Supersymmetric Yang-Mills with domain wall fermions

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Lattice gauge theory for LHC physics

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Supersymmetry (SUSY) on the lattice

- Much effort devoted toward formulating supersymmetric lattice theories
  - fascinating theoretical challenge
  - potential role of SUSY in beyond the standard model physics
  - desire to better understand various nonperturbative aspects of SUSY theories via numerical simulation

SUSY on the lattice

- Naive lattice discretization explicitly breaks SUSY:

(supercharges) (generator of translations)

$$\{\bar{Q}, Q\} \sim P$$

- Undesirable SUSY violating operators may arise through radiative corrections
- Fine-tuning of operators is required to reach SUSY limit

SUSY on the lattice

N=1 SYM is the only four-dimensional SUSY theory that can be studied using conventional lattice discretizations and without-fine tuning

- Only relevant SUSY violating operator allowed by gauge and hypercubic symmetries is a gluino mass term
- Chiral symmetry realized on lattice implies SUSY restoration in continuum limit (e.g., overlap or domain wall fermions)

N=1 super Yang-Mills (SYM)

$$L = \frac{1}{g^2}\left[\bar{\lambda}\gamma_\mu D_\mu \lambda + \frac{1}{4} v_{\mu\nu} v_{\mu\nu}\right] \ , \qquad \bar{\lambda} = \lambda^T C$$

- One of the simplest of SUSY gauge theories in terms of field content
  - 1 vector field and 1 adjoint Majorana fermion (gluino)
  - a single input parameter, the gauge coupling (g)
- Anomalous $U(1)_R$ (chiral) symmetry: $\lambda \to e^{-i\alpha\gamma_5}\lambda \Rightarrow \theta \to \theta - 2N_c\alpha$
- Z_{2N_c} subgroup of $U(1)_R$ survives at quantum level
  - partition function invariant for: $\alpha = \frac{\pi k}{N_c} \ , \qquad k = 0, \ldots, 2N_c - 1$

N=1 SYM

$$L = \frac{1}{g^2}\left[\bar{\lambda}\gamma_\mu D_\mu \lambda + \frac{1}{4}v_{\mu\nu}v_{\mu\nu}\right] \ , \qquad \bar{\lambda} = \lambda^T C$$

- Gluino condensation: $\langle \bar{\lambda}\lambda \rangle \neq 0$
  - discrete chiral symmetry breaking: $Z_{2N_c} \rightarrow Z_2$
- Confinement
  - colorless bound states (glue-glue, glue-gluino, gluino-gluino)
- No SUSY breaking (non-vanishing Witten index)

N=1 SYM

$$L = \frac{1}{g^2}\left[\bar{\lambda}\gamma_\mu D_\mu \lambda + \frac{1}{4} v_{\mu\nu} v_{\mu\nu}\right] \ , \qquad \bar{\lambda} = \lambda^T C$$

- Why study N=1 SYM via numerical simulations?
  - natural starting point: simplest of SUSY theories, can simulate with well understood lattice actions
  - test exact theoretical predictions about SYM (e.g., discrete chiral symmetry breaking)
  - access to nonperturbative aspects of SYM, for which less is known (e.g., spectrum)

Numerical studies of N=1 SYM

- Wilson
  - DESY-Muenster-Roma Collaboration (hep-lat/0112007 and references therein, arXiv:0811.1964)
- Domain wall fermions
  - G. T. Fleming, et al. (hep-lat/0008009)
  - M. G. Endres (arXiv:0810.0431, arXiv:0902.4267)
  - J. Giedt, et. al. (arXiv:0807.2032, arXiv:0810.5746)

Simulation details

- Wilson gauge action (fund. rep.) + DWF (adj. rep.)
- SU(2) gauge group
- Simulations performed:
  - using modified Columbia Physics System (CPS)
  - on QCDOC at Columbia University and New York Blue (BlueGene/L) at Brookhaven National Laboratory

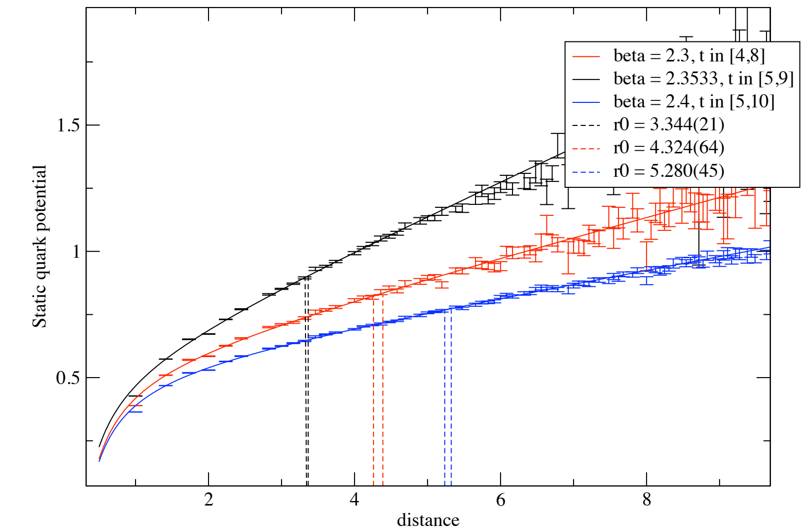
Simulation parameters

$L^3 x T$	β	L_s	m_f
$16^3 x 32$	2.3	16	0.01
			0.02
			0.04
		20	0.01
			0.02
			0.04
		24	0.01
			0.02
			0.04
		28	0.01
			0.02
			0.04
		32	0.02
		40	0.02
		48	0.02
	2.353	28	0.02
	2.4	28	0.02

$L^3 x T$	β	L_s	m_f
$8^3 x 8$	2.3	16	0.02
		20	0.02
		24	0.02

- Static potential
- Residual mass
- Gluino condensate
- Dirac spectrum
- Topological charge
- Spectrum

