lambda_U^2)$$

- Doesn't explain the increase in m_π as we go from L=16 to L=24
- In fact, that behavior is at odds w/ a variational argument, as was pointed out to me by R. Brower (periodically extend L=12 pion wavefunction, same E, but we expect there will be a lower E state by variational analysis – expand basis on larger lattice).
- There may be something topological happening at the L=16 to L=24 transition.
- Same behavior seen at stronger coupling…

Stronger coupling

L	ma	$m_q a$	$m_\pi a$	$m_\rho a$	$f_\pi a$	Ra^2
8	-1.31	0.0253(2)	0.1433(6)	0.1530(9)	0.738(9)	19.55(8)
12	-1.31	0.015236(63)	0.1215(17)	0.1547(24)	0.4598(29)	13.83(14)
16	-1.31	0.01214(16)	0.1075(15)	0.1531(25)	0.406(10)	13.854(76)
24	-1.31	0.00800(11)	0.1254(42)	0.1770(59)	0.1743(46)	8.25(25)

Table 1: Quantities of interest for the $\beta = 2.05$, $L^3 \times 32$ PBC lattice, with unimproved Wilson fermions.

- Decrease in m_q much larger – but note behavior of pion!
- Splitting $m_\rho - m_\pi$ larger
- Falloff with 1/L murkier since m_q changes by factor of 3

Comparison to pheno. fit

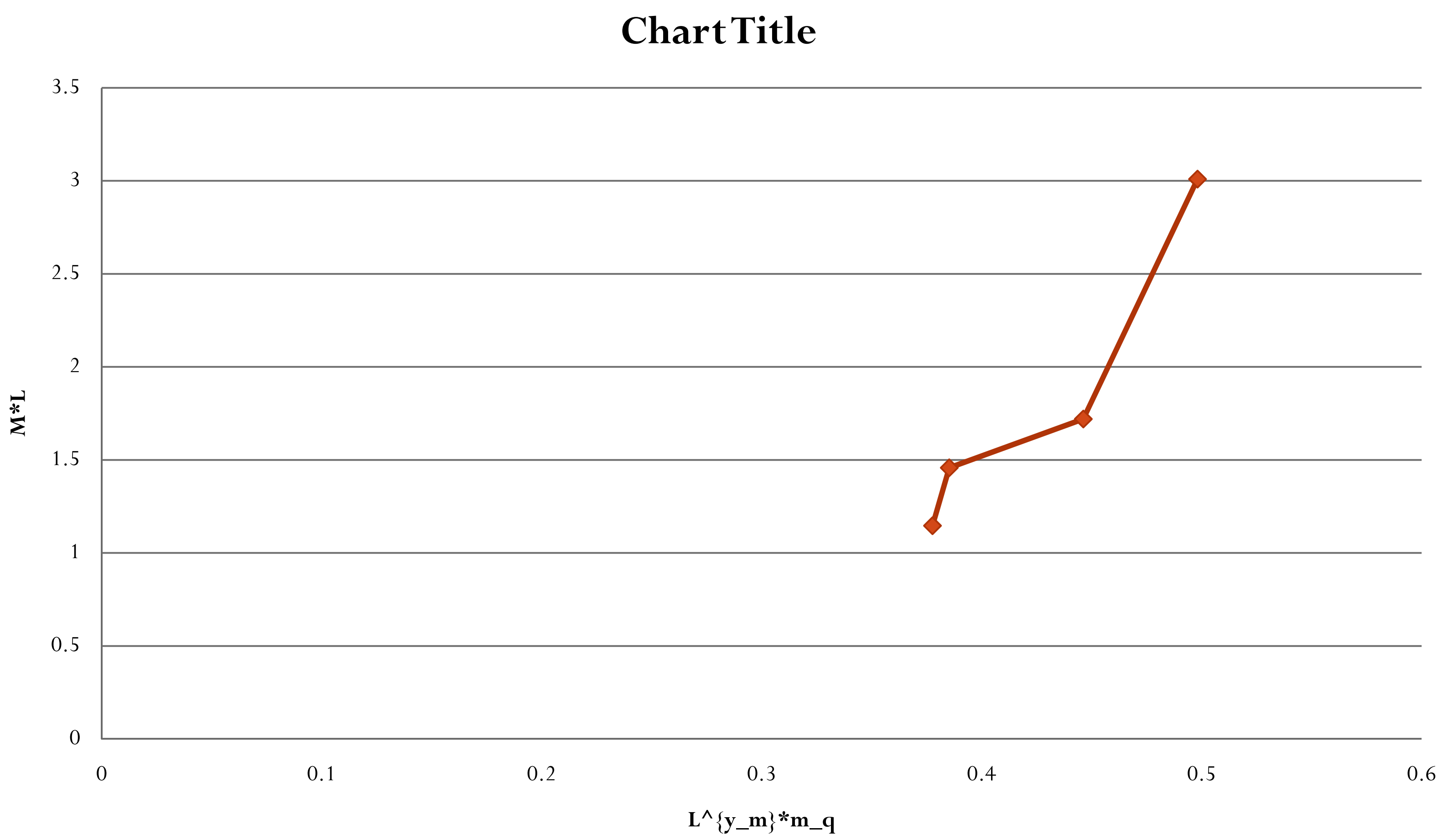
- In this $\beta = 2.05$ data, m_q L = const.