Summary of basic measurements



- Static potential:
  - $r_0/L \approx 0.18$ (β=2.3) to 0.30 (β=2.4)
- Residual mass:
  - $(m_f + m_{res})\, r_0 \sim 0.35$ (β=2.4) - 0.5 (β=2.3) for L_s=28
  - m_{res} is 5-10 times larger than m_f
- Gluino condensate in the chiral limit:
  - 20-25% systematic errors due to large residual chiral symmetry breaking

Dirac spectrum

- Check if DWFs are working correctly for chosen parameters
  - are low modes bound to fifth dimension boundaries?
  - does Dirac spectrum reflect expected continuum properties?
  - do the matrix elements of the chirality operator make sense?
- Alternative approach for estimating residual mass
- Determine gluino condensate via the Banks-Casher relation

Topological charge

- Check topological ergodicity
- Check consistency of topological charge and near zero-modes of Dirac operator

Continuum (Euclidean) Dirac operator

Operator:	$\gamma_\mu D_\mu$	$D = \gamma_\mu D_\mu + m_g$	$D_H = \gamma_5 D$
Eigenvalue:	$\pm i\lambda$	$\pm i\lambda + m_g$	$\pm\lambda_H = \sqrt{\lambda^2 + m_g^2}$
Eigenstate:	$\lvert \pm \lambda\rangle$	$\lvert \pm \lambda\rangle$	$\lvert\pm\lambda_H\rangle \propto \sqrt{-i\lambda + m_g}\,\lvert\lambda\rangle \pm \sqrt{i\lambda + m_g}\,\lvert-\lambda\rangle$

- Zero-modes unpaired and eigenstates of chirality operator
- Eigenvalues come in positive/negative pairs

$$\gamma_\mu^\dagger = \gamma_\mu \qquad \{\gamma_5\ , \gamma_\mu D_\mu\} = 0$$

- Eigenvalues are two-fold degenerate

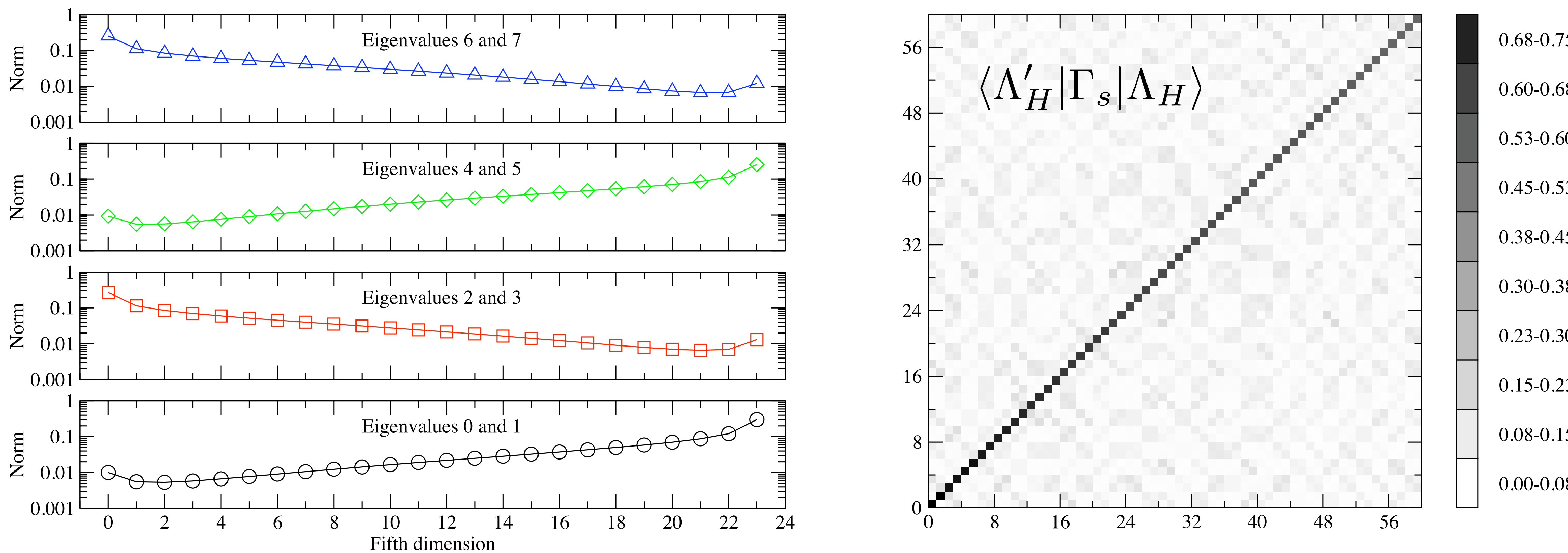
$$C^\dagger D C \quad = \quad D^*$$

Eigenvalue measurement details

- Performed eigenvalue studies of the Hermitian DWF Dirac operator (D_H)
  - eigenvalues are two-fold degenerate
  - ± pairing of eigenvalues absent at finite lattice spacing
- Measurements performed on 8^4 lattices
- Obtained lowest 64 eigenvalues (Λ_H) on each of 150 gauge field configurations

Low modes of D_H

β=2.3, m_f=0.2 and L_s=24



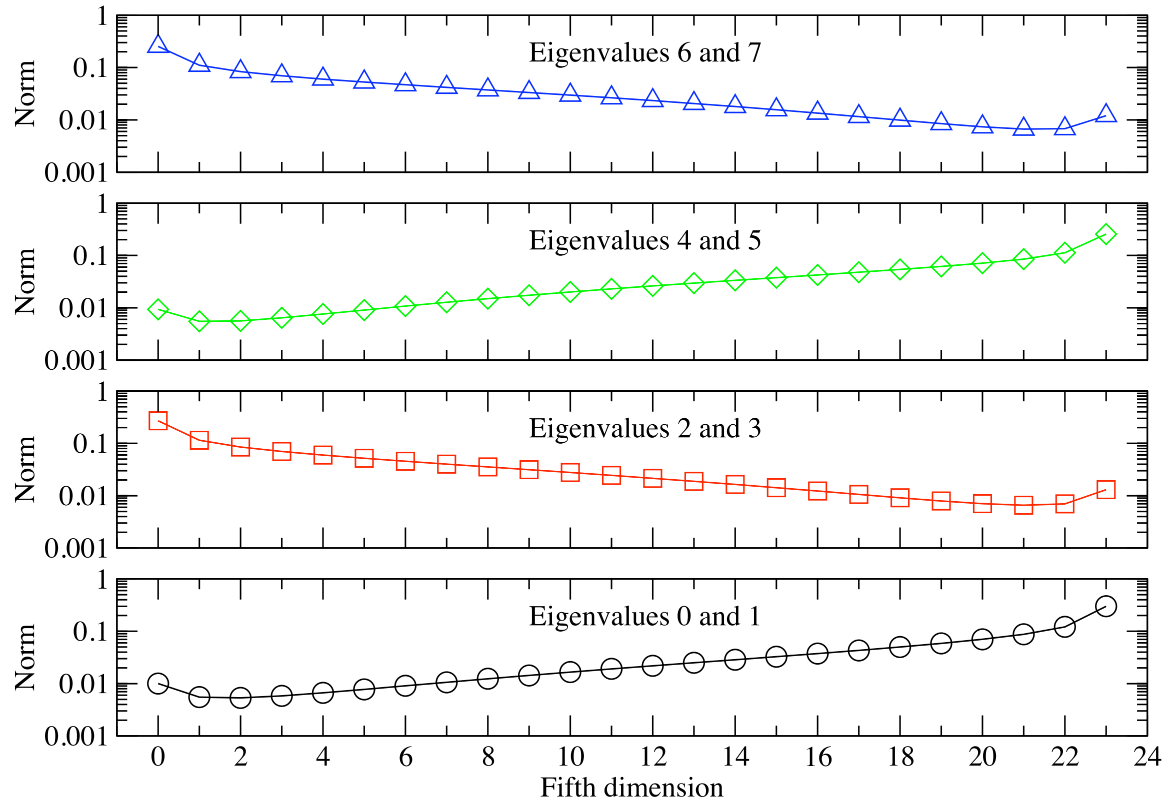
$$\mathcal{N}_{\Lambda_H}(s) = \sum_x \Psi^\dagger_{\Lambda_H}(x,s)\Psi_{\Lambda_H}(x,s)$$

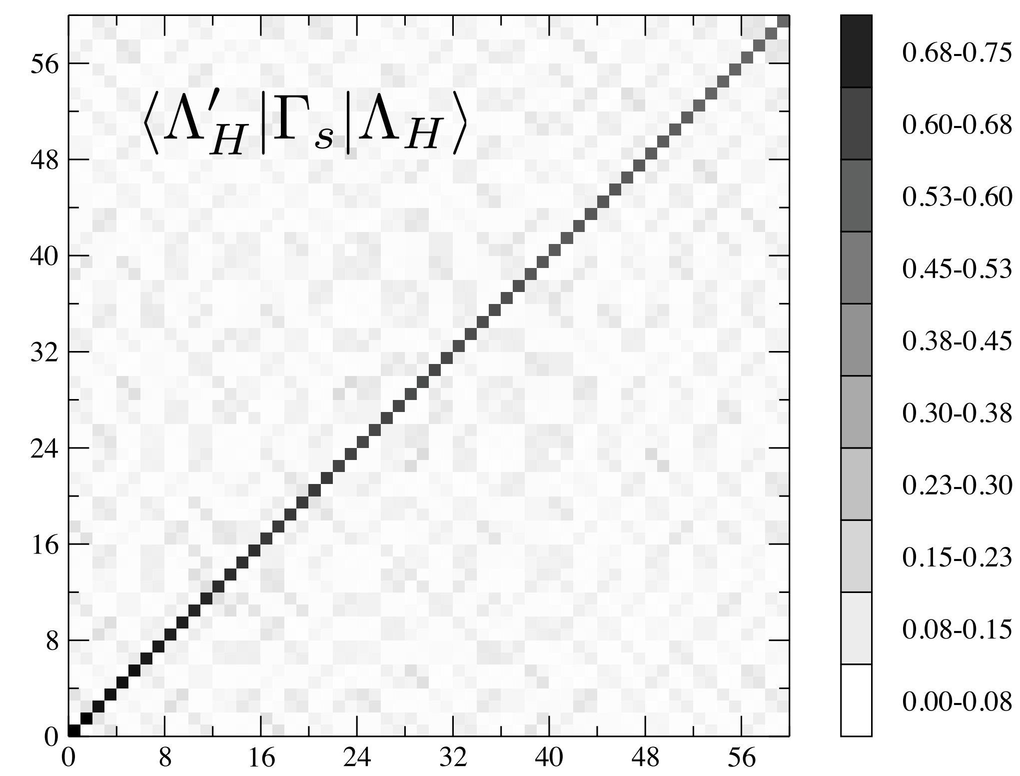
$$D_H \Psi_{\Lambda_H} = \Lambda_H \Psi_{\Lambda_H}$$

$$\langle \Lambda'_H | \Gamma_s | \Lambda_H \rangle = \sum_{x,s} sgn\left(\frac{L_s - 1}{2} - s\right) \Psi^\dagger_{\Lambda'_H}(x,s)\Psi_{\Lambda_H}(x,s)$$

Low modes of D_H

β=2.3, m_f=0.2 and L_s=24





- Modes exponentially localized on left/right walls
- Modes have definite chirality, consistent with: $m_g >> \lambda$
  - in continuum theory: $\langle \lambda'_H | \gamma_5 | \lambda_H \rangle = \frac{1}{\lambda_H} \left[m_g \delta_{\lambda'_H, \lambda_H} + i|\lambda| \delta_{-\lambda'_H, \lambda_H} \right]$

Extracting information from Λ_H

- Fit valence mass dependence of eigenvalues using reparameterized Taylor expansion:

$$\Lambda_H^2(m_v) = n_5^2\left[\lambda^2 + (m_v + \delta m)^2\right] + \mathcal{O}(m_v^3)$$

T. Blum, et al., hep-lat/0007038

- Can show:

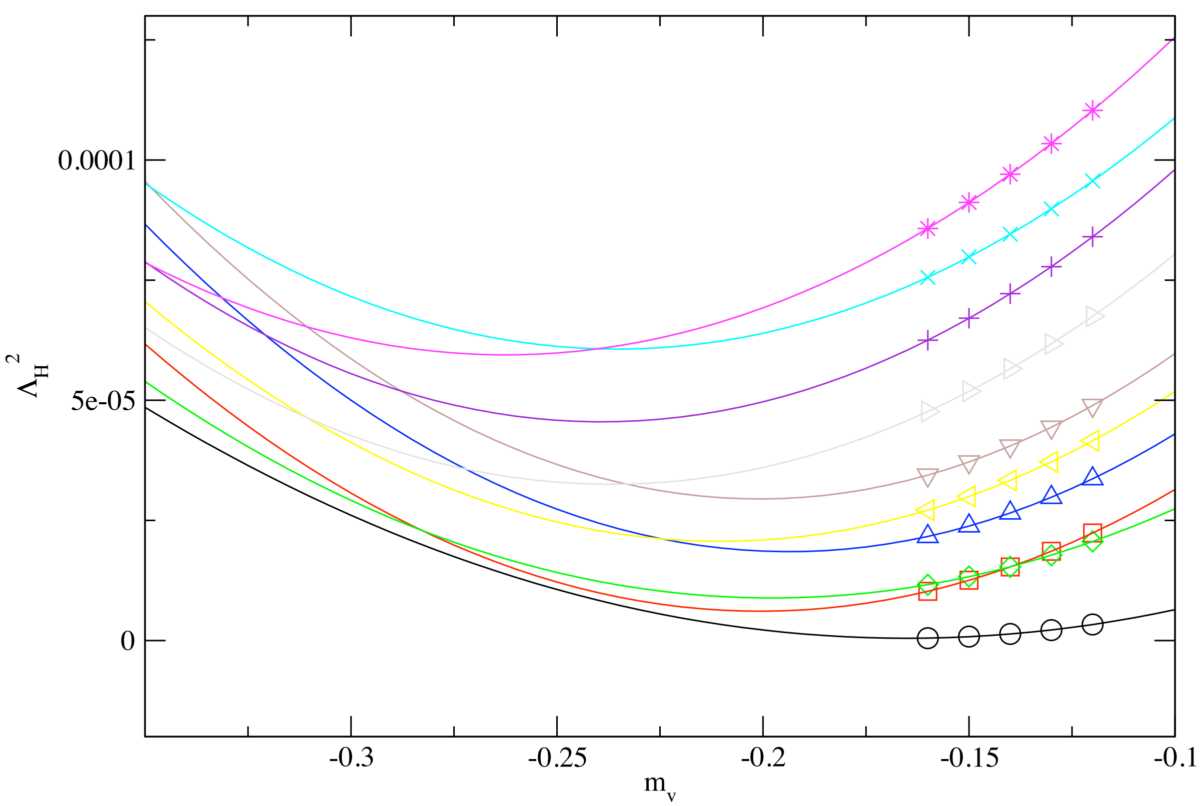
$$\langle\bar{\psi}\psi\rangle = \frac{1}{12V}\sum_i\left\langle\frac{m_f + \delta m_i}{\lambda_i^2 + (m_f + \delta m_i)^2}\right\rangle + \mathcal{O}(m_v^3)$$

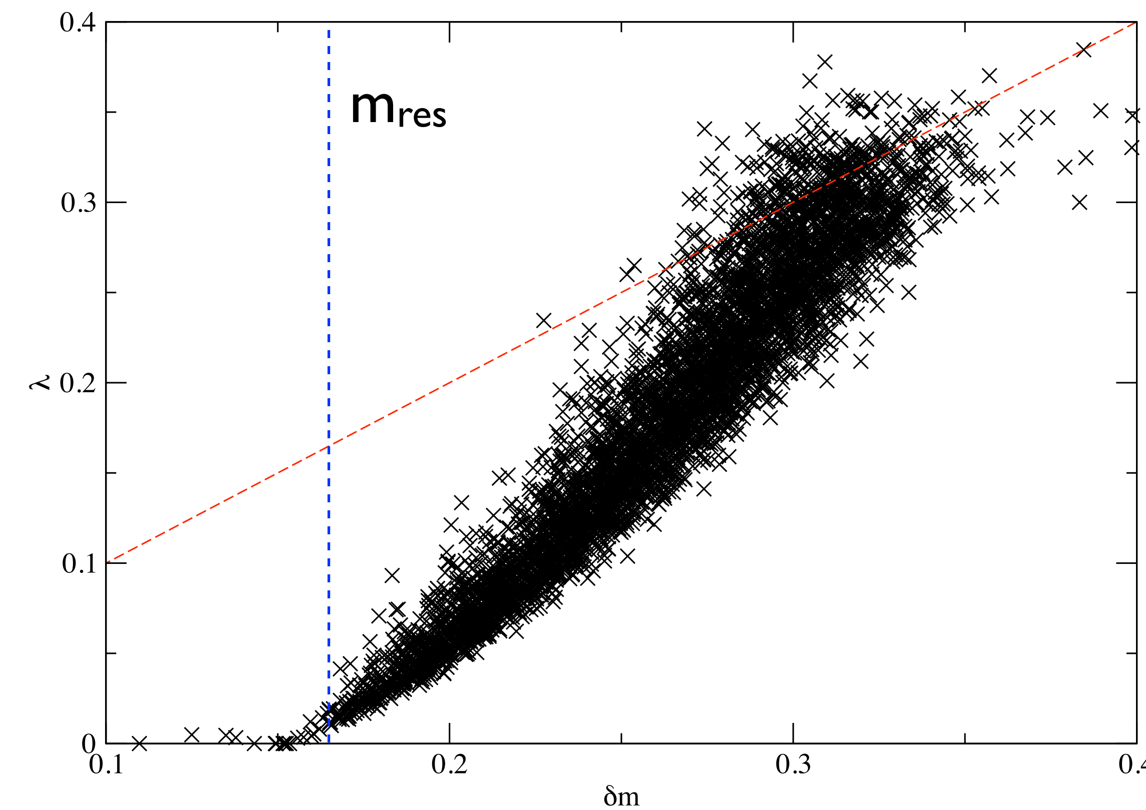
"effective 4D eigenvalue of D-slash"