$$f_\pi = c m_q^\delta L^{\delta-1}, \quad m_\pi = c' m_q (m_q L)^{-\delta}$$

- But we do not see the 3-fold decrease in m_π
- Seems we need a more general form.

- Next, look carefully at two features of $\beta = 2.05$ results:

Finite size scaling with $y_m = 1.3$



Constituent mass, V-P splitting

L	$m_q a$	m_π / m_q	$(m_\rho - m_\pi)/m_\pi$
8	0.0253(2)	5.664(51)	0.0677(77)
12	0.015236(63)	7.97(12)	0.273(27)
16	0.01214(16)	8.86(17)	0.424(31)
24	0.00800(11)	15.68(57)	0.411(67)

Table 1: Pion mass and rho-pion splitting enhancement as m_q and $1/L$ are decreased, for the $\beta = 2.05$, $L^3 \times 32$ PBC lattice, with unimproved Wilson fermions.

- As m_q gets lighter, constituent quark mass becomes as much as 8 times the current quark mass $\rightarrow$ strong coupling effects, light quark limit
- The vector-pseudoscalar splitting becomes noticeable in this limit

Attempt to generalise: Suppose ∃ IRFP

- 3 dimensionful quantities: m_q, a, L
- W/o loss of gen.

$$f_\pi = m_q F(m_q a, \frac{1}{m_q L})$$

- f_π will be finite (nonzero) if any of 3 limits are taken separately (with other 2 quantities held fixed):

$$\lim_{a\to 0} f_\pi, \quad \lim_{L\to\infty} f_\pi, \quad \lim_{m_q\to 0} f_\pi$$

- A nonvanishing result for the last limit is b/c the Pauli term is generated radiatively at finite *a*.

Explaining $f_\pi \sim 1/L$

- We cannot take $m_q \to 0$ and $L \to \infty$ simultaneously b/c the CFT needs an IR cutoff, to avoid IR singularities.
- If $m_q L \gg 1$, then $L \gg 1/m_q$ and m_q is the cutoff.
- If the $m_q \to 0$ lim. of f_π exists, then we need

$$F(m_q a, \frac{1}{m_q L}) \sim \frac{1}{L}$$

- We should have a power series in $1/(m_q L)$, so long as m_q is finite. (There is an IR cutoff.) Thus:

$$f_\pi = m_q F(m_q a, 0) + \frac{1}{L} F'(m_q a, 0) + \frac{1}{m_q L^2} F''(m_q a, 0) + \cdots$$