reflects residual chiral symmetry breaking due to finite L_s

- Use λ and δm distributions to extract condensate and m_{res}

β=2.3, m_f=0.2 and L_s=24

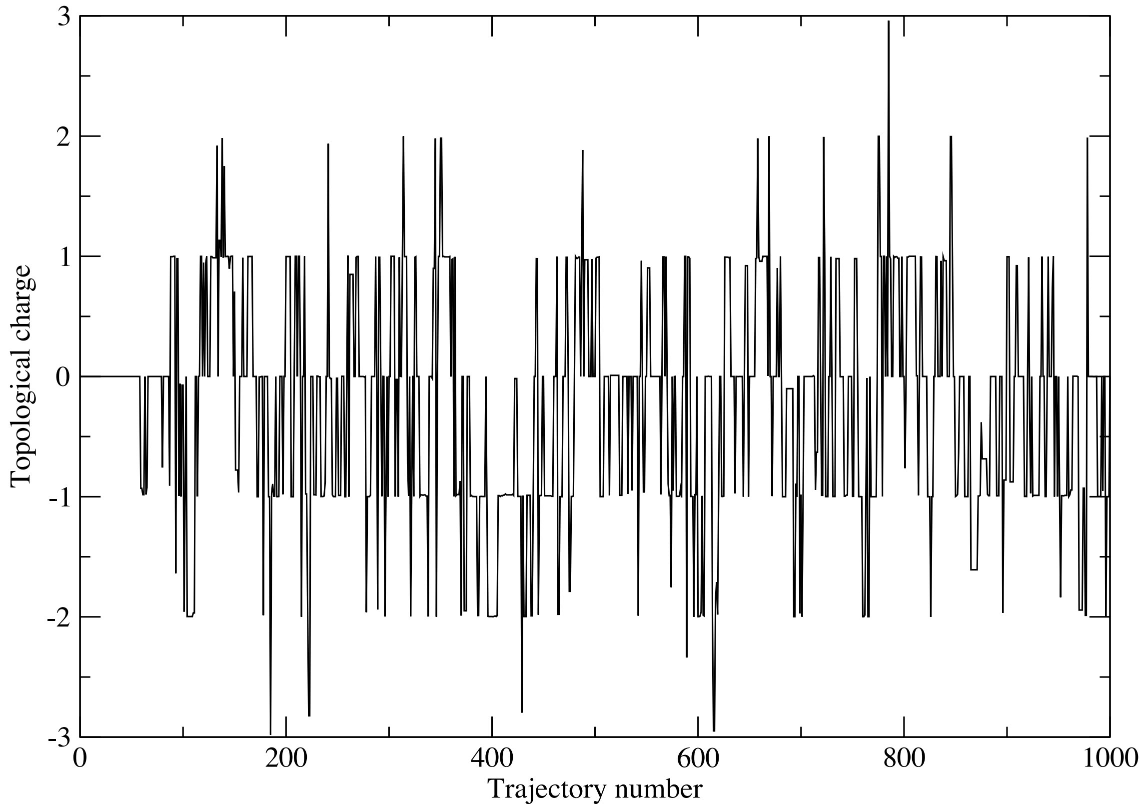


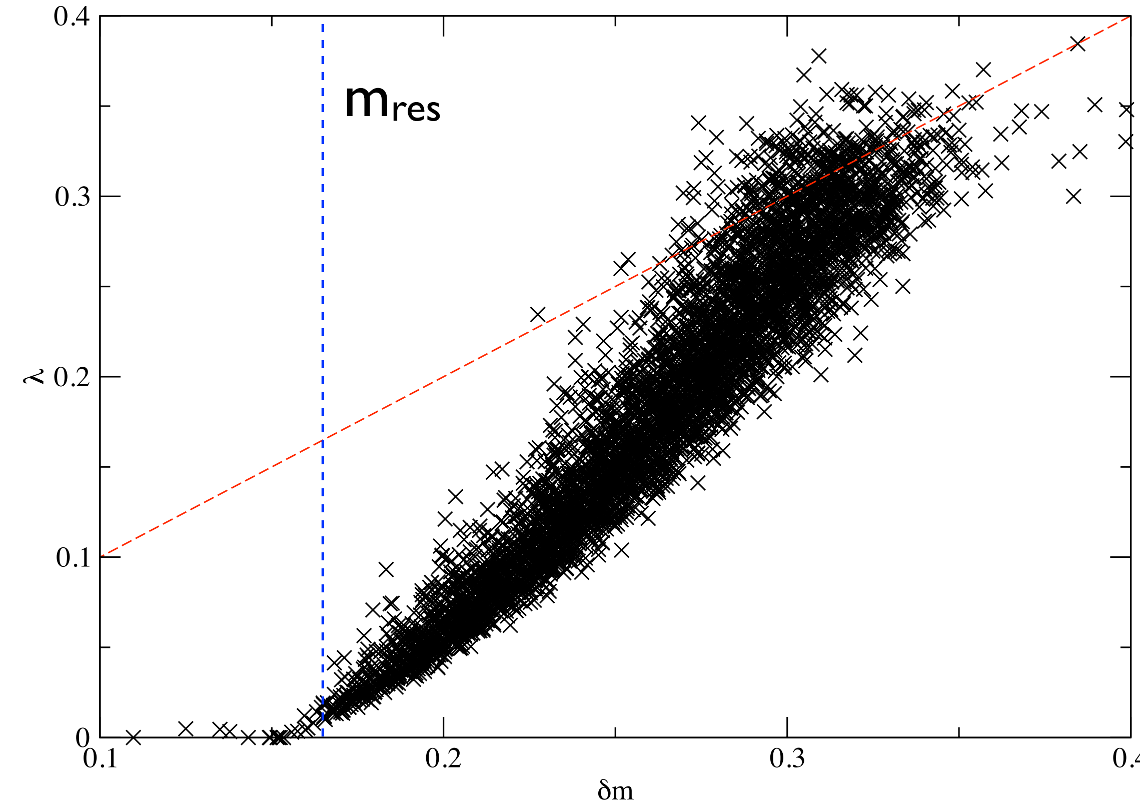


- No evidence for $\pm\Lambda_H$ pairing suggests large lattice spacing
- $m_f + \delta m >> \lambda$, consistent with "physical" γ_5 results
- Residual mass cannot be extracted from δm distribution
- Few near nero-modes

Near zero-modes and topological charge

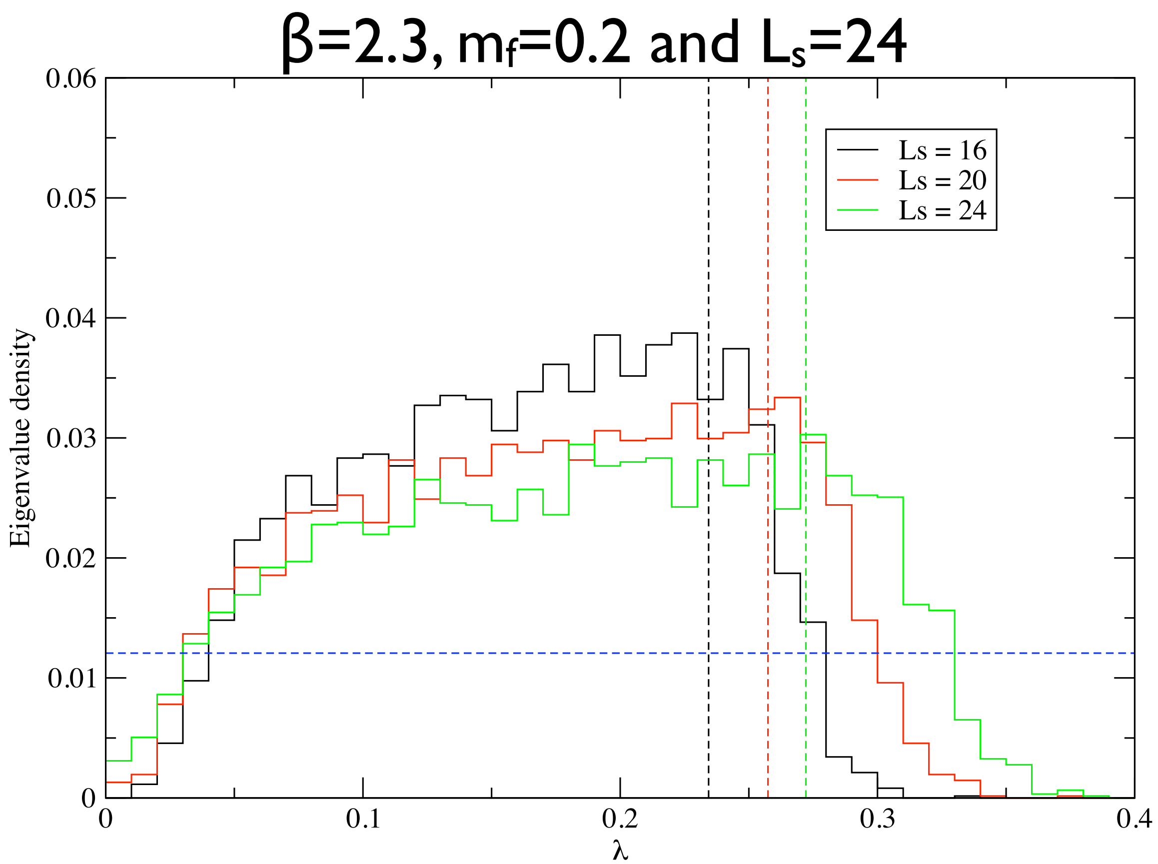
β=2.3, m_f=0.2 and L_s=24





- Gluonic definition of topological charge for same ensemble suggest that there is topological ergodicity
- Nonzero topological charge is poorly reflected in analysis of eigenvalues

Banks-Casher relation for DWFs



$$\rho'(\lambda; m_g) = \left\langle \sum_i \delta(\lambda - \lambda_i) \right\rangle$$