- Our f_π data is explained if the 2^{nd} term dominates.
- It may be that the difference operator used in forming f_π suppresses the first term.

m_π leads to contradictions

- By similar arguments, we can write: $m_\pi = m_q G(m_q a, \frac{1}{m_q L})$

$$m_\pi = m_q G(m_q a, 0) + \frac{1}{L} G'(m_q a, 0) + \frac{1}{m_q L^2} G''(m_q a, 0) + \cdots$$

- From this perspective, it is difficult to understand the very mild dependence on m_q and L seen in the data.
- It seems to want a singularity

$$G(m_q a, 0) \sim \frac{1}{m_q a}$$

- The ρ data is similarly difficult to explain.
- An $m_\pi \sim 1/a$ dependence may be just a result of having broken conformality badly and the fact that with a CFT we don't have a clean separation between UV and IR.

Clover tests

- We have also tested the clover propagator, which includes the Pauli term

$$aF_{\mu\nu}\bar{\psi}\sigma_{\mu\nu}\psi$$

  on the unimproved Wilson sea configs. we have.
- It shows that we can get m_π a < 0.05.
- Is it a mismatch systematic error or a real reduction in m_π?
- Does it suggest we need a "perfect action" to adequately study an IRFP on the lattice?
- Does it rule out the interpretation that pions, rhos might not be part of the CFT spectrum?

Part I Conclusion

- The MWTC gauge theory might flow to an IRFP.
- This would be consistent with the result of Hietanen, Rummukainen, Tuominen [0904.0864].
- They saw forward/backward running
- That is supposedly smoking gun for IRFP,
- But lattice artifacts need to be brought under control
- One mystery to solve in our data in such an interpretation:

  Why do we not see a large decrease in m_π, m_ρ ?
- More points in m_q, L will clarify behavior further – in progress.

Part II: N=4 SYM head-on w/ overlap

- Can preserve $SU(4)_R$ symmetry
- Limits number of counterterms
- SU(2), SU(3): $m_\phi^2, Z_\phi, \lambda_1, \lambda_2$
- SU(N>3): $m_\phi^2, Z_\phi, \lambda_1, \ldots, \lambda_4$
- On small lattices at weak coupling, we can start the scan with one-loop counterterms
- Overlap perturbation theory calculation in progress
- Beginning with 4d Wess-Zumino model as a warm-up for all the methods (student, Chen Chen)
- Goal: GPGPU implementation in next 1-3 years.

Symmetries of the overlap N=4 SYM lattice action

This action possesses an exact $SU(4)_R$ symmetry, with the scalars transforming as in the continuum and the fermions transforming according to

$$\begin{aligned}
\delta\psi/i\epsilon &= (T\hat{P}_L - T^*\hat{P}_R)\psi\,,\\
\delta\Psi/i\epsilon &= (T+T^*)\gamma_5 D\psi + (TP_L - T^*P_R)\Psi,\\
\delta\bar{\psi}/i\epsilon &= \bar{\psi}\,(T^*P_L - TP_R) + \bar{\Psi}(T+T^*)\gamma_5\,,\\
\delta\bar{\Psi}/i\epsilon &= -\bar{\Psi}\,(TP_L - T^*P_R)\,.
\end{aligned}$$

Here $\hat{P}_{L/R} \equiv \frac{1}{2}(1\pm\hat{\gamma}_5) = 1/2(1\pm\gamma_5(1-2D))$ are the lattice modified chiral projection operators, T is the generator of $SU(4)_R$ in the fundamental ($\mathbf{4}$) and we have suppressed the $SU(4)_R$ indices.