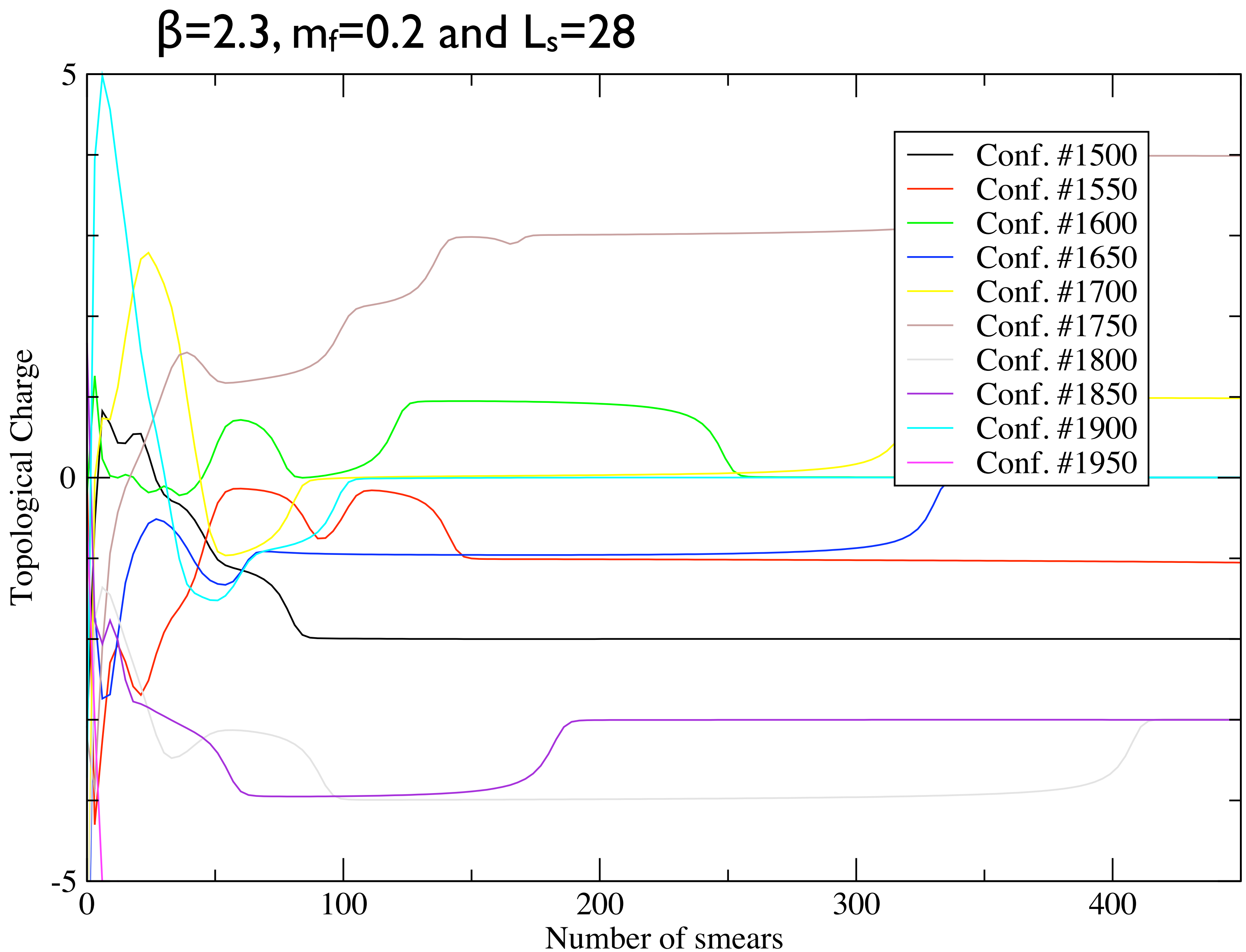
chiral limit extrap. of <$\overline{\psi}\psi$>
(normalized appropriately)

- Large disagreement in density of eigenvalues and chiral limit value of the condensate
- Moderate L_s dependence of $\rho(\lambda)$ for $\lambda \sim 0.1 - 0.2$
- Spectrum too distorted by large residual chiral symmetry breaking effects

Topological charge

- Smoothed gauge fields using APE smearing (smearing coefficient equal to 0.45)
- Measured topological charge using two methods:
  - 5-loop improved (5li) definition of charge, $O(a^4)$ classically improved field strength tensor Ph. de Forcrand, M. Garcia Perez and I.-O. Stamatescu
  - fermionic definition based on stochastic estimate of $\langle\psi\gamma_5\psi\rangle$

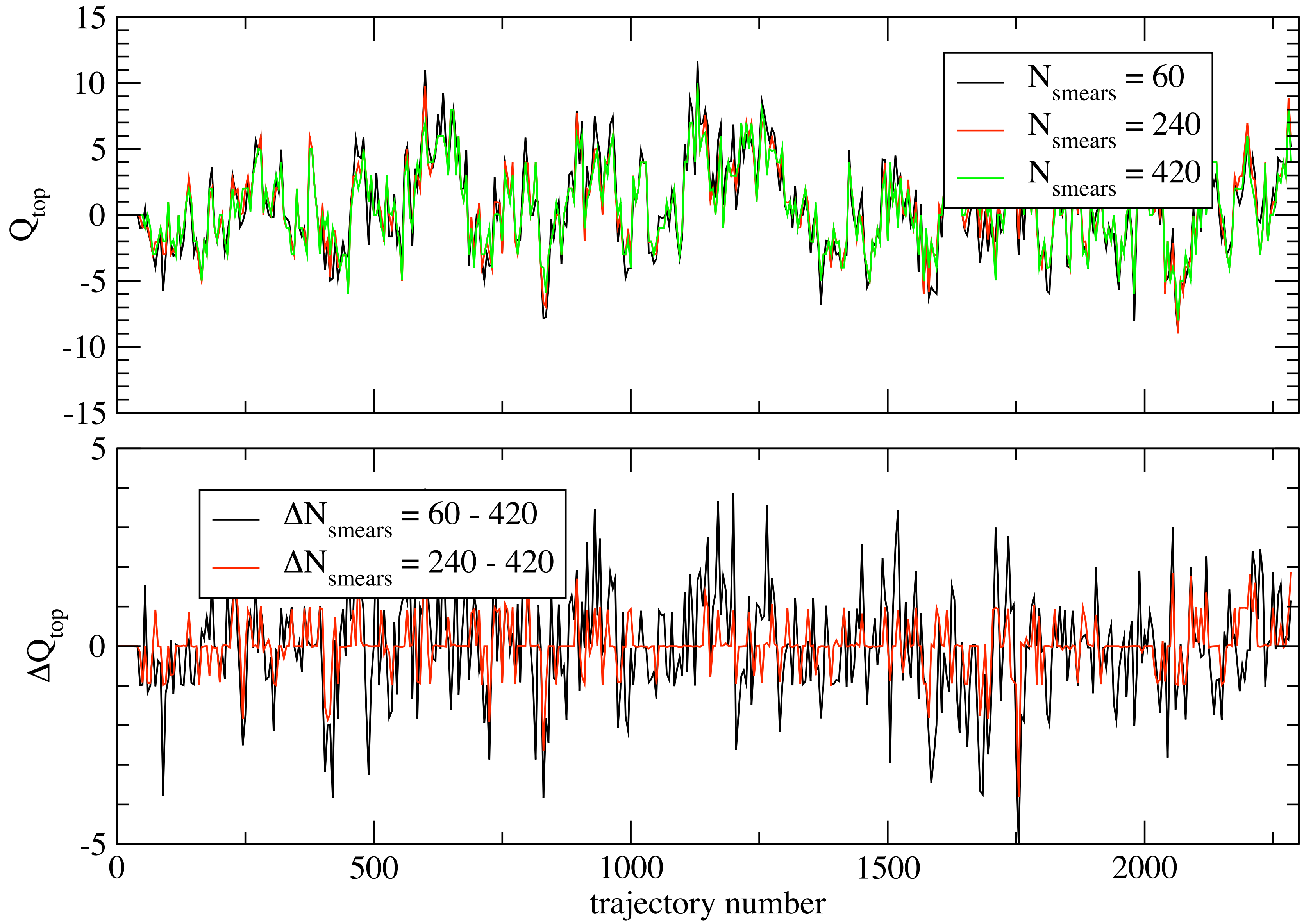
Topological charge



- Q_{top} fluctuation are a result of smearing away of small instantons
- Adequate for qualitative results, but should probably consider cooled lattices in future studies

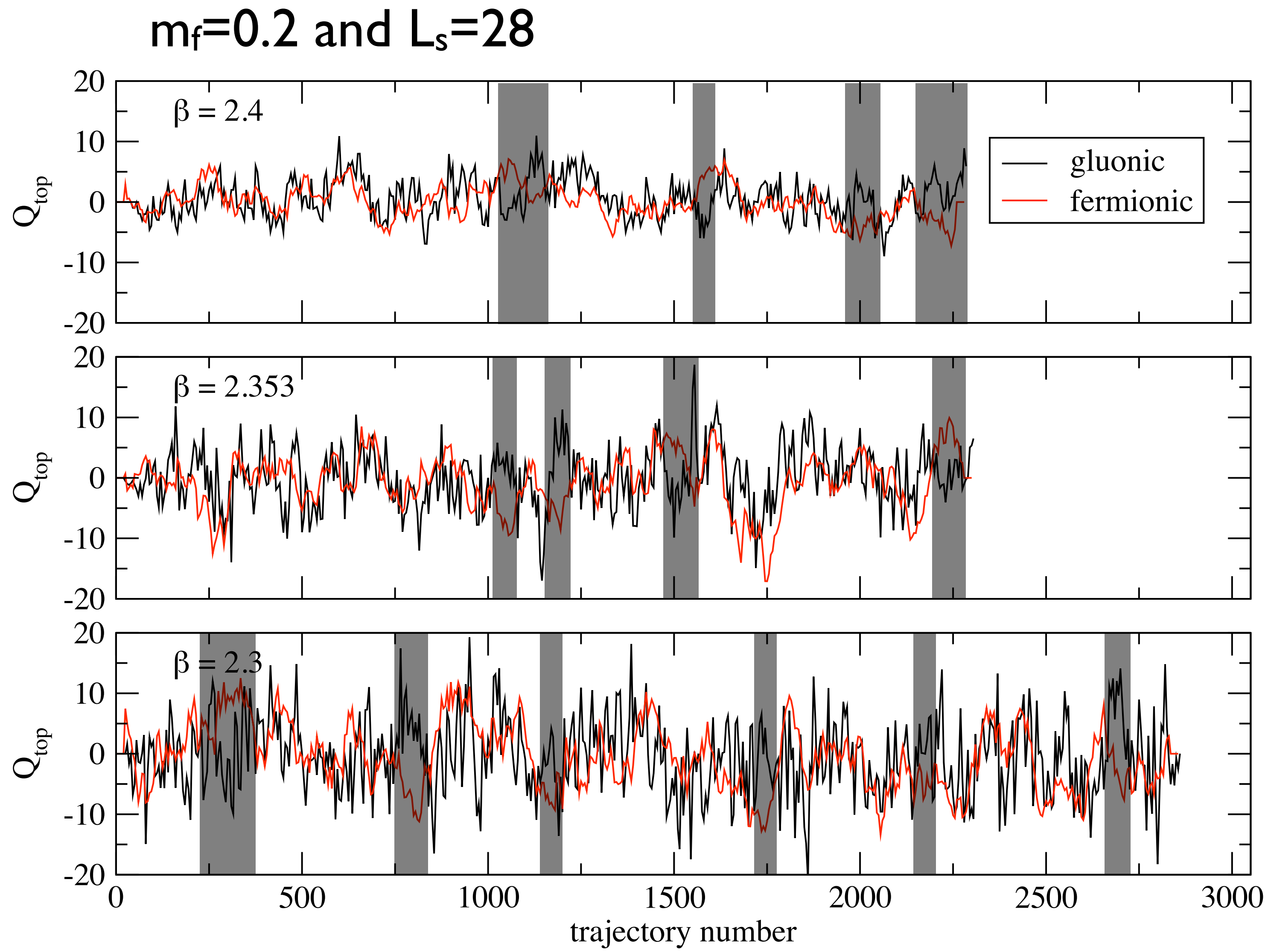
Topological charge

β=2.3, m_f=0.2 and L_s=28



- Qualitative picture of topological charge is independent of N_{smear}
- Variation in topological charge with N_{smear} ~ 1- 2 on average

Topological charge



- Gluonic and fermionic definitions seem track reasonably well with each other at weakest couplings
  - hints of an index theorem at weaker coupling?

Summary of results

- Current DWF SYM results are generally disappointing due to large residual chiral symmetry breaking
  - large systematic errors in chiral limit extrapolations
  - evident from Dirac spectrum studies and matrix elements of the "physical" chirality operator
- Substantial reduction in residual chiral symmetry breaking is crucial for reliable study of N=1 SYM
- Nonetheless, these simulations are important for understanding the relationship between the input parameters of the lattice action and physical quantities, which will guide future studies.

Future tasks and directions

- Combine improved action, larger L_s and weaker coupling to achieve smaller residual chiral symmetry breaking
  - Iwasaki gauge action
  - AuxDet DWFs recently implemented in Columbia Physics System (D. Renfrew) and modified for use with adjoint representation fermions (MGE)
- AuxDet simulations will be performed on New York Blue and QCDOC at Brookhaven National Laboratory in the near future
  - parameter tuning in progress... stay tuned.