Here, Ψ is an auxiliary fermion

E.g., the quartic terms allowed by symmetry

The quartic interaction terms in the SU(2) and SU(3) case are:

$$\lambda_1 \mathrm{Tr}\phi_m\phi_n\phi_m\phi_n + \lambda_2 \mathrm{Tr}\phi_m\phi_m\phi_n\phi_n.$$

SUSY corresponds to

$$\lambda_1 = 1/g^2, \quad \lambda_2 = -1/g^2.$$

In the case of $SU(N_c > 3)$, two more four quartic terms should be included

$$\lambda_3 \mathrm{Tr}\phi_m\phi_n \mathrm{Tr}\phi_m\phi_n + \lambda_4 \mathrm{Tr}\phi_m\phi_m \mathrm{Tr}\phi_n\phi_n.$$

One benefit: reality

It is easy to see that the fermion measure is real. In the field space (ψ, Ψ) the fermion matrix has the 2×2 block form:

$$\mathcal{M} = \begin{pmatrix} D + M_Y & M_Y \\ M_Y & M_Y - 1 \end{pmatrix}, \quad M_Y = y\sqrt{2}\left(\phi^{ij} P_L - (\phi^{ij})^* P_R\right).$$

Since $\gamma_5 D^\dagger \gamma_5 = D$ and similarly for M_Y, we have

$$(\det \mathcal{M})^* = \det \mathcal{M}^\dagger = \det \gamma_5 \mathcal{M}^\dagger \gamma_5 = \det \mathcal{M}.$$

The sign of the determinant may fluctuate.

Multicanonical reweighting

One replaces S with

$$S_{MCRW} = S + W[\mathcal{O}_1, \mathcal{O}_2, \ldots], \tag{1}$$

where $W[\mathcal{O}_1, \mathcal{O}_2, \ldots]$ is a carefully engineered function of some small set of observables. For instance in the $\mathcal{N} = 4$ SYM case W will be a function of $\int d^4x\ \phi^2$, the distinct quartic terms $\int d^4x\ \phi^4$ and the kinetic term $\int d^4x\ (D\phi)^2$. The (reweighted) expectation value of an observable in the distribution corresponding to S_{MCRW} is:

$$\langle \mathcal{O} \rangle = \frac{\sum_{C \in F(n)} \mathcal{O}_C\ \exp W[\mathcal{O}_1^C, ...]}{\sum_{C \in F(n)} \exp W[\mathcal{O}_1^C, ...]}. \tag{2}$$

Conclusions: Part II

- Difficult but probably possible to study N=4 SYM on small lattices by this method (in near future)
- Challenging project to engineer the multicanonical function $W[O_1,\ldots]$: bootstrap approach seems best – automate it
- Present stage: code development and perturbative starting points.
- Complementary: Work with S. Catterall, E. Dzienkowski, A. Joseph to find 1-loop counterterms for the twisted N=4 SYM approach – seems to have some amazing nonrenormalization properties (talk by Catterall)

Postdoctoral position at RPI

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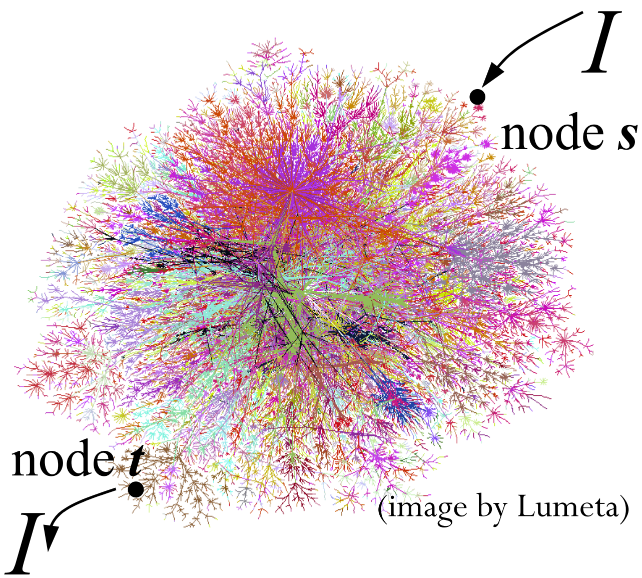

(image by Lumeta)